ROBUST CONTROL METHODS

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Preface

The purpose of these lecture notes is to present modern feedback control methods based on H_2- and H_∞-optimal control theory in a concise way, requiring only a moderate mathematical background.

The notes are based on a course first given in the fall 1997 at the Department of Engineering at Åbo Akademi University. I am grateful for the feedback from those who endured the course: Tom Fredman, Tom Nyman, Jan Ramstedt, Fredrik Smeds, Jan Torrkulla and Matias Waller. I am also grateful to PhD student Rasmus Nyström for his careful and critical reading of the manuscript and many valuable comments.

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H.T.

Chapter 1

Introduction – Fundamental issues of control and important application examples

1.1 The role of feedback

Consider the dynamical system depicted in Figure 1.1. Here G denotes a plant described by

$$z = Gu \tag{1.1}$$

where u denotes a manipulated input, and z is the controlled output variable. The signals $u(t)$ and $z(t)$ are functions of time t. The mapping G represents a dynamical system which relates the input signal $u(t)$ and the output signal $z(t)$, and it is in general described by a differential equation.

Typically, the objective is to manipulate the input $u(t)$ in such a way that the variable $z(t)$ follows, or is at least close to, a given reference signal $r(t)$. In principle, this could be achieved by solving the equation

$$Gu = r \tag{1.2}$$

for u as a function of time, given the desired reference value $r(t)$ for $z(t)$. Although (1.2) may not be exactly solvable for u, one could always solve the equation approximately,

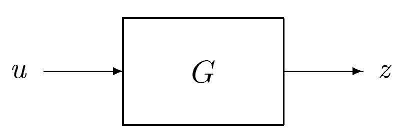

Figure 1.1: Dynamical system.

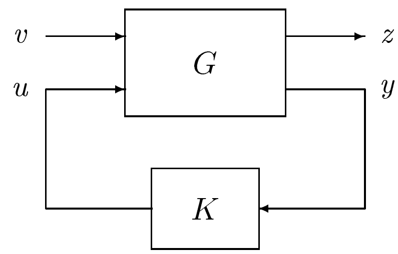

Figure 1.2: Feedback control system (v: disturbance, y: measured output, K: controller).

provided the relation G is known.

In practice, however, such a procedure seldom works! A fundamental reason for this is that there are always various kinds of uncertainties present in practical situations:

- unknown *disturbances* affecting the process,
- incompletely known plant dynamics, i.e., *model uncertainties.*

In order to compensate for the uncertainties the natural approach is to use *measurements* (y) from the process, and let the manipulated variable u be a function of the measured process outputs y as well. This leads to the concept of *feedback control* (Figure 1.2).

Feedback can be viewed as a means to reduce the effect of uncertainty. In particular, the use of feedback makes it possible to manage with a cruder process model. This nice property of feedback explains why control engineers tend to work with surprisingly simple, often linear, models, even when models built from first principles may be extremely complex.

Example 1.1. A PI controller provides an example of how feedback is applied to reduce the uncertainty of low-frequency disturbances. A PI controller can be tuned to eliminate steady-state offsets for unknown load changes using a very crude model of the process dynamics. Notice in particular, that zero steady-state offset is achieved although the magnitude of the constant disturbance is completely unknown.

Apart from uncertainties caused by disturbances and modeling uncertainty, another reason for applying feedback is due to the fact that

- feedback provides the *only way to stabilize an unstable plant.*

One might think that if one has a perfect plant model, and selects the input in such a way that the variables of the model remain bounded, then the same input would stabilize the real plant as well. However, for an unstable plant, even infinitesimal discrepancies between plant and model and in the initial state will cause the variables of the model and plant to diverge, and stabilization without feedback is therefore not achievable in practice.

The reasons for using feedback as described above are reflected in some of the fundamental problems of feedback control.

