Chapter 7

Robust performance problems

In the previous chapters we have presented synthesis methods for optimal H_2 and H_∞ control problems, and studied the robust stabilization problem with respect to both unstructured and structured norm-bounded uncertainties. In realistic practical controller design problems the controller is in most cases required to satisfy both performance and robustness criteria. This leads in general to more complex design procedures than the synthesis methods described in the previous chapters. In this chapter some approaches which are used will be described.

We consider the control system in Figure 7.1. Here Δ denotes a norm-bounded uncertainty, which may be structured or unstructured. Denote the closed-loop transfer function from v_P to z_P by $F_P(P,K,\Delta)$, such that

$$z_P = F_P(P,K,\Delta)v_P \tag{7.1}$$

holds for the control system in Figure 7.1. The control objective is to minimize a performance measure $J(F_P(P,K,\Delta))$ related to the closed-loop transfer function. Typically, the cost $J(F_P)$ denotes the H_∞ norm or (the square of) the H_2 norm. In contrast to the optimal control problems studied in chapters 3 and 4, the actual value of the cost depends on the uncertainty Δ, which is unknown. Therefore, some assumptions on the uncertainty should be made in the formulation of an optimal control problem for the uncertain plant. A minimum requirement of any controller for the uncertain plant in Figure 7.1 is, of course, that it be robustly stabilizing. There are two main formulations of optimal control problems for uncertain plants: the robust performance problem and nominal performance optimization subject to robust stability.

In the robust performance problem the *worst cost* obtained in the assumed uncertainty set is minimized. The general robust performance problem is defined as follows.

Robust performance problem.
Find a robustly stabilizing controller for the uncertain plant depicted in Figure 7.1, which minimizes the worst-case cost $J_{worst}(P,K)$ defined as

$$J_{worst}(P,K) = \sup_{\Delta}\Big\{J(F_P(P,K,\Delta)) : \|\Delta\|_\infty \le \delta\Big\} \tag{7.2}$$

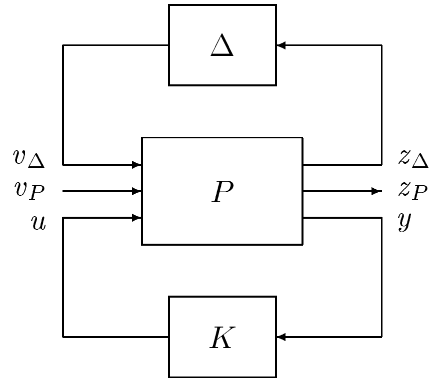

Figure 7.1: Control system for robust performance problem.

(unstructured uncertainties), or

$$J_{worst}(P, K) = \sup_{\Delta}\Big\{J(F_P(P, K, \Delta)) : \Delta \in \mathbf{\Delta}_s(\delta)\Big\} \tag{7.3}$$

(structured uncertainties), where $\mathbf{\Delta}_s(\delta)$ denotes the structured norm-bounded uncertainty set (6.8).

A controller designed for robust performance which achieves a given performance bound such that $J_{worst}(P, K) < \gamma$, guarantees a cost less than γ for all norm-bounded uncertainties. There are systematic controller synthesis procedures for both the robust H_∞ and the robust H_2 performance problems. The robust H_∞ performance problem can be shown to be equivalent to a μ-synthesis problem. The robust H_2 performance problem is more complex, as it mixes two different system norms; the H_2-norm associated with performance, and the H_∞ norm associated with robustness. This leads to a mixed H_2/H_∞ problem, for which special solution methods have been developed.

The worst-case nature of the costs in (7.2) and (7.3) may lead to a conservative design for 'average uncertainties', which are not worst-case with respect to the cost $J(F_P)$. For this reason, an alternatve formulation of the optimal control problem for uncertain plants may be stated as follows.

Nominal performance problem subject to robust stability.
Find a robustly stabilizing controller for the uncertain plant depicted in Figure 7.1, which minimizes the nominal cost $J(F_P(P, K, 0))$ defined for the nominal plant with $\Delta = 0$.

