

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0$$

$$0 \leq t \leq T, 0 \leq x \leq l$$

$$u(x, 0) = u_0(x)$$

$$u(0, t) = 0$$

$$u(l, t) = 0$$

$$\frac{\partial u}{\partial t}(x, t) = 0$$

$$x_i = i h_x \quad i = \overline{0, \bar{I}} \quad h_x = \frac{l}{\bar{I}}$$

$$t_k = k h_t \quad k = \overline{0, \bar{K}} \quad h_t = \frac{T}{\bar{K}}$$

$$\frac{\partial^2 u}{\partial x^2}(x_i, t_k) \approx \frac{u(x_{i+1}, t_k) - 2u(x_i, t_k) + u(x_{i-1}, t_k))}{h_x^2}$$

$$\frac{\partial^2 u}{\partial t^2}(x_i, t_k) \approx \frac{u(x_i, t_{k+1}) - 2u(x_i, t_k) + u(x_i, t_{k-1}))}{h_t^2}$$

$$\frac{\partial u}{\partial t}(x_i, t_k) \approx \frac{u(x_i, t_{k+1}) - u(x_i, t_k)}{h_t} \approx \frac{u(x_i, t_{k+1}) - u(x_i, t_{k-1}))}{2h_t}$$

$$\frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h_x^2} - \frac{1}{v^2} \frac{u_i^{k+1} - 2u_i^k + u_i^{k-1}}{h_t^2} = 0 \quad i = \overline{1, \bar{I}-1}, k = \overline{1, \bar{K}-1}$$

$$u_0^k = 0, \quad u_{\bar{I}}^k = 0, \quad k = \overline{0, \bar{K}}$$

$$u_i^0 = \begin{cases} 1, & i = \left[\frac{3l}{4h_x} \right] \\ \frac{4}{3l} i h_x, & i < \left[\frac{3l}{4h_x} \right] \\ -\frac{4}{l} i h_x + 4, & i > \left[\frac{3l}{4h_x} \right] \end{cases} \quad i = \overline{0, \bar{I}}$$

$$\frac{u_i^{k+1} - u_i^k}{h_t} = 0, \quad i = \overline{0, \bar{I}}, k = \overline{0, \bar{K}-1}$$

$$u_i^{k+1} = \frac{h_t^2 v^2}{h_x^2} (u_{i+1}^k + u_{i-1}^k) - u_i^{k-1} + 2 \left(\frac{h_t^2 v^2}{h_x^2} - 1 \right) u_i^k, \quad i = \overline{1, \bar{I}-1}$$

$$u_i^1 = u_i^0, \quad i = \overline{0, \bar{I}}$$