Nasa Formal Methods (NFM) 2024, Moffett Field, California (USA)

Quantitative Input Usage Static Analysis

Denis Mazzucato, Marco Campion, and Caterina Urban

4 June 2024

Aircraft Landing Alarm System 🚨

Aircraft Landing Alarm System 🚨

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
    risk = 0
else if landing_coeff > 5 then
    risk = 3
else
    risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
  risk = 0
else if landing_coeff > 5 then
  risk = 3
else
  risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
    risk = 0
else if landing_coeff > 5 then
    risk = 3
else
    risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
  risk = 0
else if landing_coeff > 5 then
  risk = 3
else
  risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
  risk = 0
else if landing_coeff > 5 then
  risk = 3
else
  risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
    risk = 0
else if landing_coeff > 5 then
    risk = 3
else
    risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
    risk = 0
else if landing_coeff > 5 then
    risk = 3
else
    risk = floor(landing_coeff) - 2
```

Aircraft Landing Alarm System 🚨

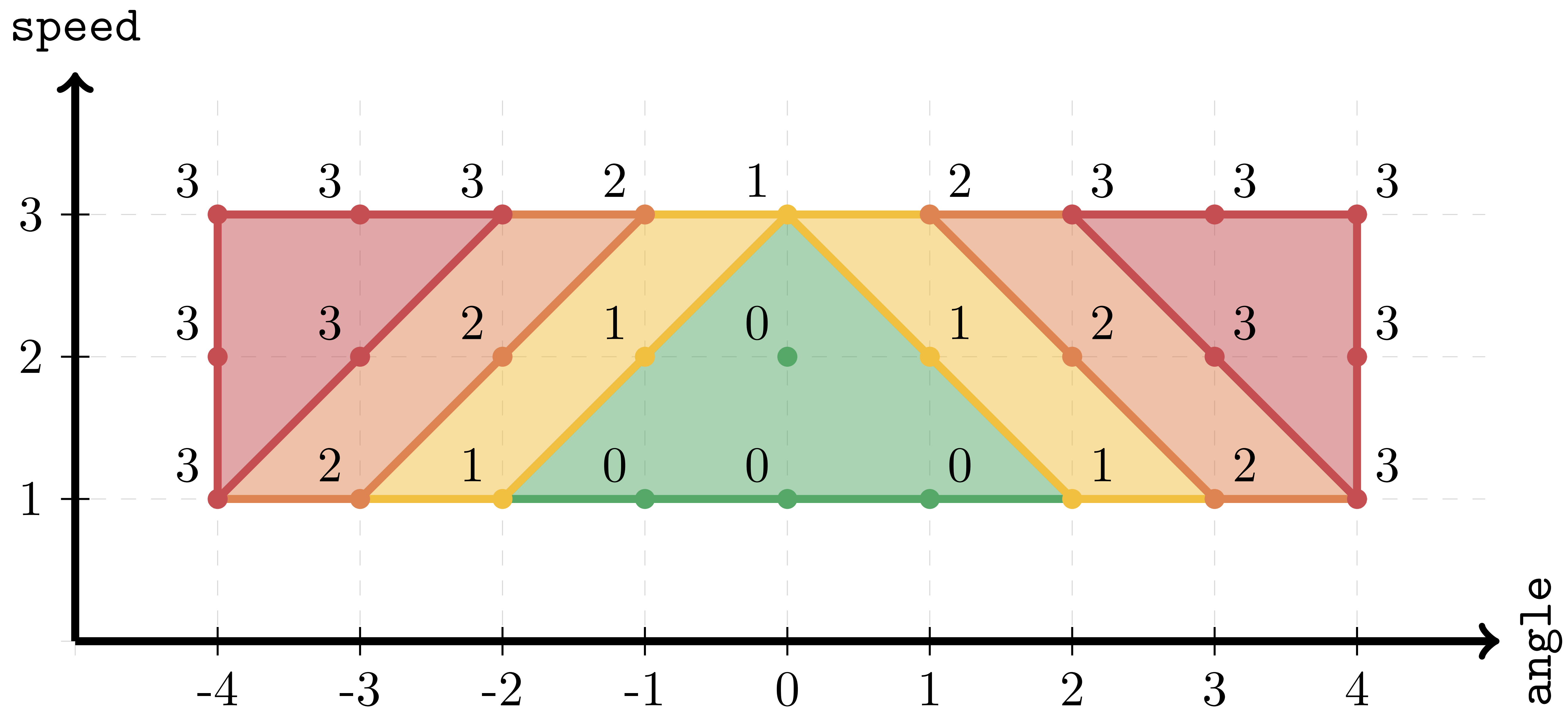
risk = 0

risk = 3

0 < risk < 3

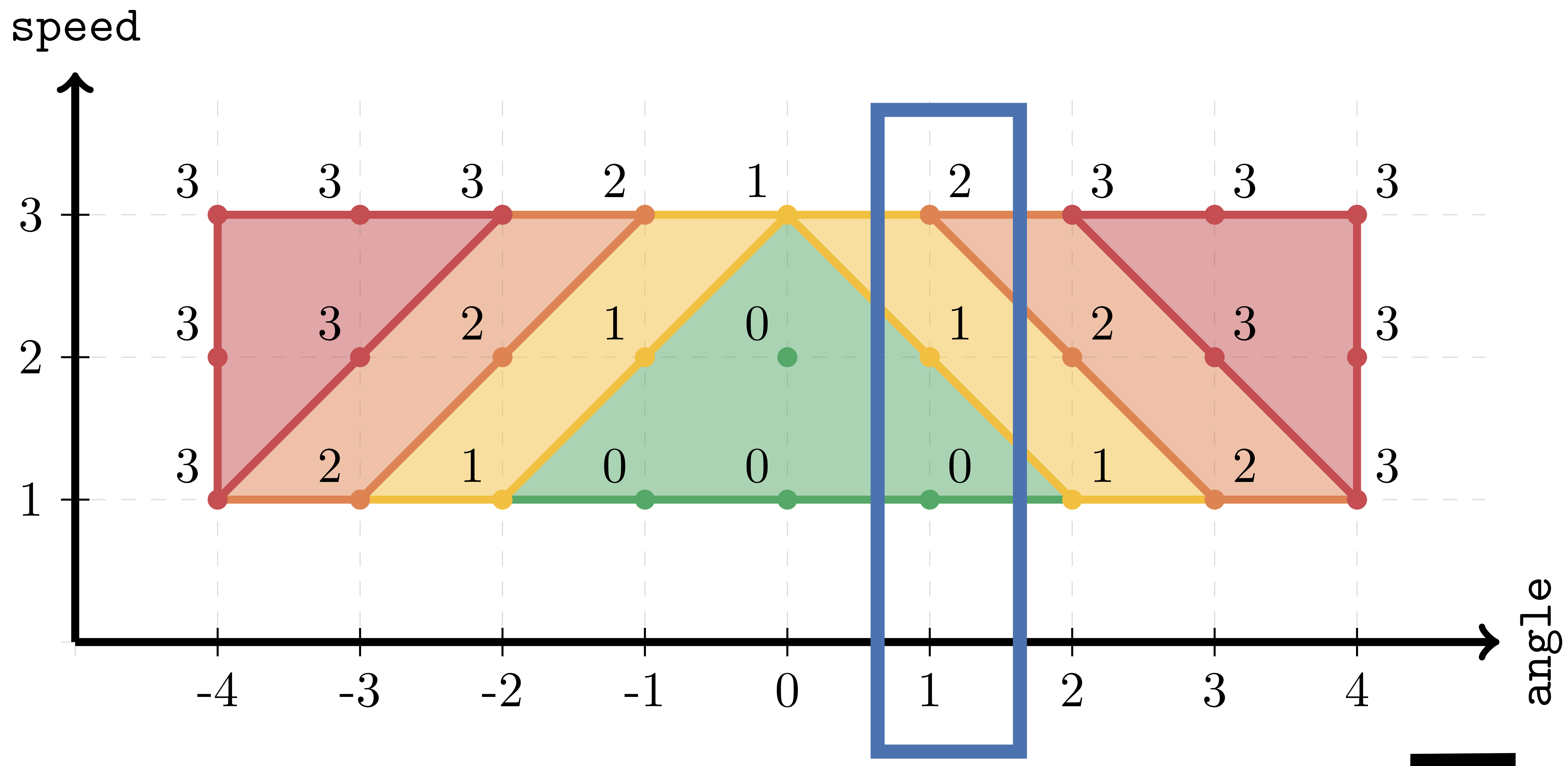
Aircraft Landing Alarm System 🚨

- risk = 0
- risk = 3
- 0 < risk < 3



Aircraft Landing Alarm System 🚨

- risk = 0
- risk = 3
- 0 < risk < 3

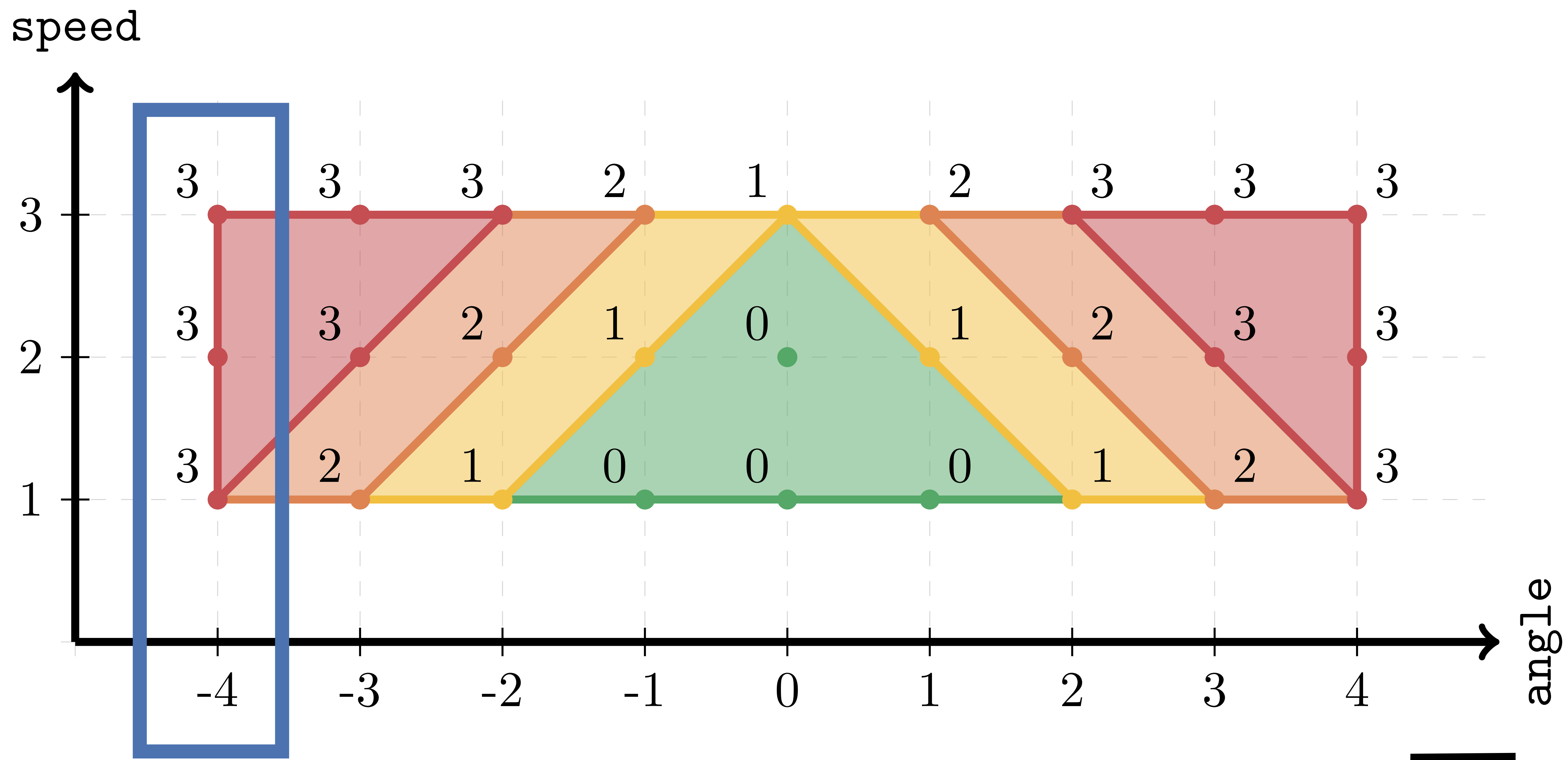


Aircraft Landing Alarm System 🚨

risk = 0

risk = 3

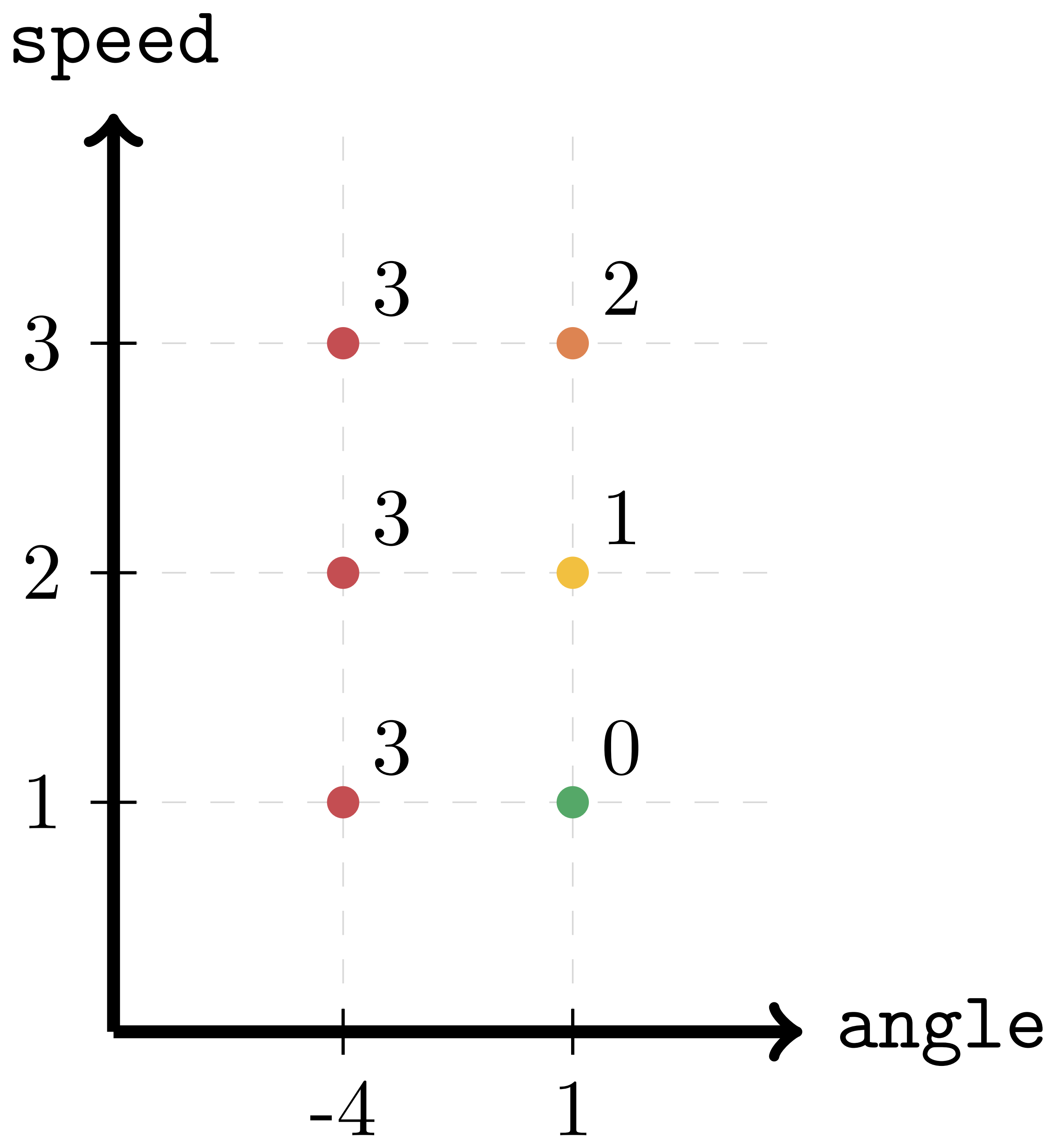
0 < risk < 3



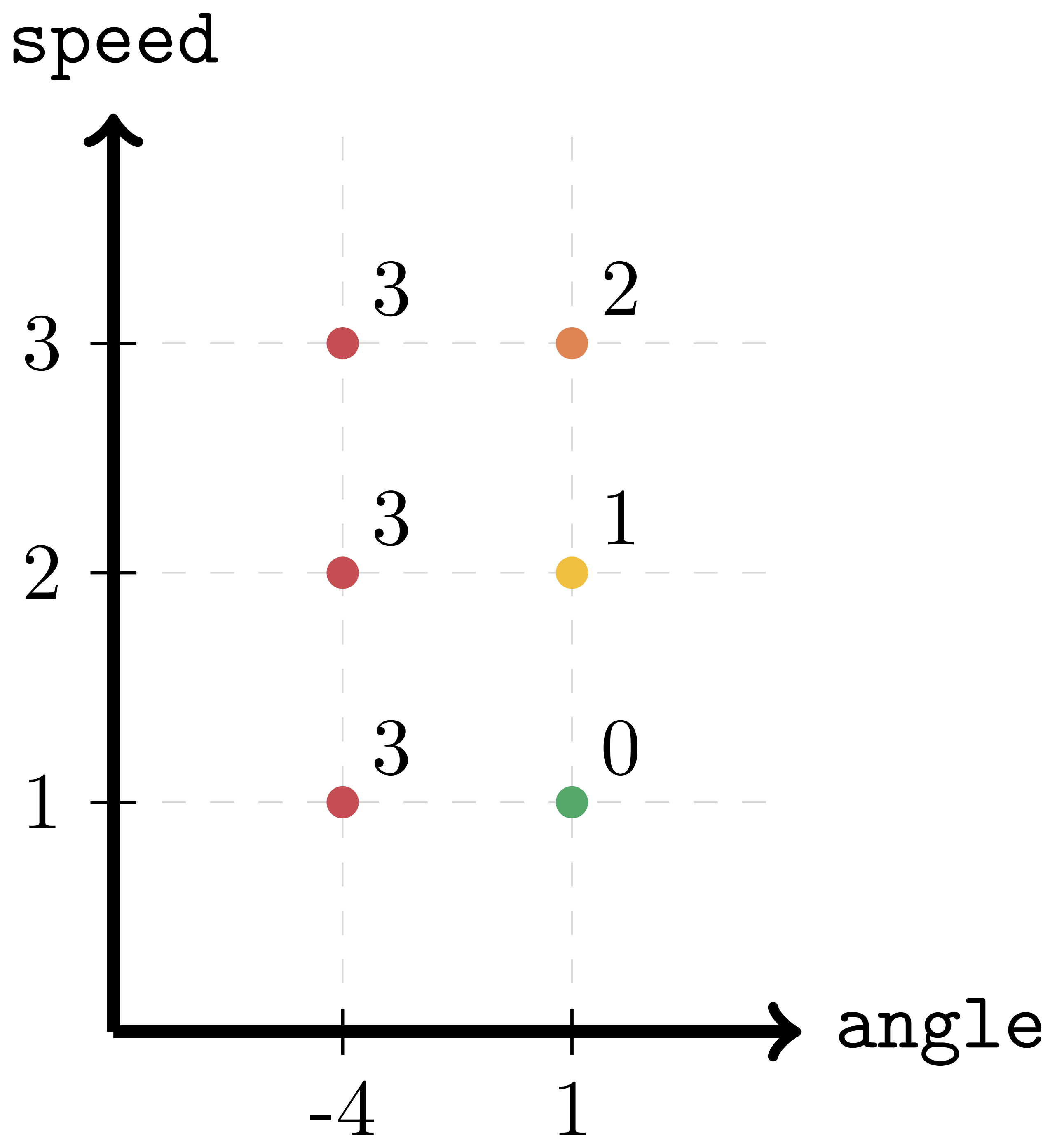
