The third homework
Bifurcations

Task. Consider the following nonlinear system:

1. Variant.
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = rx_2 + x_1 - x_1^2 + x_1 x_2;$$

2. Variant.
$$\dot{x}_1 = -x_1,$$
$$\dot{x}_2 = rx_2 + x_2^3 - x_2^5;$$

3. Variant.
$$\dot{x}_1 = -x_1,$$
$$\dot{x}_2 = rx_2 - x_2^3 + x_2^5;$$

4. Variant.
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -x_1 - (x_1^2 - r)x_2;$$

5. Variant.
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -x_1 - (x_2^2 - r)x_2;$$

6. Variant.
$$\dot{x}_1 = rx_1 - x_1 x_2,$$
$$\dot{x}_2 = x_1 x_2 - x_2;$$

7. Variant.
$$\dot{x}_1 = -rx_1 + x_1^2 x_2,$$
$$\dot{x}_2 = -x_2 + x_1^2 x_2;$$

8. Variant.
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -x_1 + (r + \cos x_2)x_2;$$

9. Variant.
$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -x_1 + (r + \sin x_2)x_2;$$

10. Variant.
$$\dot{x}_1 = x_2^3 + x_1 x_2,$$
$$\dot{x}_2 = x_1 + r;$$

- Find all the possible bifurcations in the system. Determine the type of equilibrium points for all values of bifurcation parameter r.

- Draw the phase portraits of linearized system and nonlinear system for each type of the equilibrium point.