Experiment 2:
Measurement of g

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Lab performed on: April 24, 2018
Lab section: Lab 6- Tuesday 11am
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Partners' names:

2. Derivations:

D- distance between the lower photogate and the sensor mat

T_2- time taken to travel D (lower photogate and sensor mat)

d- distance between the lower and upper photogates

T_1- time taken to travel d (lower and upper photogates)

- Distance traveled is divided by the time taken to find the average velocity:
  (1) Average velocity to travel d: $v_d = \frac{d}{T_1}$
  (2) Average velocity to travel D: $v_D = \frac{D}{T_2}$
- By using the relationship between velocity, the equation $v = v_0 + at$, acceleration (which is gravity in this experiment) and time can be obtained (initial velocity when dropping the ball in the experiment is 0). Plugging in equations (1) and (2), we get:
  (3) $g = \frac{v}{t} = \frac{v_D - v_d}{t}$ where $t = t_2 - t_1$
- To derive t_1 and t_2 (times when ball traveled half of distance d and D, which are points where instantaneous velocity is equal to average velocity for each period):
  (4) $t_1 = \frac{T_1}{2}$
  (5) $t_2 = \frac{T_2}{2} + T_1 = \frac{T_2}{2} + \frac{2T_1}{2} = \frac{T_2 + 2T_1}{2}$
- From equations (4) and (5), we get t:
  (6) $t = t_2 - t_1 = \frac{T_2 + 2T_1}{2} - \frac{T_1}{2} = \frac{T_2 + T_1}{2}$
- By plugging in equation (6) into (3), we get:
  (7) $g = \frac{v_D - v_d}{\frac{T_2 + T_1}{2}} = \frac{2(v_D - v_d)}{T_2 + T_1}$
- By plugging in equations (1) and (2) into (7), we get the equation 2.1 from the lab manual:
  (8) $g = \frac{2}{T_1 + T_2}\left(\frac{D}{T_2} - \frac{d}{T_1}\right)$

3. Plots:

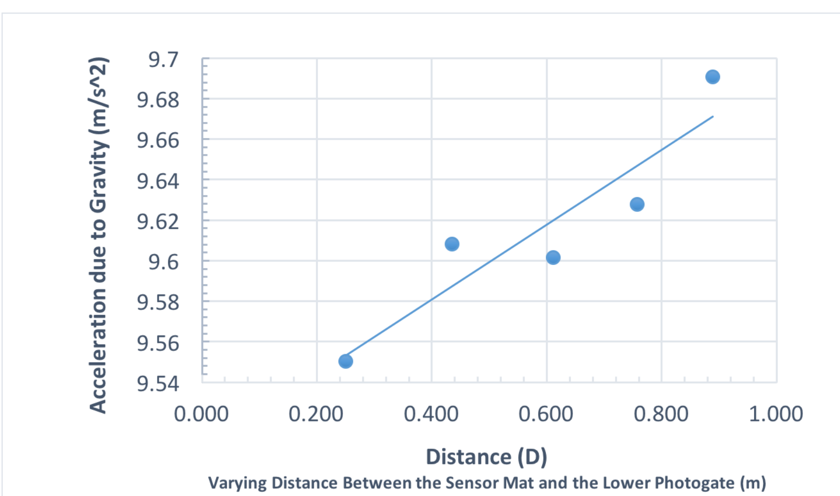

Figure 1— Acceleration due to gravity (g) vs. Distance (D). This graph shows the relationship between the acceleration due to gravity (g, measured in m/s^2) and the varying distance the ball traveled from the lower photogate to the sensor mat (D, measured in m).

The equation of the fit line is:
y = (0.18±0.04 m/s^2)x ± (9.51±0.03)

Since g is considered as a constant near the surface of the Earth, I would expect g to be independent of D. However, from the fit line depicted in Figure 1, acceleration is shown to be directly proportional to the distance. The coefficient of the x variable is too large to be removed by the uncertainty ±0.04.

The value of the intercept, which is supposed to be the acceleration of gravity if the coefficient of x is zero, is (9.51±0.03 m/s^2) whereas the actual value of acceleration (9.7955±0.0003) m/s^2. A way to quantitatively confirm or rule out a linear dependence on D is by conducting further experiments with different heights.

