1. $\dot{x} = \theta_1 \sin(2x^4-3) + \cos(x^2+x) + \theta_2 \sin(x-x^3) + \theta_3 x^5 + 6u$ – объект

адаптивное слежение, цель: $\lim\limits_{t\to\infty}(x_m(t) - x(t)) = \lim\limits_{t\to\infty}\varepsilon(t) = 0$

$$\varepsilon = x_m - x, \quad \dot{x}_m = -\lambda x_m + \lambda g, \quad \lambda > 0$$

$\dot{\varepsilon} = \dot{x}_m - \dot{x} = -\lambda x_m + \lambda g - \theta_1 \sin(2x^4-3) - \cos(x^2+x) - \theta_2 \sin(x-x^3) - \theta_3 x^5 - 6u$

нужно: $\dot{\varepsilon} = -\lambda\varepsilon \Rightarrow -\lambda\varepsilon = -\lambda x_m + \lambda g - \theta_1 \sin(2x^4-3) - \cos(x^2+x) - \theta_2 \sin(x-x^3) - \theta_3 x^5 - 6u$

$$\Updownarrow$$

$$u = \frac{1}{6}\left(\lambda\varepsilon - \lambda x_m + \lambda g - \theta_1 \sin(2x^4-3) - \cos(x^2+x) - \theta_2 \sin(x-x^3) - \theta_3 x^5\right)$$

настраиваемый регулятор: $u = \frac{1}{6}\left(-\lambda x + \lambda g - \hat{\theta}_1 \sin(2x^4-3) - \cos(x^2+x) - \hat{\theta}_2 \sin(x-x^3) - \hat{\theta}_3 x^5\right)$

необх. найти оценку $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$

use м-д ф-ии Ляпунова:

$\dot{x} = -\lambda x + \lambda g + \tilde{\theta}_1 \sin(2x^4-3) + \tilde{\theta}_2 \sin(x-x^3) + \tilde{\theta}_3 x^5$, где $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $i = \overline{1,3}$

тогда $\dot{\varepsilon} = \dot{x}_m - \dot{x} = -\lambda x_m + \lambda g + \lambda x - \lambda g - \tilde{\theta}_1 \sin(2x^4-3) - \tilde{\theta}_2 \sin(x-x^3) - \tilde{\theta}_3 x^5 =$

$$= -\lambda\varepsilon - \tilde{\theta}_1 \sin(2x^4-3) - \tilde{\theta}_2 \sin(x-x^3) - \tilde{\theta}_3 x^5$$

ф-я Ляпунова: $V(x) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma_1}\left(\tilde{\theta}_1 \sin(2x^4-3)\right)^2 + \frac{1}{2\gamma_2}\left(\tilde{\theta}_2 \sin(x-x^3)\right) + \frac{1}{2\gamma_3}\left(\tilde{\theta}_3 x^5\right)^2$

$V(x) > 0, \forall x$ и $V(x) \to \infty$ при $\|x\| \to \infty$

$$\begin{aligned}\dot{V}(x) = \varepsilon\dot{\varepsilon} &+ \frac{1}{\gamma_1}\tilde{\theta}_1 \sin(2x^4-3)\left[\dot{\tilde{\theta}}_1 \sin(2x^4-3) + \tilde{\theta}_1 \cos(2x^4-3)\cdot 8x^3\right] + \\ &+ \frac{1}{\gamma_2}\tilde{\theta}_2 \sin(x-x^3)\left[\dot{\tilde{\theta}}_2 \sin(x-x^3) + \tilde{\theta}_2 \cdot \cos(x-x^3)(1-3x^2)\right] + \\ &+ \frac{1}{\gamma_3}\tilde{\theta}_3 x^5\left[\dot{\tilde{\theta}}_3 x^5 + \tilde{\theta}_3 \cdot 5x^4\right] = / \dot{\tilde{\theta}}_i = -\dot{\hat{\theta}}_i, i = \overline{1,3} / =\end{aligned}$$

$$= -\lambda\varepsilon^2 + \frac{1}{\gamma_1}\left[-\dot{\hat{\theta}}_1 \cdot \tilde{\theta}_1 \cdot \sin^2(2x^4-3) + \tilde{\theta}_1^2 \sin(2x^4-3)\cos(2x^4-3)8x^3\right] - \varepsilon\tilde{\theta}_1 \sin(2x^4-3) +$$

$$+ \frac{1}{\gamma_2}\left[\ \ldots\right.$$

неверно выбрана ф-я Ляп, try another

$$V(\varepsilon) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^2 + \frac{1}{2\gamma_3}\tilde{\theta}_3^2$$

