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UNIVERSITE CATHOLIQUE DE LOUVAIN

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# Study and Analysis of Networks Flows

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## Abstract



## Acknowledgment



# Contents

<b>1</b>	<b>The Maximum Flow Problem</b>	<b>9</b>
<b>2</b>	<b>Existing Algorithms</b>	<b>11</b>
2.1	Augmenting path algorithms . . . . .	11
2.1.1	Introduction . . . . .	11
2.1.2	Ford-Fulkerson vs Edmonds-Karp . . . . .	12
2.1.3	Complexities . . . . .	12
2.2	Pre-flow algorithms . . . . .	12
<b>3</b>	<b>Data Structures</b>	<b>13</b>
<b>4</b>	<b>Improvements of Existing Algorithms</b>	<b>15</b>
<b>5</b>	<b>Implementation</b>	<b>17</b>
<b>6</b>	<b>Experimental Analysis</b>	<b>19</b>
<b>7</b>	<b>Conclusion</b>	<b>21</b>
	<b>Bibliography</b>	<b>21</b>





## Chapter 1

# The Maximum Flow Problem



## Chapter 2

# Existing Algorithms

### 2.1 Augmenting path algorithms

#### 2.1.1 Introduction

The idea behind the augmenting path algorithms is as follows : As long as there is a path from the source to the sink, we send flow along this path. And so on until there is no more path from the source to the sink.

An available path from the source to the sink is called *augmenting path* and to find it, we use the *residual graph*. A *residual graph* is a double oriented graph with the available capacities. For instance here is a graph with its *residual graph*:

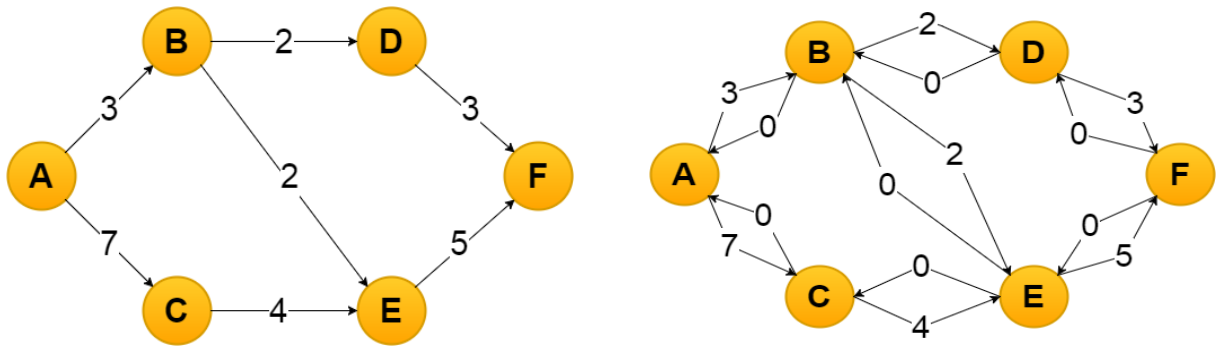


Figure 2.1: Graph with its residual graph

When an *augmenting path* is found, we send a flow equivalent to the minimum capacity of the edges of this path. We update the *residual graph* and then we look for a new augmenting path.

Here is the *residual graph* after sending 4 units of flow through the *augmenting path* A-C-E-F :

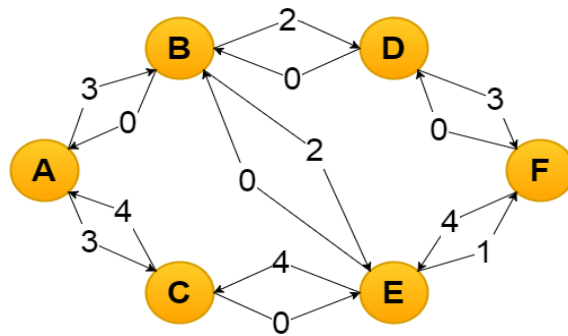


Figure 2.2: Residual graph

The pseudo-code of the augmenting path algorithm is given here :

```

Look for an augmenting path;
while There is an augmenting path do
    | Send flow through this path;
    | Update the residual graph;
    | Look for an new augmenting path;
end
  
```

### 2.1.2 Ford-Fulkerson vs Edmonds-Karp

Je cause de la différence entre FF et EK

### 2.1.3 Complexities

Petite intro pour dire que c'est polynomial

**Ford-Fulkerson**

**Edmonds-Karp**

Il faut démontrer que la complexité grand O est pas possible?

## 2.2 Pre-flow algorithms

## Chapter 3

# Data Structures



## Chapter 4

# Improvements of Existing Algorithms





## Chapter 5

# Implementation



## Chapter 6

# Experimental Analysis



## Chapter 7

## Conclusion



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# List of Figures

2.1	Graph with its residual graph . . . . .	11
2.2	Residual graph . . . . .	12



# List of Tables