The fourth homework
Discrete systems

Task 1. Consider the canonical model of sicrete system in the state space:

$$\begin{cases} x(k+1) = Ax(k) + bu(k), \\ \quad y(k) = Cx(k), \end{cases}$$

where $x \in \mathbb{R}^3$, $u \in \mathbb{R}$, $y \in \mathbb{R}$. The initial conditions are zeros.

You should write down this model in functional form "input-output" and derive the transfer function of this system.

1. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}, \quad b = \begin{bmatrix} 0.1 \\ -0.3 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

2. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.2 & 0.1 & -0.5 \end{bmatrix}, \quad b = \begin{bmatrix} 0.3 \\ 0.6 \\ 0.6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

3. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.4 & 0.2 & 0.7 \end{bmatrix}, \quad b = \begin{bmatrix} -0.5 \\ 0.4 \\ 0.4 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

4. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.7 & -0.1 & -0.3 \end{bmatrix}, \quad b = \begin{bmatrix} -0.3 \\ -0.4 \\ 0.8 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

5. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.4 & 0.1 & -0.3 \end{bmatrix}, \quad b = \begin{bmatrix} 0.4 \\ 0.5 \\ -0.6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

6. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.4 & 0.2 & -0.8 \end{bmatrix}, \quad b = \begin{bmatrix} 0.6 \\ 0.3 \\ -0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

7. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.6 & -0.6 & 0.1 \end{bmatrix}, \quad b = \begin{bmatrix} -0.5 \\ 0.3 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

8. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.4 & 0.2 & 0.7 \end{bmatrix}, \quad b = \begin{bmatrix} -0.2 \\ 0.5 \\ 0.6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

9. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.8 & -0.4 & -0.7 \end{bmatrix}, \quad b = \begin{bmatrix} 0.4 \\ 0.1 \\ -0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

10. Variant.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.1 & -0.3 & 0.8 \end{bmatrix}, \quad b = \begin{bmatrix} 0.5 \\ -0.5 \\ 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

Task 2. Consider the functional model of discrete system. The initial conditions are zeros.

You should write down this model in canonical form in the state space.

1. Variant.

$$y(k+3)+0.1y(k+2)-0.3y(k+1)-0.1y(k) = -0.6u(k+2)+0.09u(k+1)+0.01u(k).$$

2. Variant.

$$y(k+3)+0.4y(k+2)+0.2y(k+1)-0.2y(k) = -0.2u(k+2)+0.06u(k+1)+0.08u(k).$$

3. Variant.

$$y(k+3)+0.1y(k+2)-0.2y(k+1)-0.3y(k) = 0.3u(k+2)+0.01u(k+1)+0.06u(k).$$

4. Variant.

$$y(k+3)-0.8y(k+2)+0.3y(k+1)-0.1y(k) = 0.1u(k+2)-0.2u(k+1)-0.05u(k).$$

5. Variant.

$$y(k+3)+0.7y(k+2)+0.4y(k+1)-0.8y(k) = -0.2u(k+2)+0.28u(k+1)+0.08u(k).$$

6. Variant.

$$y(k+3)-0.7y(k+2)-0.2y(k+1)-0.4y(k) = 0.6u(k+2)+0.02u(k+1)+0.2u(k).$$

7. Variant.

$$y(k+3)+0.8y(k+2)-0.2y(k+1)+0.4y(k) = -0.5u(k+2)-0.18u(k+1)-0.12u(k).$$

8. Variant.

$$y(k+3)-0.1y(k+2)+0.6y(k+1)+0.6y(k) = 0.2u(k+2)+0.12u(k+1)-0.18u(k).$$

9. Variant.

$$y(k+3)+0.3y(k+2)-0.1y(k+1)-0.4y(k) = -0.6u(k+2)+0.21u(k+1)+0.2u(k).$$

10. Variant.

$$y(k+3)+0.3y(k+2)+0.1y(k+1)+0.7y(k) = 0.8u(k+2)+0.25u(k+1)+0.28u(k).$$

Task 3. The transfer function of stable continious system is given:

1. Variant.

$$W(s) = \frac{s}{s^2+2s+1}.$$

2. Variant.

$$W(s) = \frac{2s-1}{s^2+3s+2}.$$

3. Variant.

$$W(s) = \frac{s+2}{s^2+4s+5}.$$

4. Variant.

$$W(s) = \frac{s+3}{s^2+3s+1}.$$

5. Variant.
$$W(s) = \frac{2s-3}{s^2+4s+8}.$$

6. Variant.
$$W(s) = \frac{4s-1}{s^2+7s+2}.$$

7. Variant.
$$W(s) = \frac{5s+3}{s^2+5s+8}.$$

8. Variant.
$$W(s) = \frac{s-4}{s^2+2s+9}.$$

9. Variant.
$$W(s) = \frac{4s-3}{s^2+6s+3}.$$

10. Variant.
$$W(s) = \frac{s+6}{s^2+2s+8}.$$

- Plot the step response of this function in Matlab (function *stepplot*).
- Find the transfer function of corresponding discrete systems using the Euler method:
$$W_d(s) = W\left(\frac{s-1}{h}\right),$$
where $W_d(s)$ is a transfer function of discrete system, h is a discretization step.
- Find the estimate of the discretization step, for which the discrete system is stable:
$$h < \min_i \frac{2|\operatorname{Re} s_i(A)|}{|s_i(A)|^2},$$
where $s_i(A)$ are the roots of the polynomial $A(s)$, $(W(s) = B(s)/A(s))$.
- Plot the step response of the obtained discrete transfer function in Matlab for two different discretization steps: in the first case the discrete system should be stable, while in the second case it should be unstable. Compare the results with original continious system.

- Find the transfer function of corresponding discrete systems using the Tustin's method:

$$W_d(s) = W\left(\frac{2}{h}\frac{s-1}{s+1}\right),$$

  where $W_d(s)$ is a transfer function of discrete system, h is a discretization step.

- Plot the step response of the obtained discrete transfer function in Matlab. Compare the results with original continious system.