Goal: Quantify the impact of speed and angle on risk

```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
  risk = 0
else if landing_coeff > 5 then
  risk = 3
else
  risk = floor(landing_coeff) - 2
```

Goal: Quantify the impact of speed and angle on risk

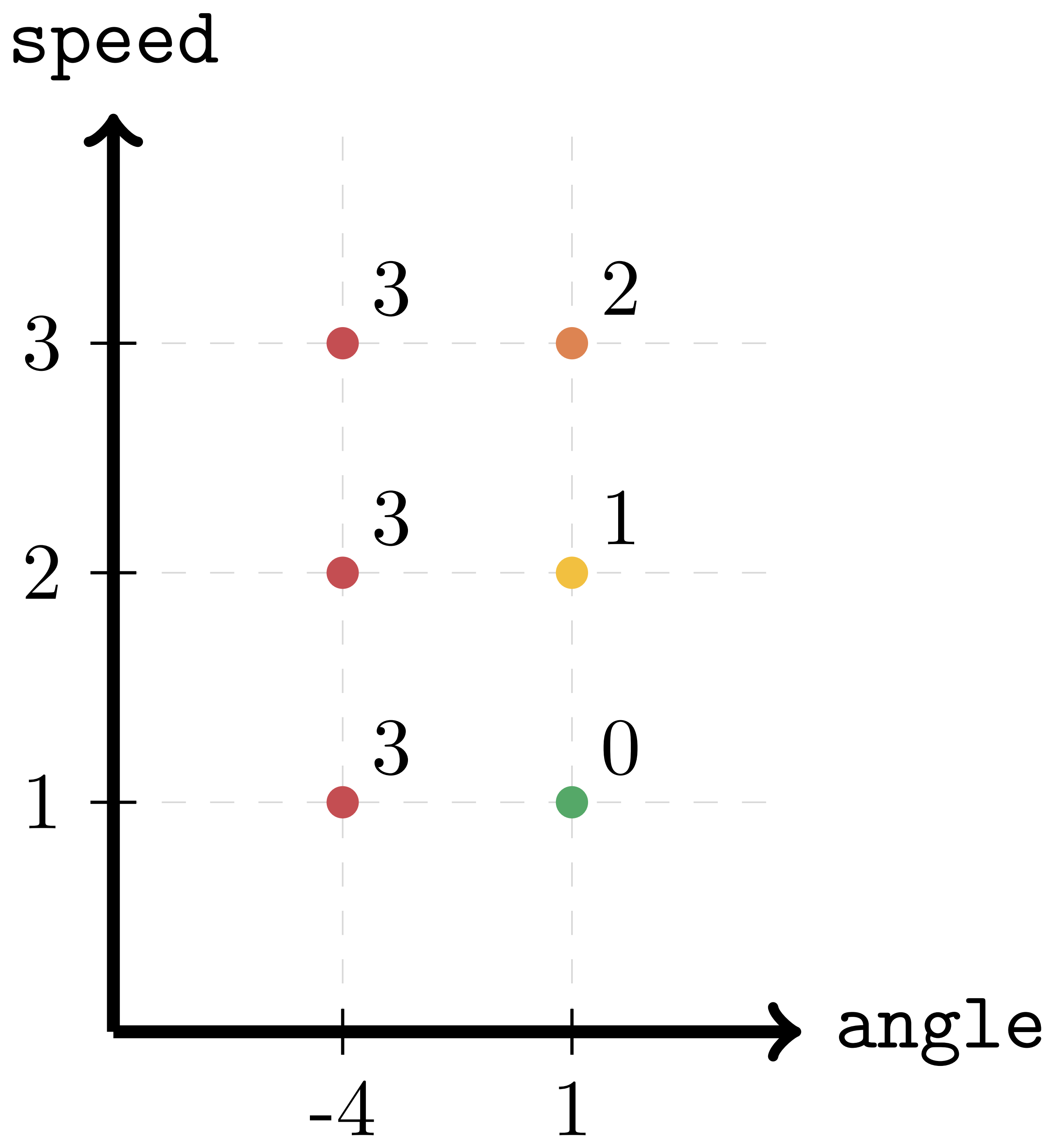


Goal: Quantify the impact of speed and angle on risk



Number of *reachable outcomes*

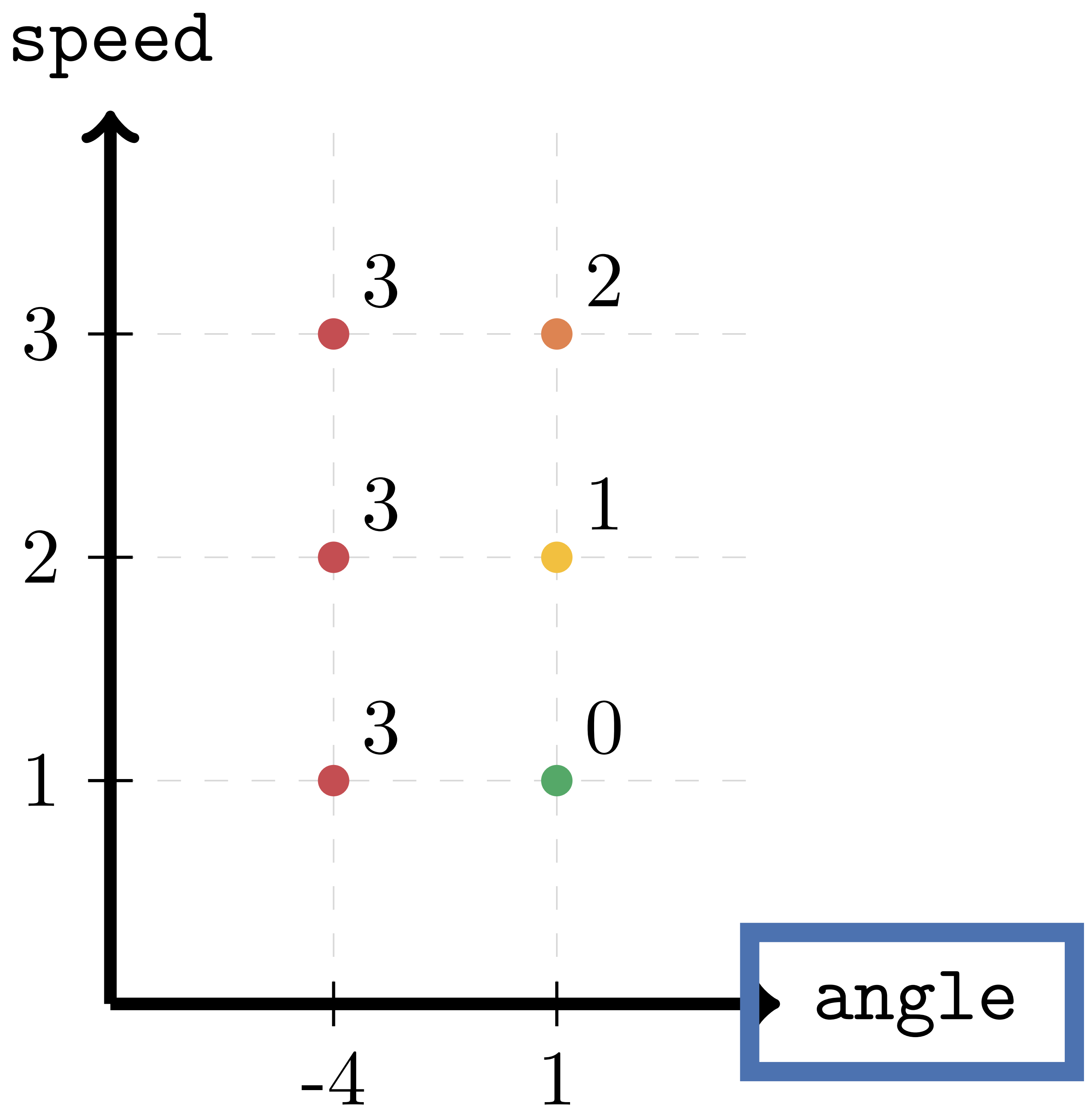
Goal: Quantify the impact of `speed` and `angle` on `risk`



Number of *reachable outcomes*

OUTCOMES

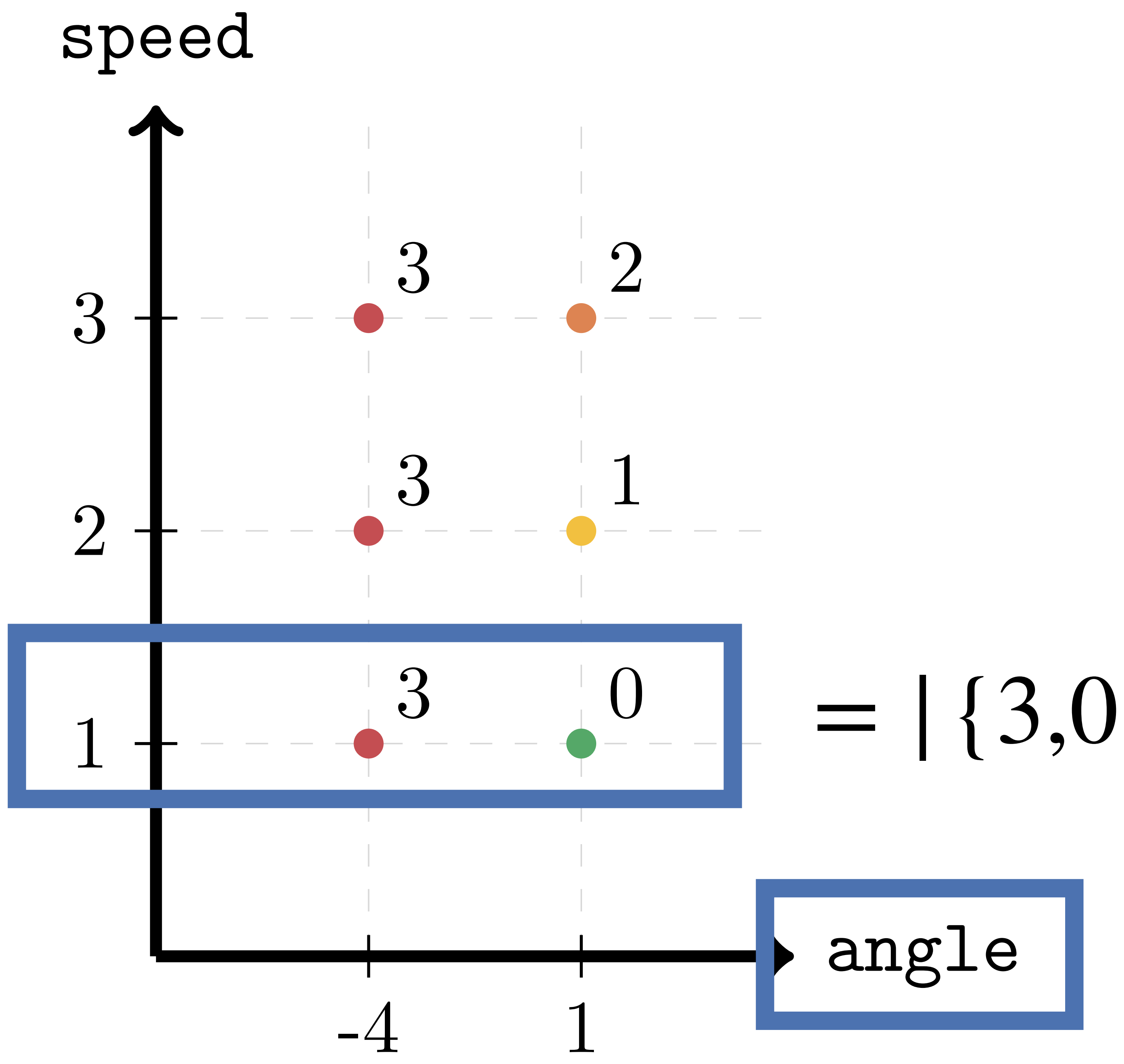
Goal: Quantify the impact of speed and angle on risk



Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of `speed` and `angle` on `risk`

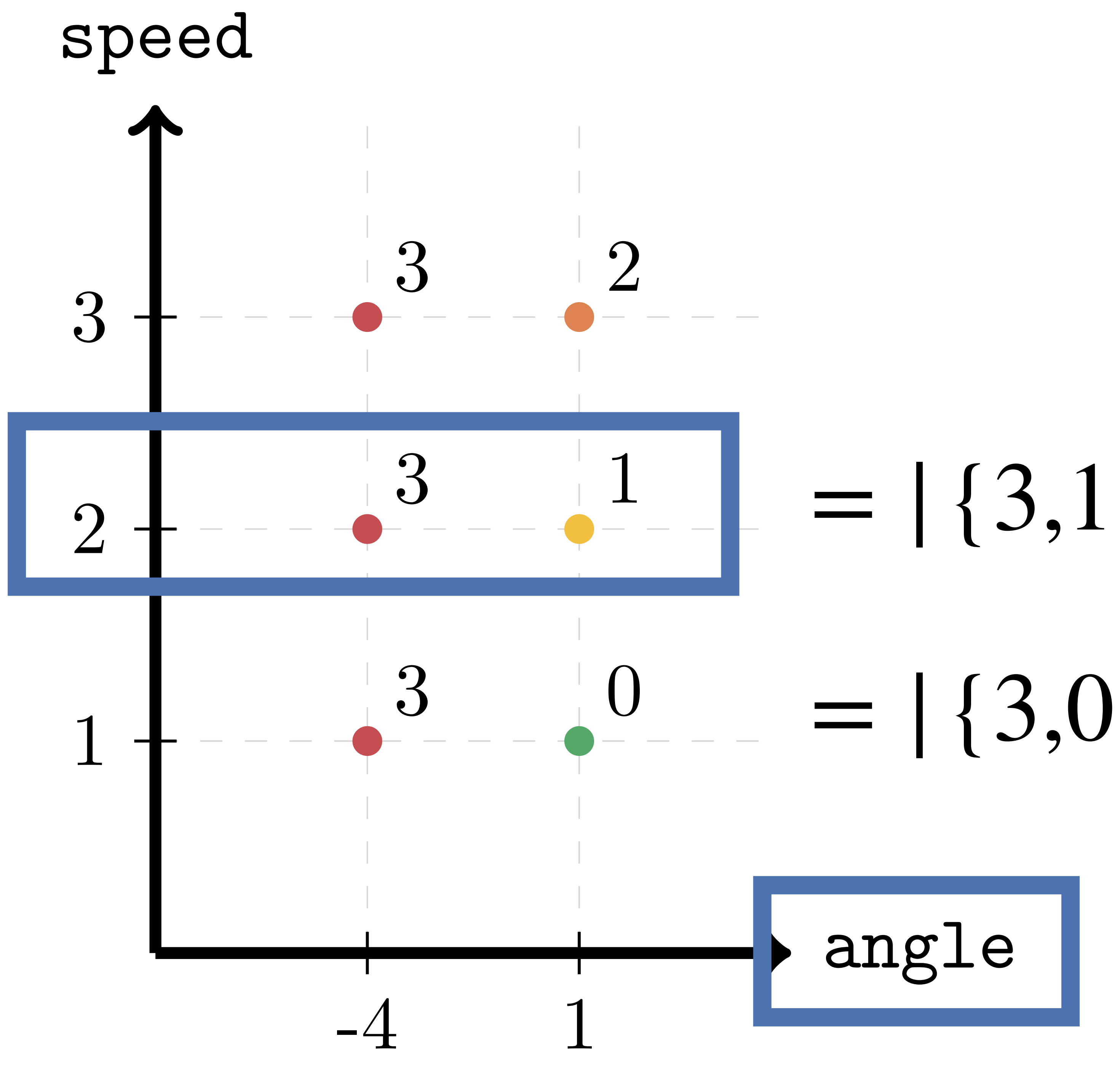


$= |\{3,0\}| \quad = 2$

Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of speed and angle on `risk`



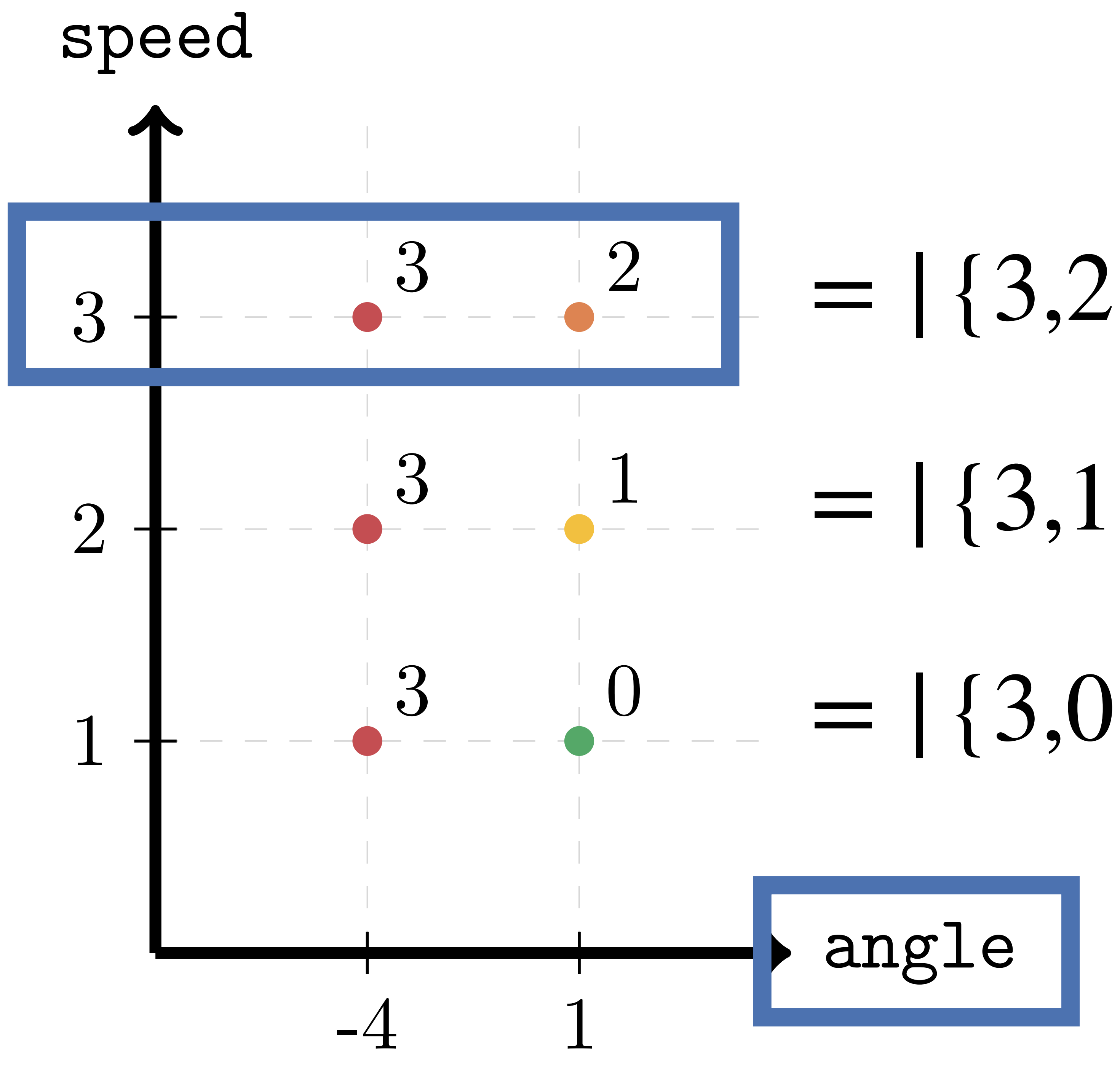
$= |\{3,1\}| \quad = 2$

$= |\{3,0\}| \quad = 2$

Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of speed and angle on `risk`



$= |\{3,2\}| \quad = 2$

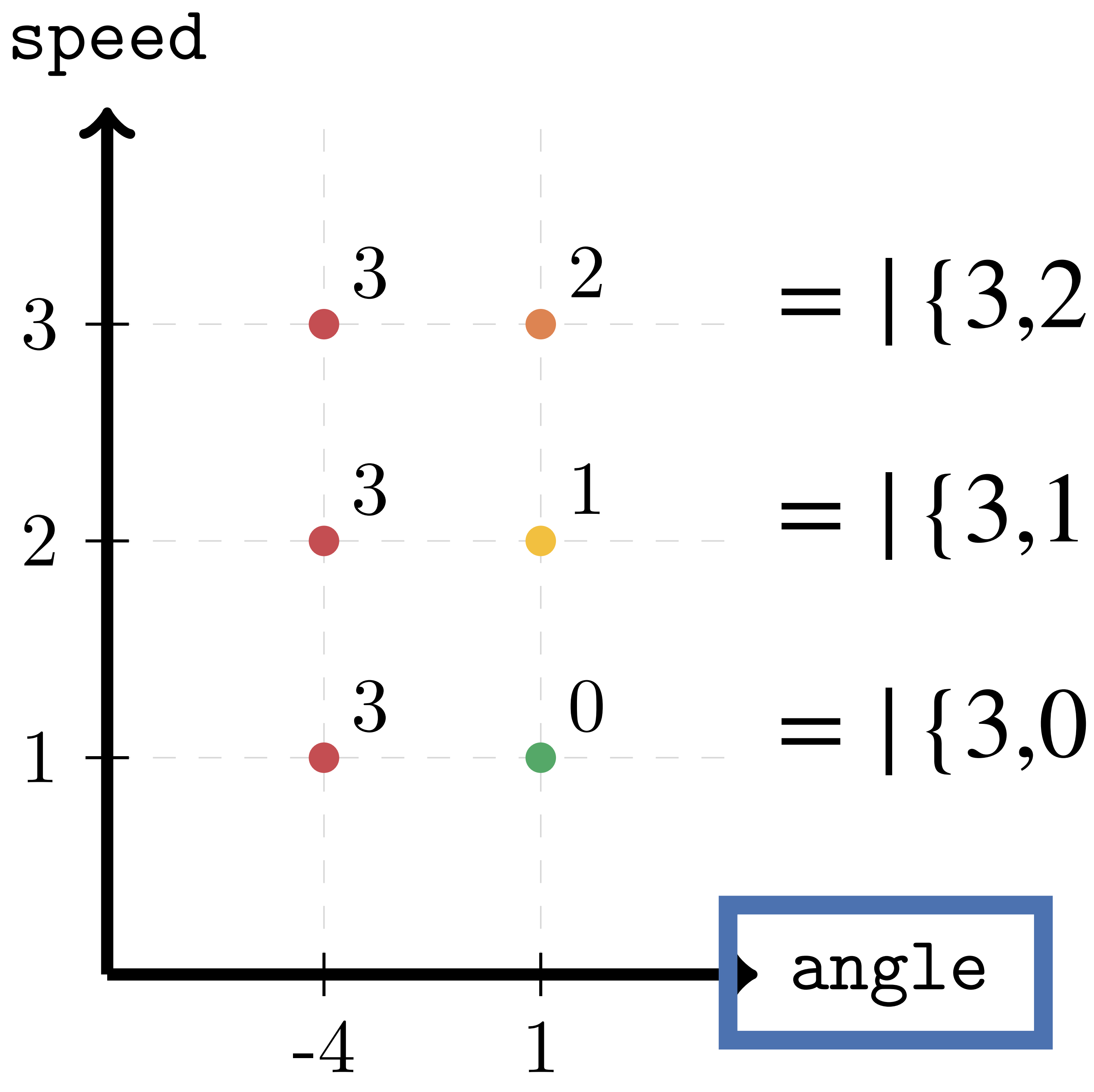
$= |\{3,1\}| \quad = 2$

$= |\{3,0\}| \quad = 2$

Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of speed and angle on `risk`



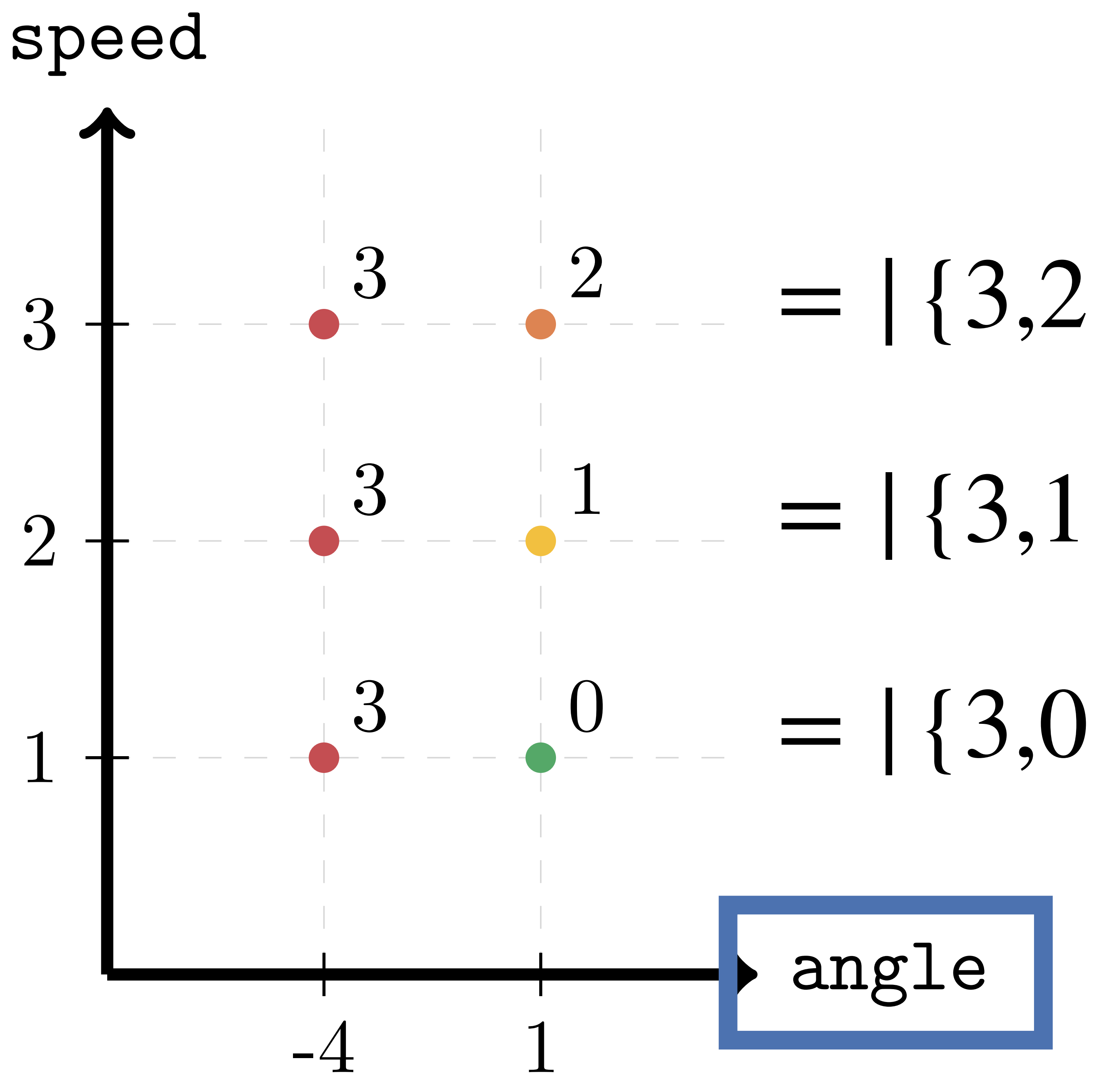
$$\max \left\{ \begin{array}{l} = |\{3,2\}| \quad = 2 \\ = |\{3,1\}| \quad = 2 \\ = |\{3,0\}| \quad = 2 \end{array} \right.$$

Number of *reachable outcomes*

OUTCOMES

Worst case scenario

Goal: Quantify the impact of speed and angle on `risk`



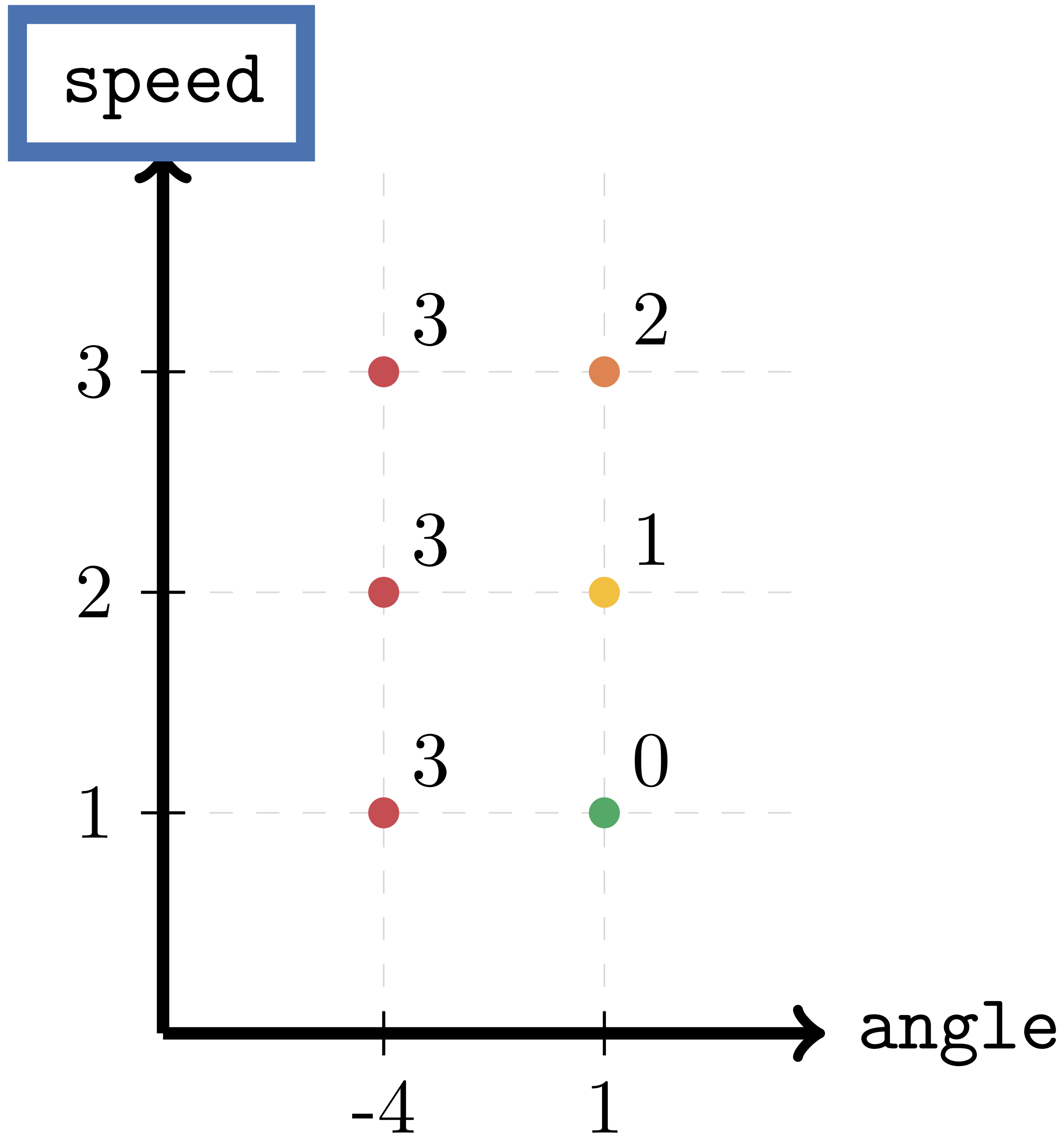
$$
\left.
