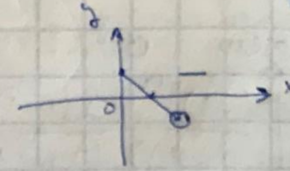
ИР. Ряды Фурье. 7 вар.

Кирбаба Д.
R3137



• $f(x) = \begin{cases} 1-x, & x \in [0,2) \\ 1, & x \in [2,3] \end{cases}$ $\quad T = 3$

$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{2\pi n}{T} x + b_n \sin\frac{2\pi n}{T} x \right)$

$a_n = \frac{2}{T} \int_0^T f(x) \cos\frac{2\pi n}{T} x \, dx$

$b_n = \frac{2}{T} \int_0^T f(x) \sin\frac{2\pi n}{T} x \, dx$

• $a_0 = \frac{2}{3} \int_0^3 f(x) dx = \frac{2}{3}$

$a_n = \frac{2}{3} \left(\int_0^2 (1-x) \cos\frac{2\pi n}{3} x \, dx + \int_2^3 \cos\frac{2\pi n}{3} x \, dx \right) =$

$= \frac{2}{3} \left(\int_0^2 \cos\frac{2\pi n}{3} x \, dx + \int_2^3 \cos\frac{2\pi n}{3} x \, dx - \int_0^2 x \cos\frac{2\pi n}{3} x \, dx \right) =$

$= \frac{2}{3} \left(\frac{3}{2\pi n} \sin\frac{2\pi n}{3} x \Big|_0^3 - \int_0^2 x \cos\frac{2\pi n}{3} x \, dx \right) =$

• $\int_0^2 \underbrace{x}_{u} \underbrace{\cos\frac{2\pi n}{3} x \, dx}_{dv} = x \frac{3}{2\pi n} \sin\frac{2\pi n}{3} x \Big|_0^2 - \int_0^2 \frac{3}{2\pi n} \sin\frac{2\pi n}{3} x \, dx =$

$= \frac{3}{\pi n} \sin\frac{4\pi n}{3} - \left(-\frac{9}{4\pi^2 n^2} \cos\frac{2\pi n}{3} x \Big|_0^2 \right) =$

$= \frac{3}{\pi n} \sin\frac{4\pi n}{3} + \frac{9}{4\pi^2 n^2} \cos\frac{4\pi n}{3} - \frac{9}{4\pi^2 n^2}$

$= \frac{2}{3} \left(\frac{3}{2\pi n} \sin 2\pi n - \frac{3}{\pi n} \sin\frac{4\pi n}{3} - \frac{9}{4\pi^2 n^2} \cos\frac{4\pi n}{3} + \frac{9}{4\pi^2 n^2} \right)$

$$a_n = \frac{2}{3}\left(\int_0^3 \cos\frac{2\pi n}{3}x\,dx - \int_0^2 x\cos\frac{2\pi n}{3}x\,dx\right) =$$

$$= \frac{2}{3}\left(\frac{3\sin(2\pi n)}{2\pi n} - \frac{9\cos\left(\frac{4\pi n}{3}\right)}{4\pi^2 n^2} + \frac{9}{4\pi^2 n^2} - \frac{3\sin\left(\frac{4\pi n}{3}\right)}{\pi n}\right) = \frac{\sin(2\pi n)}{\pi n} - \frac{3\cos\left(\frac{4\pi n}{3}\right)}{2\pi^2 n^2} + \frac{3}{2\pi^2 n^2} - \frac{2\sin\left(\frac{4\pi n}{3}\right)}{\pi n}$$

$$b_n = \frac{2}{3}\left(\int_0^2 (1-x)\,\cancel{\cos}\sin\frac{2\pi n}{3}x\,dx + \int_2^3 \cancel{\cos}\sin\left(\frac{2\pi n}{3}x\right)dx\right) =$$

$$= \frac{2}{3}\left(\int_0^3 \cos\frac{2\pi n}{3}x\,dx - \int_0^2 x\cos\frac{2\pi n}{3}x\,dx\right) =$$

$$= \frac{2}{3}\left(-\frac{3\cos(2\pi n)}{2\pi n} + \frac{3}{2\pi n} - \frac{9\sin\left(\frac{4\pi n}{3}\right)}{4\pi^2 n^2} + \frac{3\cos\left(\frac{4\pi n}{3}\right)}{\pi n}\right) =$$

$$= -\frac{\cos(2\pi n)}{\pi n} + \frac{1}{\pi n} - \frac{3\sin\left(\frac{4\pi n}{3}\right)}{2\pi^2 n^2} + \frac{2\cos\left(\frac{4\pi n}{3}\right)}{\pi n}$$

