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**must**

**page of your FINAL EXAM-paper answer pages.**

## York University

**Department of Electrical Engineering and Computer Science Lassonde School of Engineering**

**MATH 1090 B. FINAL EXAM, December 22, 2021; 9:05am-11:05am**

**Professor George Tourlakis**

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**FINAL EXAM-paper answer pages.**

By puttingmy name and student ID on this MID TERM page, I attest to the fact that my answers included here and submitted by Moodle are my own work, and that I have acted with integrity, abiding by the *Senate Policy on Academic Honesty* that the instructor discussed at the beginning of the course and *linked the full Policy to the Course Outline*.

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**FINAL EXAM-paper answer pages.**

**README FIRST! INSTRUCTIONS:**

***EXAM***. **1.TIME-LIMITED ON LINE *FINAL***

You have **120 MIN *to answer the EXAM questions***.

**ABSOLUTELY last** opportunity toupload toeClassis **BY 11:20am**, that is, **EX- ACTLY 15 min** after theOfficial End Time.

1. **Only ONE file can be uploaded per stu- dent**.
2. If you submit photographed copy **it still must be ONE file that you submit**. Either ZIP the JPEG/PNG images **OR** import them in **MS Word** and submit *ONE* Word *file* with the images attached.

|  |  |  |
| --- | --- | --- |
| **Question** | **MAX POINTS** | **MARK** |
| 1 | 3 |  |
| 2 | 3 |  |
| 3 | 5 |  |
| 1 | 3 |  |
| 2 | 5 |  |
| 3 | 5 |  |
| 4 | 5 |  |
| 5 | 5 |  |

*uploading mechanisms (15 min)* for the FI- **4.** *Using the time allotted for the* NAL EXAM-answering part is at your own **discretion**.

But also at your own **risk**.

|  |  |  |
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| TOTAL | 34 |  |

**EXAM *not* written**. **FINAL EXAM *not* uploaded = FINAL**

1. Please write your answers by hand **as you nor-**

**mally do for assignments** or use a word processor that can convert to PDF.Microsoft Word is acceptable to uploadas is (without conversion to PDF).

1. Whatever results were *proved* in class or appeared in the assignments you may use without proof, **unless I tion**. If you are not sure whether some statement has **indeed** been proved *in class*, I recommend that you **am asking you to prove them inthis Examina-** prove it in order to be “safe”.

The following are the axioms of Propositional Calculus: In what follows, *A, B, C* stand for arbitrary formulae.

Properties of *≡*

**Associativity of** *≡* ((*A ≡ B*) *≡ C*) *≡* (*A ≡* (*B ≡ C*)) (1)

**Symmetry of** *≡* (*A ≡ B*) *≡* (*B ≡ A*) (2) Properties of *⊥, T*

*T* vs. *⊥ T ≡ ⊥ ≡ ⊥* (3)

Properties of *¬*

**Introduction of** *¬ ¬A ≡ A ≡ ⊥* (4) Properties of *∨*

**Associativity of** *∨* (*A ∨ B*) *∨ C ≡ A ∨* (*B ∨ C*) (5) **Symmetry of** *∨ A ∨ B ≡ B ∨ A* (6) **Idempotency of** *∨ A ∨ A ≡ A* (7)

**Distributivity of** *∨* **over** *≡ A ∨* (*B ≡ C*) *≡ A ∨ B ≡ A ∨ C* (8)

**Excluded Middle** *A ∨ ¬A* (9)

Properties of *∧*

**Golden Rule** *A ∧ B ≡ A ≡ B ≡ A ∨ B* (10) Properties of *→*

**Implication** *A → B ≡ A ∨ B ≡ B* (11)

and

The **Primary** Boolean rules are:

*A, A ≡ B*

*B*

*C*[**p** := *A*] *≡ C*[**p** := *B*] *A ≡ B*

(*Eqn*)

(*Leib*)

The following are the Predicate Calculus Axioms:

##### *Any partial generalisation of any formula in groups* Ax1*–*Ax6 *is an axiom for Predicate*

***Calculus.***

Groups **Ax1–Ax6** contain the following schemata:

**Ax1.** Every tautology.

**Ax2.** (*∀***x**)*A → A*[**x** := *t*]*,* for any term *t*. **Ax3.** *A* *→* (*∀***x**)*A*, provided **x** is not free in *A*. **Ax4.** (*∀***x**)(*A → B*) *→* (*∀***x**)*A →* (*∀***x**)*B*.