\begin{aligned}
&= |\{3,2\}| = 2 \\
&= |\{3,1\}| = 2 \\
&= |\{3,0\}| = 2
\end{aligned}
\right\} \max = 2
$$

Number of *reachable outcomes*

OUTCOMES

Worst case scenario

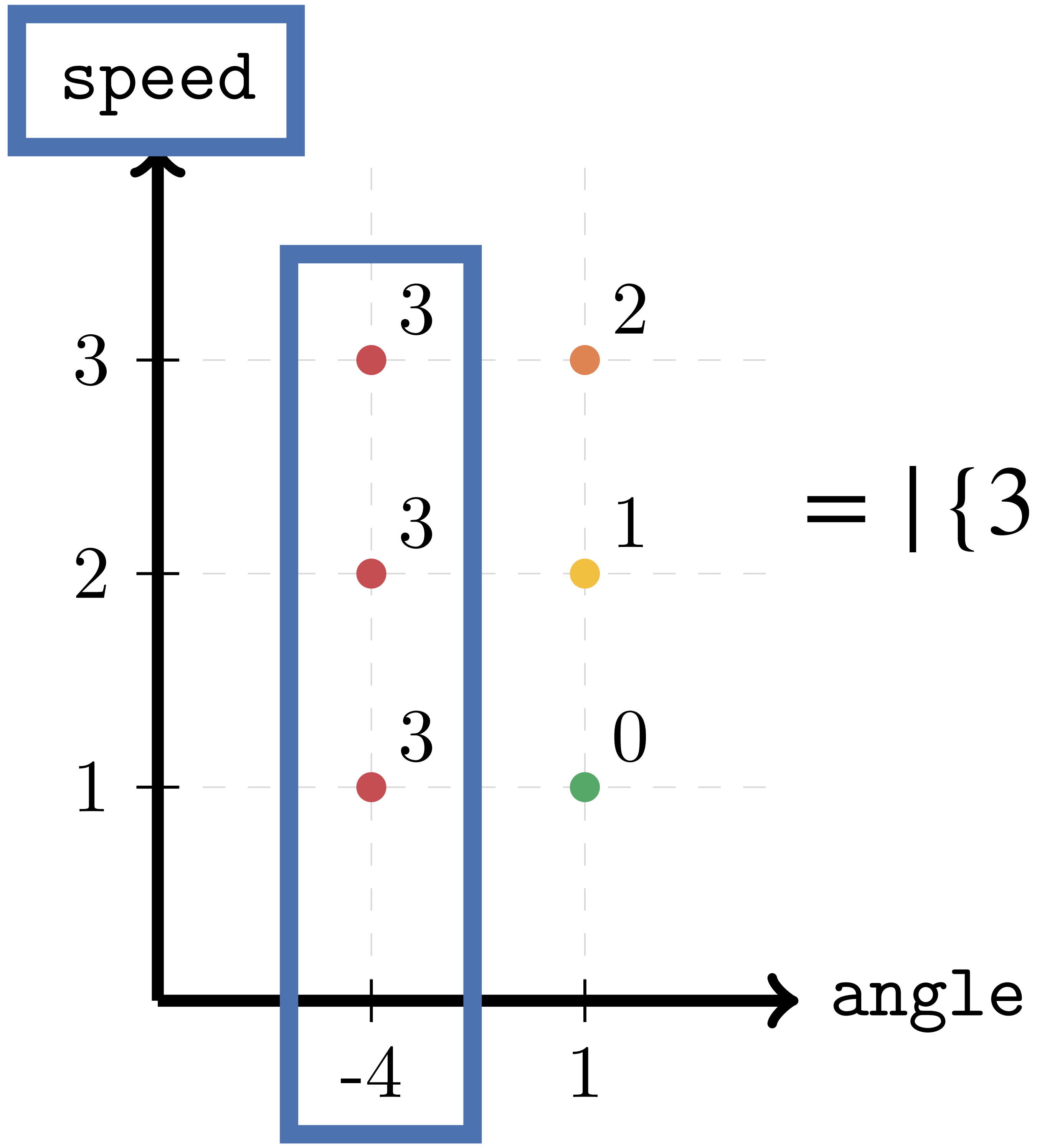
Goal: Quantify the impact of speed and angle on risk



Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of speed and angle on risk

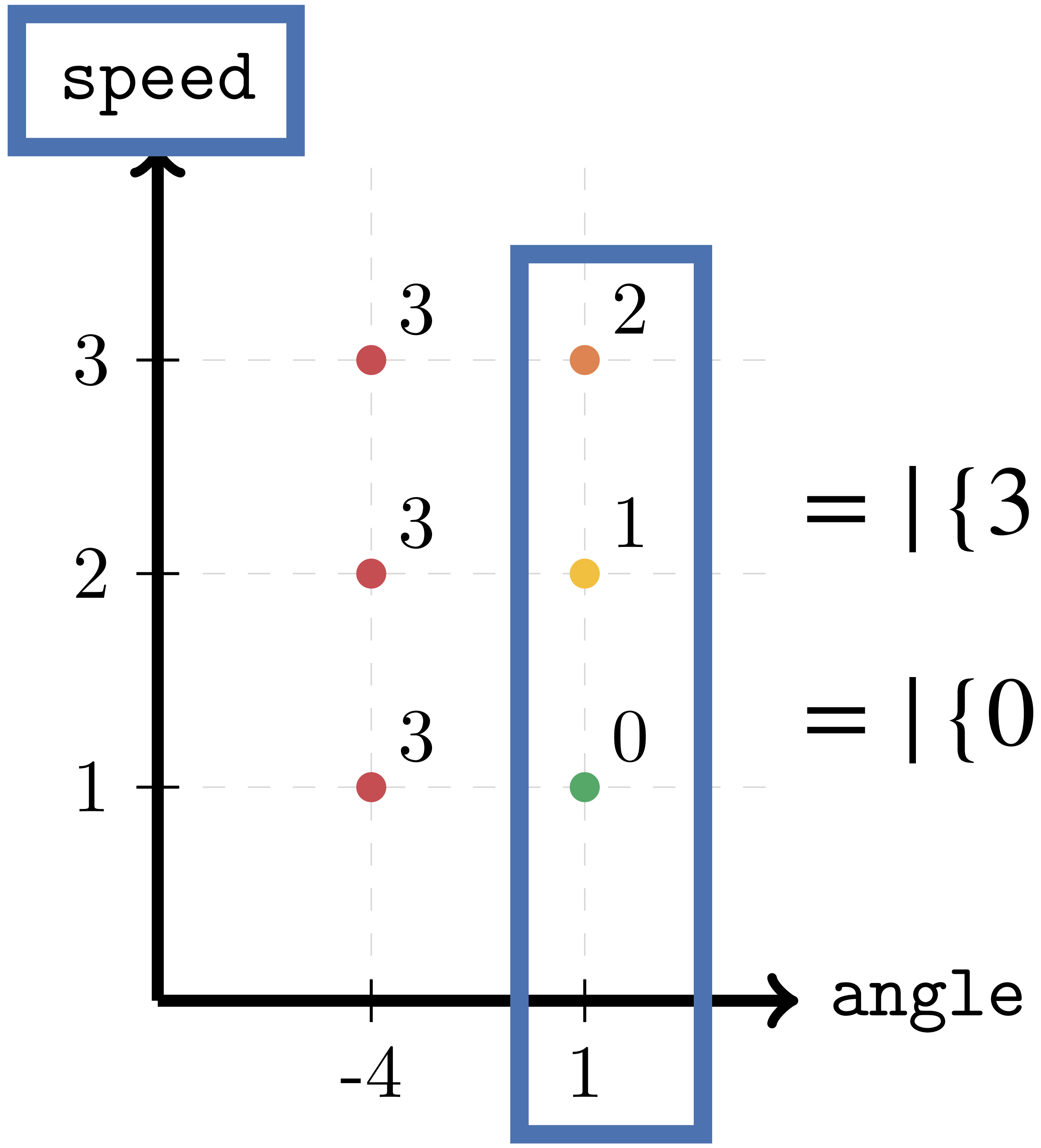


$= |\{3,3,3\}| \;\; = 1$

Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of speed and angle on risk



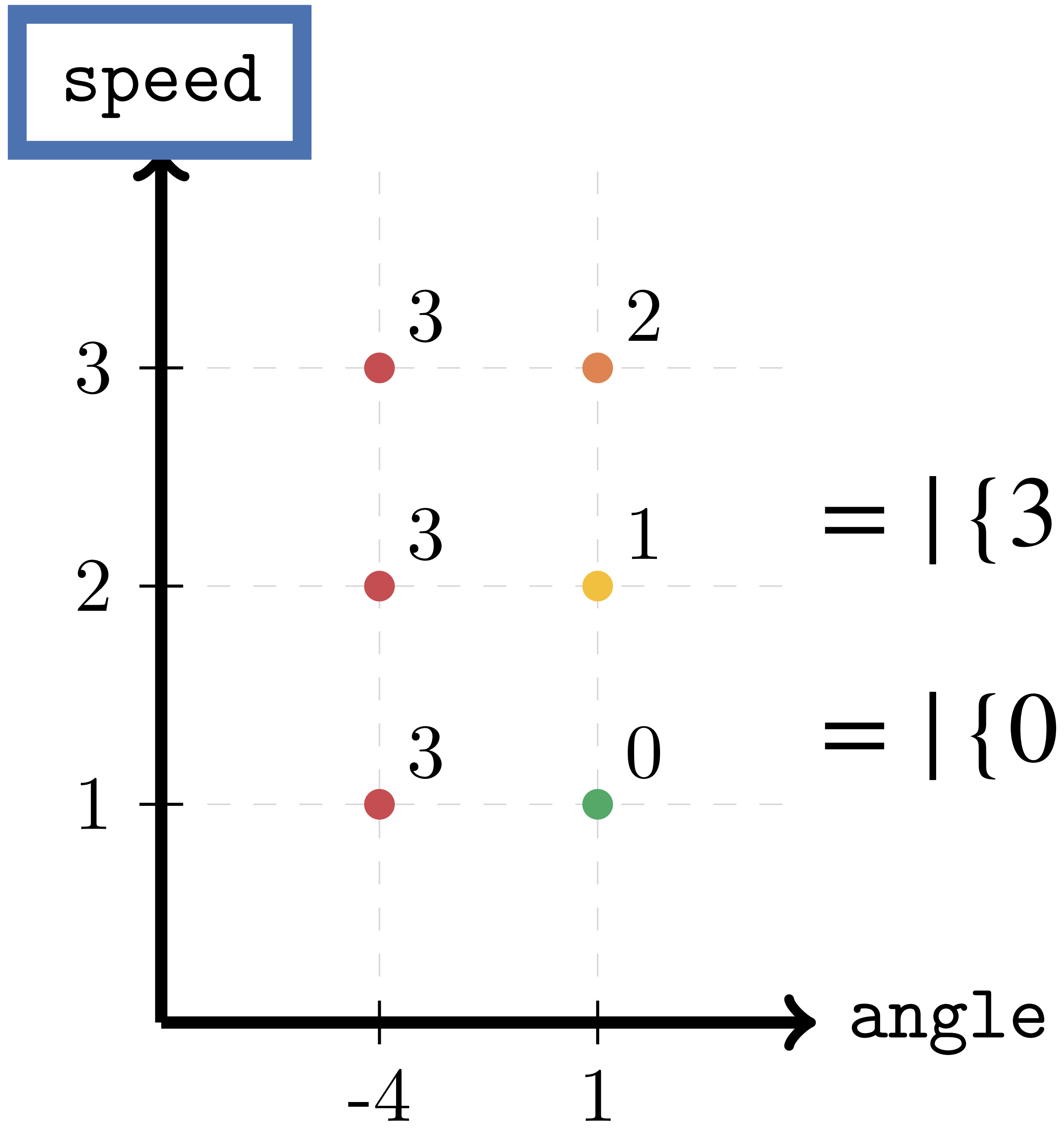
$= |\{3,3,3\}| \; = 1$

$= |\{0,1,2\}| \; = 3$

Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of `speed` and `angle` on `risk`



$$\left.\begin{array}{l} = |\{3,3,3\}| \;\; = 1 \\ = |\{0,1,2\}| \;\; = 3 \end{array}\right\} \max \;\; = 3$$

Number of *reachable outcomes*

OUTCOMES

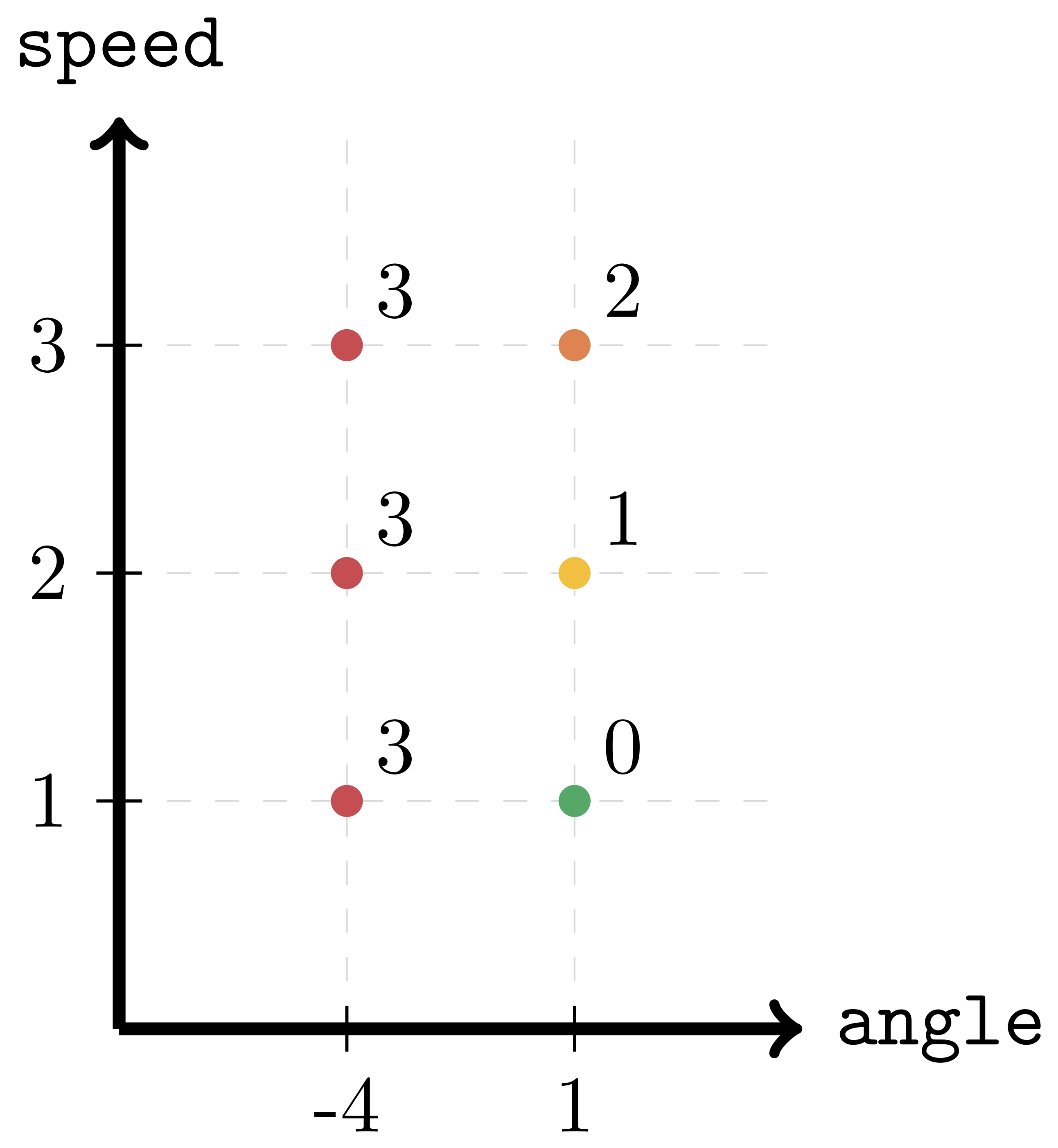
Denis Mazzucato et al. *Quantitative Input Usage Static Analysis*, NFM 2024

ENS | PSL Inria

Goal: Quantify the impact of speed and angle on `risk`

Distance of *reachable outcomes*

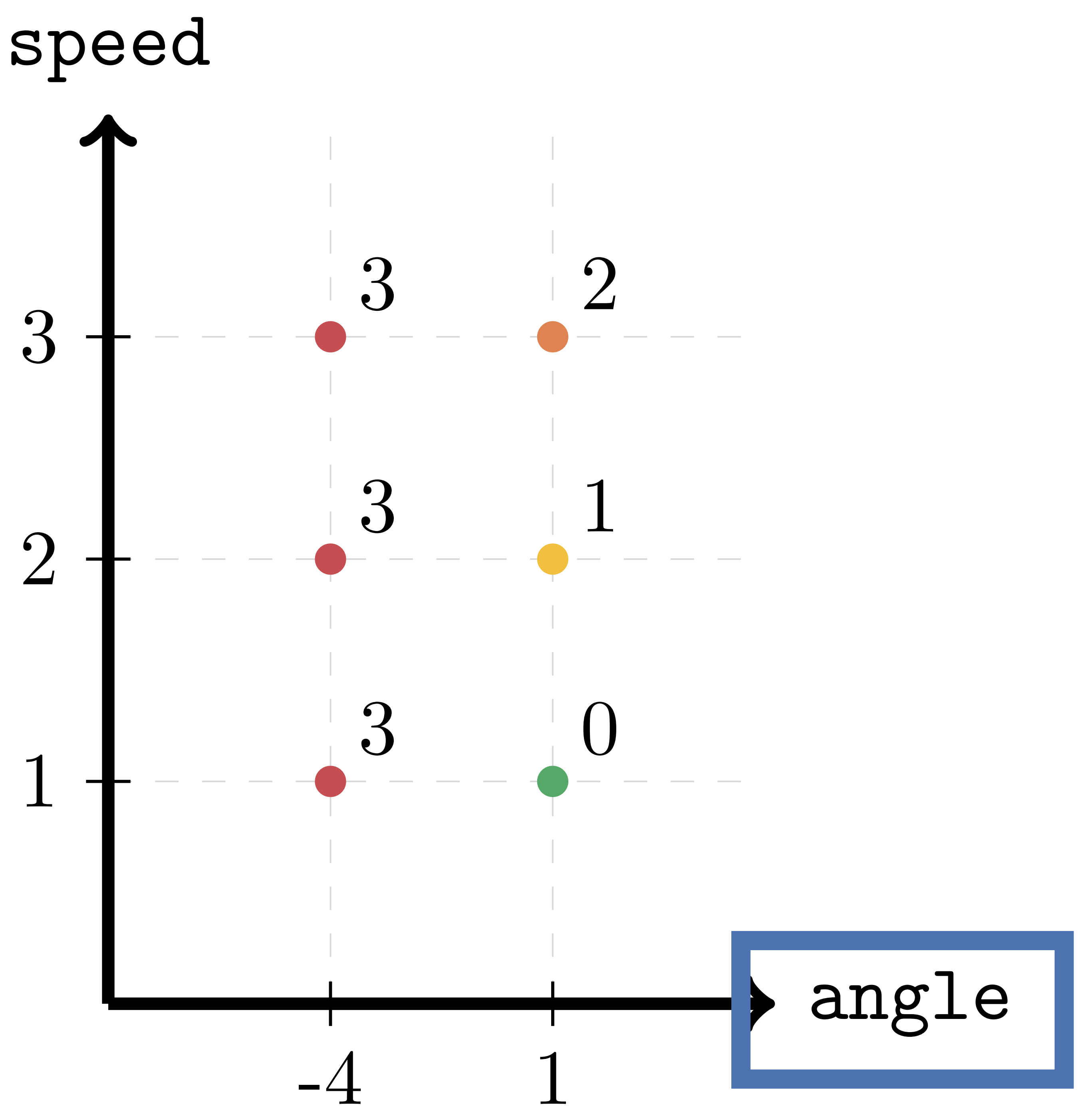
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of *reachable outcomes*

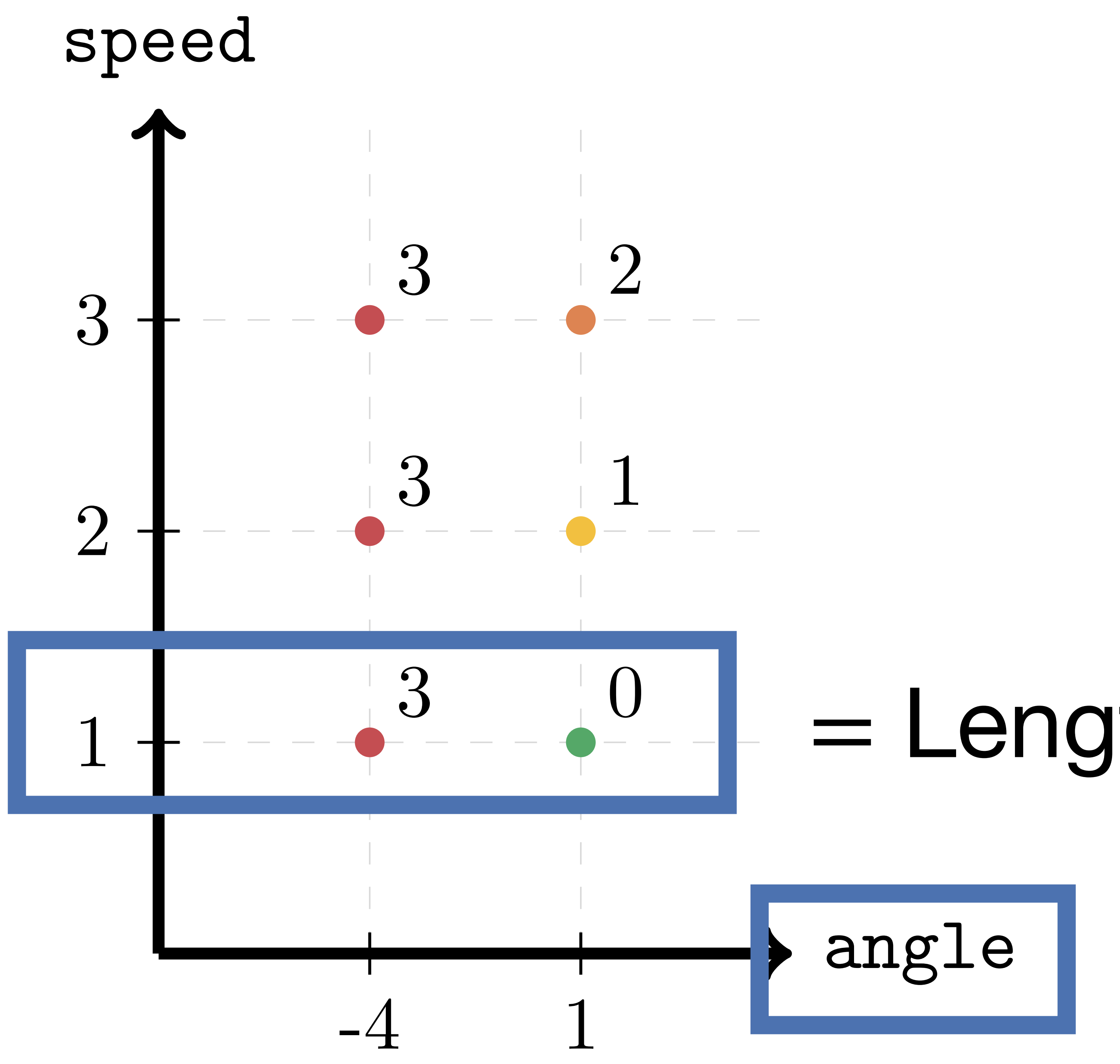
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of *reachable outcomes*