$V(\varepsilon) > 0\ \forall \varepsilon$ и $V(\varepsilon) \to \infty$ при $\|\varepsilon\| \to \infty$

$$\begin{aligned}\dot{V}(\varepsilon) &= \varepsilon \cdot \dot{\varepsilon} + \frac{1}{\gamma_1}\tilde{\theta}_1 \cdot \dot{\tilde{\theta}}_1 + \frac{1}{\gamma_2}\tilde{\theta}_2 \cdot \dot{\tilde{\theta}}_2 + \frac{1}{\gamma_3}\tilde{\theta}_3 \cdot \dot{\tilde{\theta}}_3 = \\ &= \varepsilon \cdot \left(-\lambda\varepsilon - \tilde{\theta}_1 \sin(2x^4-3) - \tilde{\theta}_2 \sin(x-x^3) - \tilde{\theta}_3 x^5\right) + \frac{1}{\gamma_1}\tilde{\theta}_1\dot{\tilde{\theta}}_1 + \frac{1}{\gamma_2}\tilde{\theta}_2\dot{\tilde{\theta}}_2 + \frac{1}{\gamma_3}\tilde{\theta}_3\dot{\tilde{\theta}}_3 \\ &= -\lambda\varepsilon^2 + \tilde{\theta}_1\left(\frac{1}{\gamma_1}\dot{\tilde{\theta}}_1 - \varepsilon\sin(2x^4-3)\right) + \tilde{\theta}_2\left(\frac{1}{\gamma_2}\dot{\tilde{\theta}}_2 - \varepsilon\sin(x-x^3)\right) + \tilde{\theta}_3\left(\frac{1}{\gamma_3}\dot{\tilde{\theta}}_3 - \varepsilon x^5\right) = \\ &= -\lambda\varepsilon^2 + \tilde{\theta}_1\left(-\frac{1}{\gamma_1}\dot{\hat{\theta}}_1 - \varepsilon\sin(2x^4-3)\right) + \tilde{\theta}_2\left(-\frac{1}{\gamma_2}\dot{\hat{\theta}}_2 - \varepsilon\sin(x-x^3)\right) + \tilde{\theta}_3\left(-\frac{1}{\gamma_3}\dot{\hat{\theta}}_3 - \varepsilon x^5\right)\end{aligned}$$

3 $\dot{\hat{\Theta}}_1 = -\gamma_1 \varepsilon \sin(2x^4-3)$, $\dot{\hat{\Theta}}_2 = -\gamma_2 \varepsilon \sin(x-x^3)$, $\dot{\hat{\Theta}}_3 = -\gamma_3 \varepsilon x^5$ — алгоритм адаптации

$\Rightarrow \dot{V} = -\lambda \varepsilon^2 < 0 \quad \forall \varepsilon \neq 0.$

След-но имеем 1) асимптотич. устойчивость в задаче стремления ошибки управления ε к нулю.

2) огранич. всех сигналов в системе

3) экспонен. стремление $\hat{\Theta} \to \Theta$ if $\int_t^{t+T} x^2(\tau)d\tau \geq \alpha I > 0$

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2. объект: $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \Theta_1 \cos(x_1) x_2^3 + \Theta_2 \sin(x_2) + \Theta_3 x_1^3 - \sin(\|x_2\|^6) + 5u \end{cases}$

цель: $\lim_{t\to\infty} \|e(t)\| = 0$, $e = x_M - x$, $\dot{x}_M = A_M x_M + b_M g$, $g(t)$ - зад. возд.

~~условие согласования:~~

$e = x_M - x$, $\dot{e} = \dot{x}_M - \dot{x} = A_M x_M + b_M g - \begin{bmatrix} x_2 \\ \Theta_1 \cos(x_1) x_2^3 + \Theta_2 \sin(x_2) + \Theta_3 x_1^3 - \sin(\|x_2\|^6) + 5u \end{bmatrix}$

$= \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} x_{M_1} \\ x_{M_2} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} g(t) - \begin{bmatrix} x_2 \\ \dots \end{bmatrix} =$

$= \begin{bmatrix} a_1 x_{M_1} + a_2 x_{M_2} + b_1 g - x_2 \\ a_3 x_{M_1} + a_4 x_{M_2} + b_2 g - \Theta_1 \cos(x_1) x_2^3 - \Theta_2 \sin(x_2) - \Theta_3 x_1^3 + \sin(\|x_2\|^6) - 5u \end{bmatrix}$

управлением можем влиять only на II компоненту вектора ошибки

$\dot{\varepsilon}_2 = a_3 x_{M_1} + a_4 x_{M_2} + b_2 g - \Theta_1 \cos(x_1) x_2^3 - \Theta_2 \sin(x_2) - \Theta_3 x_1^3 + \sin(\|x_2\|^6) - 5u$

след-но задача сводится к задаче слежения одномерным объектом (п. 1)

необх.: $\dot{\varepsilon}_2 = -\lambda \varepsilon_2 \Rightarrow u = \frac{1}{5}\Big[\lambda \varepsilon_2 + a_3 x_{M_1} + a_4 x_{M_2} + b_2 g - \hat{\Theta}_1 \cos(x_1) x_2^3 -$

$- \hat{\Theta}_2 \sin(x_2) - \hat{\Theta}_3 x_1^3 + \sin(\|x_2\|^6)\Big]$ — построенный регулятор

найдем алгоритм адаптации:

$\dot{x}_2 = \Theta_1 \cos(x_1) x_2^3 + \Theta_2 \sin(x_2) + \Theta_3 x_1^3 - \sin(\|x_2\|^6) + \lambda \varepsilon_2 + a_3 x_{M_1} + a_4 x_{M_2} + b_2 g - \hat{\Theta}_1 \cos(x_1) x_2^3 -$