Here, the nominal cost is minimized, and the uncertainty is taken into account only by requiring robust stability. A potential problem with this formulation is that even though nominal performance may be good, there is no guarantee of acceptable performance when there are uncertainties $\Delta \neq 0$. The nominal performance problems subject to robust stability are in general more complex to solve than the robust performance

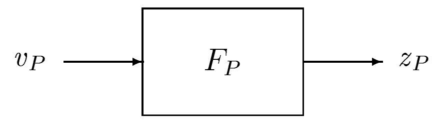

Figure 7.2: Plant with performance-related signals.

problems. In particular, they lack closed solutions, and must be solved by numerical optimization techniques.

A possible method for both avoiding too conservative design and ensuring acceptable over-all performance for all uncertainties could be to combine the robust performance and the nominal performance problems by minimizing a combination of nominal and worst-case costs. Not much has been done on this type of problems, however.

The next sections give brief discussions of the robust H_∞ and H_2 problems.

7.1 The robust H_∞ performance problem

In this section, we study the robust H_∞ performance problem defined for the uncertain plant in Figure 7.1. For convenience, it is assumed that the uncertainty belongs to the set $\mathbf{\Delta}_s(\delta)$ in equation (6.8). This implies no restriction, as an unstructured uncertainty can always be characterized by the set (6.8) with one uncertainty block ($s = 1$).

The robust H_∞ performance problem is defined as follows.

Robust H_∞ performance problem.
Find a robustly stabilizing controller for the uncertain plant in Figure 7.1, which achieves the robust H_∞ performance bound

$$\sup_{\Delta}\Big\{\|F_P(P,K,\Delta)\|_\infty : \Delta \in \mathbf{\Delta}_s(\delta)\Big\} < \delta^{-1} \tag{7.4}$$

For convenience, the H_∞-norm bound has been taken equal to the inverse of the uncertainty magnitude δ. This is not restrictive, as it can always be achieved by suitable scaling of the variables. In order to solve the robust H_∞ performance problem, consider the plant in Figure 7.2. It follows from the arguments in Chapter 5 that the plant in Figure 7.2 satisfies the H_∞-norm bound

$$\|F_P\|_\infty < \delta^{-1} \tag{7.5}$$

if and only if the uncertain plant depicted in Figure 7.3 is robustly stable with respect to norm-bounded (unstructured) uncertainties Δ_P that satisfy the norm-bound $\|\Delta_P\|_\infty \leq \delta$. Thus, an H_∞ performance problem is equivalent to a robust stability problem. Here, the uncertainty Δ_P can be considered as a fictive uncertainty associated with the H_∞-norm bound (7.5).

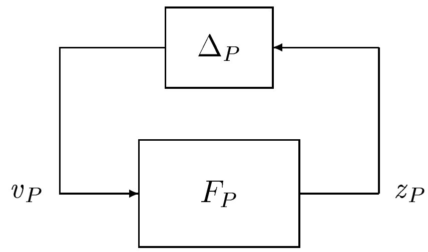

Figure 7.3: Characterization of performance as robust stability.

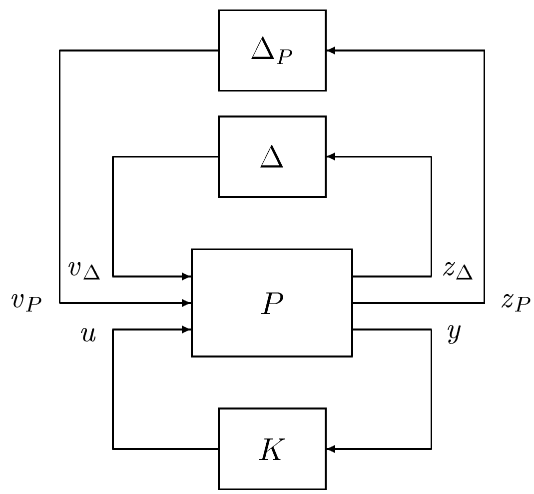

Figure 7.4: Characterization of robust performance in terms of structured uncertainty.