RANGE

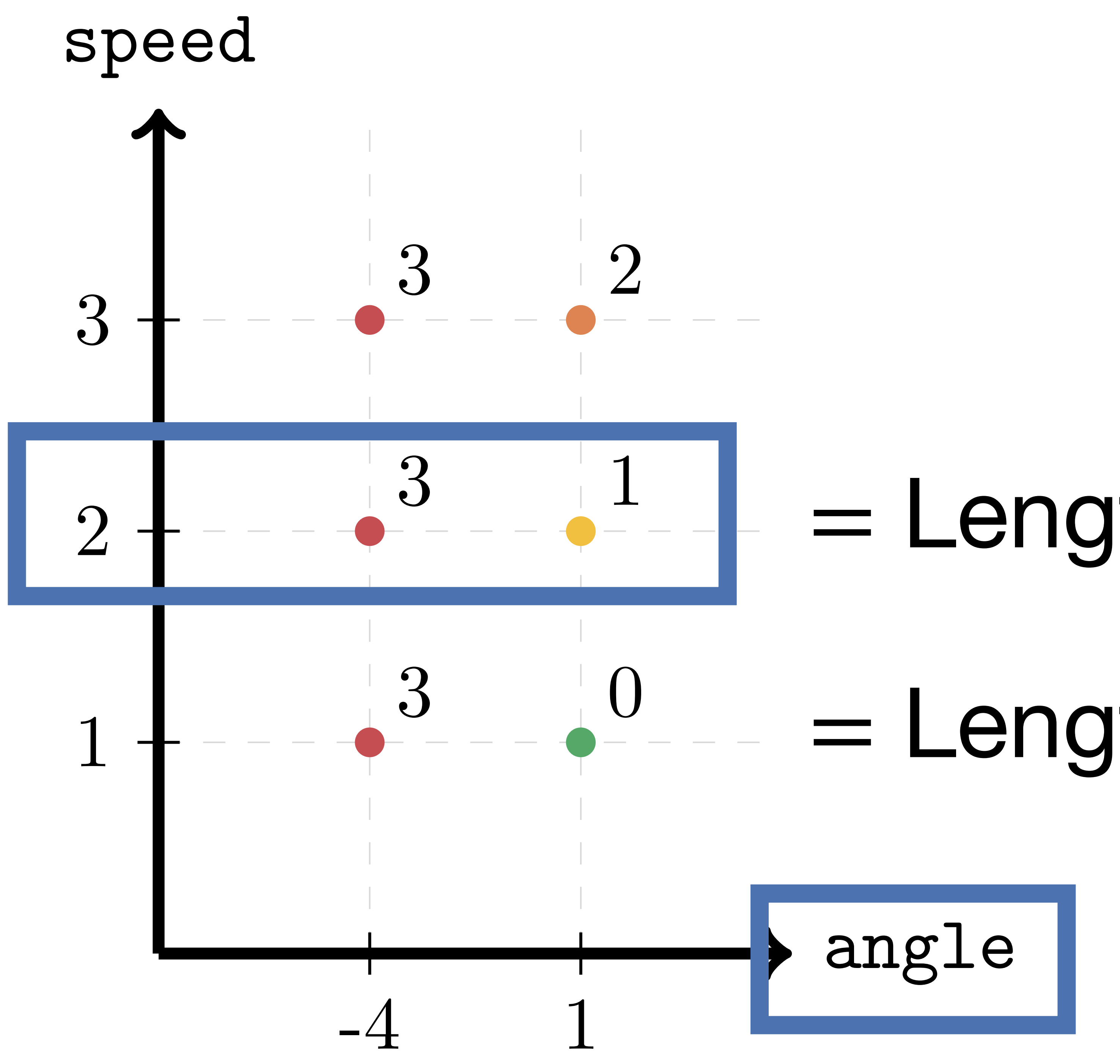


$= \text{Length}(\{3,0\}) = 3$

Goal: Quantify the impact of speed and angle on risk

Distance of *reachable outcomes*

RANGE



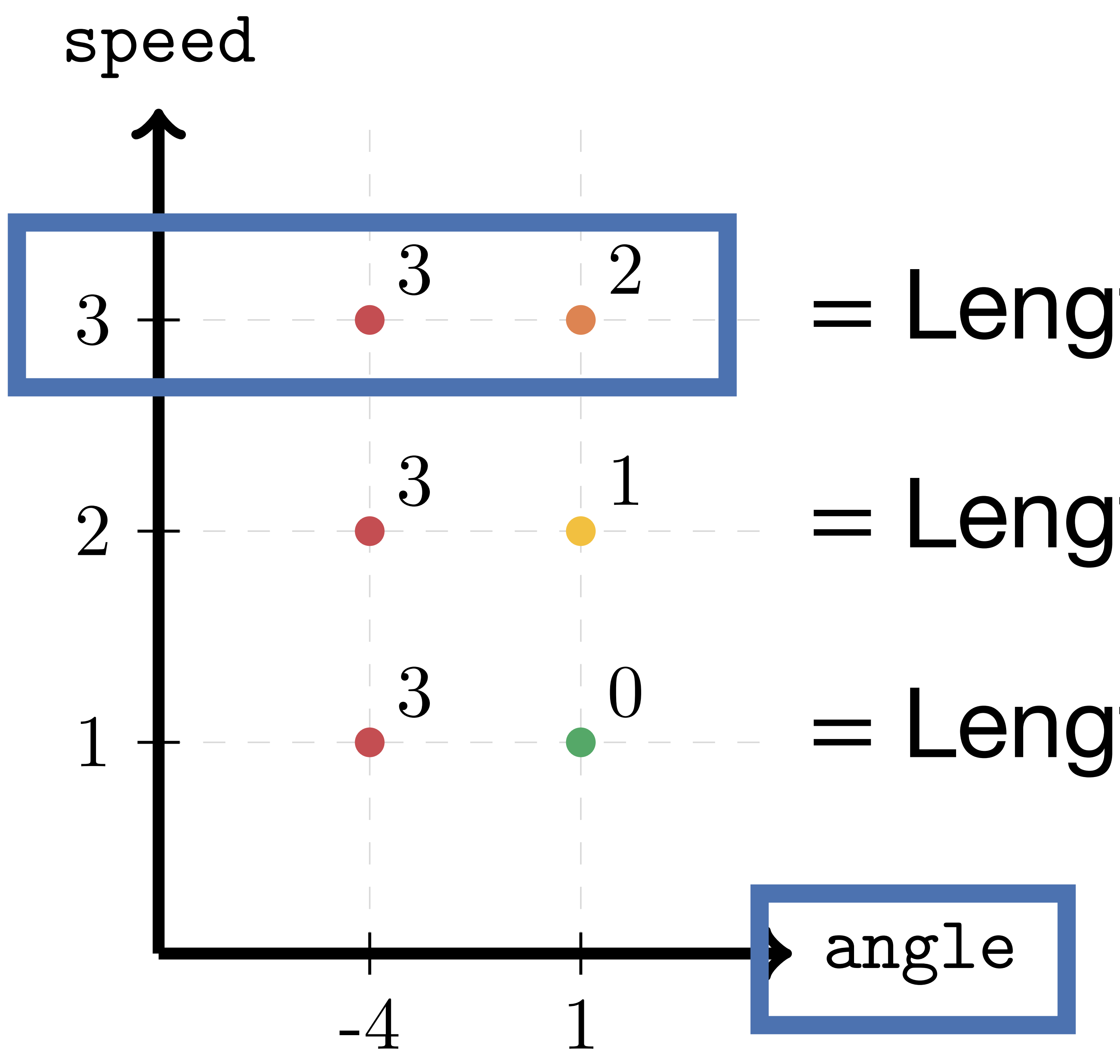
$= \text{Length}(\{3,1\}) = 2$

$= \text{Length}(\{3,0\}) = 3$

Goal: Quantify the impact of speed and angle on risk

Distance of *reachable outcomes*

RANGE



$= \text{Length}(\{3,2\}) = 1$

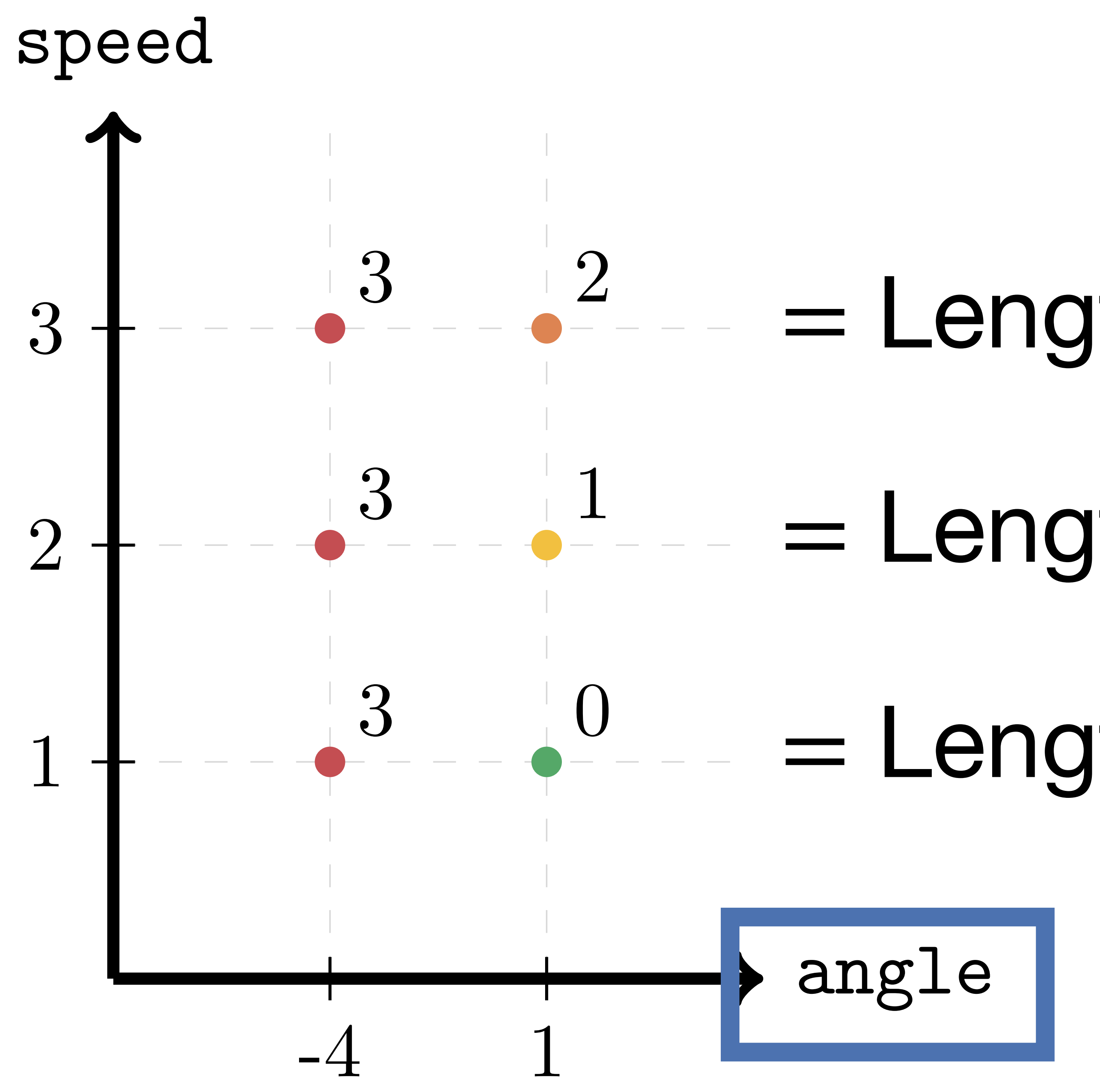
$= \text{Length}(\{3,1\}) = 2$

$= \text{Length}(\{3,0\}) = 3$

Goal: Quantify the impact of speed and angle on `risk`

Distance of *reachable outcomes*

RANGE



$= \text{Length}(\{3,2\}) = 1$

$= \text{Length}(\{3,1\}) = 2$

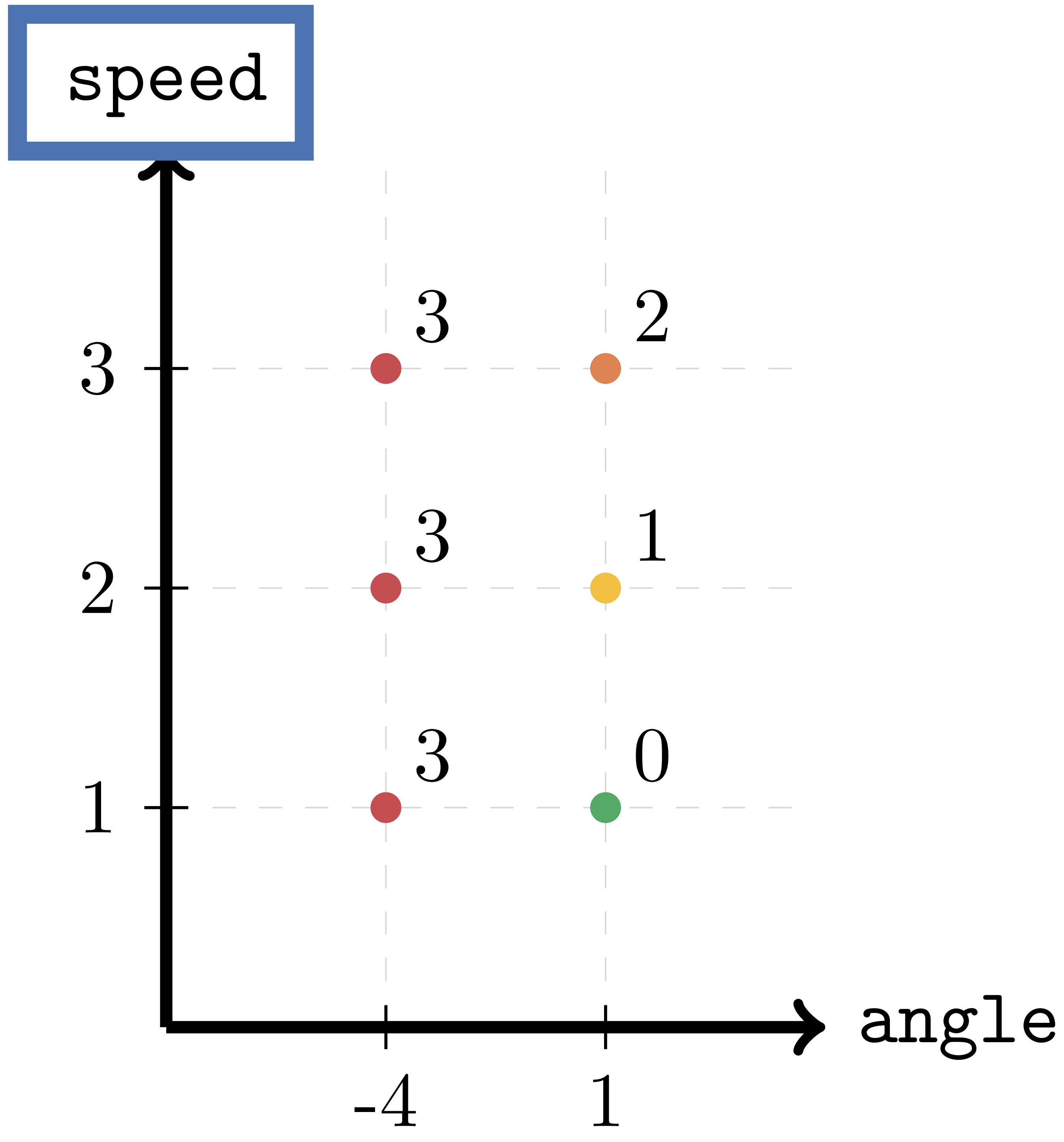
$= \text{Length}(\{3,0\}) = 3$

$\max = 3$

Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

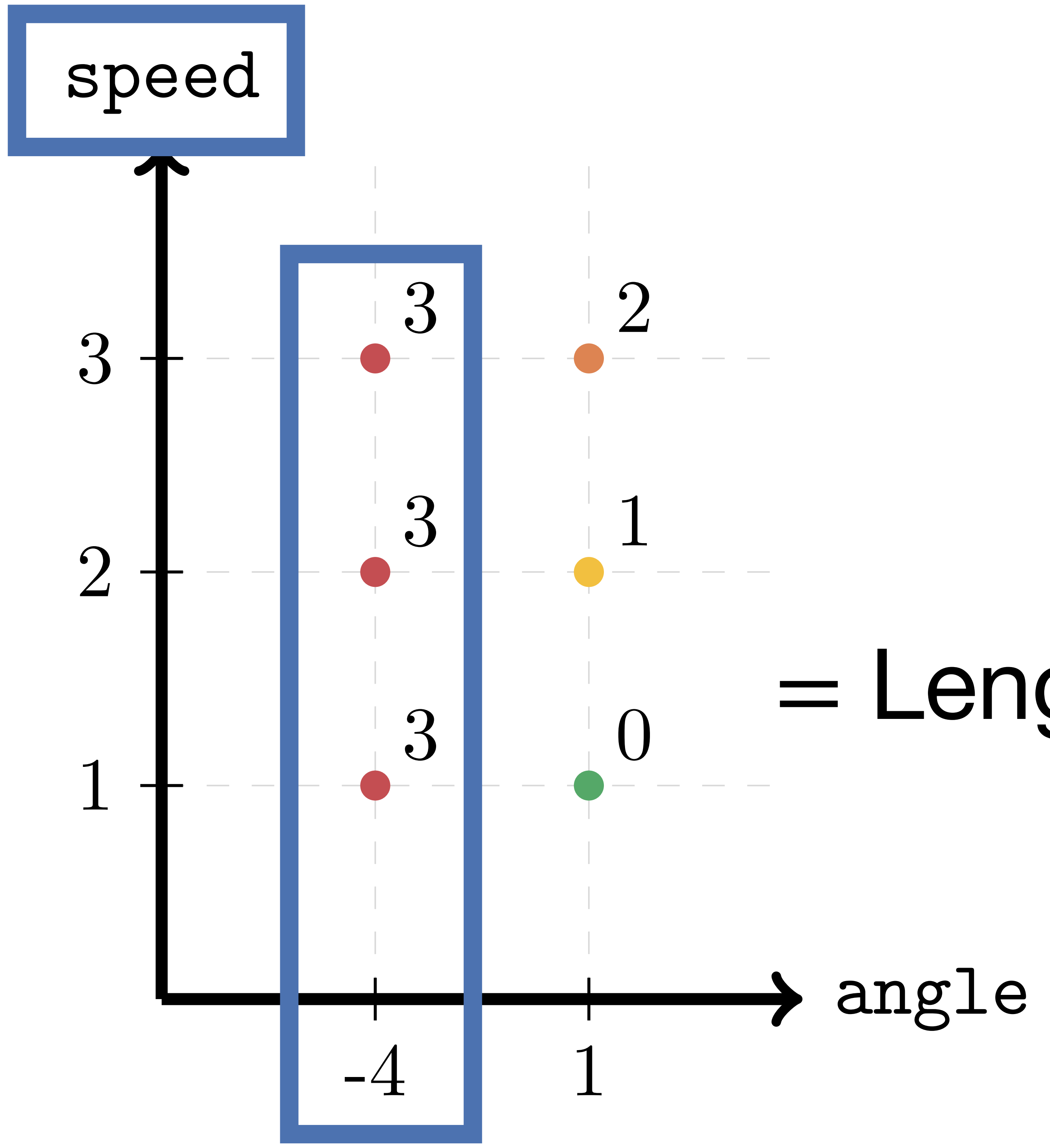
RANGE



Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

RANGE

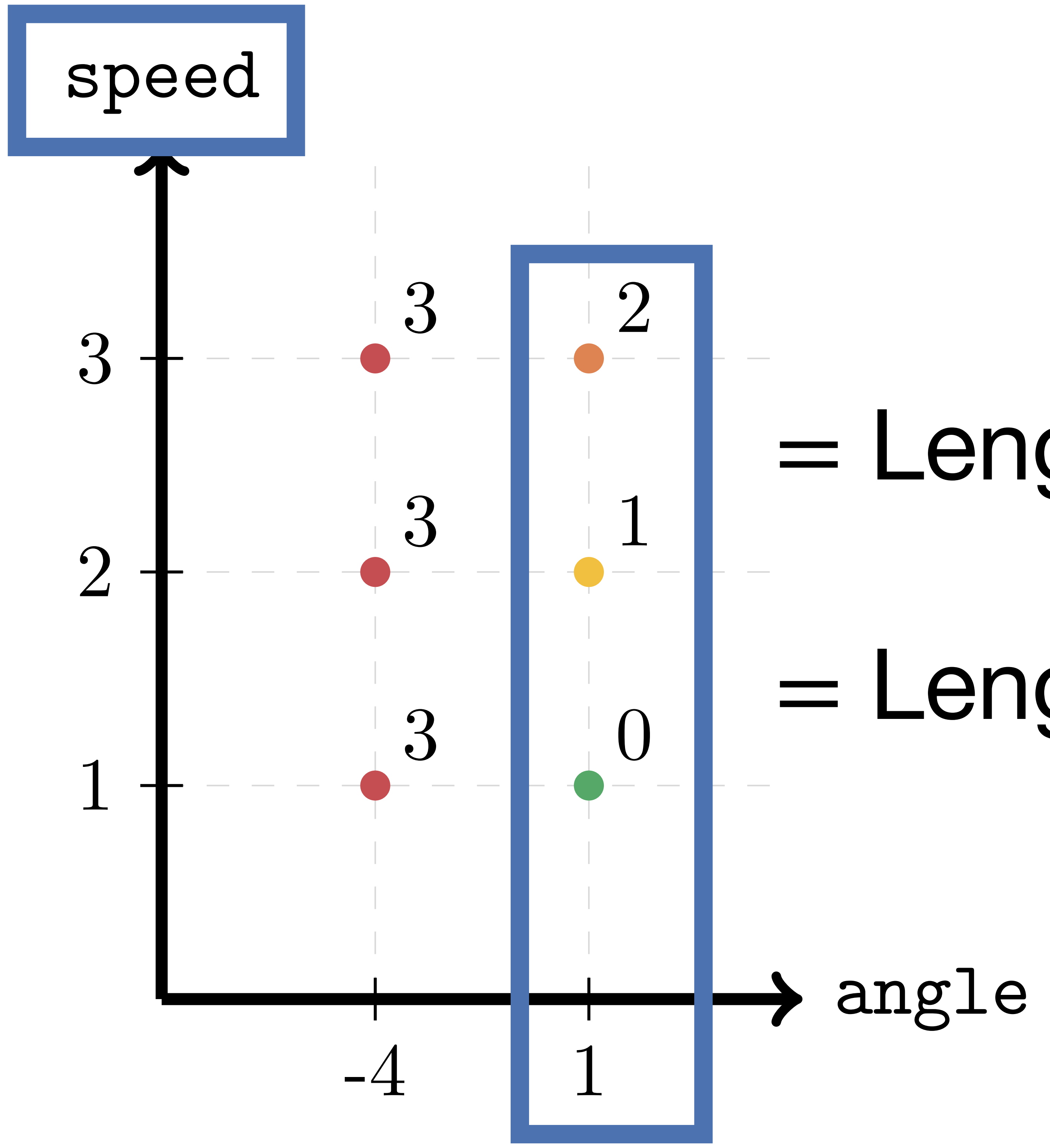


$= \text{Length}(\{3,3,3\}) = 0$

Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

RANGE



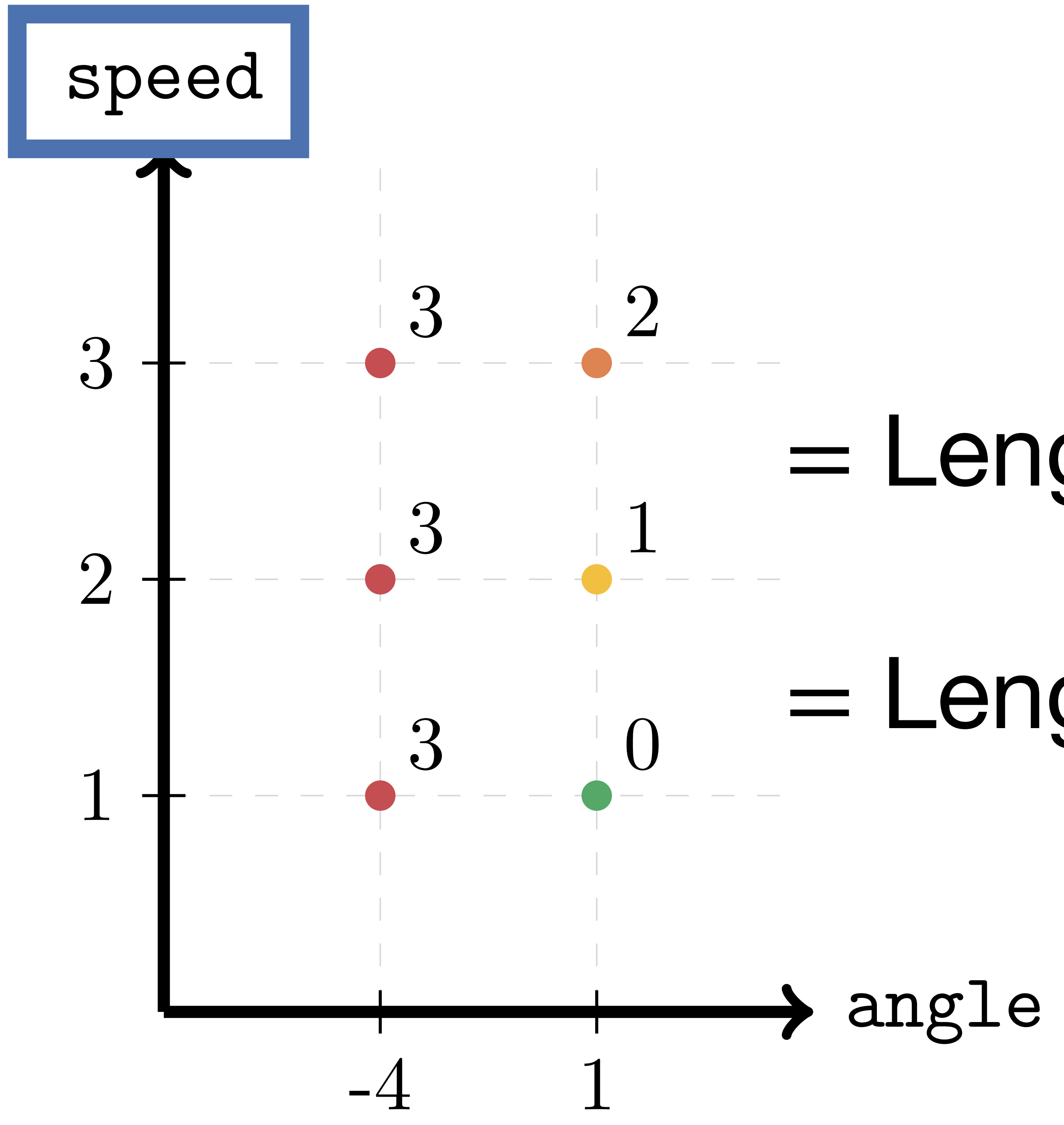
$= \text{Length}(\{0,1,2\}) = 2$

$= \text{Length}(\{3,3,3\}) = 0$

Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

RANGE



