Adaptation Algorithm (Static Error)

Static error model

$$\varepsilon = \omega^T \tilde{\theta}$$

Lyapunov function

$$V = \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} = \frac{1}{2\gamma}\left\|\tilde{\theta}\right\|^2$$

Adaptation algorithm

$$\dot{\hat{\theta}} = \gamma\omega\varepsilon$$

Parametric error model

$$\dot{\tilde{\theta}} = -\gamma\omega\omega^T\tilde{\theta}$$

$$\det(\omega\omega^T) \equiv 0$$

Adaptation algorithm with improved parametric convergence (Kreisselmeier`s scheme)

Modified model of parametric errors

$$\dot{\tilde{\theta}} = -\gamma L(s)\left[\omega\omega^T\right]\tilde{\theta}$$

where $L(s)$ is the operator with "memory" with which we use a stable minimum-phase positive transfer function.

$$L(s)\left[\omega\omega^T\right] \geq \alpha_0 I > 0,$$ if the regressor satisfies the persistent excitation condition

Lyapunov function derivative

$$\dot{V} = \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} = -\tilde{\theta}^T L(s)\left[\omega\omega^T\right]\tilde{\theta} \leq -\alpha_0\left\|\tilde{\theta}\right\|^2 = -2\gamma\alpha_0 V$$

Modified adaptation algorithm

$$\dot{\hat{\theta}} = \gamma\left(L(s)\left[\omega\varepsilon + \omega\omega^T\hat{\theta}\right] - L(s)\left[\omega\omega^T\right]\hat{\theta}\right)$$

Adaptation algorithm with improved parametric convergence (Lion`s scheme)

Modified model of parametric errors

$$\dot{\tilde{\theta}} = -\gamma \Xi^T \Xi \tilde{\theta}$$

where $\Xi = col\left(H_1(s)\left[\omega^T\right], H_2(s)\left[\omega^T\right], \cdots, H_q(s)\left[\omega^T\right]\right) \in \mathbb{R}^{q\times m}$, $q \geq m$, $H_i(s)$, $i = \overline{1,q}$ are linear independent stable minimum-phase positive transfer functions.

$$\Xi^T \Xi \geq \lambda_0 I > 0,$$ if the regressor satisfies the persistent excitation condition

Lyapunov function derivative

$$\dot{V} = \frac{1}{\gamma}\tilde{\theta}^T \dot{\tilde{\theta}} = -\tilde{\theta}^T \Xi^T \Xi \tilde{\theta} \leq -\lambda_0 \left\|\tilde{\theta}\right\|^2 = -2\gamma\lambda_0 V$$

Modified adaptation algorithm

$$\dot{\hat{\theta}} = \gamma \Xi^T \left(\bar{\Xi} - \Xi \hat{\theta}\right)$$

where $\bar{\Xi} = col\left(H_1(s)\left[\varepsilon + \omega^T \hat{\theta}\right], H_2(s)\left[\varepsilon + \omega^T \hat{\theta}\right], \cdots, H_q(s)\left[\varepsilon + \omega^T \hat{\theta}\right]\right).$