telegraph equations solved with MFEM library

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1 Introduction

This document explains the telegraph equations simulation using finite differences method based on MFEM library.

2 Theory, Finite Differences

The telegraph equations are:

$$\frac{\partial V}{\partial x} + L\frac{\partial I}{\partial t} + RI = 0 \tag{1}$$

$$\frac{\partial I}{\partial x} + C\frac{\partial V}{\partial t} + GV = 0 \tag{2}$$

1. Assume the TL is divided in N segments with node 0 to N. There is N+1 nodes. So nodes are located at $n\Delta x$.
2. Time is sampled at $k\Delta T$.
3. Assume the $\frac{\partial V(t,x)}{\partial x}$ and $\frac{\partial I(t,x)}{\partial x}$ are constant over each segment.

So at a given time $k\Delta T$ we can express the $\frac{\partial V(t,x)}{\partial x}$ and $\frac{\partial I(t,x)}{\partial x}$ as difference $\frac{V(k\Delta T,(n+1)\Delta x)-V(k\Delta T,(n-1)\Delta x)}{2\Delta x}$ and $\frac{I(k\Delta T,(n+1)\Delta x)-I(k\Delta T,(n-1)\Delta x)}{2\Delta x}$.

with the $\frac{\partial}{\partial t}$ on the left side.

$$\frac{\partial I}{\partial t} = -\frac{RI}{L} - \frac{1}{L}\frac{\partial V}{\partial x} \tag{3}$$

$$\frac{\partial V}{\partial t} = -\frac{GV}{C} - \frac{1}{C}\frac{\partial I}{\partial x} \tag{4}$$

We can write matrix equations....

$$\frac{\partial I}{\partial t} = \begin{bmatrix} Dv & Ri \end{bmatrix} \begin{bmatrix} V^k \\ I^k \end{bmatrix} \tag{5}$$

$$\frac{\partial V}{\partial t} = \begin{bmatrix} Gv & Di \end{bmatrix} \begin{bmatrix} V^k \\ I^k \end{bmatrix} \tag{6}$$

The two equations above can be written as a single matrix equation...

$$\begin{bmatrix} \frac{\partial V}{\partial t} \\ \frac{\partial I}{\partial t} \end{bmatrix} = \begin{bmatrix} GvDi \\ DvRi \end{bmatrix} \begin{bmatrix} V^k \\ I^k \end{bmatrix} \tag{7}$$

Where Dv and Di are the same derivative matrix multiplied by different terms -1/L for Dv and -1/C for Di.

$$\begin{bmatrix} 0 & -1/2h & 0 & 0 & ... & 0 & 0 \\ 1/2h & 0 & -1/2h & 0 & 0 & ... & 0 \\ 0 & 1/2h & 0 & -1/2h & 0 & ... & 0 \\ 0 & 0 & 01/2h & 0 & -1/2h & ... & 0 \\ 0 & 0 & 0 & ...1/2h & 0 & -1/2h & 0 \\ 0 & 0 & 0 & ... & 1/2h & 0 & 1/2h \\ 0 & 0 & 0 & 0 & ... & 1/2h & 0 \end{bmatrix} \tag{8}$$

Gv and Ri are identity matrix scale by either -G/C and -R/L.
For Dv the multiplier value will be $\frac{\Delta t}{Lh}$.
Using the above operator we will step in time using RK4.

$$\begin{bmatrix} V^{k+1} \\ I^{k+1} \end{bmatrix} = \begin{bmatrix} V^k \\ I^k \end{bmatrix} + \Delta t \begin{bmatrix} \frac{\partial V}{\partial t} \\ \frac{\partial I}{\partial t} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} V^{k+1} \\ I^{k+1} \end{bmatrix} = \begin{bmatrix} V^k \\ I^k \end{bmatrix} + \Delta t \begin{bmatrix} GvDi \\ DvRi \end{bmatrix} \begin{bmatrix} V^k \\ I^k \end{bmatrix} \tag{10}$$

2.1 MFEM Implementation

The program is named stltfdrk4.cpp for Single Transmission Line Transient Finite Difference runge kutta 4.

The signal injected is gaussian pulse centered at 100ns with a Tau of 20ns multiplied by a triangular window of 200ns wide centered at 100ns.

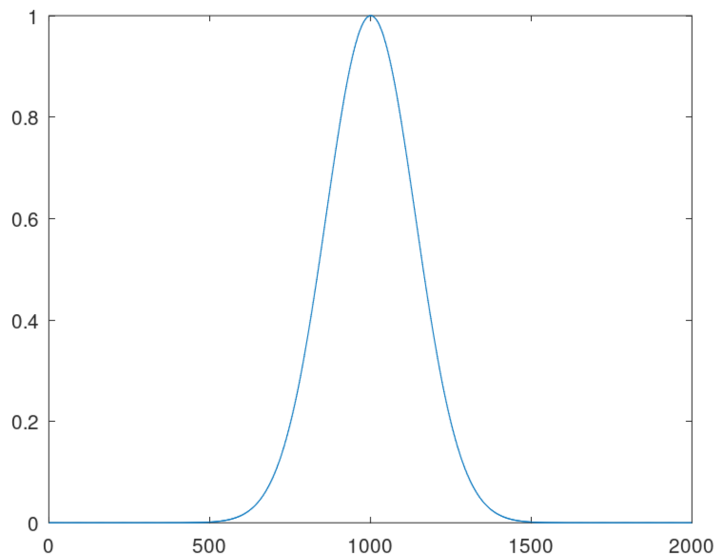

Figure 1: Test pulse

next step is the time step...