Applying the equivalence of H_∞ performance and robust stability to the closed-loop system in Figure 7.1 shows that the control system in Figure 7.1 satisfies the H_∞-norm bound

$$\|F_P(P, K, \Delta)\|_\infty < \delta^{-1} \tag{7.6}$$

if and only if the system in Figure 7.4 is robustly stable with respect to norm-bounded (unstructured) uncertainties Δ_P that satisfy the norm-bound $\|\Delta_P\|_\infty \leq \delta$. Hence, the plant in Figure 7.1 achieves the robust H_∞ performance bound (7.4), or equivalently, satisfies the H_∞-norm bound (7.6) for all $\Delta \in \mathbf{\Delta}_s(\delta)$, if and only if the system in Figure 7.4 is robustly stable with respect to all norm-bounded Δ_P which satisfy $\|\Delta_P\|_\infty \leq \delta$ *and* all $\Delta \in \mathbf{\Delta}_s(\delta)$. But this is equivalent to robust stability with respect to the set $\mathbf{\Delta}_{P,s}(\delta)$ of uncertainties defined as

$$\mathbf{\Delta}_{P,s}(\delta) = \left\{\tilde{\Delta} = \text{block diag}(\Delta, \Delta_P),\ \ \Delta \in \mathbf{\Delta}_s,\ \ \Delta_P \in H_\infty^{r_p \times r_p},\ \ \|\Delta_P\|_\infty \leq \delta\right\} \tag{7.7}$$

where it is assumed that Δ_P is $r_p \times r_p$, cf. Figure 7.5. Thus, the robust H_∞ performance

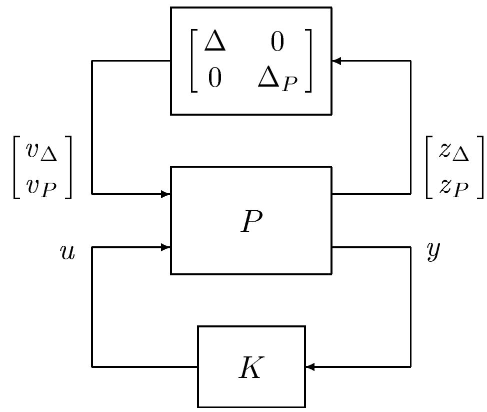

Figure 7.5: Characterization of robust performance problem.

problem is equivalent to a robust stability problem with respect to structured uncertainty belonging to the set $\mathbf{\Delta}_{P,s}(\delta)$, obtained by extending the original uncertainty set $\mathbf{\Delta}_s(\delta)$ with a performance-related uncertainty block Δ_P.

To summarize, we have the following result. Define the closed-loop transfer function $F = F(P, K)$ in Figure 7.5 from $[v_\Delta^T \;\; v_P^T]^T$ to $[z_\Delta^T \;\; z_P^T]^T$,

$$\begin{bmatrix} z_\Delta \\ z_P \end{bmatrix} = F(P, K) \begin{bmatrix} v_\Delta \\ v_P \end{bmatrix} \tag{7.8}$$

Theorem 7.1 Robust H_∞ performance.
Consider the system in Figure 7.1, where the uncertainty Δ is assumed to be in the set $\mathbf{\Delta}_s(\delta)$. The system is robustly stable and achieves the robust H_∞ performance bound (7.4) if and only if the closed-loop tranfer function $F = F(P, K)$ from $[v_\Delta^T \;\; v_P^T]^T$ to $[z_\Delta^T \;\; z_P^T]^T$ in Figure 7.5 is stable, and

$$\sup_\omega \mu_P(F(j\omega)) < \delta^{-1} \tag{7.9}$$

where the structured singular value $\mu_P(F)$ is taken to correspond to the structure of the extended uncertainty set $\mathbf{\Delta}_{P,s}(\delta)$ in (7.7).

The problem of finding a controller which achieves robust H_∞ performance can thus be solved by the procedures for the robust stabilization problem described in Chapter 6. Conditions for robust performance with respect to nonlinear and/or time-varying uncertainties are obtained in analogy with the robust stability conditions for these uncertainty classes, cf. Chapter 6.

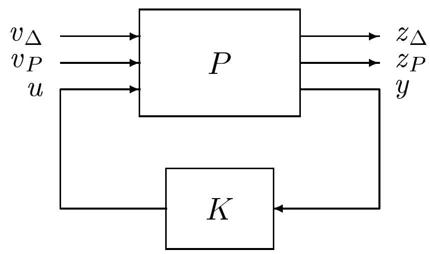

Figure 7.6: Control system for robust H_2 performance problem.