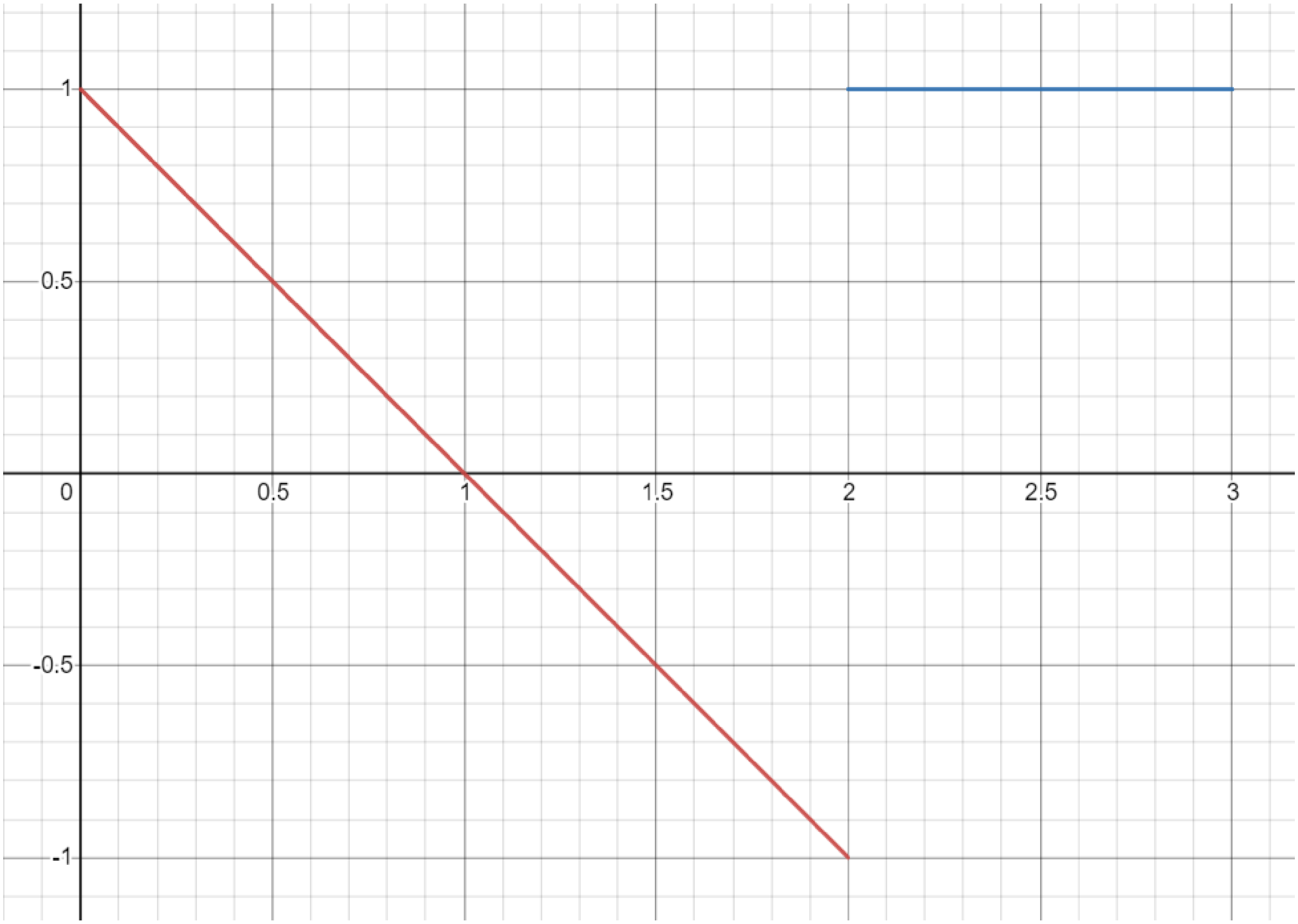
$$f(x) \sim \frac{1}{3} + \sum_{n=1}^{\infty}\left(\frac{\sin(2\pi n)}{\pi n} - \frac{2\sin\left(\frac{4\pi n}{3}\right)}{\pi n} - \frac{3\cos\left(\frac{4\pi n}{3}\right)}{2\pi^2 n^2} + \frac{3}{2\pi^2 n^2}\right)\cos\left(\frac{2\pi n}{3}x\right)$$

$$+ \left(\frac{1}{\pi n} - \frac{\cos(2\pi n)}{\pi n} + \frac{2\cos\left(\frac{4\pi n}{3}\right)}{\pi n} - \frac{3\sin\left(\frac{4\pi n}{3}\right)}{2\pi^2 n^2}\right)\sin\left(\frac{2\pi n}{3}x\right)$$

- сумма ряда Фурье

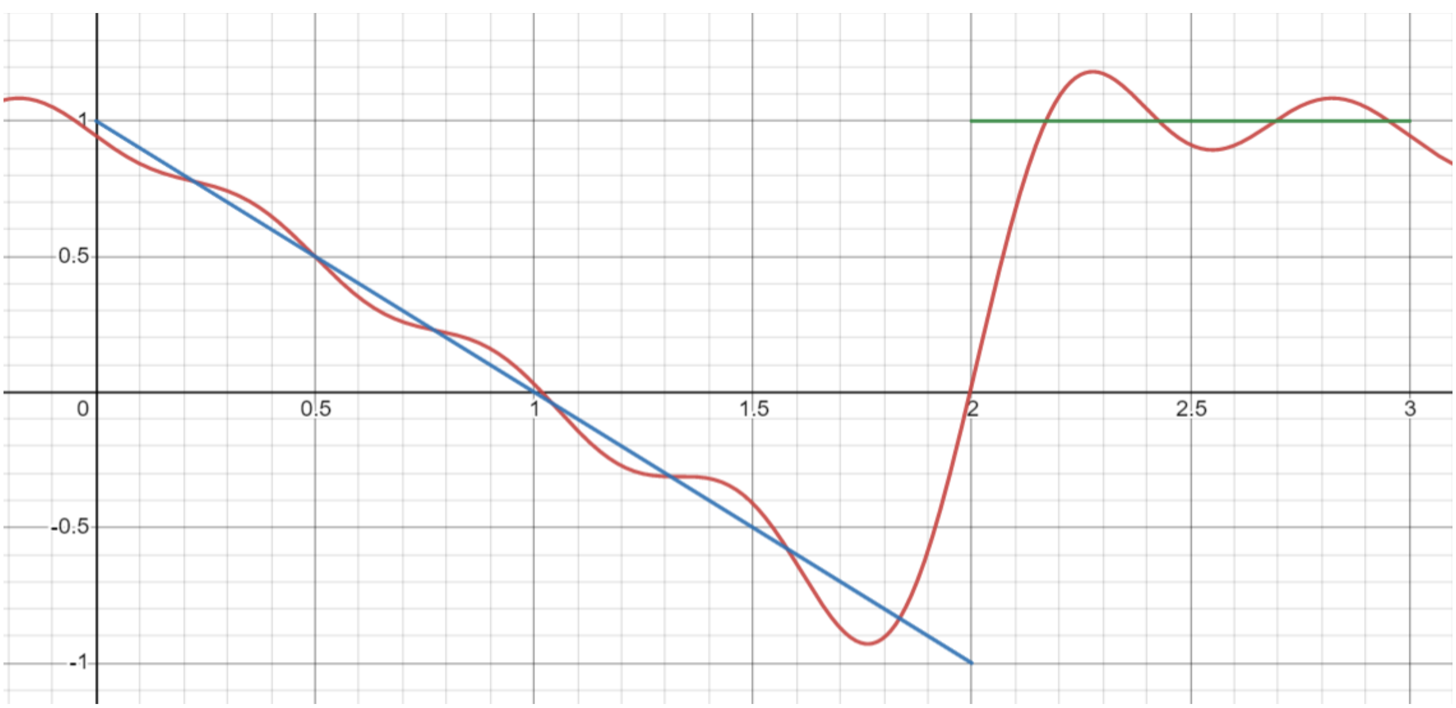
$$= \begin{cases} \text{значению ф-ии при } x \in \mathbb{R} \setminus \{2+3k\},\ k \in \mathbb{Z} \\ 0,\ x \in \{2+3k\}. \end{cases}$$

