2-7.

(a) Is $2^{n+1}$ = O($2^n$)? True

f(n) is said to be O(g(n)) if there exist constants c and k such that if l ≥ k then f(l) ≤ c × g(l)

If c = 2 and k = 1 we have 2 × $2^n$ ≥ $2^{n+1}$ for all n ≥ 1

(b) Is $2^{2n}$ = O($2^n$)? False

Let x = $2^n$ then there exist a constant c such that $2^x$ ≤ cx for all x

log($2^x$) ≤ log(cx) = x ≤ log(c) + log(x) = x – log(x) ≤ log(c)

this is a contradiction since x – log(x) is an increasing function and will eventually pass the constant c as x tends to infinity

2-23.

(a) O(n^2) worst-case means that the worst-case is bound from above by O(n^2). It does not necessarily mean that all cases must follow that complexity. Thus, there could be some inputs that are O(n).

(b) This is true since O(n^2) worst-case is only an upper bound on the worst-case. It's possible that all inputs can be done on O(n), which still follows this upper bound.

(c) Although the worst case is theta(n^2), this does not mean all cases are theta(n^2) .

(d) It's not possible since the worst-case input must follow theta(n^2), so it can't be O(n). Therefore, all cases are not O(n).

2-35.

(a) $T(n) = \sum_{i=1}^{n/2} \sum_{j=i}^{n-1} \sum_{k=1}^{j} c$

c is the constant time for printing foo where c = 1

(b) $T(n) = \sum_{i=1}^{n/2} \sum_{j=i}^{1-1} j \Rightarrow \sum_{i=1}^{n/2} \frac{(n-i)(n-1+i)}{2}$

$$\Rightarrow \sum_{i=1}^{n/2} \frac{(n-i)(n-1+i)}{2}$$

$$\Rightarrow \sum_{i=1}^{n/2} \frac{(n^2 - n + in - in + i - i^2)}{2}$$

$$\Rightarrow \sum_{i=1}^{n/2} \frac{(n^2 - n + i - i^2)}{2}$$

$$\Rightarrow \frac{1}{2} \sum_{i=1}^{n/2} (n^2 - n + i - i^2)$$

$$\frac{1}{2}\left(\frac{n^3}{2} - \frac{n^2}{2} + \frac{n(n+2)}{8} - \frac{n^3 + 3n^2 + 2n}{24}\right) \Rightarrow T(n) = \frac{n^3}{4} - \frac{n^2}{4} + \frac{n(n+2)}{16} - \frac{n^3 + 3n^2 + 2n}{48}$$

2-39.

Let,

$log_b(x) = m \Rightarrow x = b^m$ (1)

$log_a(x) = p \Rightarrow x = a^p$ (2)

$log_b(a) = q \Rightarrow a = b^q$ (3)

From (1) we can solve for b

$b = x^{1/m}$ (4)

Similarly, we can solve for b from (3)

$b = a^{1/q}$ (5)

From (4) and (5)

$x^{1/m} = a^{1/q} \Rightarrow x = a^{m/q}$ (6)

Combine (6) and (2)

$a^{m/q} = a^p \Rightarrow \frac{m}{q} = p$

Replace your known values: $\frac{log_b x}{log_b a} = log_a x$

2-47.

To identify the forgery bag, take 1 coin from the first bag, 2 coins from the second bag, 3 coins from the third bag, and so on until the tenth bag. Weigh all the coins on the scale. The total weight should be 10 grams times (1+2+3+4+5+6+7+8+9+10) = 55 = 550 grams if all the coins are the same. Subtract the total of bag from the total of 550. If the difference is 1 gram, then bag 1 is the forgery, if it's 2 gram, then bag 2 is the forgery, and so on.