

Log Worksheet

1. If $\log_{100} x = y$, express $\log_{10} x^3$ in terms of y ?

$$\log_{10} x^3 = 3 \log_{10} x$$

$$3\left(\frac{\log_{100} x}{\log_{100} 10}\right) = 3\left(\frac{y}{\frac{1}{2}}\right)$$

$$6y$$

2. Prove that $\log(n!) = O(n \log n)$.

$$n! \leq n^n, n \geq 1$$

$$\log n! \leq \log n^n, n \geq 1$$

$$\log n! \leq n \log n, n \geq 1$$

$$\text{Therefore, } \log(n!) = O(n \log n).$$

3. Prove that $\log(n!) = \Omega(n \log n)$ (difficult).

$$n! \geq \left(\frac{n}{2}\right)^{\frac{n}{2}}, n \geq 2$$

$$\log n! \geq \frac{n}{2} \log \frac{n}{2}$$

$$\log n! \geq cn \log n, c < \frac{1}{2}$$