Experiment 5:
Harmonic Oscillator Part I. Spring Oscillator.

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Abstract: 194 words
Remaining: 1,531 words

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Resonant Frequency of a Damp and Undamped Harmonic Oscillating Mass on a Spring

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Abstract

Simple harmonic motion is a fundamental principle in physics. It is possible to generalize this type of motion by applying it to oscillation. In this experiment, we are able to verify the predicted resonant frequencies for damped and undamped harmonic oscillating cases by using a mass on a string. To predict these resonant frequencies, the spring constant k = 2.77$\pm$0.08 N/m was subtracted from the tension against extension of the spring graph. The damping time τ = 4.8$\pm$0.2 s and Q-factor Q = 12.1$\pm$0.3 were calculated. The mass was set oscillating vertically with its natural frequency with small amplitude in order to obtain the experimental values. An aluminum cylinder was used to create the damping force for the undamped data set. The values of frequencies, $f_{0,e}$ (0.684$\pm$0.002 Hz) and $f_{damped,e}$ (0.68$\pm$0.01 Hz), were obtained from the sensor voltage against time plots. From these values, I was able to verify that the predicted frequencies, $f_{0,p}$ (0.683$\pm$0.002 Hz) and $f_{damped,p}$ (0.69$\pm$0.02 Hz), agree with the experimental values. Furthermore, I was able to conclude that the damping force does not have an effect on the frequency of oscillations because both of the frequencies do not significantly differ.

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Introduction

In mechanics, a harmonic oscillation is when a system is displaced from an equilibrium position and experiences a restoring force proportional to the displacement.[2] When this occurs, the system undergoes simple harmonic motion and its oscillations are undamped. On the other hand, if another force is acting on the system and is proportional to the velocity of the system, its oscillations are damped.

The purpose of this experiment was to verify the predicted resonant frequencies for damped and undamped cases. A mass was attached to a spring by a string while the end of the spring was secured to the force sensor hook by a piece of string, which allowed us to obtain the experimental value of frequency of the undamped data set. The spring constant k was obtained to predict the frequencies. For the undamped case, the freely suspended mass on the spring oscillated vertically with its natural frequency and small amplitude. For the damped case, an aluminum cylinder was used to create a damping force and the mass had magnets attached to it, which caused the damping force to act on the mass within the cylinder. The data was collected and plotted, and for the damped case, the Q-factor and damping time τ were calculated. By manually processing data for both oscillations the two values of frequencies$f_{0,e}$ and $f_{damped,e}$ were calculated for each data set. These values were then verified if the predicted frequencies agree with the values obtained from the experiment and compared to see if damping forces would have an effect on the frequency of oscillation in the system.

Methods

Five different masses were hung vertically and were attached to a spring using a string, and the spring was attached the the force sensor hook also using string. We recorded the vertical length from the lower end of the spring to the floor using a ruler and recorded its initial value. When we hung the masses and recorded the corresponding lengths in the same way. By subtracting the initial value from the other readings, we obtained the extension of the spring, which we later plotted against force.

Then, we set up the DAQ and the force sensor was connected to the PASCO. The table consisted of the recorded sensor reading in volts, since it doesn't use force values, and time in seconds. The frequency was set to 25Hz in the Controls palette, which allowed us to get sufficient data, which were recorded so that precision is not a limiting factor in the data.

After recording the data set for each partner, we repeated the procedures for the damped oscillations. This part of the experiment was set up with an aluminum cylinder as illustrated in Figure 5.1 from the Lab Manual[1] and once again, we allowed the mass to oscillate with small amplitude. The mass with magnets attached to it was connected on the spring by a string, which was used to prevent systematic uncertainties from the spring as it stretches. We made sure that the mass did not inch towards the edges of the cylinder and was oscillating only inside of it, which made the data more accurate by decreasing the error. For both the damped and undamped oscillations, each of the readings were recorded for twenty seconds.

