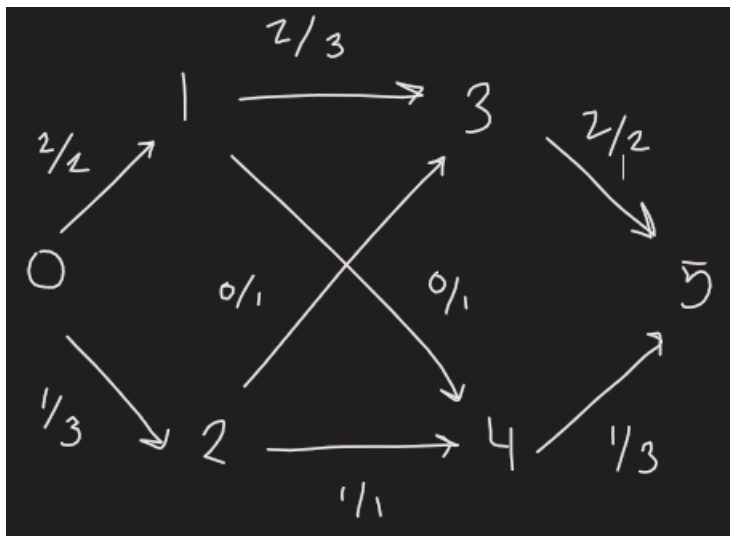


### Flow/Residual Network Worksheet

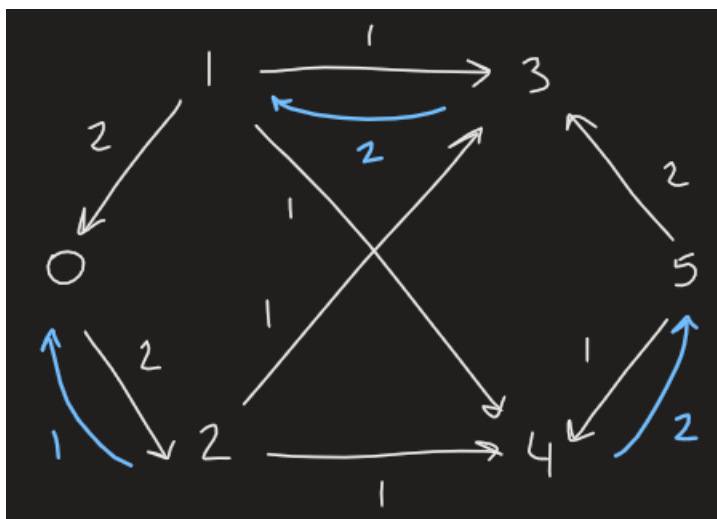
The table below shows the capacity and flow for each directed edge in a flow network with source 0 and sink 5.

Directed edge	capacity	flow
(0,1)	2	2
(0,2)	3	1
(1,3)	3	2
(1,4)	1	0
(2,3)	1	0
(2,4)	1	1
(3,5)	2	2
(4,5)	3	1

1. Draw the flow network.



2. Draw the residual network.



3. Identify an augmenting path from the source to the sink in the residual network.

$0 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 5$

## Bipartite Matching/Network Flows Worksheet

Large companies like Yahoo! and Google have enormous advertising potential due to the simple fact that million of users look at their websites everyday. By convincing people to provide some personal data or even by obtaining a user's location from their IP address, a company like Yahoo! or Google can show a user a targeted advertisement. For example, a Computer Science major from the Colorado School of Mines may see a banner ad for apartments in Golden while an investment banker in Connecticut may see a banner ad for Lincoln Town Cars instead.

Deciding which ads to show which people involves some behind-the-scenes computation. Suppose a popular website has identified  $k$  distinct demographic groups  $G_1, G_2, \dots, G_k$ . Note that these groups may overlap; for example  $G_i$  can be equal to all residents of Colorado, and  $G_j$  can be equal to all people with a computer science degree. Suppose the site has contracts with  $m$  different advertisers  $A_1, A_2, \dots, A_m$  to show exactly two copies of each ad to a subset of the  $n$  users  $U_1 \dots U_n$  of the website. Advertiser  $A_i$  wants its ads shown only to users who belong to at least one of the demographic groups in the set  $X_i \subseteq \{G_1, G_2, \dots, G_k\}$ .

Describe how to use Bipartite Matching/Network Flows to design a good advertising policy - a way to show each of the  $m$  ads to 2 users of the site so that a total of  $2m$  ads are shown to  $2m$  distinct users.

A graph can be constructed with 3 vertical columns - advertisers on the left, groups in the middle, and users on the right. Each advertiser can be linked to a relevant group, and each user can be linked to the groups they are associated with. To ensure that an advertisement is displayed to a maximum of 2 users, a capacity of 2 can be set for each advertisement connecting to the sink.