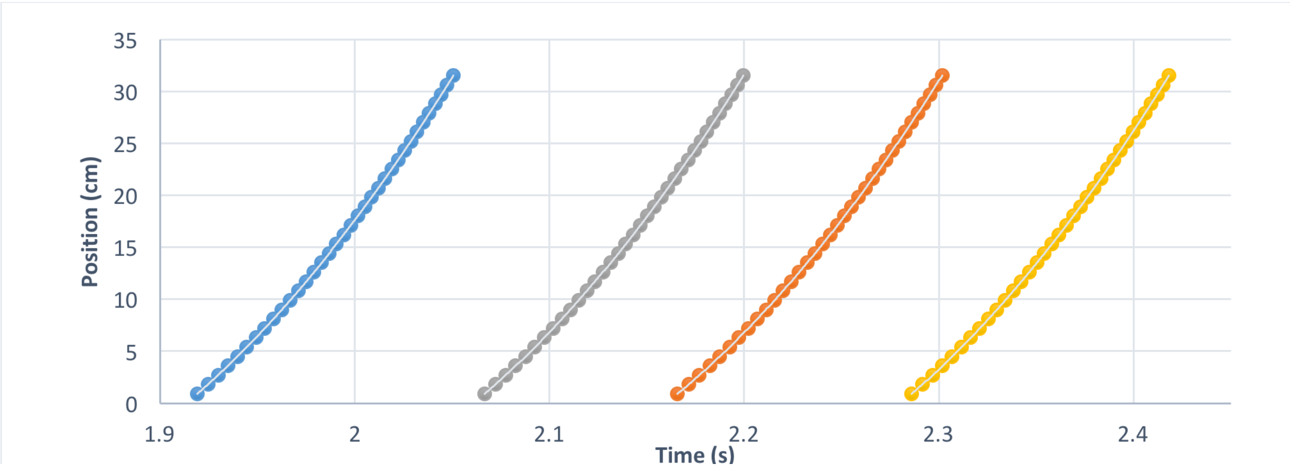

Figure 2— Relationship between position (cm) vs. time (s) for the photogate comb method. The plot illustrated the parabolic relationship between the position and time (s).
The coefficients for x^2 are: blue = 480±1 cm/s^4, grey = 400±3 cm/s^4, orange = 478.3±0.3 cm/s^4, and yellow = 477±1 cm/s^4. The quadratic fit lines have the equations of: blue y = (480±1 cm/s^4)x^2 – (1838±2)x + (1606.6±0.3) , grey y = (400±3 cm/s^4) x^2 – (1989.5±0.2)x + (1890.1±0.3), orange y = (478.3±0.3 cm/s^4)x^2 – (2096.7±0.5)x + (2104±2), and yellow y = (477±1 cm/s^4)x^2 – (2207.7±0.2)x + (2338±1).

4. Data Tables:

Data for ball drop method:

Trial	Photogate spacing *d* (m) ± 0.005	Gap to impact sensor *D* (m) ± 0.005	Measured acceleration *g* (m/s^2)
1	0.083	0.250	9.55±0.02
2	0.083	0.435	9.61±0.02
3	0.083	0.611	9.60±0.02
4	0.083	0.757	9.63±0.03
5	0.083	0.889	9.69±0.03

Table 1— Data Recorded and Calculated (Ball Drop Method). This data table shows *d*, the spacing measured between the two photogates, *D*, the spacing measured between the lower photogate and impact sensor, and *g*, the measured values of acceleration using the equation 2.1. The statistical and systematic uncertainties were included in the measurements of *g* for each distance *D*. The values of *d* and *D* share the same systematic uncertainty, while the values of *g* have varying uncertainties, which resulted in systematic uncertainty.

Data for photogate comb method:

Trial	1	2	3	4
Measured acceleration, g (m/s^2)	9.61$\pm$0.04	8.7$\pm$0.3	9.51$\pm$0.03	9.6$\pm$0.4

Table 2— Data Recorded and Calculated (Photogate Comb Method). This data table shows g, the measured acceleration in m/s^2 for the photogate comb method. The space between the successive slot edges is λ. The total length of 35 successive spacings was measured and then divided by the total number of spacing. By doing so, the value obtained for λ = 0.9000$\pm$0.0003 cm. The uncertainty was calculated by dividing the value of 0.1 mm and the number of slots measured, which resulted in systematic uncertainty.

5. Conclusion:

For the dropping ball method, the systematic uncertainty was calculated using the g_{max} and g_{min} and the expression 2($g_{max} - g_{min}$) = g's uncertainty. Using the uncertainties in d, the distance between the two photogates, and *D*, the distance between the lower photogate and the sensor mat, I was able to calculate g_{max} and g_{min}. By using the formula $\delta g = \sqrt{(\delta d)^2 + (\delta D)^2}$, I calculated the statistical error, which resulted in $\pm$ 0.0005 for the trials. Since the statistical uncertainty was smaller than the systematic uncertainty, I used the systematic uncertainty. For the photogate comb method, the uncertainty was calculated using n λ and resulted in $\pm$0.0003cm.