7.2 The robust H_2 performance problem

In the robust H_2 performance problem, the objective is to find a controller which achieves robust H_2 performance for the uncertain plant in Figure 7.1.

The robust H_2 performance problem.
Find a robustly stabilizing controller for the uncertain plant in Figure 7.1, which minimizes the worst-case cost H_2 cost

$$J_{2,worst}(P,K) = \sup_{\Delta}\Big\{\|F_P(P,K,\Delta)\|_2^2 : \|\Delta\|_\infty \leq \delta\Big\} \tag{7.10}$$

(unstructured uncertainties), or

$$J_{2,worst}(P,K) = \sup_{\Delta}\Big\{\|F_P(P,K,\Delta)\|_2^2 : \Delta \in \mathbf{\Delta}_s(\delta)\Big\} \tag{7.11}$$

(structured uncertainties), where $\mathbf{\Delta}_s(\delta)$ denotes the structured norm-bounded uncertainty set (6.8).

The problem contains a mixture of a performance-related H_2 cost and a robustness-related H_∞ cost. In contrast to the robust H_∞ performance problem, it cannot be reduced to any of the standard problems studied so far. Instead, the problem gives rise to a sort of *mixed H_2/H_∞ problem.* In fact, the problem is too hard to make an exact solution feasible. Instead, it is common to introduce an upper bound on the worst-case H_2 cost $J_{2,worst}(P,K)$ by observing that $\|\Delta\|_\infty \leq \delta$ implies

$$\|v_\Delta\|_2^2 \leq \delta^2 \|z_\Delta\|_2^2 \tag{7.12}$$

Similarly, a structured uncertainty implies a number of quadratic inequalities for the various components of v_Δ and z_Δ. It follows that an upper bound on the worst H_2 cost can be obtained by introducing a quadratically constrained H_2 cost for the plant in Figure 7.6 defined as

$$J_{rob}(P,K) = \sup_{v_\Delta = T v_P} \left\{ \sum_{k=1}^{m} \left[\int_0^\infty z_P^T(t) z_P(t) dt \quad : \quad v_P = e_k \delta(t) \right] \right\} \tag{7.13}$$

subject to

$$\sum_{k=1}^{m}\left[\int_0^\infty \left[z_\Delta^T(t)z_\Delta(t) - \delta^{-2}v_\Delta^T(t)z_\Delta(t)dt\right] \ : \ v_P = e_k\delta(t)\right] \geq 0 \tag{7.14}$$

The constraint of the form (7.14) is usually referred to in the literature as an *integral quadratic constraint* (IQC). As $v_\Delta = \Delta z_\Delta$ implies (7.12), which in turn implies (7.14), it follows that the quadratically constrained cost defined by (7.13), (7.14) gives an upper bound on the worst-case H_2 cost $J_{2,worst}(P,K)$.

The quadratically constrained cost can be evaluated by introducing the associated Lagrangian function,

$$\begin{aligned} L_{rob}(P,K,c) \ = \ \sup_{v_\Delta = T v_P} \Bigg\{ \sum_{k=1}^{m} \Bigg[\int_0^\infty \left[z_P^T(t)z_P(t) + c^2\left(z_\Delta^T(t)z_\Delta(t) - \delta^{-2}v_\Delta^T(t)v_\Delta(t)\right)\right] dt \ : \\ v_P = e_k\delta(t)\Bigg]\Bigg\} \end{aligned} \tag{7.15}$$

where $c > 0$ is a Lagrange multiplier associated with the quadratic constraint (7.14). It is straightforward to show that if the H_∞ norm from v_Δ to z_Δ is less than δ^{-1}, then c can be selected so that the cost $L_{rob}(P,K)$ is bounded, and then

$$J_{2,worst}(P,K) \leq J_{rob}(P,K) \text{ s.t. } (7.14) \leq L_{rob}(P,K,c), \ \text{ all } c > 0. \tag{7.16}$$

The upper bound in (7.16) on the worst-case H_2 cost can be used to introduce a robust H_2 (LQ) problem as follows.

