1-7.

The base case of multiply(y,0) does return 0, so the algorithm works. Now assume k > 0 and that multiply(y,z) = yz for all z < k. We need to prove for all integers k ≥ 0, for all integers y, z such that |z| = k, we have multiply(y, z) = y · z. Given c ≥ 2, we can write k uniquely as

k = rc+s where r ≥ 0 and s in {0,1,...,c-1}

Then floor(k/c) = r, k mod c = s, and r < k since c > 1. Therefore,

multiply(y,k) = multiply(cy, r) + ys = cyr + ys = yk.

1-15.

The base case when n = 1, the left side of (1) is 1/(1 · 2) = 1/2, and the right side is 1/2, so both sides are equal and (1) is true for n = 1. Let k be an integer ≥ 1 and suppose (1) is true for n = k. Then,

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2}$$

Thus, (1) holds for n = k + 1, and the proof of the induction step is complete.

1-25.

If the algorithm takes time proportional to n^2, then $1{,}000^2$ = 1,000,000 and $10{,}000^2$= 100,000,000. Dividing the latter by the former yields 100. Therefore, the sorting algorithm would take 1 minute and 40 seconds to sort 10,000 items.

If the algorithm takes time proportional to nlogn, then 1,000log1,000 = 3,000 and 10,000log10,000 = 40,000. Dividing the latter by the former yields 13.333.

1-29.

First, divide the 25 horses into groups of 5 and race the horses in each group. This totals to 5 races. Second, take the winner from each group and race those 5 horses. The winner of this race is the fastest horse overall. So far, this totals to 6 races. To keep track of the horses, we can label the 5 groups as a, b, c, d, e to correspond the horses finishing 1st, 2nd, 3rd, 4th, and 5th. Using subscripts to identify the order the horse finished in the group will be more efficient. For example, a_4 is the horse in 4th place from group a. One more race need to be conducted to find the 2nd and 3rd place horse. Race horse a_2, a_3, b_1, b_2, c_1. The horses in this race will have 2nd and 3rd place winners. This totals to 7 races are need to find the top 3 horses.