$$= \text{Length}(\{0,1,2\}) = 2$$

$$= \text{Length}(\{3,3,3\}) = 0$$

$$\max = 2$$

Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

Number of *reachable outcomes*

	RANGE	OUTCOMES
`angle`	3	2
`speed`	2	3

Automatic & Sound Static Analysis
by Abstract Interpretation

	RANGE	OUTCOMES
`angle`	3	2
`speed`	2	3

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

Smallest k permitted
by the abstraction!

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
`angle`	$3 \leq k$	$2 \leq k$
`speed`	$2 \leq k$	$3 \leq k$

Smallest k permitted by the abstraction!

1. Output Buckets

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

	Range	Outcomes
`angle`	$3 \leq k$	$2 \leq k$
`speed`	$2 \leq k$	$3 \leq k$

Smallest k permitted by the abstraction!

1. Output Buckets
2. Backward Abstract Analysis

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
`angle`	$3 \leq k$	$2 \leq k$
`speed`	$2 \leq k$	$3 \leq k$

Smallest k permitted by the abstraction!

1. Output Buckets
2. Backward Abstract Analysis
3. Abstract Implementations of RANGE and OUTCOMES

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

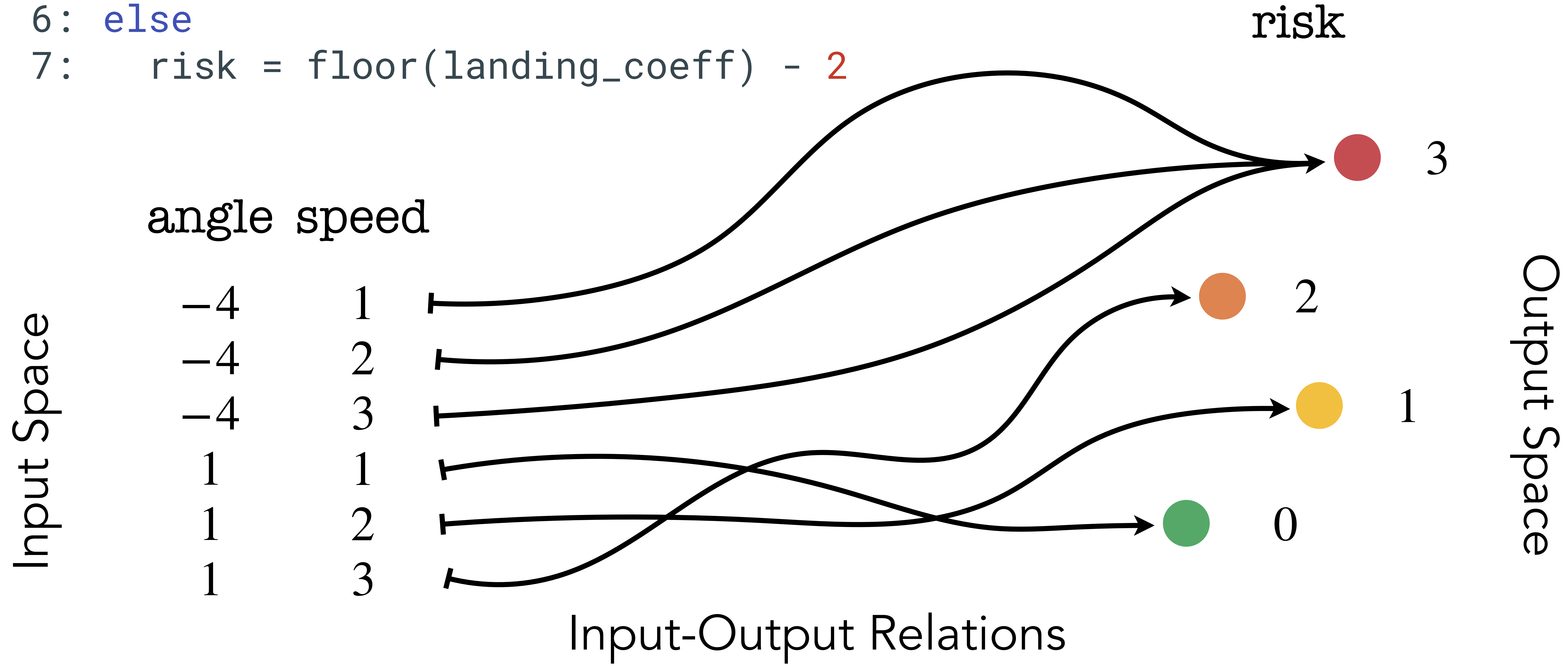
	RANGE	OUTCOMES
`angle`	$3 \leq k$	$2 \leq k$
`speed`	$2 \leq k$	$3 \leq k$

Smallest k permitted by the abstraction!