Robust H_2 performance problem.
Find a constant $c > 0$ and a stabilizing controller for the plant in Figure 7.6, such that the cost $L_{rob}(P,K,c)$ in (7.15) in minimized.

Remark 7.1.
A linear time-invariant uncertainty Δ implies that v_Δ is bounded by δz_Δ at each frequency, i.e.,

$$\|v_\Delta(j\omega)\|^2 \leq \delta^2\|z_\Delta(j\omega)\|^2, \ \text{ all } \omega \tag{7.17}$$

This inequality can be used to construct a stricter upper bound on the worst-case H_2 cost. The associated constrained cost can be evaluated in terms of a Lagrangian function with a frequency-dependent Lagrange multiplier $d(j\omega)$, cf. the references.

The scalar c which minimizes $L_{rob}(P,K,c)$ can be found by direct search. In order to study the problem of finding a controller which minimizes the cost (7.15), consider the cost for a given fixed c. For this purpose, introduce the variable

$$z = \begin{bmatrix} c z_\Delta \\ z_P \end{bmatrix} \tag{7.18}$$

and the state-space representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_0 v_P(t) + B_1 v_\Delta(t) + B_2 u(t), \ \ x(0) = 0 \\ z(t) &= C_1 x(t) + D_{12}u(t) \\ y(t) &= C_2 x(t) + D_{20} v_P(t) + D_{21} v_\Delta(t) \end{aligned} \tag{7.19}$$

The cost (7.15) then takes the form

$$J_{mxd}(P,K) = \sup_{v_\Delta = T v_P} \left\{ \sum_{k=1}^{m} \left[\int_0^\infty \left[z^T(t) z(t) - \gamma^2 v_\Delta^T(t) v_\Delta(t) \right] dt \ : \ v_P = e_k \delta(t) \right] \right\} \quad (7.20)$$

where $\gamma = c/\delta$. The cost (7.20) is called a *mixed* H_2/H_∞ *cost*, because it consists of both an H_∞ type cost, with respect to the input v_Δ, and an H_2 type cost, with respect to the input v_P.

The problem of finding a controller which minimizes the mixed H_2/H_∞ cost (7.20) can be solved in a similar way as the optimal H_2 and H_∞ control problems. In particular, the optimal controller can be obtained in terms of an optimal state-feedback controller and an associated variable transformation, and an optimal estimator defined for the transformed system.

The optimal state-feedback mixed H_2/H_∞ controller is given by the following theorem.

Theorem 7.2 Mixed H_2/H_∞-optimal state feedback control.
Consider the system (7.19). Suppose that the assumptions (A1)–(A4) in Chapter 4 hold. Assume that the control signal $u(t)$ has access to the present and past values of the state, $x(\tau), \tau \le t$. Then there exists a state-feedback controller such that the mixed H_2/H_∞ cost $J_{mxd}(P,K)$ is bounded if and only if there exists a positive (semi)definite solution to the algebraic Riccati equation

$$A^T X + XA - XB_2(D_{12}^T D_{12})^{-1} B_2^T X + \gamma^{-2} X B_1 B_1^T X + C_1^T C_1 = 0 \quad (7.21)$$

such that the matrix

$$A - B_2(D_{12}^T D_{12})^{-1} B_2^T X + \gamma^{-2} B_1 B_1^T X \quad (7.22)$$

is stable, i.e. all its eigenvalues have negative real parts.

Moreover, when these conditions are satisfied, the state-feedback controller which minimizes the cost $J_{mxd}(P,K)$ is given by

$$u(t) = K_{mxd} x(t) \quad (7.23)$$

where

$$K_{mxd} = -(D_{12}^T D_{12})^{-1} B_2^T X \quad (7.24)$$

and the minimum value of the cost is

$$\min_{u=Kx} J_{mxd}(P,K) = \mathrm{tr}(B_0^T X B_0) \quad (7.25)$$

The above result can be derived in analogy with the optimal H_∞ and H_2 problems. In particular, we have the expansion (4.13), i.e.,

