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The Obj function of the problem is given by:

$$\min \frac{1}{2} \sum (x - y_i)^2$$

By FOC, set the first derivative equal to 0:

$$\sum (x - y_i) = 0$$

Get

$$x_{\text{opt}} = \frac{1}{N} \sum y_i$$

The iteration of IGD

$$x_{k+1} = x_k - \gamma^k (x_k - y_{ik})$$

$$\gamma^k = 1/k+1$$

$$x_{k+1} = x_k - \gamma^k (x_k - y_{k+1})$$

$$x_{k+1} = x_k - \frac{1}{k+1} (x_k - y_{k+1})$$

$$x_{k+1} = ((k+1)x_k - (x_k - y_{k+1})) / (k+1)$$

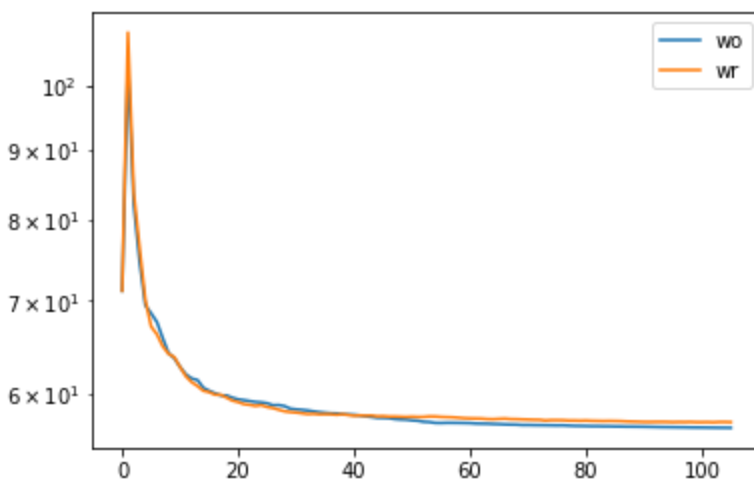
$$x_{k+1} = (kx_k + y_{k+1}) / (k+1)$$

$$(k+1)x_k = kx_k + y_{k+1}$$

$$Nx_N = (N-1)x_{N-1} + y_N$$

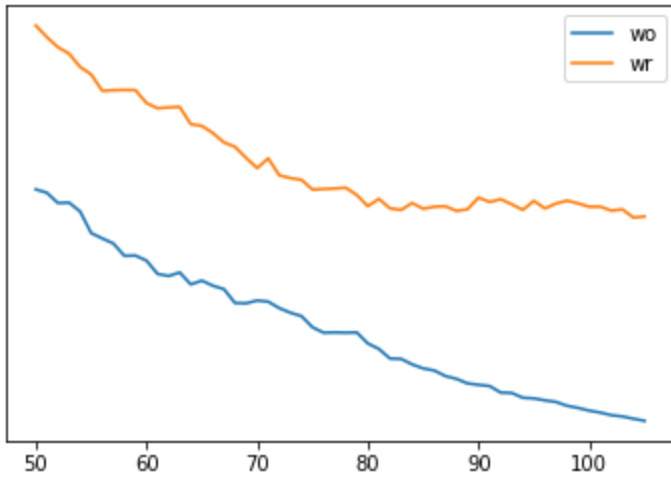
$$x_N = \frac{1}{N} \sum y_i = \mu$$

Task 1:



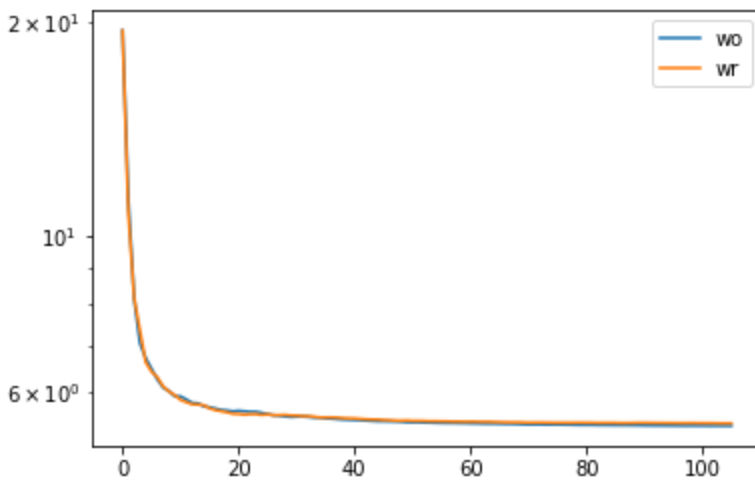
With and Without replacement(y1)

Hard to see.



