Log Worksheet

1. If $\log_{100} x = y$, express $\log_{10} x^3$ in terms of y?

$$3\ log_{10}\ x = 6y$$

$$log_{10}\ x = \frac{log_{100}\ x}{log_{100}\ 10}$$

$$= \frac{y}{1\ /\ 2} = 2y$$

2. Prove that $\log(n!) = O(n\log n)$.

$$n! \leq n^n, n \geq 1$$

$$n \times (n-1) \times (n-2) \times \ldots \times 1 \leq n \times n \times n \ldots \times n$$

$$log\ n! \leq log\ n^n$$

$$n\ log\ n, n \geq 1$$

$$c = 1$$

3. Prove that $\log(n!) = \Omega(n\log n)$ (difficult).

$$n! \geq (\frac{n}{2})^{(n/2)}$$

$$log\ n! \geq \frac{n}{2} log\ \frac{n}{2}$$

$\geq c\ n\ log\ n$ → want to find constant c

$(1-c)n\ log\ n \geq (log\ e)n$ → must satisfy n ≥ n_0

$(1-c)n\ log\ n \geq (log\ e)$ → must satisfy n > 0

$(1-c)logn \geq (1-c)logn_0$ → need positive c and n_0 to satisfy the statement below

$$(1-c)logn_0 \geq (log\ e)$$

For example, c = 0.1 and n_0 = 4

$$log\ n! = \theta(n\ log\ n)$$