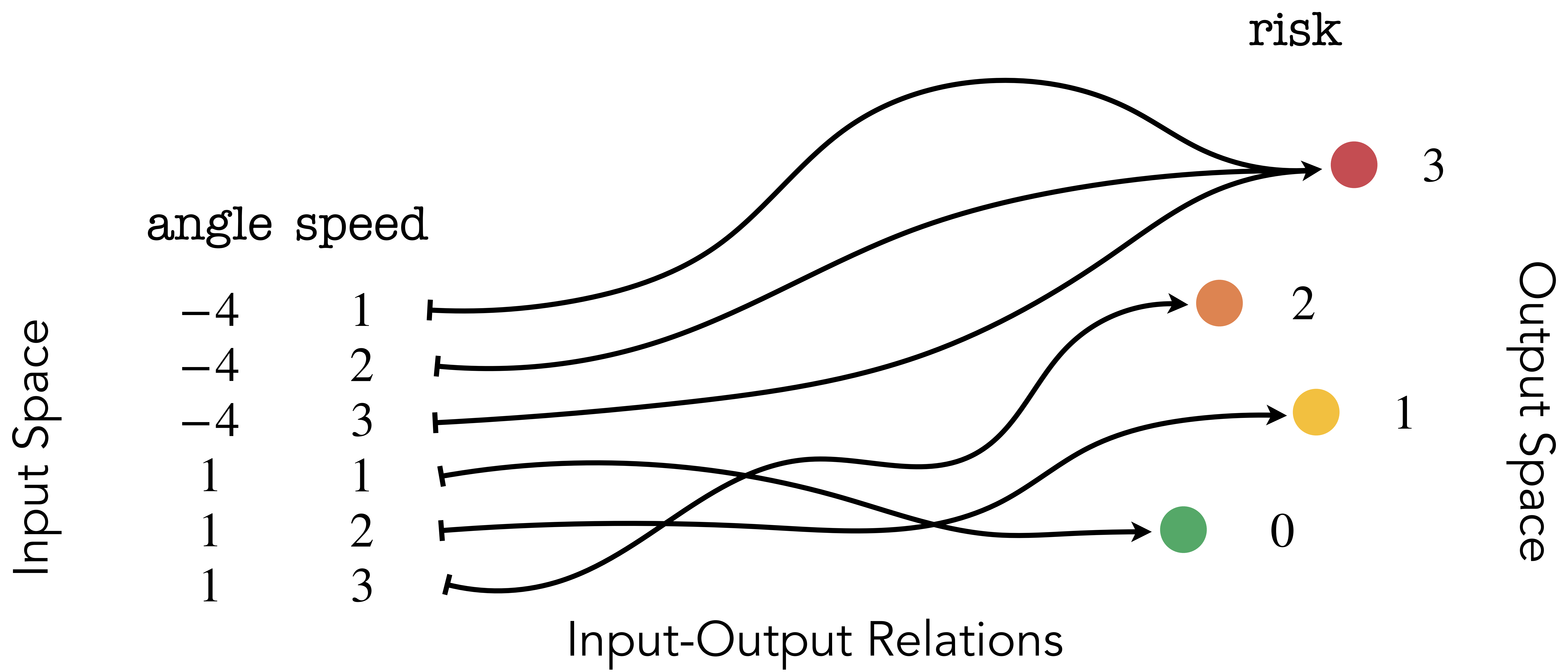
1. Output Buckets
2. Backward Abstract Analysis
3. Abstract Implementations of RANGE and OUTCOMES

$\texttt{Range}^\natural$ and $\texttt{Outcomes}^\natural$

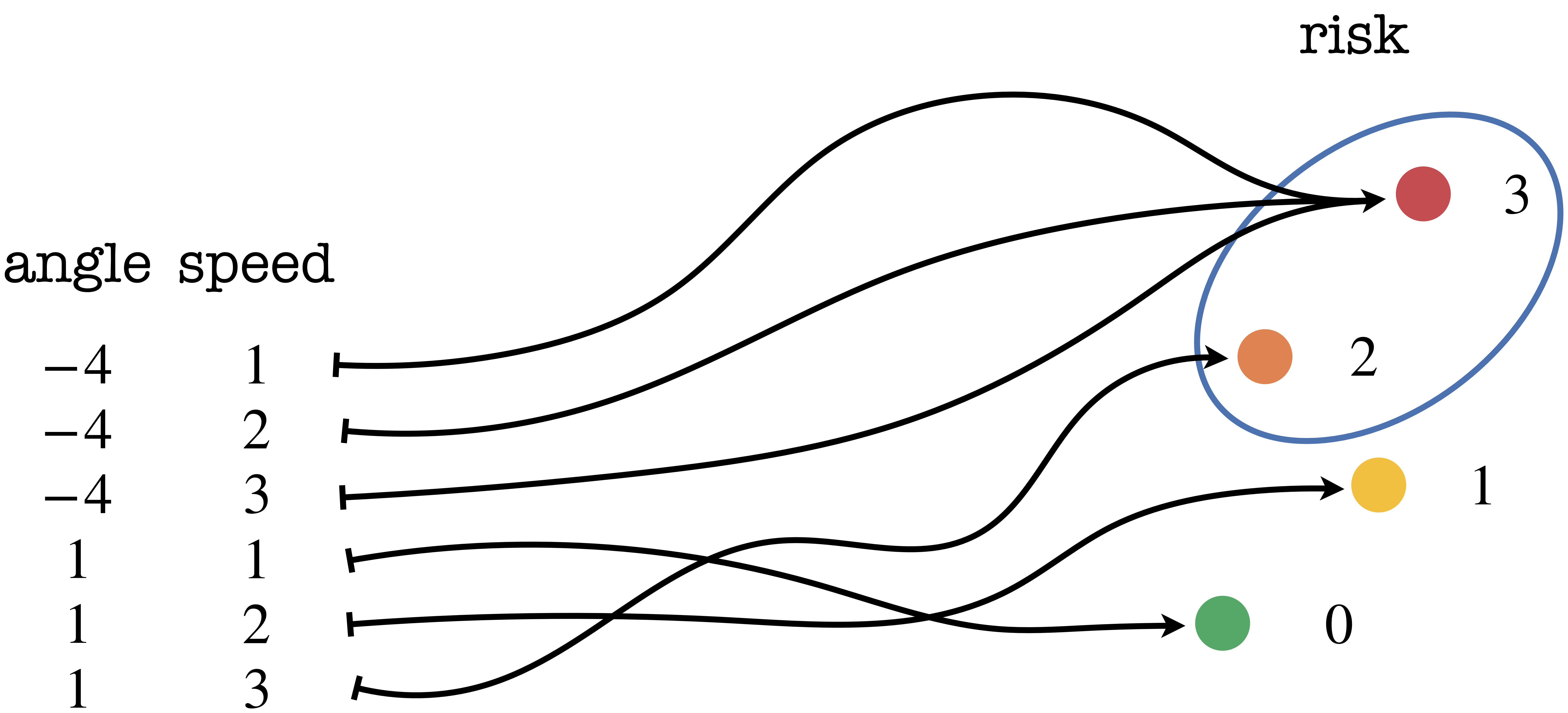
```
landing_coeff = abs(angle) + speed
if landing_coeff < 2 then
  risk = 0
else if landing_coeff > 5 then
  risk = 3
else
  risk = floor(landing_coeff) - 2
```



Intuition: OUTCOMES for the variable **angle**

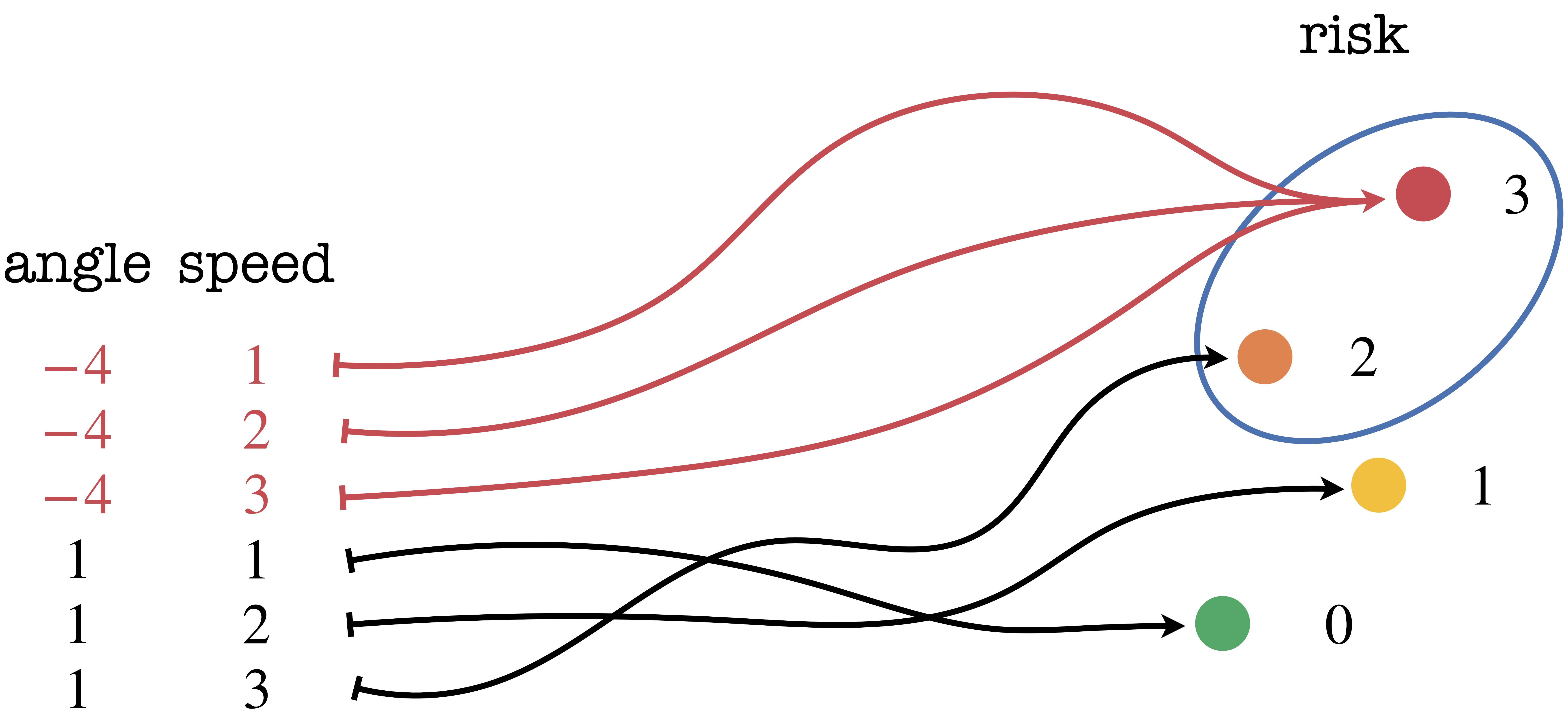


Intuition: OUTCOMES for the variable **angle**



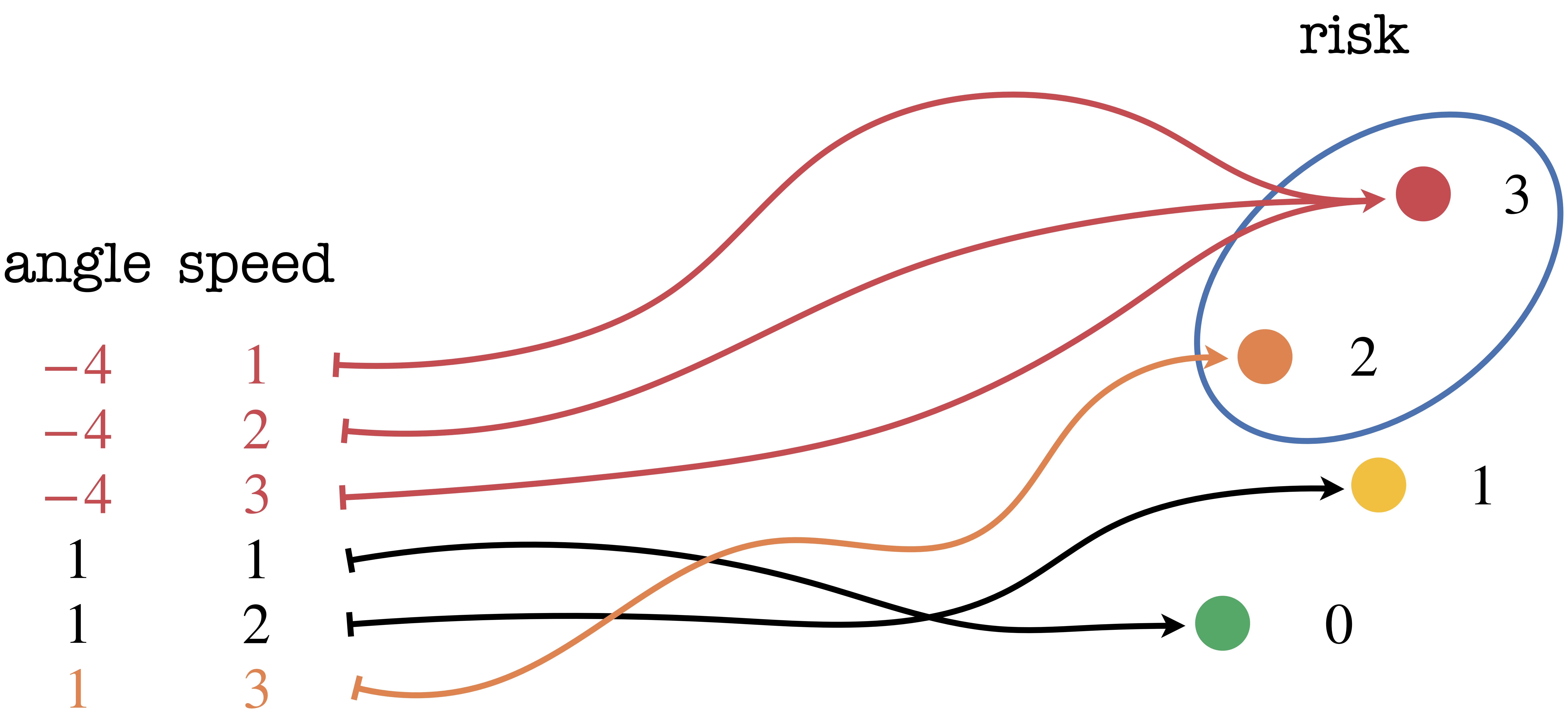
Input-Output Relations

Intuition: OUTCOMES for the variable **angle**



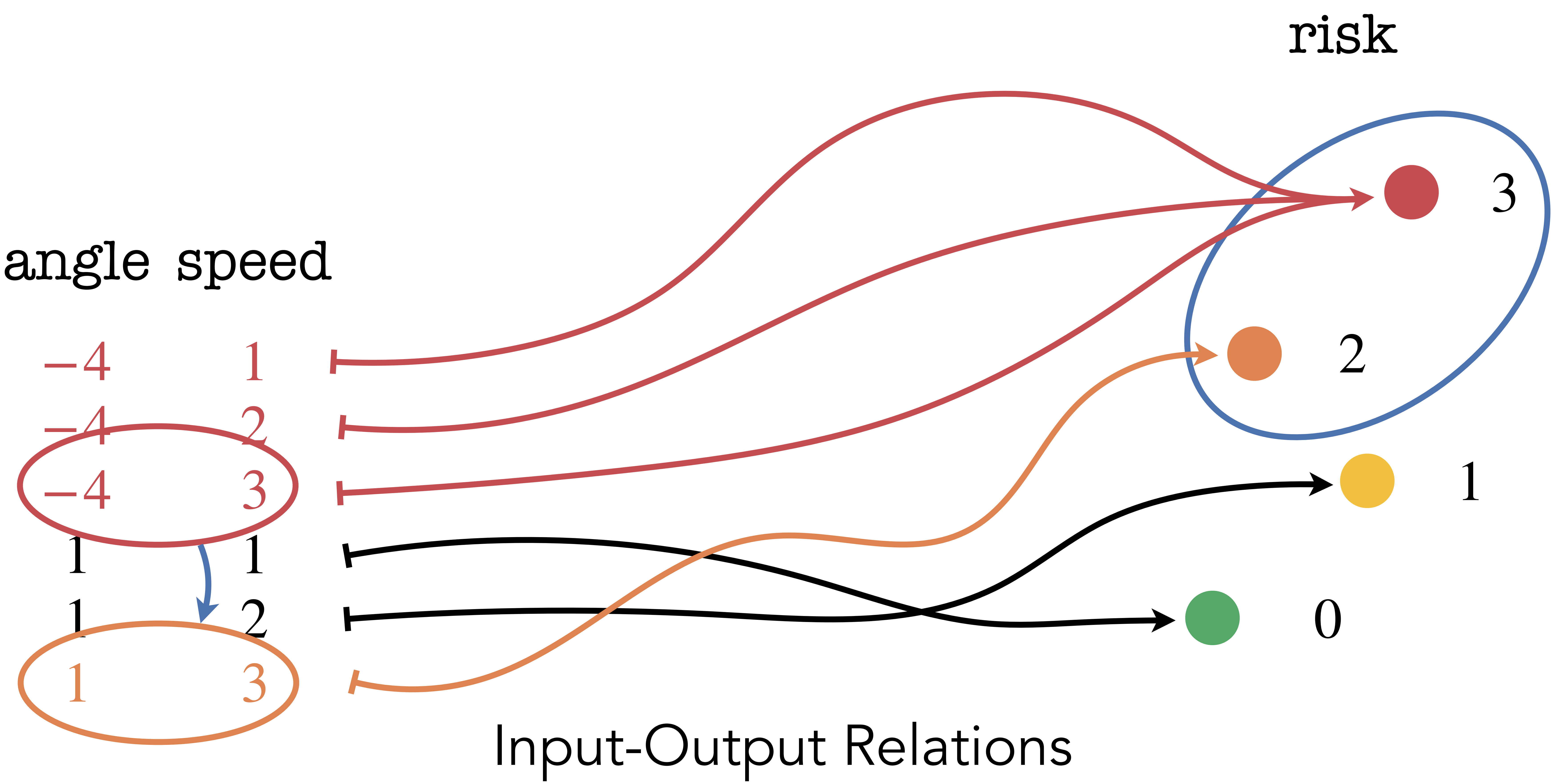
Input-Output Relations

Intuition: OUTCOMES for the variable **angle**

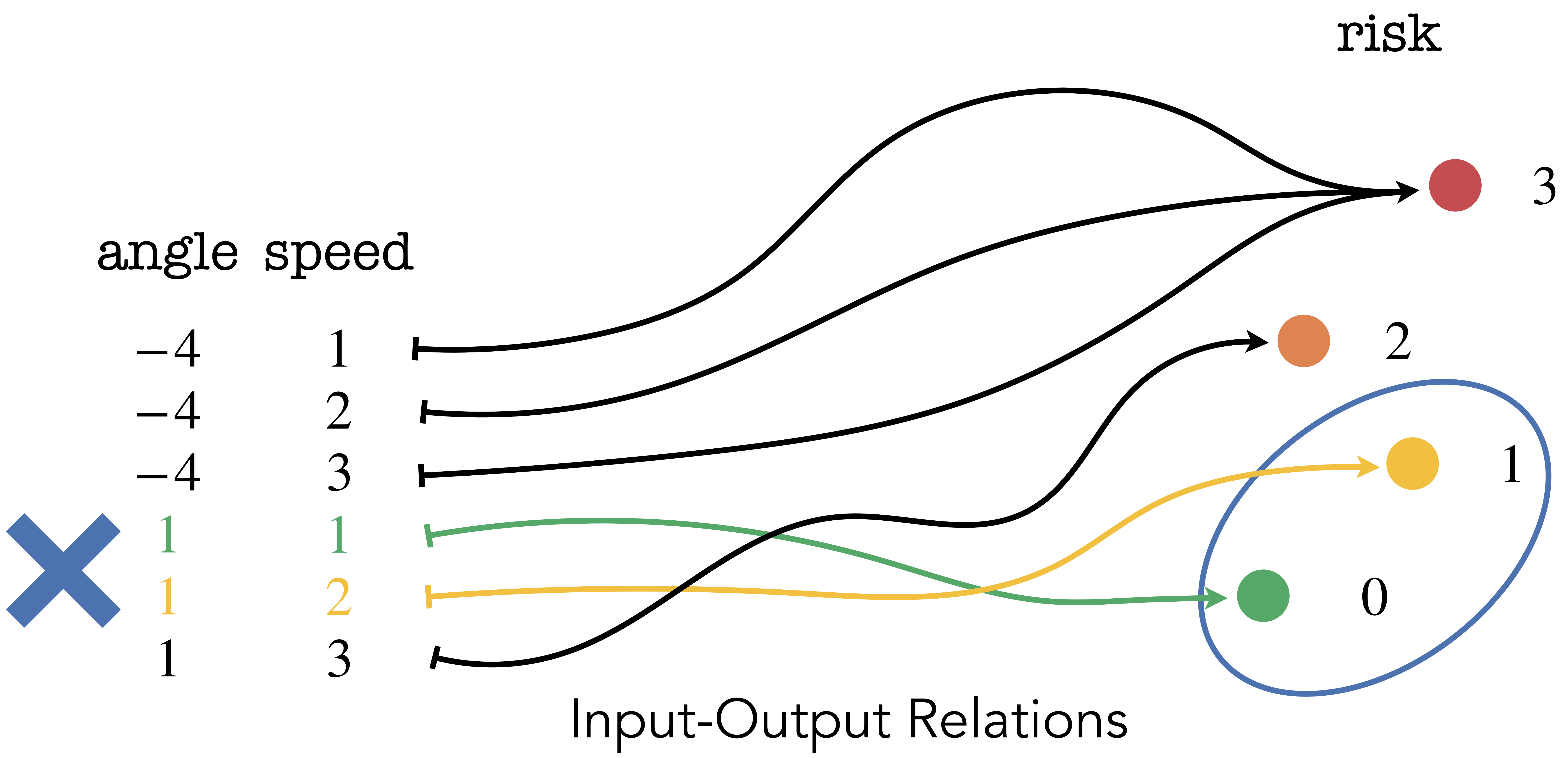


Input-Output Relations

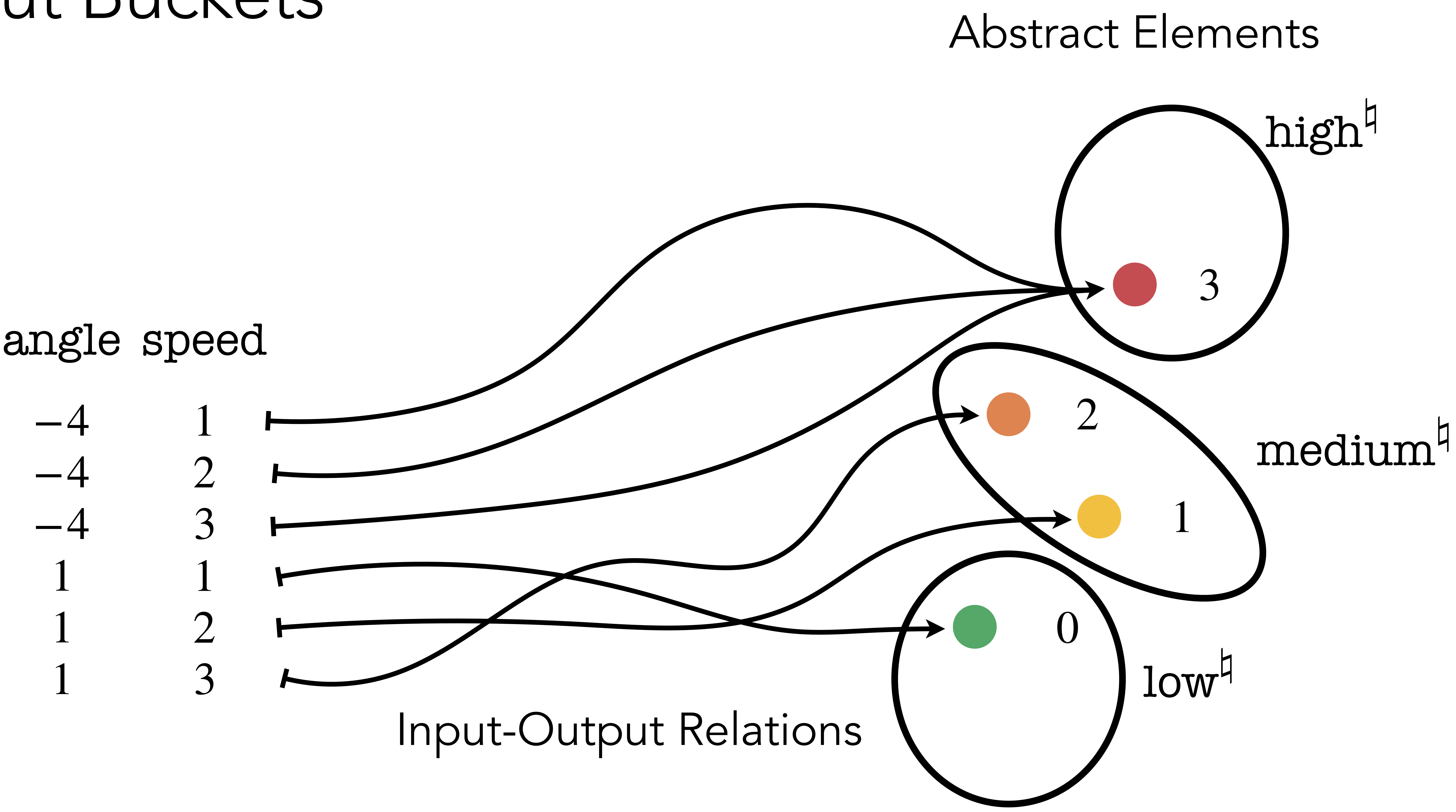
Intuition: OUTCOMES for the variable **angle**



Intuition: OUTCOMES for the variable **angle**



1) Output Buckets

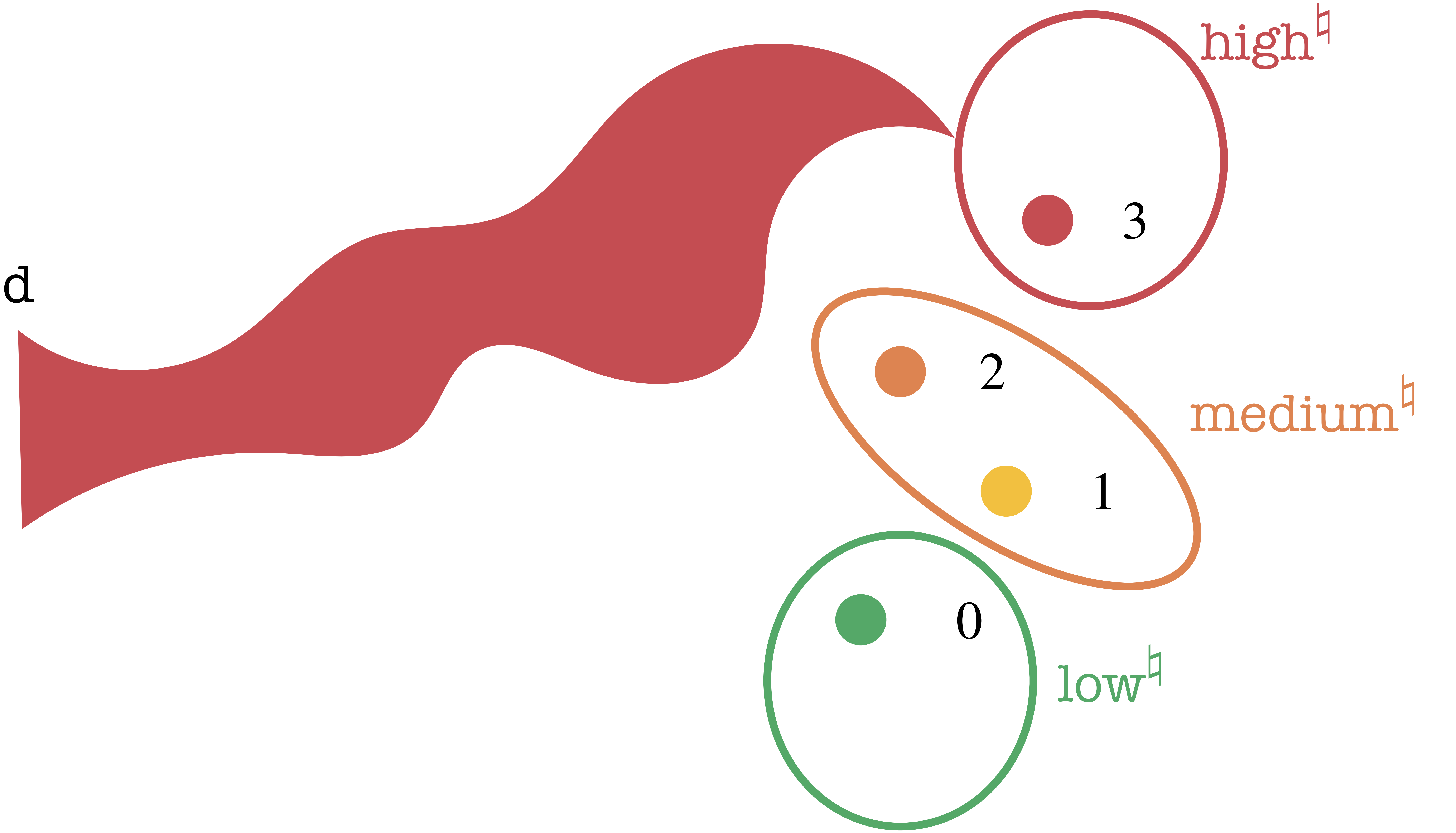


2) Backward Abstract Analysis

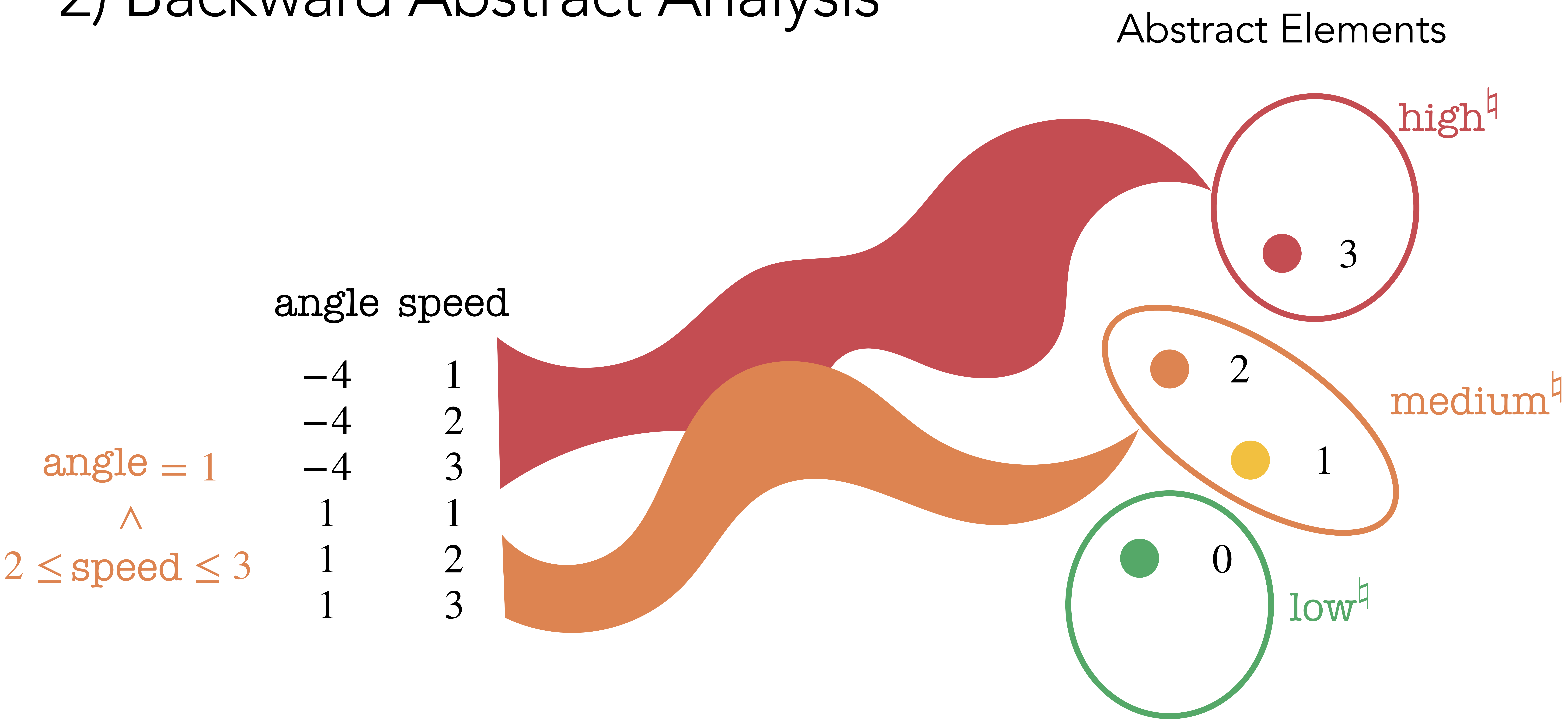
Abstract Elements

$$\texttt{angle} = -4 \;\wedge\; 1 \leq \texttt{speed} \leq 3$$

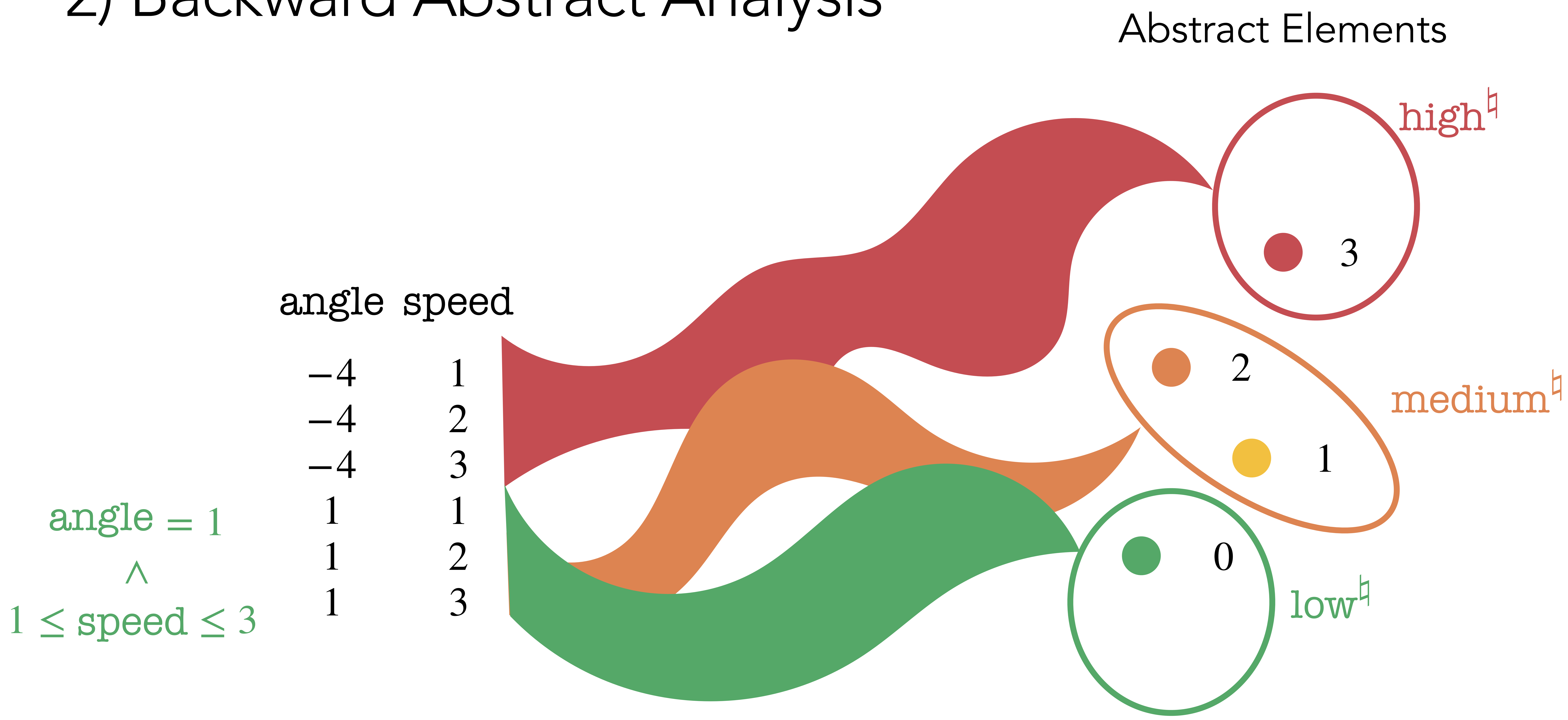
angle	speed
−4	1
−4	2
−4	3
1	1
1	2
1	3



2) Backward Abstract Analysis



2) Backward Abstract Analysis



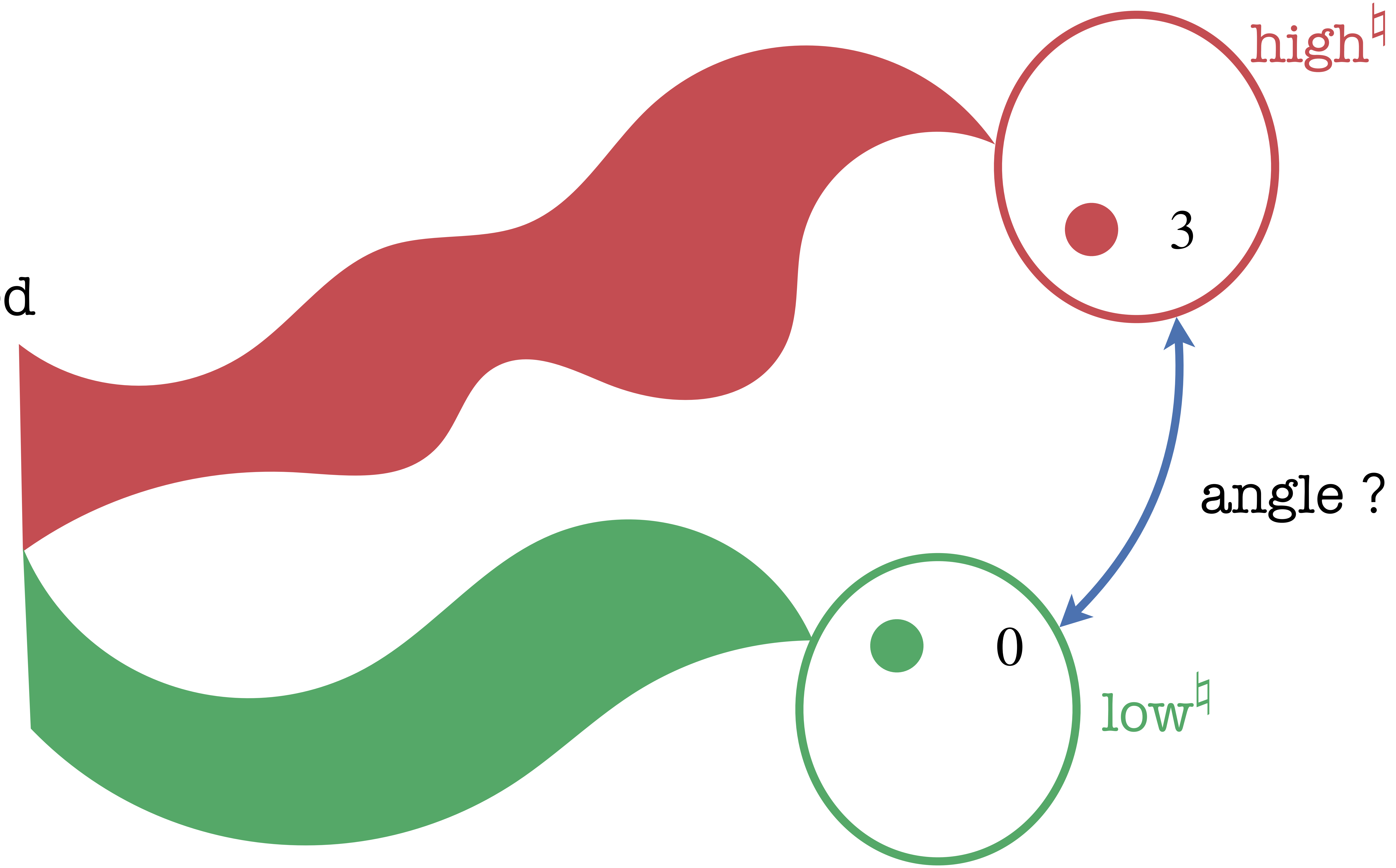
2) Backward Abstract Analysis

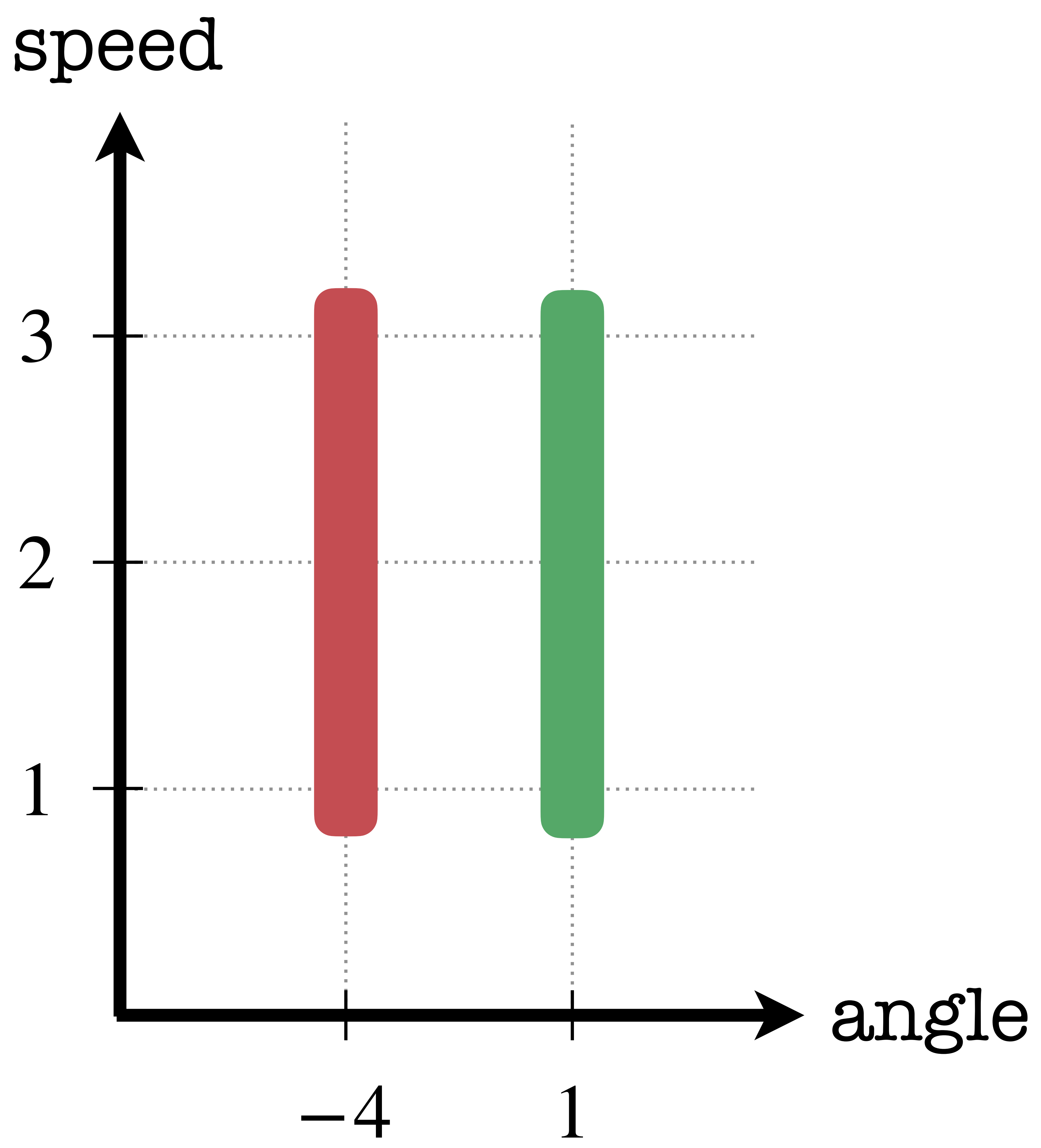
Abstract Elements

$$\text{angle} = -4 \;\wedge\; 1 \leq \text{speed} \leq 3$$

$$\text{angle} = 1 \;\wedge\; 1 \leq \text{speed} \leq 3$$

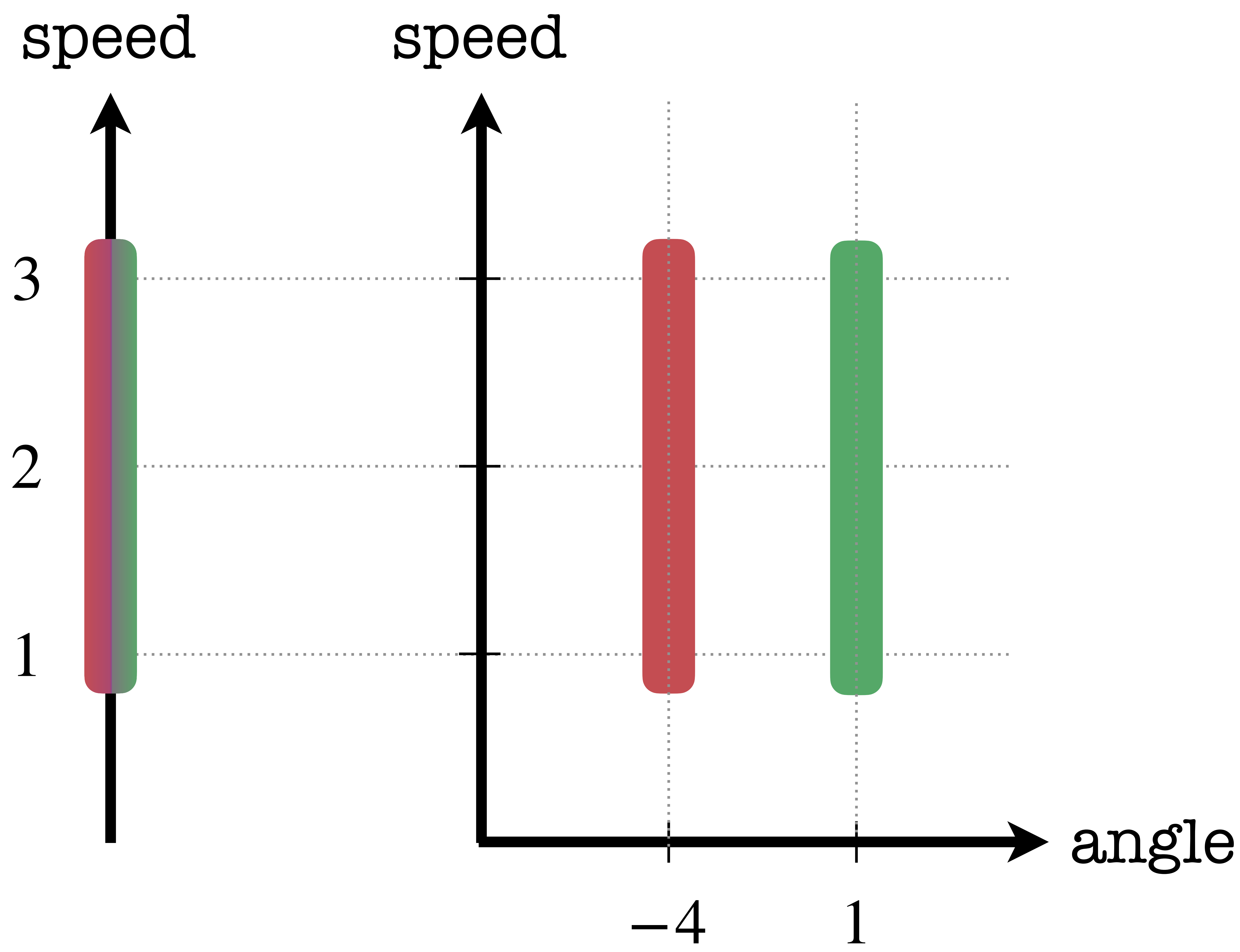
angle	speed
-4	1
-4	2
-4	3
1	1
1	2
1	3





$$\text{angle} = -4 \\ \wedge \\ 1 \leq \text{speed} \leq 3$$

$$\text{angle} = 1 \\ \wedge \\ 1 \leq \text{speed} \leq 3$$



$$\text{angle} = -4 \;\wedge\; 1 \leq \text{speed} \leq 3$$

$$\text{angle} = 1 \;\wedge\; 1 \leq \text{speed} \leq 3$$

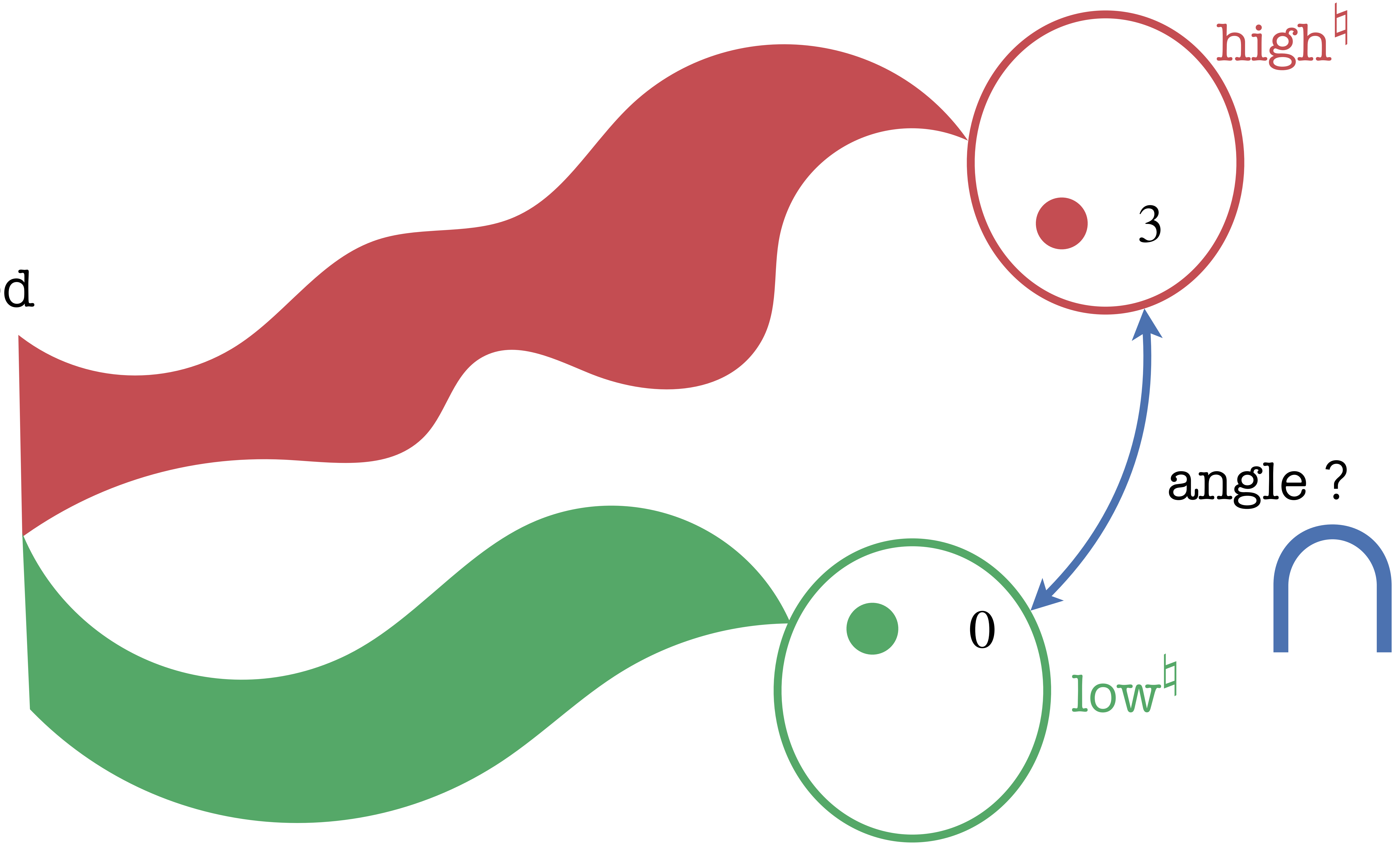
2) Backward Abstract Analysis

Abstract Elements

$\mathtt{angle} = -4$
$\land$
$1 \leq \mathtt{speed} \leq 3$

$\mathtt{angle} = 1$
$\land$
$1 \leq \mathtt{speed} \leq 3$

angle	speed
−4	1
−4	2
−4	3
1	1
1	2
1	3



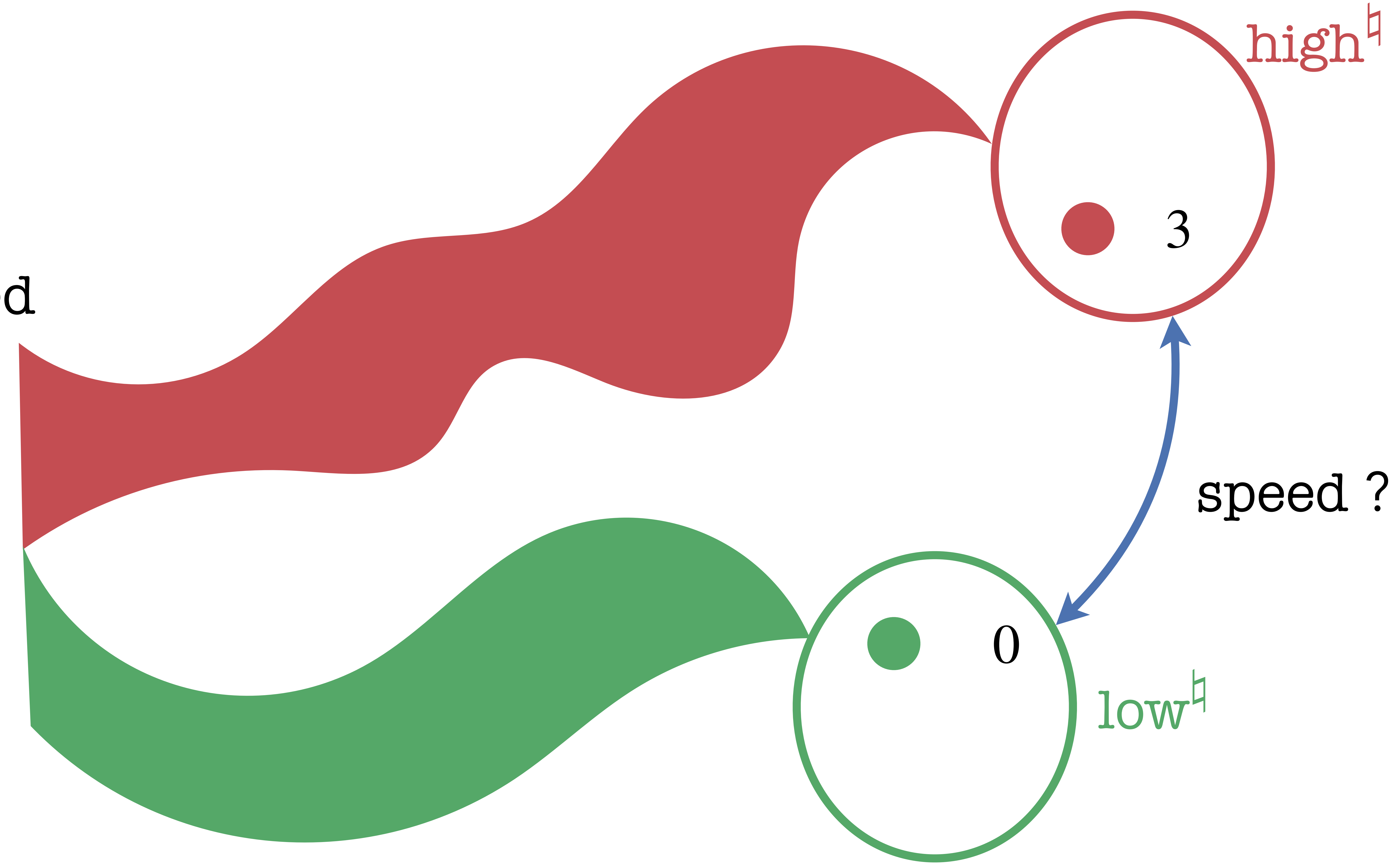
