The first homework
Stability of linear systems

Task 1. Consider the canonical model in the state space:

$$\begin{cases} \dot{x} = Ax + bu, \\ y = Cx, \end{cases}$$

where $x \in \mathbb{R}^3$, $u \in \mathbb{R}$, $y \in \mathbb{R}$. The initial conditions are zeros.

You should write down this model in functional form "input-output" and derive the transfer function of this system.

1. Variant.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

2. Variant.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

3. Variant.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

4. Variant.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

5. Variant.

$$A = \begin{bmatrix} -1 & -3 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

6. Variant.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 2 & -3 \\ 1 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

7. Variant.

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 1 & -2 \\ 0 & -1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

8. Variant.

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 0 \\ -1 & 2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

9. Variant.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & -1 \\ -2 & 2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

10. Variant.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

Task 2. Consider the functional model. The initial conditions are zeros. You should write down this model in canonical form in the state space.

1. Variant.

$$\dddot{y} - 4\ddot{y} + 6\dot{y} - 6y = 2\dot{u} + u.$$

2. Variant.

$$\dddot{y} + 2\ddot{y} - \dot{y} - 2y = -\ddot{u} + 2\dot{u} - 8u.$$

3. Variant.

$$\dddot{y} - 4\ddot{y} + 3\dot{y} - 4y = \ddot{u} - 2\dot{u} + u.$$

4. Variant.

$$\dddot{y} - 4\ddot{y} + 4\dot{y} - y = \ddot{u} - 2\dot{u} - u.$$

5. Variant.
$$\dddot{y} - 2\ddot{y} - \dot{y} - 2y = -2\dot{u} - 2u.$$
6. Variant.
$$\dddot{y} + \ddot{y} + 3\dot{y} - 3y = \ddot{u} + \dot{u} - u.$$
7. Variant.
$$\dddot{y} - \ddot{y} - 6\dot{y} + 6y = \ddot{u} + \dot{u} - 10u.$$
8. Variant.
$$\dddot{y} - 4\ddot{y} + 3\dot{y} - y = 2\ddot{u} - 6\dot{u} + 4u.$$
9. Variant.
$$\dddot{y} + \ddot{y} + 3\dot{y} - 3y = \ddot{u} + 5u.$$
10. Variant.
$$\dddot{y} - 4\ddot{y} + 4\dot{y} + 2y = \ddot{u} - 3\dot{u} + 2u.$$

Task 3. Consider the system in the state space:
$$\dot{x} = Ax + Bu,$$
where $x \in \mathbb{R}^3$, B is an identity matrix of the dimension 3×3,

1. Variant.
$$A = \begin{bmatrix} -1 & -3 & -4 \\ 0 & 2 & -5 \\ 0 & -1 & 1 \end{bmatrix}.$$
2. Variant.
$$A = \begin{bmatrix} -2 & 8 & 2 \\ 3 & -4 & 6 \\ 3 & 5 & -4 \end{bmatrix}.$$
3. Variant.
$$A = \begin{bmatrix} -3 & 1 & 5 \\ 3 & -4 & 1 \\ 3 & 0 & -4 \end{bmatrix}.$$
4. Variant.
$$A = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 3 & 1 \\ 3 & -2 & 6 \end{bmatrix}.$$
5. Variant.
$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 3 \\ -4 & 2 & 1 \end{bmatrix}.$$

6. Variant.

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 4 & 2 \\ 2 & -2 & 3 \end{bmatrix}.$$

7. Variant.

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 4 & 5 \\ 1 & -3 & 1 \end{bmatrix}.$$

8. Variant.

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & -2 & 7 \\ 1 & 2 & 5 \end{bmatrix}.$$

9. Variant.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & -1 \\ 4 & 2 & -3 \end{bmatrix}.$$

10. Variant.

$$A = \begin{bmatrix} -2 & 4 & 2 \\ -3 & 4 & 2 \\ 2 & 2 & -2 \end{bmatrix}.$$

- You should simulate this system in Matlab (for this purpose function *ode45* is used with nonzero initial conditions), plot the solution for each vector components (use *plot* function). You can use different line parameters like *'color'*, *'linewidth'* and line type. Use function *hold on* before the drawing and *hold off* after it to save all lines in the figure. Command *grid on* is used for the grid, while commands *xlabel, ylabel* are used for the axis labels. To make sure that this system is unstable find its eigenvalues (*eig*). Use Help to find the description of all functions.

- You should find the gain bound k^* such that for all $k < k^*$ regulator $u = kx$ ensures the stability of the closed-loop system. Find the eigenvalues of the closed-loop system and plot its solution.