Optimal control against disturbances.
The disturbance attenuation problem is dealt with by optimal control methods, where some measure of the magnitude of the output is minimized subject to appropriate assumptions on the disturbances. A standard procedure is the so-called *Linear Quadratic Gaussian* (LQG) control problem, where the sum of the output variances are minimized subject to the assumption that the disturbances are characterized as stochastic processes. The cost function which is minimized is then

$$J_2 = \lim_{t_f \to \infty} E[\frac{1}{t_f} \int_0^{t_f} z^T(t) z(t) dt] \tag{1.3}$$

The theory of LQG contol was developed during the 1960's, and it was successfully applied for example to aerospace problems. It turns out that the cost J_2 is equal to a particular norm of the closed-loop transfer function, the H_2 norm. The LQG control problem therefore consists of minimizing the H_2 norm of the closed-loop transfer function. Therefore one also speaks about *H_2-optimal control.*

In the frequency domain, the H_2 norm of a stable scalar transfer function $G(s)$ is defined as

$$\|G\|_2 = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega \right)^{1/2} \tag{1.4}$$

This expression shows that the H_2 norm is a measure of the average of the square of the gain taken over all frequencies.

Robust control of uncertain plants.
Robustness against model uncertainties is classically handled by phase and gain margins to ensure closed-loop stability in spite of modeling errors. The classical methods are, however, not readily generalized to multivariable plants, and they do not handle the problem of simultaneously achieving good performance against disturbances as well as robustness against model uncertainties. The modern approach to design controllers which are robust against model uncertainties is provided by the so-called H_∞ *control theory.* This theory has been developed largely during the 1980's. In this approch, it is assumed that the plant is represented as

$$G = G_0 + \Delta \tag{1.5}$$

where G_0 is the 'nominal' plant model (often linear), and Δ represents a model uncertainty, cf. Figure 1.3. The uncertainty is unknown but assumed to belong to some kind of uncertainty set $\mathcal{B}$, i.e., $\Delta \in \mathcal{B}$. For example, even though the nominal model may be linear, the uncertainty Δ may be nonlinear. In this way nonlinearities around a nominal linearized model G_0 valid at an operating point may be captured in the uncertainty Δ. The control system in Figure 1.3 is called *robustly stable* with respect to the uncertainty class $\mathcal{B}$ if it is stable for all $\Delta \in \mathcal{B}$.

For an important and realistic class of bounded uncertainties to be discussed later, it turns out that the condition that the closed loop is stable for all possible $\Delta \in \mathcal{B}$ is equivalent to a bound on the so-called H_∞ norm of the closed-loop transfer function.

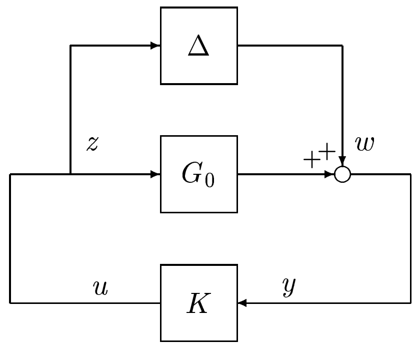

Figure 1.3: Control of uncertain plant.

The H_∞ norm of the stable scalar transfer function $G(s)$ is defined as

$$\|G\|_\infty = \max_\omega |G(j\omega)| \tag{1.6}$$

Thus, the H_∞ norm is the largest gain of the system taken over all frequencies.

It is easy to see why the H_∞ norm is related to robust stability. For the uncertain plant in Figure 1.3, we have

$$z = T_{zw}w, \;\; T_{zw} = (I - KG_0)^{-1}K \tag{1.7}$$

From elementary control theory we know that the closed loop in Figure 1.3 is stable if

$$|\Delta(j\omega)T_{zw}(j\omega)| < 1, \;\; \text{all } \omega \tag{1.8}$$

Another way to state the stability requirement is that

$$|T_{zw}(j\omega)| < 1/l(\omega), \;\; \text{all } \omega \tag{1.9}$$

implies that the closed-loop system is stable for all uncertainties Δ which satisfy the bound

$$|\Delta(j\omega)| \leq l(\omega), \;\; \text{all } \omega \tag{1.10}$$

In fact, it can be shown that the condition (1.9) is also necessary for robust stability, i.e., if (1.9) does not hold, then there exists an uncertainty Δ that satisfies (1.10) and which makes the closed loop unstable. The connection to the H_∞ norm is obtained by observing that the condition (1.9) is equivalent to

$$\|lT_{zw}\|_\infty < 1 \tag{1.11}$$

Hence, the closed-loop system in Figure 1.3 is robustly stable with respect to the uncertainty class defined by (1.10) if and only if the closed-loop transfer function satisfies the H_∞ norm bound (1.11).