Analysis

To obtain the expression for Q-factor in terms of τ, k, and m, I used equations 5.7 and 5.11 from the Lab Manual[1]: $Q = \frac{\tau}{2}\sqrt{\frac{k}{m}}$

Mass of oscillating weight: m = 174.0$\pm$0.5g

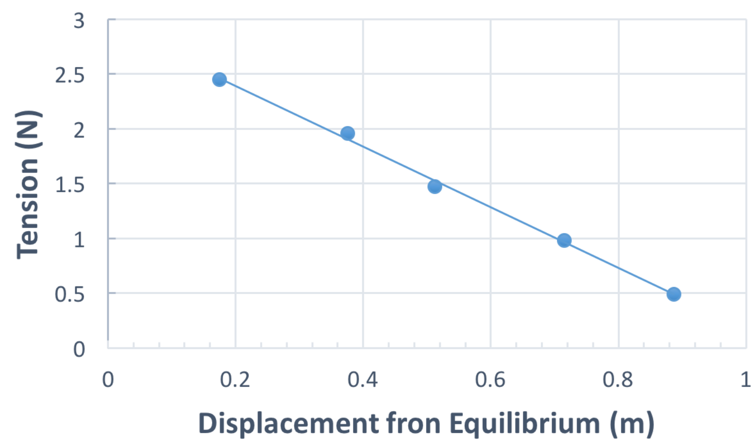

Figure 1— Spring Constant Determination. Five different masses were hung and the distance from the bottom of the spring to the floor was measured. To obtain the displacement from the equilibrium, the initial distance to the floor (with no mass hanging on the spring) was deducted from the readings. Tension, which was created by the weight of the masses, was plotted against its corresponding displacement. The slope of the fit line represents the spring constant k and is equal to 2.77±0.08 N/m. The equation of the fit line is y = (-2.77±0.08)x + (2.94±0.05)

I was able to calculated the predicted frequency of the undamped oscillations by using the equation 5.1 from the Lab Manual[1]. I derived the uncertainty by using the equations ii.23 and ii.24, which resulted in $\delta f_{0,p} = \frac{|f_{0,p,best}|}{2}\sqrt{(\frac{\delta m}{|m_{best}|})^2 + (\frac{\delta k}{|k_{best}|})^2}$. The predicted values of the free oscillation frequency, $f_{0,p}$, is 0.683±0.002 Hz.

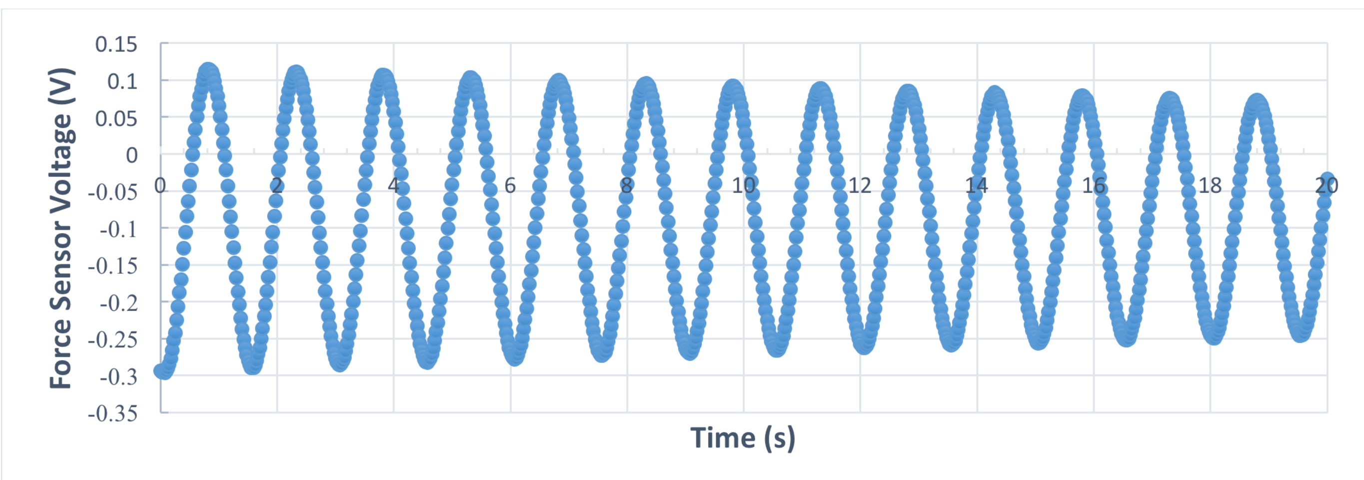

Figure 2— Undamped Oscillatory Motion of the Mass. The force sensor voltage collected the data at the frequency 25Hz and the values were plotted against its corresponding time. The times at which the maxima occurred were identified from the graph, which allowed the frequency to be calculated.