$$\begin{aligned} \int_0^\infty [z(t)^T z(t) - \gamma^2 v_\Delta(t)^T v_\Delta(t)] dt \ = \ & \int_0^\infty \Big[[u(t) - u^0(t)]^T D_{12}^T D_{12} [u(t) - u^0(t)] \\ & - \gamma^2 [v_\Delta(t) - v_\Delta^0(t)]^T [v_\Delta(t) - v_\Delta^0(t)] \Big] dt + x(0)^T X x(0) \quad (7.26) \end{aligned}$$

where $u^0(t) = K_{mxd}x(t)$ and $v_\Delta^0(t) = \gamma^{-2}B_1^TXx(t)$. The optimal controller and minimum cost then follow in analogy with Theorems 3.1 and 4.1.

From the expansion (7.26) it follows that the mixed H_2/H_∞ optimal output feedback controller can be constructed in terms of the optimal state-feedback law (7.24) and an optimal estimator for the transformed system (cf. (4.22))

$$\begin{aligned}
\dot{x}(t) &= \tilde{A}x(t) + B_0v_P(t) + \tilde{B}_1\tilde{v}_\Delta(t) + \tilde{B}_2u(t), \\
\tilde{z}(t) &= \tilde{C}_1x(t) + \tilde{D}_{12}u(t) \\
y(t) &= \tilde{C}_2x(t) + D_{20}v_P(t) + \tilde{D}_{21}\tilde{v}_\Delta(t)
\end{aligned} \tag{7.27}$$

where $\tilde{z}(t) = (D_{12}^TD_{12})^{1/2}[u(t) - u^0(t)]$, $\tilde{v}_\Delta(t) = v_\Delta(t) - v_\Delta^0(t)$, and the matrices $\tilde{A}$, etc, are defined by (4.23).

In analogy with the optimal H_2 and H_∞ problems, the expansion (7.26) reduces the optimal output feedback control problem to a mixed H_2/H_∞ estimation problem for the transformed system (7.27). The mixed H_2/H_∞-optimal estimation problem consists of finding a stable estimator $\hat{\tilde{z}} = Fy$ which minimizes the H_2/H_∞ cost defined for the estimation error,

$$\begin{aligned}
J_{mxd,e}(F) \; = \; \sup_{\tilde{v}_\Delta = Tv_P} & \left\{ \sum_{k=1}^{m} \left[\int_0^\infty \left[[\tilde{z}(t) - \hat{\tilde{z}}(t)]^T[\tilde{z}(t) - \hat{\tilde{z}}(t)] - \gamma^2\tilde{v}_\Delta^T(t)\tilde{v}_\Delta(t) \right] dt \;: \right. \right. \\
& \left. \left. v_P = e_k\delta(t) \right] \right\}
\end{aligned} \tag{7.28}$$

In contrast to the H_2 and H_∞ optimal estimation problems, the mixed H_2/H_∞ estimation problem lacks a closed solution. Instead, we have the following result.

Theorem 7.3 H_2/H_∞**-optimal estimator.**
Consider the system (7.27). There exists a stable estimator $\hat{\tilde{z}} = Fy$ which achieves a bounded cost $J_{mxd,e}(F)$ if and only if there exists a matrix L such that all eigenvalues of the matrix

$$\tilde{A} + L\tilde{C}_2 \tag{7.29}$$

have negative real parts, and such that the algebraic Riccati equation

$$(\tilde{A} + L\tilde{C}_2)^TP + P(\tilde{A} + L\tilde{C}_2) + \gamma^{-2}P(\tilde{B}_1 + L\tilde{D}_{21})(\tilde{B}_1 + L\tilde{D}_{21})^TP + \tilde{C}_1^T\tilde{C}_1 = 0 \tag{7.30}$$

has a symmetric positive (semi)definite solution such that the matrix

$$\tilde{A} + L\tilde{C}_2 + \gamma^{-2}(\tilde{B}_1 + L\tilde{D}_{21})(\tilde{B}_1 + L\tilde{D}_{21})^TP \tag{7.31}$$

is stable, i.e. all its eigenvalues have negative real parts.