**Ax5.** For *each* object variable **x**, the formula **x** = **x**.

**Ax6.** For any terms *t, s*, the schema *t* = *s →* (*A*[**x** := *t*] *≡ A*[**x** := *s*])*.*

There is ONLY ONE **Primary** First-Order rule; MODUS PONENS (*MP*)

*A, A → B*

*B*

(*MP* )

In predicate calculus the most natural proofs are Hilbert-style.

The following **metatheorems** hold for ***both*** Propositional and Predicate Calculus:

* 1. *Redundant T.* Γ *€ A* iff Γ *€ A ≡ T*
  2. *Cut Rule. A ∨ B, ¬A ∨ C € B ∨ C*
  3. *Deduction Theorem.* If Γ*, A € B*, then Γ *€ A → B*
  4. *Proof by contradiction.* Γ*, ¬A € ⊥* iff Γ *€ A*
  5. *Post’s Theorem.* (Also called “tautology theorem”, or even “completeness of Propositional Calculus theorem”)

If *|*=taut *A*, then *€ A*.

***Also***: If Γ *|*=taut *A* for finite Γ, then also Γ *€ A*.

* 1. *Proof by cases. A → B, C → D € A ∨ C → B ∨ D*

**Also the special case:** *A → B, C → B € A ∨ C → B*

**The Existential Quantifier** *∃*

(*∃***x**)*A stands for ¬*(*∀***x**)*¬A*

therefore (*∃***x**)*A ≡ ¬*(*∀***x**)*¬A* is a tautology, hence an absolute theorem.

##### *Useful facts from Predicate Calculus (proved in class—you may use them without proof):*

We ***know*** that WL (*not stated here; you should know this rule well!* ) is a **derived rule** useful in *Equa- tional proofs within predicate calculus*.

* + - More “rules” and (meta)theorems.

1. *“Renaming the Bound Variable”.*

. Σ

If **z** does not occur in (*∀***x**)*A* as either free or bound, then *€* (*∀***x**)*A ≡* (*∀***z**) *A*[**x** := **z**] If **z** does not occur in (*∃***x**)*A* as either free or bound, then *€* (*∃***x**)*A ≡* (*∃***z**) *A*[**x** := **z**]

. Σ

1. *∀ over ◦ distribution, where “ ◦” is “ ∨” or “→”.*

*€ A ◦* (*∀***x**)*B ≡* (*∀***x**)(*A ◦ B*), **provided x** is not free in *A*

*∃ over ∧ distribution*

1. *∀ over ∧ distribution.*

*∃ over ∨ distribution.*

*€ A ∧* (*∃***x**)*B ≡* (*∃***x**)(*A ∧ B*), **provided x** is not free in *A*

*€* (*∀***x**)*A ∧* (*∀***x**)*B ≡* (*∀***x**)(*A ∧ B*)

*€* (*∃***x**)*A ∨* (*∃***x**)*B ≡* (*∃***x**)(*A ∨ B*)

1. *∀ commutativity (symmetry).*

*€* (*∀***x**)(*∀***y**)*A ≡* (*∀***y**)(*∀***x**)*A*

1. *Specialisation. “Spec”* (*∀***x**)*A € A*[**x** := *t*], for any term *t*.
2. *Dual of Specialisation. “Dual Spec” A*[**x** := *t*] *€* (*∃***x**)*A*, for any term *t*.
3. *Generalisation. “Gen”* If Γ *€ A* and if, moreover, the formulae in Γ have **no free x occurrences**, then also Γ *€* (*∀***x**)*A*.
4. *Monotonicity.* If Γ *A B* so that the formulae in Γ have **no free x occurrences**, then we can infer

