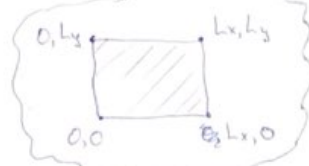


$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0, \quad 0 \leq t \leq T, 0 \leq x \leq L_x, 0 \leq y \leq L_y$$

$$u(x, 0, t) = u(0, y, t) = u(x, L_y, t) = u(x, y, L_x, t) = 0$$

$$u(x, y, 0) = \exp\left(-\frac{(x-x_0)^2 + (y-y_0)^2}{r^2}\right)$$



$$\frac{\partial u}{\partial t} = 0 \quad \text{в начальный момент } t=0$$

$$\begin{aligned} x_i &= i h_x & i &= \overline{0, I}, & h_x &= \frac{L_x}{I} \\ y_j &= j h_y & j &= \overline{0, J}, & h_y &= \frac{L_y}{J} \\ t_k &= k h_t & k &= \overline{0, K}, & h_t &= \frac{T}{K} \end{aligned}$$

$$\frac{\partial^2 u(x_i, y_j, t_k)}{\partial x^2} \approx \frac{u(x_{i+1}, y_j, t_k) - 2u(x_i, y_j, t_k) + u(x_{i-1}, y_j, t_k))}{h_x^2}$$

$$\frac{\partial^2 u(x_i, y_j, t_k)}{\partial y^2} \approx \frac{u(x_i, y_{j+1}, t_k) - 2u(x_i, y_j, t_k) + u(x_i, y_{j-1}, t_k))}{h_y^2}$$

$$\frac{\partial^2 u(x_i, y_j, t_k)}{\partial t^2} \approx \frac{u(x_i, y_j, t_{k+1}) - 2u(x_i, y_j, t_k) + u(x_i, y_j, t_{k-1}))}{h_t^2}$$

$$\frac{\partial u(x_i, y_j, t_k)}{\partial t} \approx \frac{u(x_i, y_j, t_{k+1}) - u(x_i, y_j, t_k)}{h_t}$$

x_0, y_0 - координаты
Горизонтальная

$$\frac{u_{i,j,k}^k - 2u_{i,j,k}^k + u_{i-1,j,k}^k}{h_x^2} + \frac{u_{i,j,k}^k - 2u_{i,j,k}^k + u_{i,j-1,k}^k}{h_y^2} - \frac{1}{v^2} \cdot \frac{u_{i,j,k+1}^{k+1} - 2u_{i,j,k}^k + u_{i,j,k-1}^{k-1}}{h_t^2} = 0, \quad \begin{matrix} i = \overline{1, I-1} \\ j = \overline{1, J-1} \\ k = \overline{1, K-1} \end{matrix}$$

$$u_{i,j,k}^0 = u_{i,j}^0 = u_{i,j}^0 = u_{i,j}^0 = 0, \quad i = \overline{0, I}, j = \overline{0, J}, k = \overline{0, K}$$

$$u_{i,j}^0 = \exp\left(-\frac{(i h_x - x_0)^2 + (j h_y - y_0)^2}{r^2}\right), \quad i = \overline{0, I}, j = \overline{0, J}$$

$$u_{i,j}^1 = u_{i,j}^0, \quad i = \overline{0, I}, j = \overline{0, J}$$

$$u_{i,j}^{k+1} = \frac{r^2 h_t^2}{h_x^2} \left(\dots \right) + \frac{r^2 h_t^2}{h_y^2} \left(\dots \right) + 2u_{i,j}^k - u_{i,j}^{k-1} =$$

$$= r^2 h_t^2 \left(\frac{1}{h_x^2} \left(u_{i+1,j}^k + u_{i-1,j}^k \right) + \frac{1}{h_y^2} \left(u_{i,j+1}^k + u_{i,j-1}^k \right) \right) - u_{i,j}^{k-1} + 2 \left(1 - \frac{r^2 h_t^2}{h_x^2} \left(\frac{1}{h_x^2} + \frac{1}{h_y^2} \right) \right) u_{i,j}^k$$