When these conditions are satisfied, the estimator F_L defined as

$$\begin{aligned}
\dot{\hat{x}}(t) &= \tilde{A}\hat{x}(t) - L[y(t) - \tilde{C}_2\hat{x}(t)], \;\; \hat{x}(0) = 0 \\
\hat{\tilde{z}}(t) &= \tilde{C}_1\hat{x}(t)
\end{aligned} \tag{7.32}$$

(ignoring the known input u) achieves the bounded cost

$$J_{mxd,e}(F_L) = \mathrm{tr}\left[(B_0 + LD_{20})^T P(B_0 + LD_{20})\right] \tag{7.33}$$

Moreover, the estimator structure (7.32) is optimal, i.e.,

$$\inf_F J_{mxd,e}(F) = \inf_L J_{mxd,e}(F_L) \tag{7.34}$$

where the infimum on the left-hand side is taken with respect to all stable estimators F.

A detailed proof of the theorem can be found in the references. Here it will suffice to notice that introducing $\tilde{x} = x - \hat{x}$, the estimation error is given by

$$\begin{aligned}\dot{\tilde{x}}(t) &= (\tilde{A} + L\tilde{C}_2)\tilde{x}(t) + (B_0 + LD_{20})v_P(t) + (\tilde{B}_1 + L\tilde{D}_{21})\tilde{v}_\Delta(t) \\ \tilde{z}(t) - \hat{\tilde{z}}(t) &= \tilde{C}_1\tilde{x}(t)\end{aligned} \tag{7.35}$$

The cost expression (7.33) then follows in analogy with the expression (7.25).

Finally, the optimal state-feedback result of Theorem 7.2 and the estimator result of Theorem 7.3 give the following characterization of the mixed H_2/H_∞-optimal output feedback controller.

Theorem 7.4 Mixed H_2/H_∞-optimal control problem.
Consider the system (7.19). Suppose that the assumptions (A1)–(A4) of Chapter 4 hold. Then there exists a controller $u = Ky$ which achieves a bounded cost $J_{mxd}(P, K)$ if and only if the conditions of Theorems 7.2 and 7.3 are satisfied.

When the conditions are satisfied, the controller $u = K_L y$ defined by

$$\begin{aligned}\dot{\hat{x}}(t) &= \tilde{A}\hat{x}(t) + \tilde{B}_2 u(t) - L[y(t) - \tilde{C}_2\hat{x}(t)] \\ u(t) &= K_{mxd}\hat{x}(t)\end{aligned} \tag{7.36}$$

achieves the bounded cost

$$J_{mxd}(P, K_L) = \mathrm{tr}(B_0^T X B_0) + \mathrm{tr}[(B_0 + LD_{20})^T P(B_0 + LD_{20})] \tag{7.37}$$

Moreover, the controller structure (7.36) is optimal, i.e.,

$$\inf_K J_{mxd}(P, K) = \inf_L J_{mxd}(P, K_L) \tag{7.38}$$

where the infimum on the left-hand side is taken with respect to all stabilizing controllers K.

By Theorem 7.4 the mixed H_2/H_∞ optimal controller can be found by minimizing the cost (7.37), or (7.33), with respect to the estimator gain matrix L. In contrast to the H_2 and H_∞ problems, there exists no closed-form expression for the minimizing gain. The minimization problem can, however, be transformed to a *convex optimization problem* by a certain parameter transformation. The convex problem has no local minima, and the solution can therefore be found by using globally convergent convex optimization methods, which are guaranteed to find the optimum.

7.3 Notes and references

The connection between robust H_∞ performance and the structured singular value is due to Doyle, Wall and Stein (1982).

The mixed H_2/H_∞ optimal control problem and its relation to the robust H_2 performance problem has been discussed for example by Stoorvogel (1993) and Zhou *et al.* (1994). Khargonekar and Rotea (1991) have presented the convex optimization procedure to the mixed H_2/H_∞ problem. Their treatment applies to an H_2/H_∞ state-feedback problem, which is a dual version of the estimation problem in Theorem 7.3 and the cost (7.33). The procedure can, however, be applied to the estimation problem in a straightforward way by taking simple matrix substitutions.

Optimization-based methods for a number of robust performance problems have also been studied by Pensar (1995).

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