

$$\binom{n}{k} + \binom{n}{k+1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} = \frac{n!}{k!(n-k-1)(n-k)} + \frac{n!}{k!(k+1)(n-k-1)!}$$

$$= \frac{n!}{k!(n-k-1)!} \left[\frac{1}{n-k} + \frac{1}{k+1} \right]$$

$$= \frac{n!}{k!(n-k-1)!} \left[\frac{k+1+n-k}{(n-k)(k+1)} \right]$$

$$= \frac{n!(n+1)}{(k+1)!(n-k)!} = \frac{(n+1)!}{(k+1)!(n-k)!} = \binom{n+1}{k+1}$$