Counterexample Worksheet

The discovery of an algorithm often begins with a sudden insight into the problem. Sometimes (unfortunately), an idea that seems very intuitive at first glance, turns out not to be correct on further thought. Figuring out that something is not correct by finding counterexamples is a useful skill. Among other things, it deepens your understanding of the problem.

Find counterexamples for the following propositions:

1. **Proposition**: a + b > min(a,b)

   a and b can be both less than zero

2. **Proposition**: the shortest route in a road network between two points is one with the fewest turns.

   A route with a staircase-function might be the shortest if it is the most direct access route. Other routes with fewer turns might require travelling redundant distances.

3. **Proposition**: being up by a queen in a game of chess guarantees a win!

   You can sacrifice a queen to gain an advantage like more material, a better position, or checkmate. You win when you checkmate the king.

Insertion-Sort/Execution-Counter Worksheet

Assume array A is indexed from 1 to n.

INSERTION_SORT(A, n)
1.**for** $j \leftarrow 2$ **to** n **do**
  2.$key \leftarrow A[j]$
  3.$i \leftarrow j - 1$;
  4.**while** $i > 0$ **and** $A[i] > key$ **do**
  5.$A[i+1] \leftarrow A[i]$
  6.$i \leftarrow i - 1$
  7.$A[i+1] \leftarrow key$

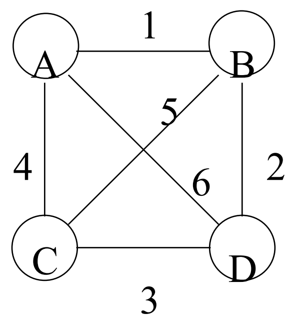
Instance 1 : [4, 3, 2, 1]
Instance 2 : [1, 4, 2, 3]
Instance 3 : [5, 4, 3, 2, 1]
Instance 4: [1, 2, 3, 4, 5]

	# Times Executed			
Line No	Instance 1	Instance 2	Instance 3	Instance 4
L1	4	4	5	5
L2	3	3	4	4
L3	3	3	4	4
L4	2+3+4=9	1+2+2=5	2+3+4+5=14	3+2+1+0=6
L5	1+2+3=6	0+1+1=2	1+2+3+4=10	2+1+0-1=2
L6	6	2	10	2
L7	3	3	4	4
Total	34	22	51	27

List any observations.

L1 – L3, and L7 are executed a number of times that only depends on the length of the array, and not the order of the elements.

TSP Worksheet



1. Show all possible TSP tours in the graph and compute their cost; for example, one TSP tour is A–B–C–D–A and its cost is 1 + 5 + 3 + 6 = 15.

   ABCDA = 15, BACDB = 10, CABDC = 10, DABCD = 15

   ABDCA = 10, BADCB = 15, CADBC = 17, DACBD = 17

   ACBDA = 17, BCADB = 17, CBADC = 15, DBACD = 10

   ACDBA = 10, BCDAB = 15, CBDAC = 17, DBCAD = 17

   ADCBA = 15, BDACB = 17, CDABC = 15, DCABD = 10

   ADBCA = 17, BDCAB = 10, CDBAC = 10, DCBAD = 15

2. How many distinct tours are there when you account for the same tour being counted multiple times?

$$\frac{n!}{n} = \frac{(n-1)!}{2} = \frac{3!}{2} = 3$$