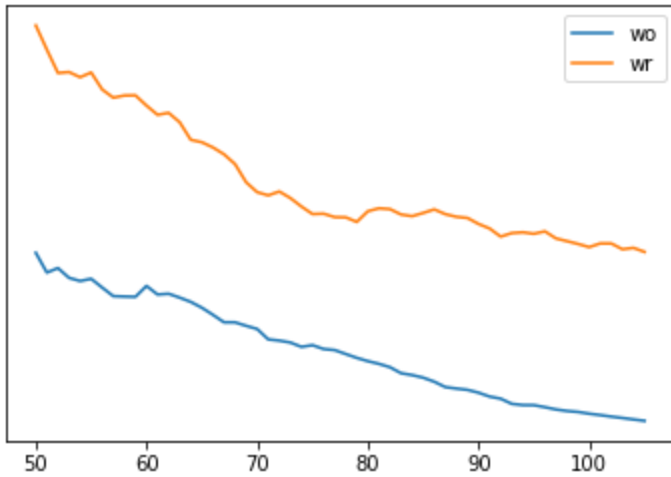
With and Without replacement(y1, last 50)

The ordering without replacement is better than with replacement, because without replacement converges to the answer in the end.



With and Without replacement(y2)

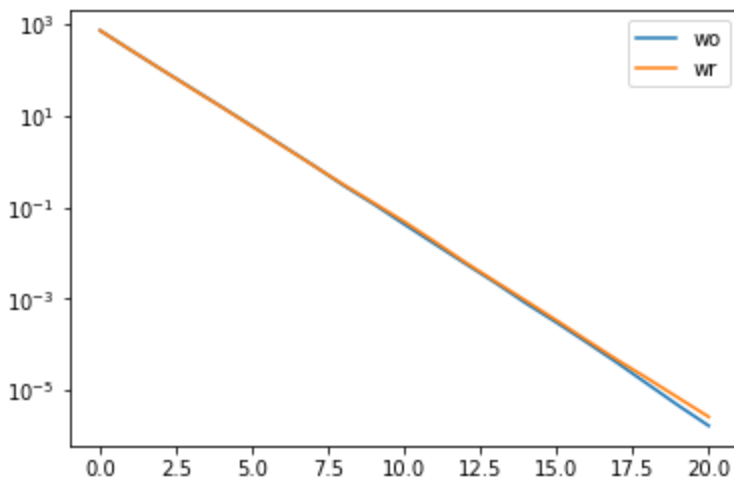
Hard to see.



With and Without replacement(y2, last 50)

The ordering without replacement is better than with replacement, because without replacement converges more to the answer in the end.

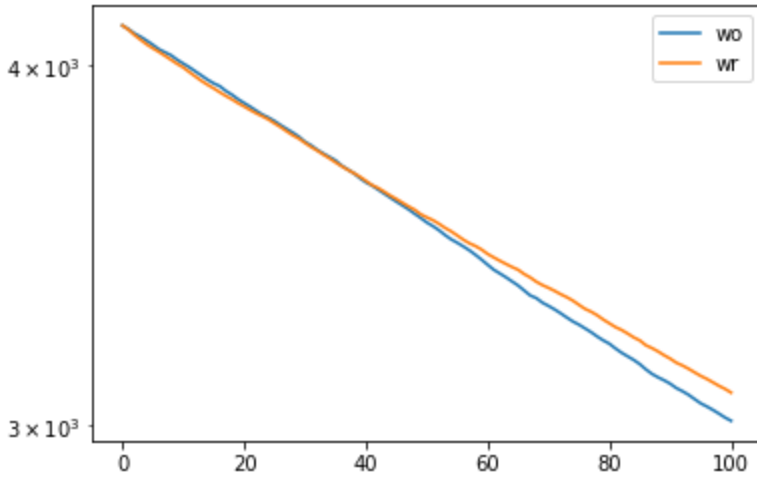
Task2:



With and Without replacement(beta=np.random.uniform(0.5, 1.0, 20))

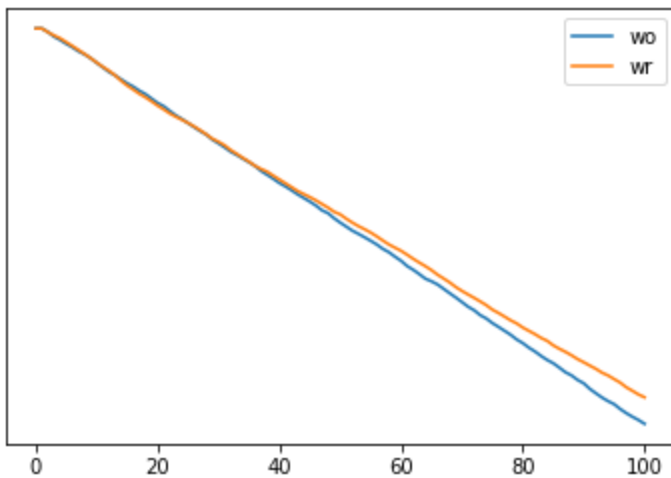
The ordering without replacement is better than with replacement, because without replacement converges more to the answer in the end.

Task3:



With and Without replacement

The ordering without replacement is better than with replacement, because without replacement converges more to the answer in the end.



The Histories of $\|x_k - x^*\|$

The ordering without replacement is better than with replacement, because without replacement converges more to the answer in the end.