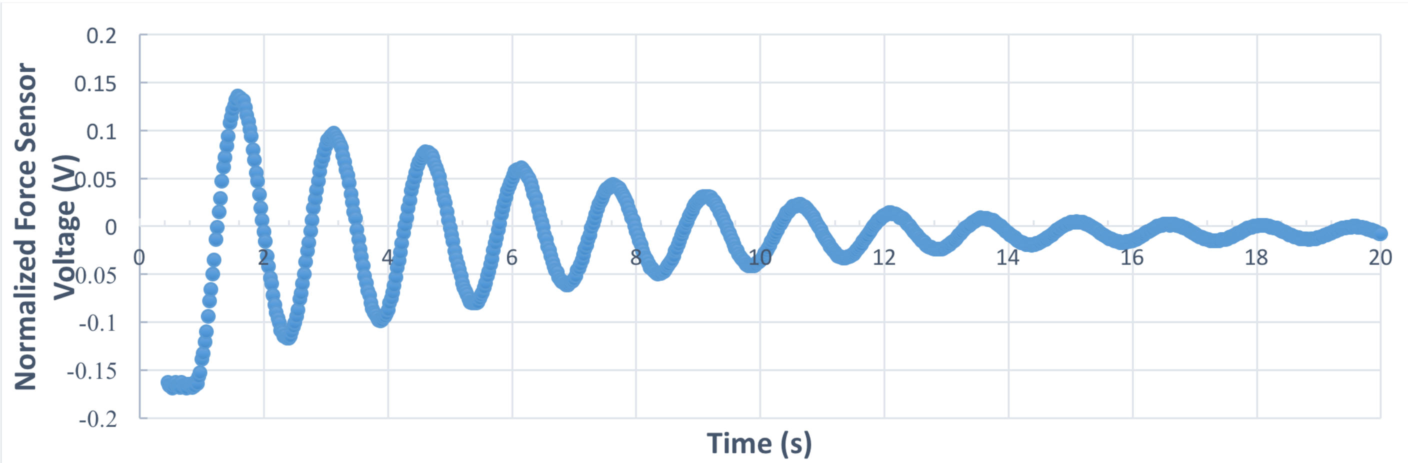

Figure 3— Damped Oscillatory Motion of the Mass. The force sensor voltage collected the data at the frequency 25Hz and the values were normalized and plotted against its corresponding time. The times at which the maxima occurred were identified from the graph, which allowed the frequency to be calculated.

To determine the frequencies of both oscillations, I created Figures 2 and 3 for each oscillation data. There were maxima values in the voltages from manually identifying the times. I recorded the ties at which the first nine maxima occurred. I took the reciprocal of the time difference between the nth maxima and the n-1 maxima, which were calculated for n = 2, 3, 4, 5, 6, 7, 8, and 9. I calculated the average of the frequencies and set it equal to f_{best} in order to estimate the uncertainty. Then, I calculated the difference between the highest and lowest frequencies, which is double of the uncertainty. For the undamped oscillations, $f_{0,e}$ = 0.684$\pm$0.002 Hz. For the damped oscillations, $f_{damped,e}$ = 0.68$\pm$0.01 Hz.

The predicted frequency for the undamped case $f_{0,p}$ is 0.683$\pm$0.002 Hz and the frequency $f_{0,e}$ = 0.684$\pm$0.002 Hz. The results of these values agree because the range of the uncertainty in the predicted frequency, 0.683 to 0.687 Hz overlaps that of the uncertainty of the experimentally obtained value 0.63 to 0.698 Hz. This suggests that the experiment with undamped oscillating mass was accurate and proves the formula used to calculated the prediction.

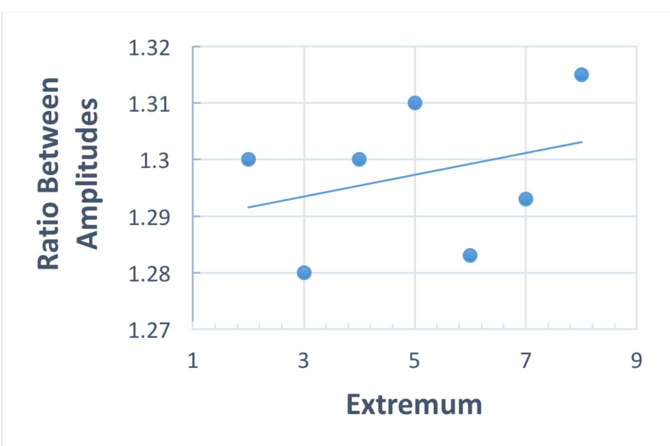

Figure 4— Ratios of Successive Extrema for the Damped Oscillation. Eight successive maxima were manually identified from Figure 2 and their ratios were calculated and plotted against the extremum number. The slope of the fit line is 0.004$\pm$0.005 so there is no upward or downward trend since the slope of the value cancels out with its uncertainties. The y-intercept of the fit line is 1.34$\pm$0.02 and represents the best value for the ratio between successive maxima. Using these values, it is possible to calculate the damping time τ.

