

# Cards Solitaire

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## Abstract

Project.

## 1 Description of the game

### 1.1 The cards set

Consider a set of playing cards form by  $s$  *suits* (palos in Spanish) having  $m$  cards each. Thus, the size of the set is  $n = s \times m$ .

For instance, the Spanish cards set (*baraja española*) is made of four suits (*espadas, bastos, oros, copas*) and each suit has 12 cards (hence,  $n = 48$ ). The French cards set is made of four suits (*spades, clubs, diamonds, hearts*) and each suit has 13 cards (hence,  $n = 52$ ).

For reasons that will become apparent next, we restrict ourselves to the case in which  $n - 1$  is a multiple of 3. Clearly, the french set satisfies this condition ( $51 = 17 \times 3$ ), while the Spanish set does not satisfy it. Interestingly, in most games played with the Spanish set, two cards of each suit are removed, so in this reduced set, the condition is satisfied.

We name the suits  $0, 1, \dots, s-1$ . The cards of each suit are ranked  $0, 1, \dots, m-1$ . Thus, can identify each card as a pair  $(i, j)$ , where  $0 \leq i < s$  denotes the suit and  $0 \leq j < m$  denotes the rank. For convenience, we will refer to card  $(i, j)$  with a unique index  $k = 4 * i + j$ .

### 1.2 The solitaire rules

The game goes as follows. We take one card (i.e, typically, the first card of first suit, which in the French set is the ace of spades) out of the set and leave it, face up, on the table. This card is the bottom of what we call the *stack*. We shuffle the rest of the set and put them face up on the table in piles of *three* (each sequence of three consecutive cards in the shuffled set form a pile).

The only allowed move of the game is to take one card from the top of a pile and add it to the top of the stack. In order for a card to be placed in stack, it must be a rank higher or lower than the top card on the stack.

The game ends if there are no more top cards that can be moved to the stack. The game is won if all of the cards end up in the stack.

Figure 1 shows an example with the French set. On the top, there is the initial configuration: the stack and the 17 piles. On the bottom, there is a winning final stack. You can easily verify that the final stack can be build by a sequence of valid moves.

	4♦	7♥	7♠	3♦	5♠	T♣	6♠	J♣
	9♠	9♥	J♥	4♠	K♦	Q♦	T♠	T♦
A♠	8♠	5♦	2♥	5♣	T♥	3♣	8♣	A♥

  

J♠	9♦	7♦	2♣	3♥	7♣	3♠	6♦	9♣
A♣	Q♠	K♠	Q♥	5♥	K♣	8♥	J♦	2♦
2♠	K♥	Q♣	4♥	6♣	6♥	A♦	4♣	8♦

  

A♠-2♣-3♠-4♦-5♠-6♠-7♠-8♥-9♠-8♠-9♣-T♠-J♠-  
 Q♥-J♥-T♣-J♣-Q♦-K♦-A♣-2♠-3♥-2♦-3♣-4♥-5♥-  
 6♣-7♥-8♣-7♣-6♦-7♦-8♦-9♥-T♥-9♦-T♦-J♦-Q♠-  
 K♠-A♥-K♥-Q♣-K♣-A♦-2♥-3♦-4♠-5♣-6♥-5♦-4♣

Figure 1:

Note that there is no guarantee, in general, of a winning strategy. It will depend on the result of the shuffling phase.

## 2 The project

The purpose of the project is to write a SAT-based solitaire player that wins the game if it is possible to do so. The input of your system will be a shuffled set (i.e, a random permutation of the numbers  $1, \dots, n$ ) and the output will be a solution of the solitaire (i.e, a new permutation of the numbers, that can be constructed from the input by valid moves) or a message reporting that there is no solution. Your solver should abort if the execution is not completed within 5 minutes.

You have to hand in in due time the following items:

- A brief description of your SAT model clearly indicating the variables and the clauses that constitute the formula with which you call the SAT solver
- The program(s) that transforms the solitaire (i.e, the input) into a CNF formula and the CNF model into a sequence of cards (i.e, the output).
- We will provide you with a set of solitaire random instances of different sizes. You must hand in a table indicating, for each instance, if your solver completed the execution before the time limit. If so, you must report if the instance is solvable or not, along with the CPU time required to find it out.