*∀ € →*

Γ *€* (*∀***x**)*A →* (*∀***x**)*B*

1. *Introduction; a special case of Monotonicity.* If Γ *A B* so that neither the formulae in Γ nor

*∀ ∀ € →*

*A* have **any free x occurrences**, then we can infer

Γ *€ A →* (*∀***x**)*B*

1. Finally, the *Auxiliary Hypothesis Metatheorem.* If Γ ( **x**)*A*, and if **y** is a variable that ***does not*** occur as either free or bound variable in any of *A* or *B* or the formulae of Γ —that is, it isfresh — then Γ*, A*[**x** := **y**] *€ B* implies Γ *€ B*

*€ ∃*

## Semantics facts

|  |  |
| --- | --- |
| Propositional Calculus | Predicate Calculus |
| (**Boolean Soundness**) *€ A* implies *|*=taut *A*  (Post) *|*=taut *A* implies *€ A* | *€ A* does **NOT** imply *|*=taut *A*  However, *|*=taut *A* implies *€ A*, **and**  (**Pred. Calc. Soundness**) *€ A* implies *|*= *A* |

 **CAUTION!** The above facts/tools are only *a fraction* of what we have covered in class. They are ***very important and very useful***, and that is why they are listed for your reference here.

You can also use *without proof* ***ALL*** the things we have covered (such as the absolute theorems known as “*∃*-definition”, “de Morgan’s laws”, etc.).

#### But these —the unlisted ones— are up to you to remember and to correctly state!

*Whenever in doubt of whether or not a “tool” you are about to use was indeed covered in*

*class,* ***prove*** *the validity/fitness of the tool before using it!* 

**Boolean Logic 1.** (3 MARKS) Suppose *€ A* and *€ B*. Does it follow that

*€ A ≡ B*?

If yes, give a proof.

If not, use soundness to justify your “NO”. Post’s theorem is NOT allowed.

**Boolean Logic 2.** (3 MARKS) Suppose *€ A ≡ B*. Does it follow that *€ A*

and *€ B*?

### If yes, give a proof.

If not, use soundness to justify your “NO”. Post’s theorem is NOT allowed.

**Boolean Logic 3.** (5 Marks) Prove **by Resolution**:

*€* .*X →* (*Y → Z*)Σ *→* .(*X → Y* ) *→* (*X → Z*)Σ

## Caution: 0 Marks gained ifanyother technique is used.In particular, Post’s theorem is NOT allowed.

###  A proof by resolution

* 1. MUST useproof by contradiction , and
  2. *It cannot/must not* be “preloaded” with *a long Equational or Hilbert proof* only to conclude with *ONE CUT*.

Such a proof, *if correct*,loses half the points . 

**Predicate Logic 1.** (3 MARKS) **True** or **False** and **WHY —No correct “WHY” = 0 MARKS**:

For any formula *A*, we have *€* (*∀***x**)(*∀***z**)(*A ≡ A*).

**Predicate Logic 2.** (5 MARKS) Use an **Equational** proof to establish the

### *∃*-version of the one-point-rule:

If **x** is not free in *t* then

*€* (*∃***x**)(**x** = *t ∧ A*) *≡ A*[**x** := *t*]

### *Hint.* Use the *∀*-version of the one-point rule as agiven .

**Predicate Logic 3.** (5 MARKS) Using soundness of predicate logic show why the converse of Ax4 [(p.3)](#_bookmark0)

(*∀***x**)(*A → B*) *→* (*∀***x**)*A →* (*∀***x**)*B* (*Ax*4) is not a theorem.

**Predicate Logic 4.** (5 MARKS) Prove *€* (*∀***x**)*A ∨* (*∀***x**)*B →* (*∀***x**)(*A ∨ B*).

### **Predicate Logic 5.** (5 MARKS) You must use the technique of the “auxil- iary hypothesis metatheorem” in the proof that you are asked to write here. Any other proof (***even IF correct*** ) will *MAX* at 0 MARKS.

For any formulas *A, B*, and *C* show that

*€* (*∃***x**).*A ∨ B → C*Σ *→* (*∃***x**)(*A → C*) *∧* (*∃***x**)(*B → C*)