From Figure 4, I was able to conclude that there is no overall increasing or decreasing trend, so I used the reciprocal of the y-intercept value c = 1.34$\pm$0.02 as the ratio between the amplitudes used in the equation 5.12[1]. To calculate the damping time τ, I used the equation 5.13[1]. Next, I used equations ii.23 and ii.24[1]: $\delta\tau = |\tau_{best}|\sqrt{(\frac{\delta f_{damped,e}}{|f_{damped,e,best}|})^2 + (\frac{\delta c}{|c_{best}|})^2}$ to calculate the uncertainty. τ was calculated to be 4.8$\pm$0.2 s. Then, I used the equation I derived for Q in terms of τ, k, and m which resulted in Q = 12.1$\pm$0.3. The formula for the uncertainty was $\delta Q = |Q_{best}|\sqrt{(\frac{\delta\tau}{|\tau_{best}|})^2 + \frac{1}{4}(\frac{\delta k}{|k_{best}|})^2 + \frac{1}{4}(\frac{\delta m}{|m_{best}|})^2}$, which was derived from ii.23 and ii.24[1] $\delta f_{damped,p} = |f_{damped,p}|\sqrt{(\frac{\delta f_{0,e}}{|f_{0,e}|})^2 + (\frac{\delta Q}{|Q_{best}|})^2}$, which resulted in 0.69$\pm$0.02. The value of $f_{damped,p}$ agrees with the value of $f_{damped,e}$ because the range of values produced by the predicted value 0.68 to 0.73 Hz consists of the range of values produced by the experimental value 0.67 to 0.71 Hz. In addition, the value of the damped frequency agrees with the value for the undamped one because the difference of the best values for both oscillations is less than their uncertainties.

Extra Credit

Using equation 5.14[1], I used another method to obtain an estimate for Q. The uncertainty was calculated by $\delta Q = |Q_{best}|\sqrt{(\frac{\delta f_{0,p}}{|f_{0,p,predicted}|})^2 + (\frac{\Delta f}{\Delta f_{best}})^2}$ from ii.23[1]. The value resulted in Q=13.8$\pm$0.2, which was not consistent with my initial value of Q = 12.1$\pm$0.3 because Q was not large enough for the approximation and the FFT was not sensitive enough because the width of the rectangles was too large.

Conclusion

The objective of the experiment was to verify the predicted frequencies of damped and undamped cases. The damping force was set up using an aluminum cylinder with magnets attached to the oscillating mass. Furthermore, this experiment checked if the damping force had an effected on the frequency of the oscillations in the system. By conducting the experiment and obtaining the values $f_{0,e}$ = 0.684$\pm$0.002 Hz and $f_{damped,e}$ = 0.68$\pm$0.01 Hz, I was able to verify the predicted frequencies of each oscillations of of the mass on the spring $f_{0,p}$ = 0.683$\pm$0.002 Hz and $f_{damped,p}$ = 0.69$\pm$0.02 Hz. I then used the spring constant obtained from the slop of the fit line k = 2.77$\pm$0.08 N/m, and Q = 12.1$\pm$0.3 to calculate the predicted values. I drew a conclusion that the predicted frequencies values with experimental values and that the damping force does not have an effect on the frequency of the oscillations since both frequencies are smaller than the uncertainties. One possible source of systematic uncertainty for the damped case is not being able to deduce the amplitude of the oscillations. The spring can become disfigured if the amplitude is large, which will cause a systematic error in the spring constant. The aluminum cylinder with the height equaled to twice the maximum appropriate amplitude should be taken to avoid this error. This would allow the experiment to have more accurate measurements.

References

1. Campbell, W. C. et al. Physics 4AL: Mechanics Lab Manual (ver. August 31, 2017). (Univ. California Los Angeles, Los Angeles, California).
2. Young, H. D., Freedman, R. A. & Ford, A. L. *Sears and Zemansky's University Physics with Modern Physics.* 238 (Pearson, 2015).