$- \hat{\Theta}_2 \sin(x_2) - \hat{\Theta}_3 x_1^3 + \sin(\|x_2\|^6) = \tilde{\Theta}_1 \cos(x_1) x_2^3 + \tilde{\Theta}_2 \sin(x_2) + \tilde{\Theta}_3 x_1^3 + \lambda \varepsilon_2 + a_3 x_{M_1} +$

$+ a_4 x_{M_2} + b_2 g$

$\dot{\varepsilon}_2 = \dot{x}_{M_2} - \dot{x}_2 = \cancel{a_3 x_{M_1}} + \cancel{a_4 x_{M_2}} + \cancel{b_2 g} - \tilde{\Theta}_1 \cos(x_1) x_2^3 - \tilde{\Theta}_2 \sin(x_2) - \tilde{\Theta}_3 x_1^3 - \lambda \varepsilon_2 - \cancel{a_3 x_{M_1}}$

$- \cancel{a_4 x_{M_2}} - \cancel{b_2 g} = -\tilde{\Theta}_1 \cos(x_1) x_2^3 - \tilde{\Theta}_2 \sin(x_2) - \tilde{\Theta}_3 x_1^3 - \lambda \varepsilon_2$

ф-я Ляпунова: $V = \frac{1}{2}\varepsilon_2^2 + \frac{1}{2\gamma_1}\tilde{\Theta}_1 + \frac{1}{2\gamma_2}\tilde{\Theta}_2 + \frac{1}{2\gamma_3}\tilde{\Theta}_3$

$\dot{V} = \varepsilon_2 \dot{\varepsilon}_2 + \frac{1}{\gamma_1}\tilde{\Theta}_1 \dot{\tilde{\Theta}}_1 + \frac{1}{\gamma_2}\tilde{\Theta}_2 \dot{\tilde{\Theta}}_2 + \frac{1}{\gamma_3}\tilde{\Theta}_3 \dot{\tilde{\Theta}}_3 \ominus$

$= \varepsilon_2 (\cancel{a_3 x_{M_1} + a_4 x_{M_2} + b_2 g} - \tilde{\theta}_1 \cos(x_1) x_2^3 - \tilde{\theta}_2 \sin(x_2) - \tilde{\theta}_3 x_1^3 - \lambda \varepsilon_2) +$

$\cancel{a_3 x_{M_1}} - \frac{1}{\gamma_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1 - \frac{1}{\gamma_2} \tilde{\theta}_2 \cdot \dot{\hat{\theta}}_2 - \frac{1}{\gamma_3} \tilde{\theta}_3 \cdot \dot{\hat{\theta}}_3 =$

$$\text{] } \underbrace{\dot{\hat{\theta}}_1 = -\gamma_1 \varepsilon_2 \cos(x_1) x_2^3, \quad \dot{\hat{\theta}}_2 = -\gamma_2 \cdot \varepsilon_2 \cdot \sin(x_2), \quad \dot{\hat{\theta}}_3 = -\gamma_3 \cdot \varepsilon_2 \cdot x_1^3}_{\text{алгоритм адаптации}}$$

$= -\lambda \varepsilon_2^2 < 0, \ \forall \varepsilon \neq 0$

След-но данный настраиваемый регулятор:

$$u = \frac{1}{5}\left[\lambda \varepsilon_2 + a_3 x_{M_1} + a_4 x_{M_2} + b_2 g - \hat{\theta}_1 \cos(x_1) x_2^3 - \right.$$
$$\left. - \hat{\theta}_2 \sin(x_2) - \hat{\theta}_3 x_1^3 + \sin(\|x_2\|^6)\right]$$

с алгоритмом адаптации:

$$\dot{\hat{\theta}}_1 = -\gamma_1 \varepsilon_2 \cos(x_1) x_2^3$$
$$\dot{\hat{\theta}}_2 = -\gamma_2 \varepsilon_2 \sin(x_2)$$
$$\dot{\hat{\theta}}_3 = -\gamma_3 \varepsilon_2 \cdot x_1^3$$

обеспечивают:

1) $\varepsilon_2 \xrightarrow[t \to \infty]{} 0$ - ошибка II компоненты

2) ограниченность сигнала $\hat{\theta}$. в р $\hat{\theta}$ экспон. стр. к θ, if x удовлетворяет условию неисчезающего возбуждения.

3) $\cancel{\forall}$ $\dot{e}_1 = \dot{x}_{M_1} - \dot{x}_1 = a_1 x_{M_1} + a_2 x_{M_2} + b_1 g - x_2$, т.к.

$e_2 \xrightarrow[t \to \infty]{} 0 \Rightarrow x_{M_2} - x_2 \xrightarrow[t \to \infty]{} 0$, то если $\begin{cases} a_1 = 0 \\ a_2 = 1 \\ b_1 = 0 \end{cases}$, то.

$\dot{e}_1 = x_{M_2} - x_2 \xrightarrow[t \to \infty]{} 0 \Rightarrow$ ошибка I компоненты асимптот. $\to 0$.