Disturbance rejection and loop-shaping using H_∞ control.
Even though robustness to modeling uncertainties is one of the most important motivations for the H_∞ control problem, it can also be applied to the disturbance rejection problem. The cost function then takes the form of a worst-case cost. This will be discussed later. The H_∞ control problem is also well suited for loop shaping. In many control problems, the closed-loop behaviour can be specified by requiring that various closed-loop transfer functions satisfy given bounds. For example, one could require that the transfer function $T_{zv}(s)$ from the disturbance v to the output z for the system in Figure 1.2 satisfies the frequency domain bound

$$|T_{zv}(j\omega)| < 1/|W(j\omega)|, \ \text{all } \omega \tag{1.12}$$

This can be stated in terms of an H_∞ norm bound as

$$\|W(j\omega)T_{zv}(j\omega)\|_\infty < 1 \tag{1.13}$$

It follows that loop-shaping problems involving design specifications of the form (1.12) can be solved efficiently using H_∞ control theory. What makes the H_∞ control problem better suited for the robust stability problem and for loop-shaping than the H_2 (LQ) control problem is due to the fact that the H_∞ norm bound (1.13) guarantees (1.12) for all frequencies. On the other hand, the H_2 norm (1.4) only gives an average measure taken over all frequencies, and therefore it cannot guarantee that the frequency domain bound holds at all frequencies.

Realistic controller design requires both robustness against model uncertainty (characterized by an H_∞ norm bound), as well as good disturbance rejection properties (characterized by an H_2- or H_∞-norm). Today controller design based on optimal (LQG or H_2) and robust H_∞ control is the standard procedure for control systems where high demands on control quality exists. Some (fairly) good software is available in `MATLAB` toolboxes.

As most of the literature and the software deals with continuous-time controllers, the treatment will focus on this case. In many respects, this is the natural approach since the controlled processes mostly operate in continuous time. However, it should be kept in mind that as the controllers are normally implemented digitally, the implementation phase will involve a second step, consisting of a discretization of the continuous-time controller. An alternative would be to apply discrete-time or sampled-data control theory directly. However, the software presently available for discrete-time controller synthesis is not as extensive as that for the continuous-time case. Therefore, controllers are still mostly first designed in continuous time.

The standard controller design procedures are restricted to linear or weakly nonlinear processes. The design of optimal and robust controllers for nonlinear plants is a significantly more complex problem. There are, however, some simpler appealing suboptimal techniques to modify the linear optimal and robust control methods to nonlinear systems. In these methods the nonlinear system is described as a linear parameter varying (LPV) system, and an optimal robust controller is calculated using the estimated maximum rate of change of the system parameters. This approach can be regarded as a rigorous way of designing optimal and robust gain-scheduled controllers.

Remark 1.1. Note that a number of control methods which have gained popularity in industry, such as fuzzy control and neural control, do not explicitly address any of the fundamental control issues in a quantitative way at all. This is the main reason why control people do not usually take these methods seriously. Naturally, these methods will still often work at least in not too demanding applications. They also appeal to many people with a non-control background and their functioning can more easily be explained to process operators and the media. It should also be remembered that the ability to analyze a particular method quantitatively is in no way a prerequisite for successful applications of the technique!

1.2 Some application examples for advanced control

It is important to note that in most practical control systems the control quality is neither a very critical issue nor is it very sensitive to the controller used. It is often sufficient to design a controller which eliminates steady state offsets and achieves an acceptable closed-loop behavior. In these cases classical PI-tuning methods are often sufficient.

However, higher demands on the efficiency of processes and technical equipment have introduced a number of applications where classical tuning methods are not sufficient for good performance. At the same time, the availability of inexpensive computing power has made the application of more complex controllers feasible in practice. This development has given rise to a number of new highly challenging application areas for control. In order to give an appreciation of the type of applications involved, we give a brief list of some of the problem areas.

