The sixth homework
The decriptor method

Task. Consider the system with an arbitrary constant delay. h:

$$\dot{x}(t) = Ax(t) + A_1 x(t-h),$$

where $x \in \mathbb{R}^2$.

1. Variant.
$$A = \begin{pmatrix} 1 & -2 \\ 1 & 2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -3 & -2 \\ -2 & -5 \end{pmatrix}.$$
2. Variant.
$$A = \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -4 & 0 \\ -1 & -5 \end{pmatrix}.$$
3. Variant.
$$A = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -5 & 1 \\ -2 & -4 \end{pmatrix}.$$
4. Variant.
$$A = \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -2 & 1 \\ -2 & -6 \end{pmatrix}.$$
5. Variant.
$$A = \begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -4 & -1 \\ 2 & -4 \end{pmatrix}.$$
6. Variant.
$$A = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -4 & -2 \\ 2 & -6 \end{pmatrix}.$$
7. Variant.
$$A = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -3 & 1 \\ 0 & -5 \end{pmatrix}.$$
8. Variant.
$$A = \begin{pmatrix} -1 & 2 \\ -1 & 3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -2 & -1 \\ 1 & -4 \end{pmatrix}.$$
9. Variant.
$$A = \begin{pmatrix} -1 & -3 \\ 2 & 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -4 & 1 \\ 2 & -3 \end{pmatrix}.$$

10. Variant.

$$A = \begin{pmatrix} -2 & 2 \\ -3 & 2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -5 & 0 \\ -2 & -3 \end{pmatrix}.$$

11. Variant.

$$A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -3 & 1 \\ -2 & -5 \end{pmatrix}.$$

12. Variant.

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -4 & -2 \\ 1 & -5 \end{pmatrix}.$$

13. Variant.

$$A = \begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -7 & 1 \\ 2 & -5 \end{pmatrix}.$$

- Simulate this system in Matlab (use function *dde23*) with different delays of h. Check that for some delays h system will be stable, while for others — unstable.

- Using the descriptor method, you should find the maximum delay at which this system is stable. Matrix inequalities can be solved using the Matlab (Yalmip package, SeDuMi solver).

- Design the $u(t) = Kx(t)$ controller such that the closed system is stable at any h delays.

  $$\dot{x}(t) = Ax(t) + A_1x(t-h) + Iu(t) = (A+K)x(t) + A_1x(t-h)$$

  will be stable at any h delays. To check the stability of the closed system, use the method of Lyapunov-Krasovsky f functions.