2) Backward Abstract Analysis

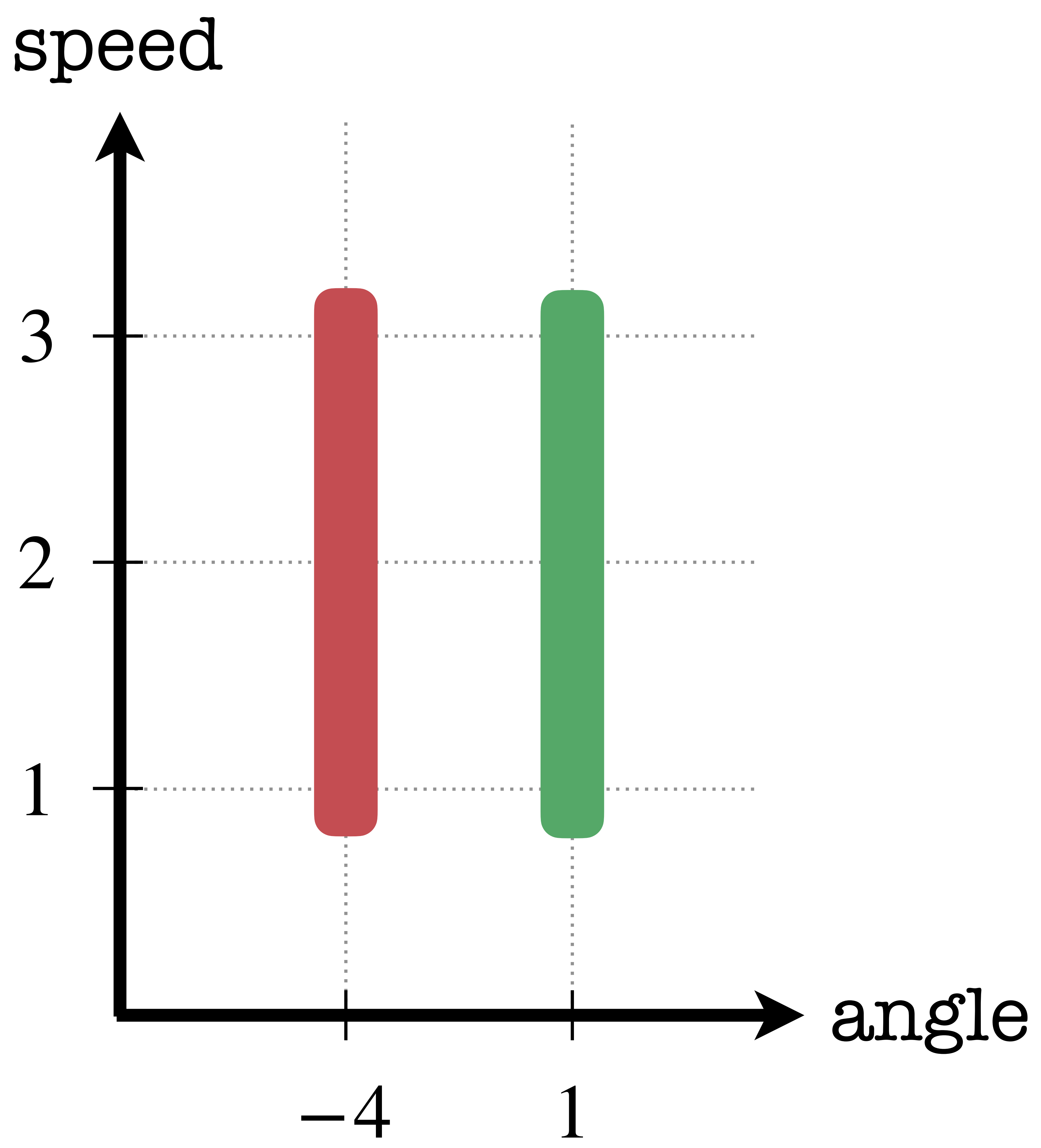
Abstract Elements

$\text{angle} = -4$
$\wedge$
$1 \leq \text{speed} \leq 3$

$\text{angle} = 1$
$\wedge$
$1 \leq \text{speed} \leq 3$

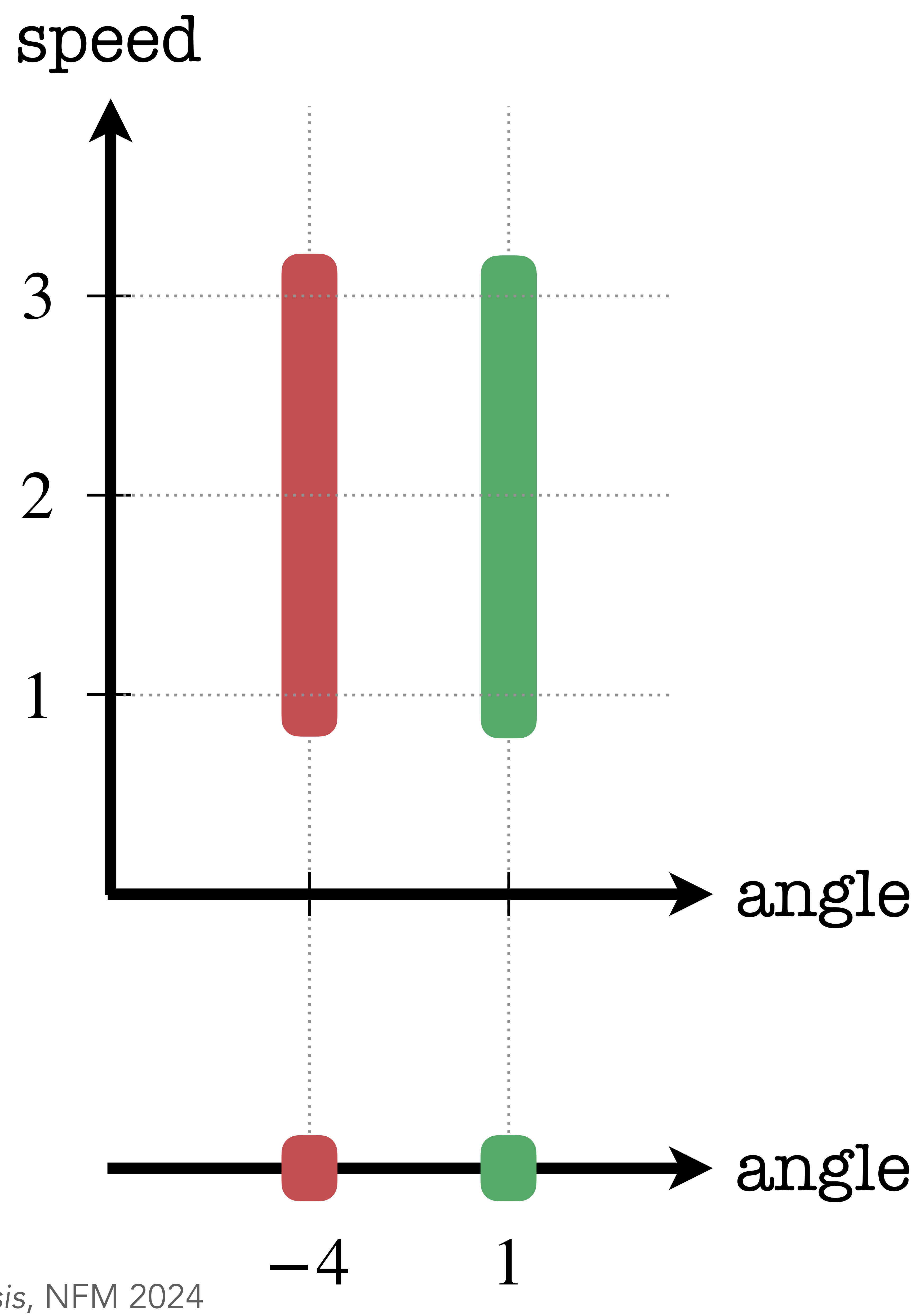
angle	speed
-4	1
-4	2
-4	3
1	1
1	2
1	3





$$\text{angle} = -4 \\ \wedge \\ 1 \leq \text{speed} \leq 3$$

$$\text{angle} = 1 \\ \wedge \\ 1 \leq \text{speed} \leq 3$$



$$\text{angle} = -4 \;\wedge\; 1 \leq \text{speed} \leq 3$$

$$\text{angle} = 1 \;\wedge\; 1 \leq \text{speed} \leq 3$$

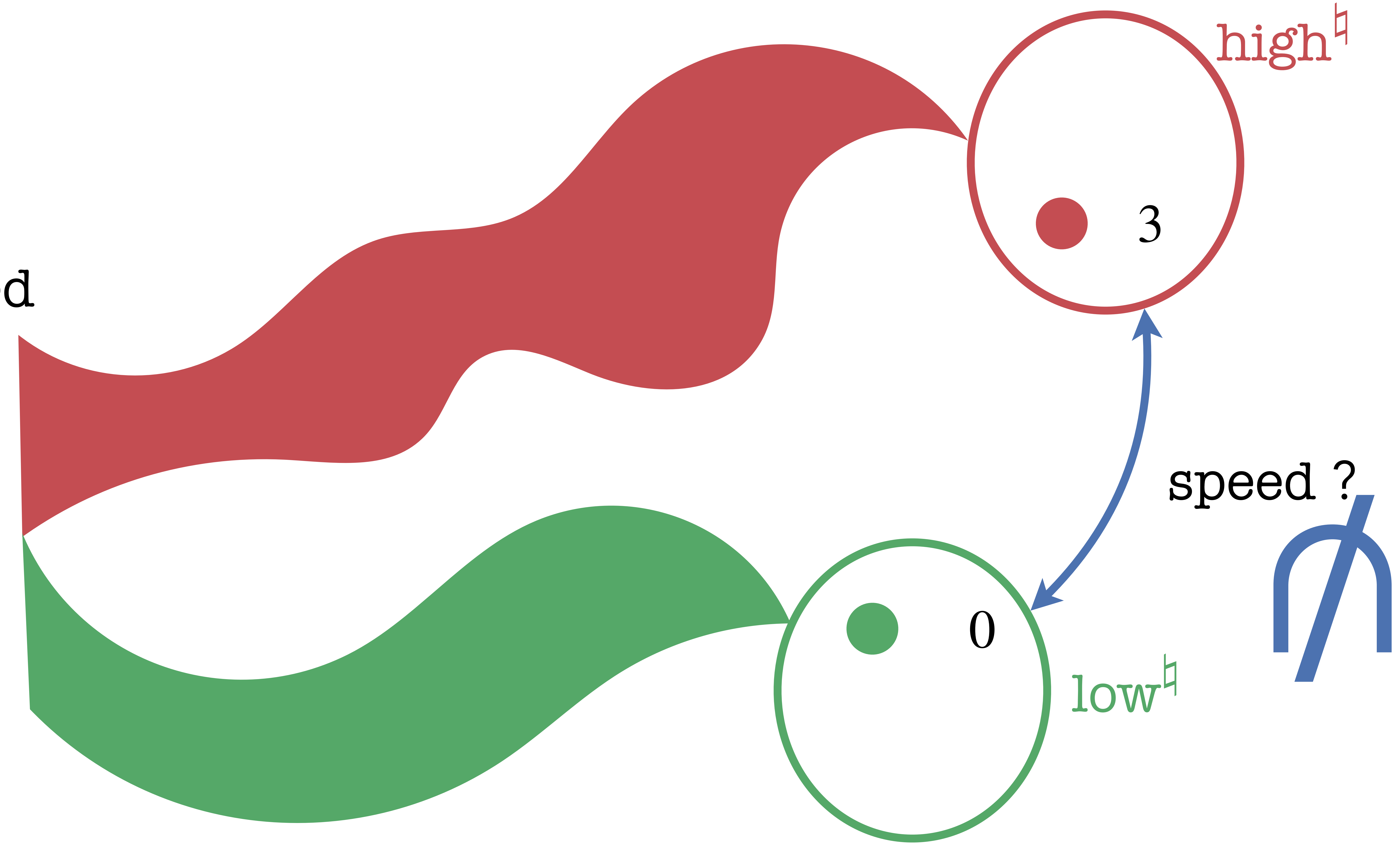
2) Backward Abstract Analysis

Abstract Elements

$\color{red}{\text{angle} = -4 \wedge 1 \leq \text{speed} \leq 3}$

$\color{green}{\text{angle} = 1 \wedge 1 \leq \text{speed} \leq 3}$

angle	speed
-4	1
-4	2
-4	3
1	1
1	2
1	3



3) Abstract Implementation of Impact Definitions

Combinations	$\mathtt{high}^\natural$ $\mathtt{medium}^\natural$	$\mathtt{high}^\natural$ $\mathtt{low}^\natural$	$\mathtt{low}^\natural$ $\mathtt{medium}^\natural$	$\mathtt{high}^\natural$ $\mathtt{medium}^\natural$ $\mathtt{low}^\natural$
`angle`	$\cap$	$\cap$	$\cap$	$\cap$
`speed`			$\cap$	

Abstract Impact **Outcomes**$^\natural$

Combinations	high$^\natural$ medium$^\natural$	$\boxed{\text{high}^\natural \text{ low}^\natural}$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	$\cap$	$0, 3$	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact **Outcomes**$^\natural$

Combinations	high$^\natural$ medium$^\natural$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	$\cap$	2	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact **Outcomes**$^\natural$

Combinations	high$^\natural$ medium$^\natural$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	1,2, 3	2	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact **Outcomes**$^\natural$

Combinations	$\boxed{\text{high}^\natural \ \text{medium}^\natural}$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	$\boxed{3}$	2	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact **Outcomes**$^\natural$

Combinations	high$^\natural$ medium$^\natural$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	3	2	3	4
speed			3	

Abstract Impact **Outcomes**$^\natural$