График исходной функции:

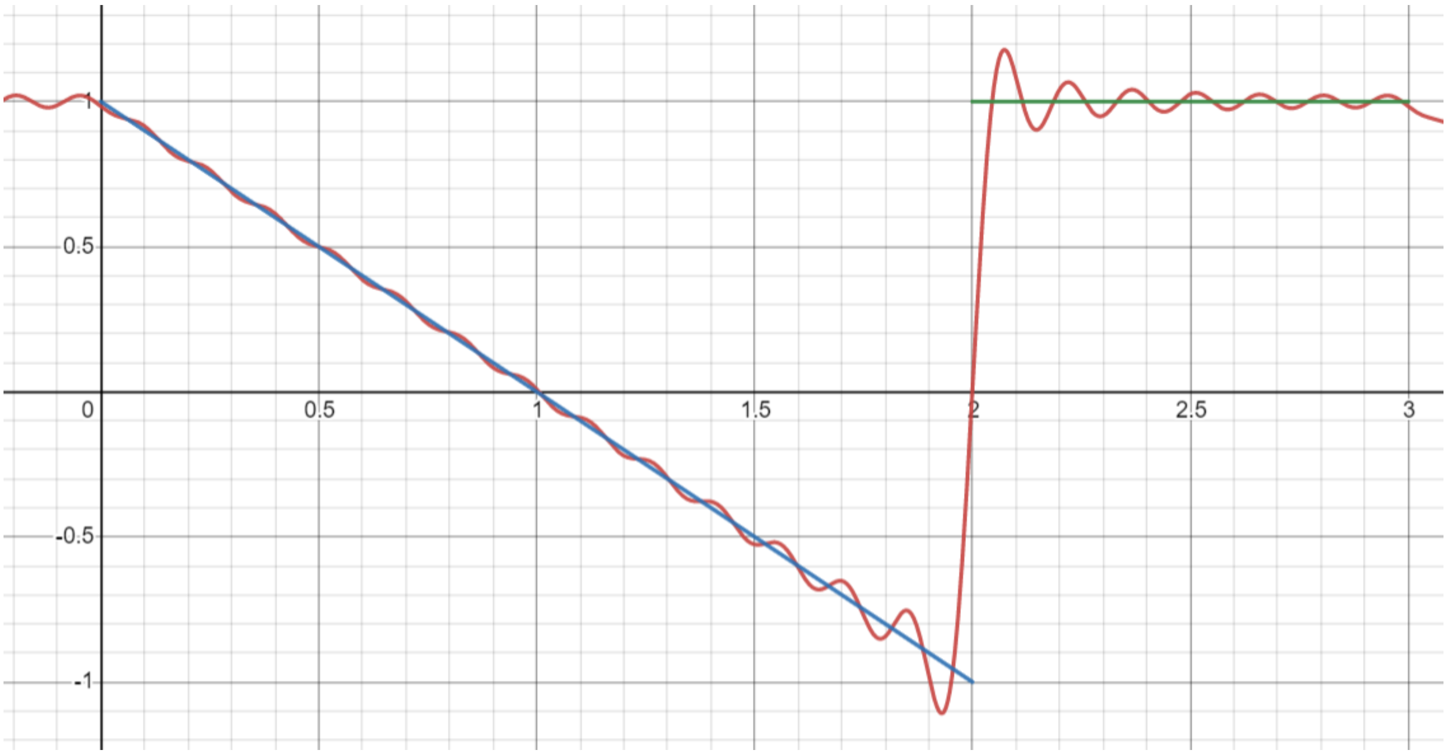


Общий тригонометрический ряд Фурье:

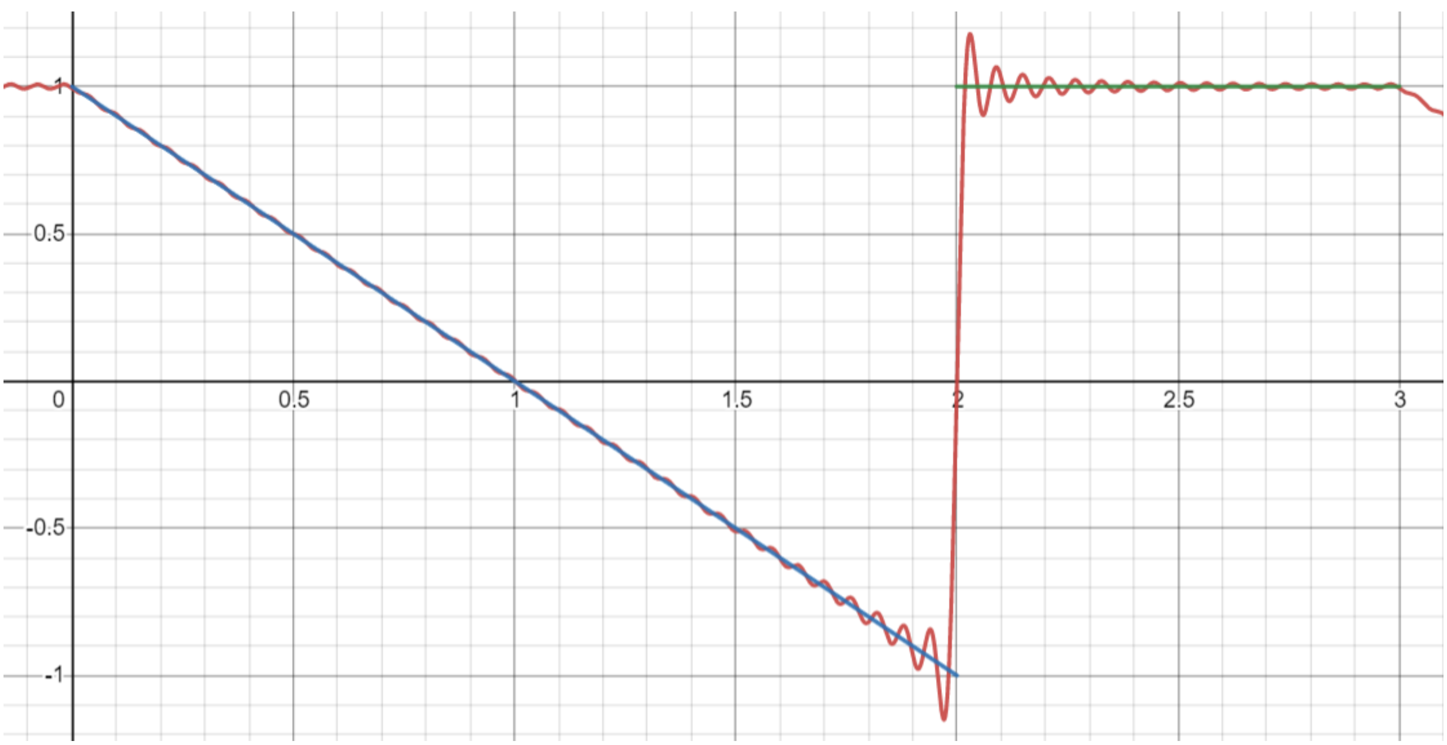
S_5



S_{20}

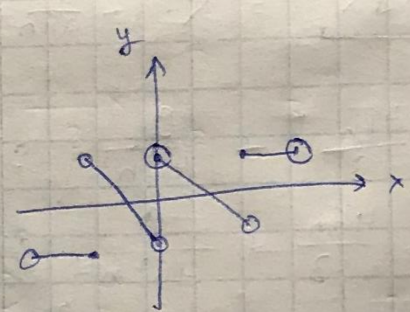


S_{50}



2) Ряд Фурье по синусам

$$f(x)=\begin{cases} -1, & x\in(-3,-2] \\ -1-x, & x\in(-2,0) \\ 1-x, & x\in(0,2) \\ 1, & x\in[2,3) \end{cases}$$



$T=6$

$$b_n=\frac{2}{6}\int_{-3}^{3} f(x)\sin\frac{2\pi n}{6}x\,dx = \frac{1}{3}\Bigg(\int_{-3}^{-2}-\sin\frac{\pi n}{3}x\,dx +$$

$$+\int_{-2}^{0}(-1-x)\sin\frac{\pi n}{3}x\,dx + \int_{0}^{2}(1-x)\sin\frac{\pi n}{3}x\,dx + \int_{2}^{3}\sin\frac{\pi n}{3}x\,dx\Bigg)$$

$$\bullet \int_{-3}^{-2}-\sin\frac{\pi n}{3}x\,dx = -\frac{3}{\pi n}\left(-\cos\frac{\pi n}{3}x\right)\Bigg|_{-3}^{-2} =$$

$$=\frac{3}{\pi n}\cos\frac{2\pi n}{3}-\frac{3}{\pi n}\cos\pi n = \frac{3}{\pi n}\cos\frac{2\pi n}{3}-\frac{3}{\pi n}\cos\pi n$$

$$\cdot \int_{-2}^{0}(-1-x)\sin\frac{\pi n}{3}x\,dx = -\int_{-2}^{0}\sin\frac{\pi n}{3}x\,dx - \int_{-2}^{0}x\sin\frac{\pi n}{3}x\,dx =$$

$$= \frac{3}{\pi n}\cos\frac{\pi n}{3}x\Big|_{-2}^{0} - \left(x\left(-\frac{3}{\pi n}\cos\frac{\pi n}{3}x\right)\Big|_{-2}^{0} + \frac{3}{\pi n}\int_{-2}^{0}\cos\frac{\pi n}{3}x\,dx\right) =$$

$$= \frac{3}{\pi n} - \frac{3}{\pi n}\cos\frac{2\pi n}{3} - \left(\frac{6}{\pi n}\cos\frac{2\pi n}{3} + \frac{9}{\pi^2 n^2}\sin\frac{2\pi n}{3}\right) =$$

$$= \frac{3}{\pi n} - \frac{9}{\pi n}\cos\frac{2\pi n}{3} - \frac{9}{\pi^2 n^2}\sin\frac{2\pi n}{3}$$