- *Chemical process control systems.*
  Distillation, chemical reactors, cements kilns, pH control, basis weight control in paper machines, etc. The multivariable nature of the plants, with a coupling of several loops, often sets high demands on the controller tuning for simultaneous good performance for all loops. Chemical processes are also typically subject to random disturbances and large modeling uncertainties. Also, the objective is typically to control a quality variable, whose variations should be made as small as possible in spite of process disturbances and model uncertainties.
- *Various mechanical systems.*
  Important examples of mechanical systems which set high requirements on the controller are for example:
  - *Active suspension systems*, which are used to complement traditional passive suspension for example in cars and trains. In this way it is possible to achieve a response which is not achievable by passive methods.
  - *Elevator control.* The position of elevators requires well-designed control system in order to give a smooth and efficient ride in spite of varying loads (number of passengers).
  - *Positioning systems in ships.* The problem of keeping a ship positioned at sea is

important in various offshore operations, and consists of a highly demanding multivariable control problem.

- *Magnetic bearings*, in which a bearing is suspended frictionlessly by magnetic forces. In this way it is possible to achieve very high rotation speeds not possible with traditional bearings. On the other hand, the position of the bearing must be controlled very accurately in order to avoid vibrations at high speeds.

- *Engine control.* Higher demands on efficiency and pollution control have recently motivated the study of high-performance control systems for engine control. Applications in this area include spark ignition engines, diesel engines as well as jet engines. The systems often exhibit difficult dynamics due to underdamped modes, nonlinear dynamics etc.

- *Active vibration control.*

  Active suppression of vibration is being introduced in order to suppress vibration in various machinery and constructions. For example, high-rise buildings in earth-quake areas, like Japan, are nowadays equipped with active vibration control systems. A well-designed active controller can outperform the best passive vibration suppression techniques.

  Vibrating structures are infinite-dimensional dynamical systems (with an infinite number of vibrating modes and resonance frequencies), and the controller is therefore in practice always based on a finite-dimensional approximate model.

- *Active control of acoustic noise.*

  This is a novel application of active control in which antinoise is applied to suppress acoustic noise. It can be used as a complement to traditional passive noise suppression in particular at low frequencies, for which passive noise absorption is inefficient. Commercial products presently include: headsets with active noise suppression, suppression of noise in the interior of some (luxury) cars and airplanes.

1.3 Scope of course

In this course we study modern methods for designing controllers with specified optimality and robustness properties. We will cover H_2-optimal and LQG control, and robust H_∞-optimal control, including the so-called μ-synthesis procedure for structured uncertainties. We will also consider mixed H_2/H_∞ control problems, in which a mixed H_2 (performance) and H_∞ (robustness) criterion is minimized in order to achieve robust performance.

It is important to notice that optimal and robust control theory addresses the solution of certain well-defined control problems, which are related to quantitatively defined optimality and robustness criteria. The controller designed with these methods is only as good as the design specifications allow. In controller design, it is therefore essential to state the problem correctly in a way which corresponds to a good design for the system to be controlled. In this course, the problem of applying optimal and robust control theory to controller design will also be discussed.

Presently, the best, and for many problems the only, available software which implements the optimal and robust controller design techniques can be found in `MATLAB`s *Robust Control Toolbox* and *μ-Synthesis Toolbox*. These will be applied in this course to solve controller design problems.

1.4 Literature

Optimal and robust control is a rather difficult problem area, and although a number of advanced texts have been written on the subject, there are surprisingly few textbooks which cover the theory and design on an elementary level. The major reason for this is that many of the methods have been developed fairly recently. The focus of most books in the area has therefore been to provide a comprehensive coverage of the state of the art in robust control, rather than a pedagogical introduction to the field. Excellent books belonging to this class are the one by Zhou *et al.* (1996), together with its simplified and condensed version (Zhou and Doyle 1998), and the one by Green and Limebeer (1995). These books can be highly recommended to anyone who wants to have a thorough exposition of the field. Unfortunately, these texts require a level of mathematical sophistication which makes them accessible only to the most mathematically inclined control engineers.

More elementary introductory texts which can be recommended are the books by Maciejowski (1989), Morari and Zafiriou (1989), Doyle *et al.* (1992) and Skogestad and Postlethwaite (1996). These books focus on the use of robust (H_∞) control techniques to solve various control design problems. However, they do not discuss the techniques to solve the H_∞-optimal or other robust control problems.

The robust H_∞-optimal control theory has now matured to stage where it can be presented in a fairly accessible way, which is not more difficult than for other well-establised fields, such as classical optimal control theory. However, no textbook which provides an accessible introduction to the controller synthesis techniques appears to be available. This course will be an attempt in this direction.

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