The ball drop method is less precise because of the greater statistical uncertainty in *d* and *D* than in that of n λ. It is also less accurate because the values calculated from that experiment differed more from the actual value of g, which is (9.7955$\pm$0.0003) m/s^2 compared to the values calculated in the photogate come experiment, since there was a bigger systematic error in the ball drop experiment.

6. Extra Credit:

My lab partners and I experimented with a rubber band that was cut in an attempt to make it a string. By doing this, we tried to keep the comb as vertical as it drops in order get a more accurate measurement of g. We looped string through the first spacing of the comb and held it above the photogate and dropped it at rest. This allowed the comb to remain vertical more consistently throughout the fall. The best trial from this string method was 10.2 m/s^2. This method was precise but not accurate because of a systematic error. There was some tension in the string that held it up to be vertical that was not taken into consideration while conducting the experiment, so the value that we obtained for acceleration was larger than the actual value of g. This method would not be appropriate when trying to measure the actual value of g.

Presentation mini-report:

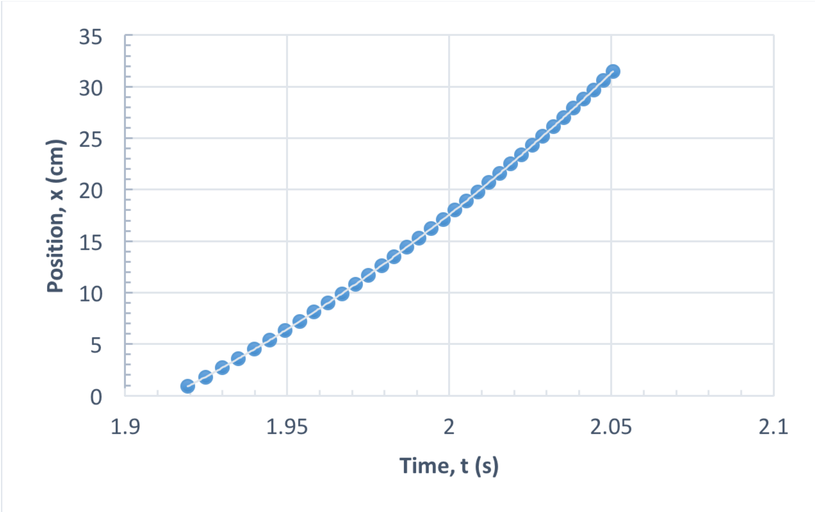

Figure 3— Relationship between position (cm) vs. time (s) for the best trial of the photogate comb method. This plot shows the data for the best trial using the photogate comb method. The comb was dropped vertically, and its corresponding time and position resulted in a parabolic fit with the coefficient t^2= (480$\pm$1) cm/s^4.

Figure 3 illustrates the data collected from the best trial conducted during the photogate comb method experiment. During the experiment, the comb was dropped at rest through the photogates, which tracked the position and time of it in cm and s. To obtain the value of λ, which is the space between the successive slots, the measured length of 35 spacings was divided by the total number of spacings. The value of λ was 0.9000$\pm$0.0003 cm. The position is plotted in cm and was calculated by n λ. This data resulted in a quadratic fit with coefficient t^2= (480$\pm$1) cm/s^4 and the value of acceleration due to gravity was 9.61$\pm$0.04 m/s^2.

From the first kinematic equation, $y = y_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$, we can determine the relationship between the distance and time. If the initial time and velocity v_0 and t_0 are zero (since the time has not started to count when the comb is released from rest), the equation becomes $y = \frac{1}{2}at^2$. We note that in this equation, a is the value of g because it is the acceleration due to gravity and the comb is free falling. From the coefficient of t^2 we can obtain g, which ultimately becomes (9.61$\pm$0.04) m/s^2. The range that the value of g results in, which is 9.57 m/s^2 to 9.65 m/s^2 does not fit within the range of the actual value of g given by Knudsen1-238[1] (9.7955$\pm$0.0003) m/s^2. This is because the uncertainty is systematic, so it lessens the accuracy of falling into the range of the actual value of g.
(255 words)

References:
[1] Campbell, W.C. et al. Physics 4AL: Mechanics Lab Manual (ver. August 31, 2017). (Univ. California Los Angeles, Los Angeles, California).