The fifth homework
Time-delay systems

Task 1. Consider the following time-delay system:

$$\dot{x}(t) = -\operatorname{sgn} x(t-h), \quad t \geq 0, h > 0, \tag{1}$$

where

$$\operatorname{sgn} x = \begin{cases} 1, & x > 0, \\ -1, & x < 0, \\ 0, & x = 0, \end{cases}$$

$h = 2$ is a constant delay, $x(t) = \phi(t)$ while $t \in [-h, 0]$.

Draw the plot of the system solution (**??**) using a step method:

1. Variant.

$$\phi(t) = \begin{cases} (t+2)^2, & t \in [-2, -1), \\ -t, & t \in [-1, 0]. \end{cases}$$

2. Variant.

$$\phi(t) = \begin{cases} -t-1, & t \in [-2, -1), \\ -(t+1)^2, & t \in [-1, 0]. \end{cases}$$

3. Variant.

$$\phi(t) = \begin{cases} 0.5, & t \in [-2, -1), \\ -t-0.5, & t \in [-1, 0]. \end{cases}$$

4. Variant.

$$\phi(t) = \begin{cases} -0.5, & t \in [-2, -1.5), \\ t+1, & t \in [-1.5, 0]. \end{cases}$$

5. Variant.

$$\phi(t) = \begin{cases} (t+2)^2 - 1, & t \in [-2, -1), \\ 0.5(t+1), & t \in [-1, 0]. \end{cases}$$

6. Variant.

$$\phi(t) = \begin{cases} -t - 1.5, & t \in [-2, -1), \\ t + 0.5, & t \in [-1, 0]. \end{cases}$$

7. Variant.

$$\phi(t) = \begin{cases} t + 1.5, & t \in [-2, -1), \\ -t - 0.5, & t \in [-1, 0]. \end{cases}$$

8. Variant.

$$\phi(t) = (t + 1)^2 - 1.$$

9. Variant.

$$\phi(t) = \begin{cases} -1, & t \in [-2, -1), \\ 2t + 1, & t \in [-1, 0]. \end{cases}$$

10. Variant.

$$\phi(t) = -(t + 1)^2 + 1.$$

Task 2. Consider the system with time-varying delay $\tau(t)$:

1. Variant.
$$\dot{x}(t) = -3x(t) - 0.3x(t - \tau(t)).$$

2. Variant.
$$\dot{x}(t) = -x(t) + 0.1x(t - \tau(t)).$$

3. Variant.
$$\dot{x}(t) = -2x(t) - 0.1x(t - \tau(t)).$$

4. Variant.
$$\dot{x}(t) = -3x(t) - 0.2x(t - \tau(t)).$$

5. Variant.
$$\dot{x}(t) = -x(t) - 0.2x(t - \tau(t)).$$

6. Variant.
$$\dot{x}(t) = -3x(t) + 0.1x(t - \tau(t)).$$

7. Variant.
$$\dot{x}(t) = -2x(t) + 0.1x(t - \tau(t)).$$

8. Variant.
$$\dot{x}(t) = -3x(t) + 0.3x(t - \tau(t)).$$

9. Variant.
$$\dot{x}(t) = -2x(t) + 0.2x(t - \tau(t)).$$

10. Variant.
$$\dot{x}(t) = -x(t) - 0.1x(t - \tau(t)).$$

Desing the Lyapunov function and prove the system stability using the Razumikhin method.

Task 3. Consider the system with a constant delay h:

$$\dot{x}(t) = Ax(t) + A_1 x(t - h),$$

where $x \in \mathbb{R}^2$.

1. Variant.

$$A = \begin{pmatrix} -2 & 1 \\ -4 & -3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, \quad h = 3.$$

2. Variant.

$$A = \begin{pmatrix} -3 & -1 \\ 1 & -3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix}, \quad h = 2.$$

3. Variant.

$$A = \begin{pmatrix} -4 & 1 \\ -2 & -4 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}, \quad h = 3.$$

4. Variant.

$$A = \begin{pmatrix} -5 & -2 \\ 1 & -2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}, \quad h = 1.$$

5. Variant.

$$A = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix}, \quad h = 2.$$

6. Variant.

$$A = \begin{pmatrix} -4 & -2 \\ 1 & -1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -2 & 0 \\ -1 & -1 \end{pmatrix}, \quad h = 1.$$

7. Variant.

$$A = \begin{pmatrix} -1 & -2 \\ 3 & -4 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix}, \quad h = 2.$$

8. Variant.

$$A = \begin{pmatrix} -2 & 1 \\ 0 & -3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \quad h = 3.$$

9. Variant.

$$A = \begin{pmatrix} -4 & -2 \\ 1 & -2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 1 & 1 \\ -1 & -3 \end{pmatrix}, \quad h = 2.$$

10. Variant.

$$A = \begin{pmatrix} -3 & 0 \\ -1 & -3 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}, \quad h = 1.$$

- Plot the solution of this system in Matlab (use function *dde23*).
- Prove the stability of this system using Lyapunov-Krasovskii method. Linear matrix inequalities can be solved using Matlab (package Yalmip, solver SeDuMi).