$$\cdot \int_{0}^{2}(1-x)\sin\frac{\pi n}{3}x\,dx = \int_{0}^{2}\sin\frac{\pi n}{3}x\,dx - \int_{0}^{2}x\sin\frac{\pi n}{3}x\,dx =$$

$$= -\frac{3}{\pi n}\cos\frac{\pi n}{3}\Big|_{0}^{2} - \left(x\left(-\frac{3}{\pi n}\cos\frac{\pi n}{3}x\right)\Big|_{0}^{2} + \frac{3}{\pi n}\int_{0}^{2}\cos\frac{\pi n}{3}\,dx\right) =$$

$$= -\frac{3}{\pi n}\cos\frac{2\pi n}{3} + \frac{3}{\pi n} - \left(-\frac{6}{\pi n}\cos\frac{2\pi n}{3} + \frac{9}{\pi^2 n^2}\sin\frac{\pi n x}{3}\Big|_{0}^{2}\right) =$$

$$= \frac{3}{\pi n}\cos\frac{2\pi n}{3} + \frac{3}{\pi n} - \frac{9}{\pi^2 n^2}\sin\frac{2\pi n}{3}$$

$$\cdot \int_{2}^{3}\sin\frac{\pi n}{3}x\,dx = -\frac{3}{\pi n}\cos\frac{\pi n}{3}x\Big|_{2}^{3} =$$

$$= -\frac{3}{\pi n}\cos\pi n + \frac{3}{\pi n}\cos\frac{2\pi n}{3}$$

$$b_n = \frac{1}{3}\Big(\frac{3}{\pi n}\cancel{\cos\frac{2\pi n}{3}} - \frac{3}{\pi n}\cos\pi n - \frac{3}{\pi n}\cos\pi n + \frac{3}{\pi n}\cancel{\cos\frac{2\pi n}{3}} +$$

$$+ \frac{3}{\pi n}\cancel{\cos\frac{2\pi n}{3}} + \frac{3}{\pi n} - \frac{9}{\pi^2 n^2}\sin\frac{2\pi n}{3} + \frac{3}{\pi n} - \frac{9}{\pi n}\cancel{\cos\frac{2\pi n}{3}} -$$

$$- \frac{9}{\pi^2 n^2}\sin\frac{2\pi n}{3}\Big) = \frac{1}{3}\left(-\frac{6}{\pi n}\cos\pi n + \frac{6}{\pi n} - \frac{18}{\pi^2 n^2}\sin\frac{2\pi n}{3}\right) =$$

$$= \frac{2}{\pi n} - \frac{2}{\pi n}\cos\pi n - \frac{6}{\pi^2 n^2}\sin\frac{2\pi n}{3} + \frac{4}{\pi n}\cos\left(\frac{2\pi n}{3}\right)$$

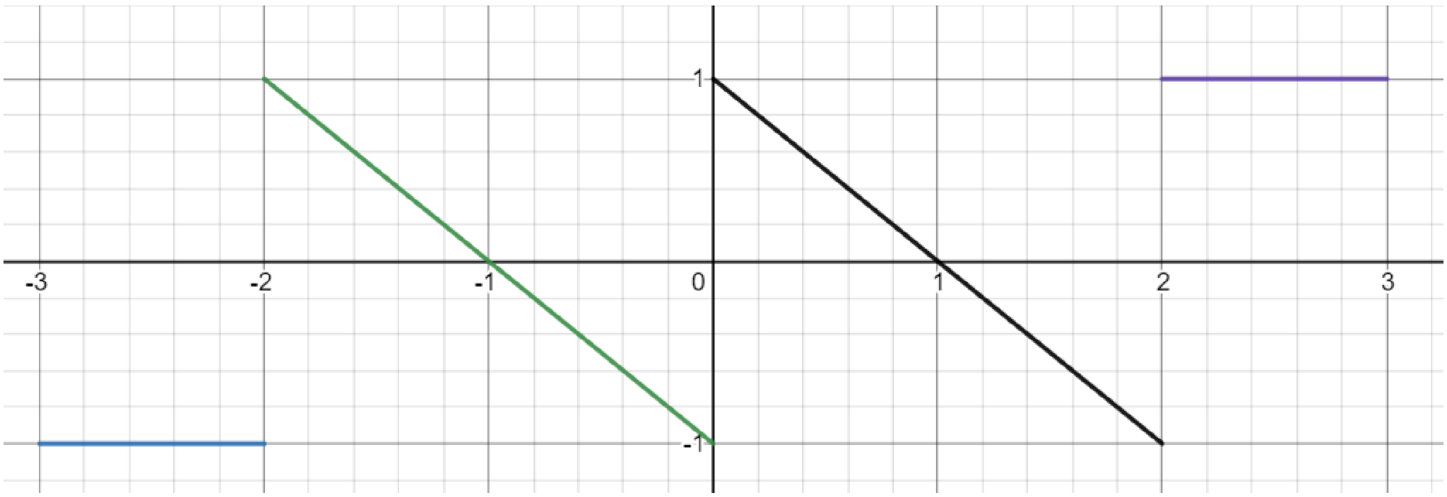
$$F(x) \sim \sum_{n=1}^{\infty}\left(\frac{2}{\pi n} - \frac{2}{\pi n}\cos\pi n - \frac{6}{\pi^2 n^2}\sin\frac{2\pi n}{3} \ast\right)\sin\frac{\pi n}{3}x$$

$$\ast\ \frac{4}{\pi n}\cos\left(\frac{2\pi n}{3}\right)$$

• сумма ряда Фурье по синусам:

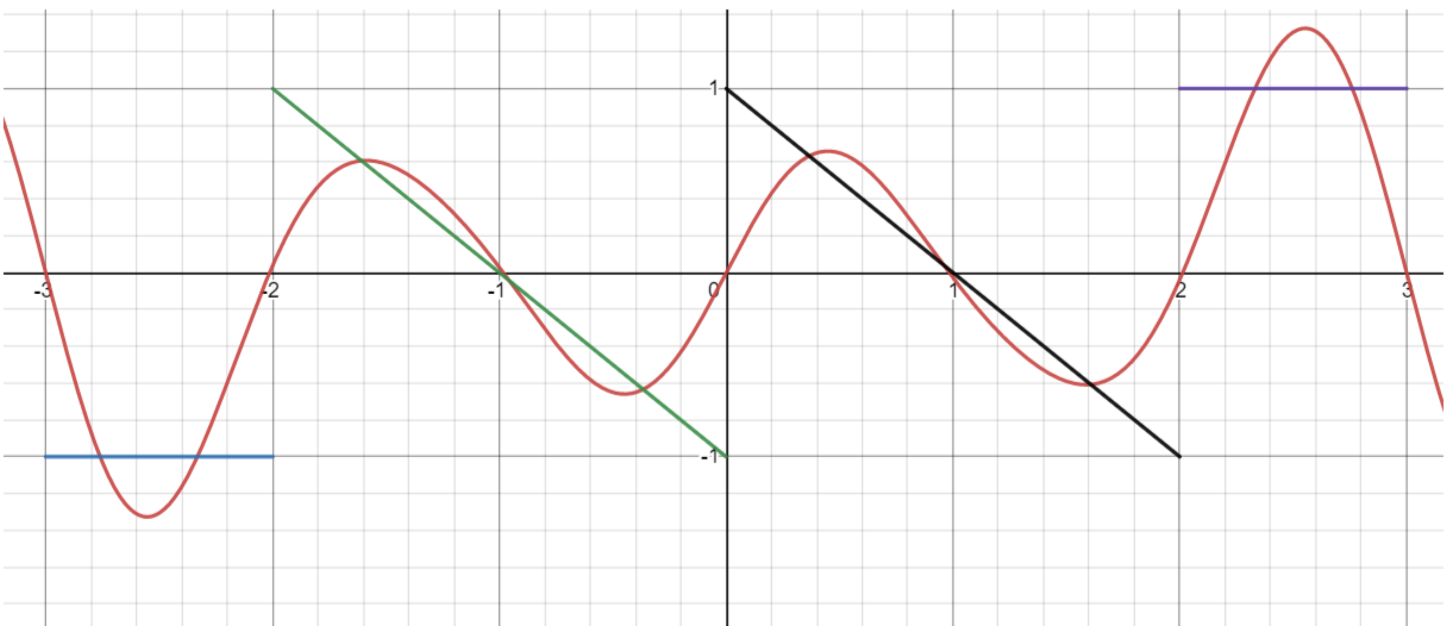
$$= \begin{cases} \text{знач. ф-ии при } x \in \mathbb{R} \setminus \{-3+6k;\ -2+6k;\ 6k;\ 2+6k\} \\ 0,\ x \in \{-3+6k;\ -2+6k;\ 6k;\ 2+6k\} \end{cases}$$

График нечётной исходной функции:

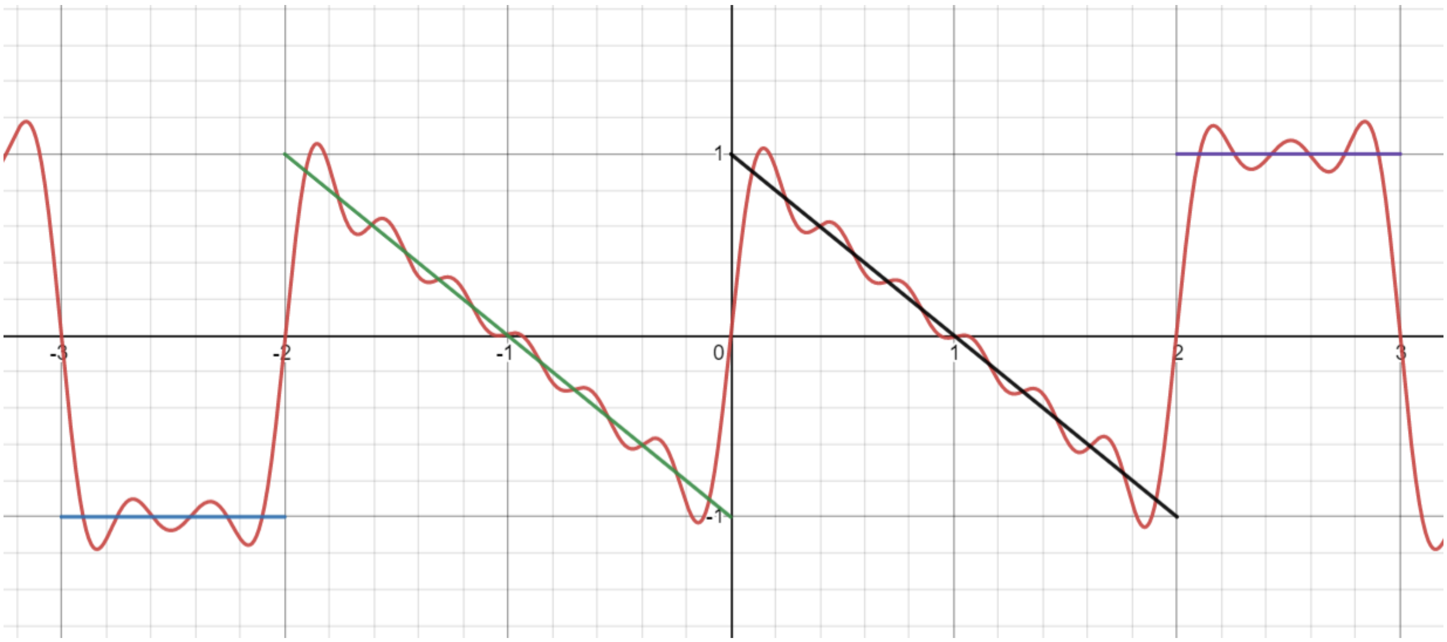


Ряд Фурье по синусам:

S_5



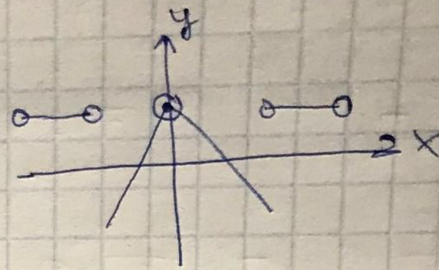
S_{20}



S_{50}

3) Ряд Фурье по косинусам.

$$f(x)=\begin{cases}1, & x\in(-3,-2)\cup(2,3)\\ 1-x, & x\in[0,2]\\ x+1, & x\in[-2,0)\end{cases}$$



$T=6$

$$a_0=\frac{1}{3}\cdot 2\int_0^3 f(x)\,dx=\frac{2}{3}$$

$$a_n=\frac{1}{3}\cdot 2\int_{-3}^{3} f(x)\cdot\cos\frac{\pi n}{3}x\,dx=$$

$$=\frac{1}{3}\left(\int_{-3}^{-2}\cos\frac{\pi n}{3}x\,dx+\int_{2}^{3}\cos\frac{\pi n}{3}x\,dx+\int_{-2}^{0}\cos\frac{\pi n}{3}x\,dx+\right.$$

$$\left.+\int_{-2}^{0}x\cos\frac{\pi n}{3}\,dx-\int_{0}^{2}x\cos\frac{\pi n}{3}x\,dx+\int_{0}^{2}\cos\frac{\pi n}{3}x\,dx\right)=$$

$$=\frac{1}{3}\left(\cancel{-\frac{3}{\pi n}\sin\frac{2\pi n}{3}}+\frac{3}{\pi n}\sin\pi n+\frac{3}{\pi n}\sin\pi n\cancel{-\frac{3}{\pi n}\sin\frac{2\pi n}{3}}+\right.$$

$$\cancel{+\frac{6}{\pi n}\sin\frac{2\pi n}{3}}-\frac{6}{\pi n}\sin\frac{2\pi n}{3}+\frac{9}{\pi^2 n^2}-\frac{9}{\pi^2 n^2}\cos\frac{2\pi n}{3}-$$

$$\left.-\frac{6}{\pi n}\sin\frac{2\pi n}{3}-\frac{9}{\pi^2 n^2}\cos\frac{2\pi n}{3}+\frac{9}{\pi^2 n^2}\right)=$$

$$=\frac{2}{\pi n}\sin\pi n-\frac{4}{\pi n}\sin\frac{2\pi n}{3}-\frac{6}{\pi^2 n^2}\cos\frac{2\pi n}{3}+\frac{6}{\pi^2 n^2}$$

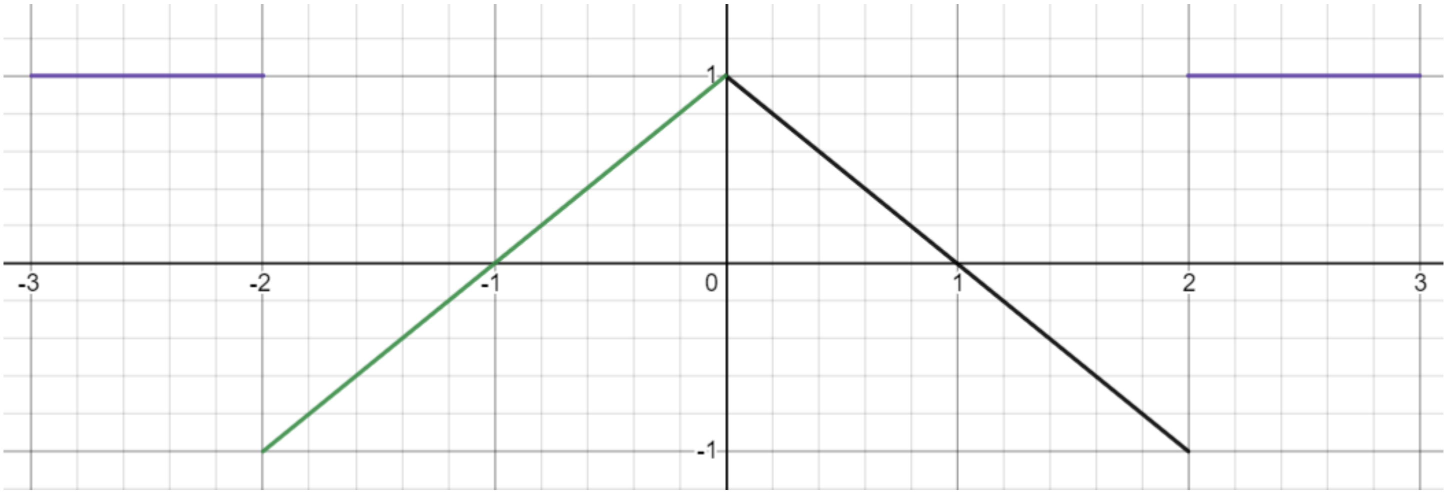
$$f(x)\sim\frac{1}{3}+\sum_{n=1}^{\infty}\left(\frac{2}{\pi n}\sin\pi n-\frac{4}{\pi n}\sin\frac{2\pi n}{3}-\frac{6}{\pi^2 n^2}\cos\frac{2\pi n}{3}+\frac{6}{\pi^2 n^2}\right)$$

$$\cdot\cos\frac{\pi n}{3}x$$

· сумма ряда Фурье по косинусам:

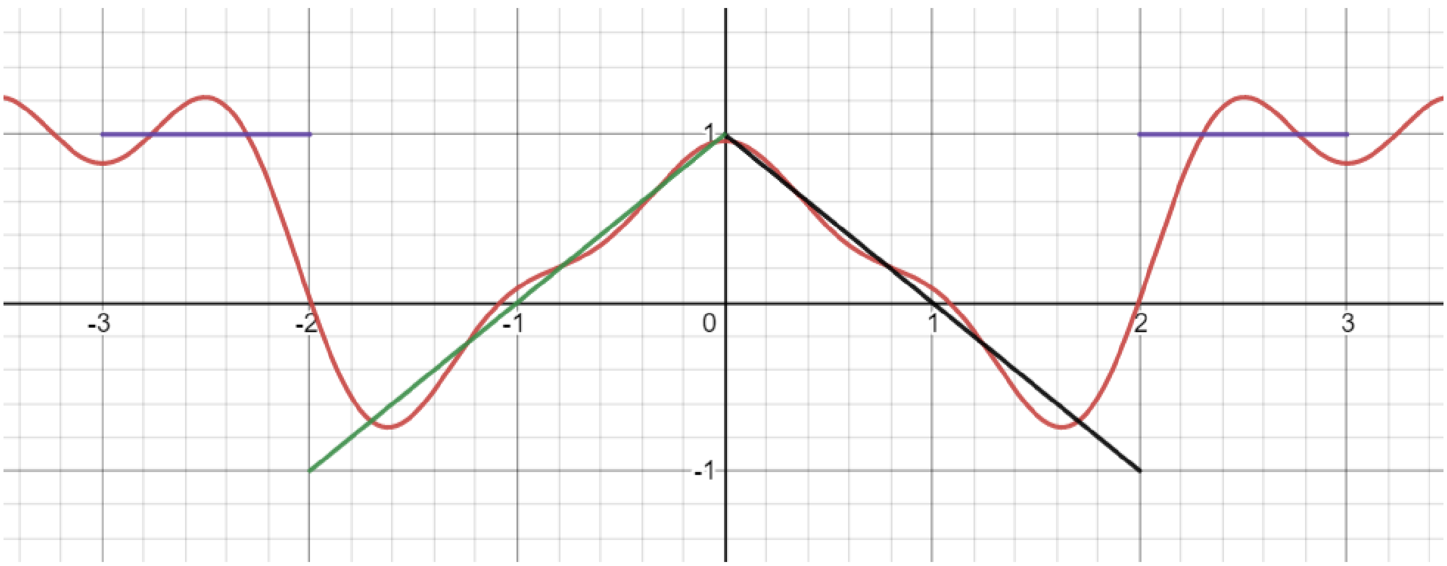
$$=\begin{cases}\text{ф-ии}, & \text{if } x\in\mathbb{R}\setminus\{-2+6k;\,2+6k\}\\ 0, & \text{if } x\in\{-2+6k;\,2+6k\}\end{cases}$$

График чётной исходной функции:

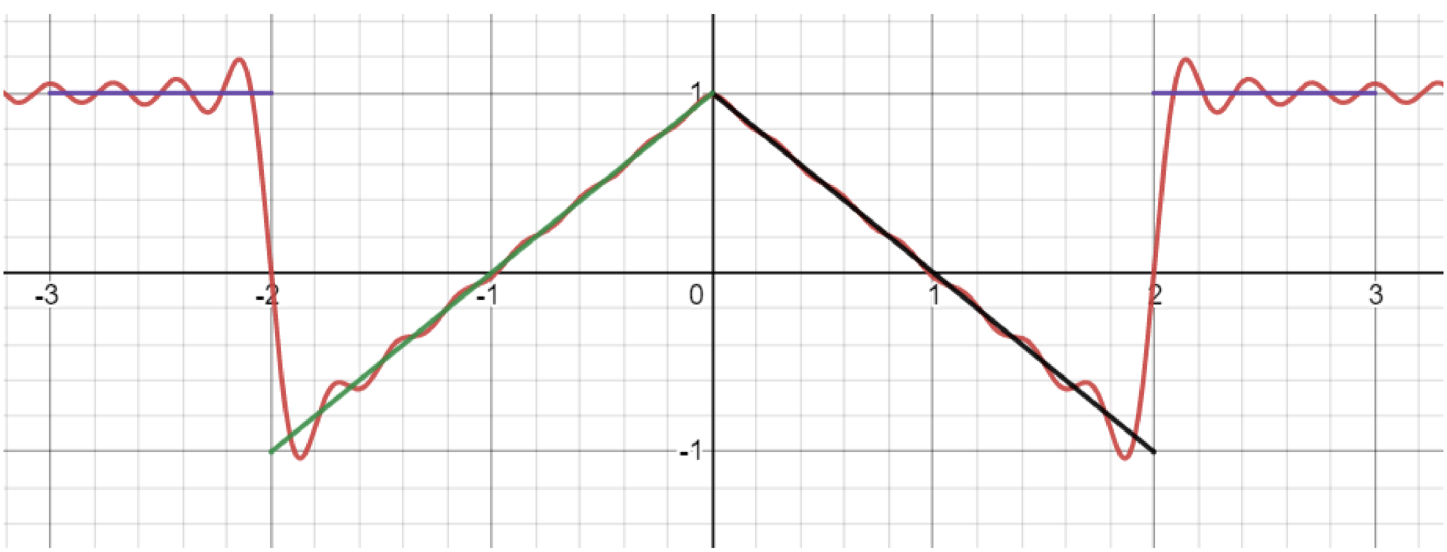


Ряд Фурье по косинусам:

S_5



S_{20}



S_{50}