Combinations	high$^\natural$ medium$^\natural$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$	Outcomes$^\natural$
angle	3	2	3	4	$\Longrightarrow$ 4
speed			3		$\Longrightarrow$ 3

Abstract Impact Range$^\natural$

Combinations	high$^\natural$ medium$^\natural$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	$\cap$	$\cap$	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact $\texttt{Range}^\natural$

Combinations	$\texttt{high}^\natural$ $\texttt{medium}^\natural$	$\texttt{high}^\natural$ $\texttt{low}^\natural$	$\texttt{low}^\natural$ $\texttt{medium}^\natural$	$\texttt{high}^\natural$ $\texttt{medium}^\natural$ $\texttt{low}^\natural$
$\texttt{angle}$	$\cap$	$\boxed{0, 3}$	$\cap$	$\cap$
$\texttt{speed}$			$\cap$	

Abstract Impact Range$^\natural$

Combinations	high$^\natural$ medium$^\natural$	high$^\natural$ low$^\natural$	low$^\natural$ medium$^\natural$	high$^\natural$ medium$^\natural$ low$^\natural$
angle	$\cap$	3	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact Range$^\natural$

Combinations	$\text{high}^\natural$ $\text{medium}^\natural$	$\text{high}^\natural$ $\text{low}^\natural$	$\text{low}^\natural$ $\text{medium}^\natural$	$\text{high}^\natural$ $\text{medium}^\natural$ $\text{low}^\natural$
angle	1,2, 3	3	$\cap$	$\cap$
speed			$\cap$	

Abstract Impact $\mathtt{Range}^\natural$

Combinations	$\mathtt{high}^\natural$ $\mathtt{medium}^\natural$	$\mathtt{high}^\natural$ $\mathtt{low}^\natural$	$\mathtt{low}^\natural$ $\mathtt{medium}^\natural$	$\mathtt{high}^\natural$ $\mathtt{medium}^\natural$ $\mathtt{low}^\natural$
$\mathtt{angle}$	$\boxed{2}$	3	$\cap$	$\cap$
$\mathtt{speed}$			$\cap$	

Abstract Impact $\texttt{Range}^\natural$

Combinations	$\texttt{high}^\natural$ $\texttt{medium}^\natural$	$\texttt{high}^\natural$ $\texttt{low}^\natural$	$\texttt{low}^\natural$ $\texttt{medium}^\natural$	$\texttt{high}^\natural$ $\texttt{medium}^\natural$ $\texttt{low}^\natural$
`angle`	2	3	2	3
`speed`			2	

Abstract Impact $\mathtt{Range}^\natural$

Combinations	$\mathtt{high}^\natural$ $\mathtt{medium}^\natural$	$\mathtt{high}^\natural$ $\mathtt{low}^\natural$	$\mathtt{low}^\natural$ $\mathtt{medium}^\natural$	$\mathtt{high}^\natural$ $\mathtt{medium}^\natural$ $\mathtt{low}^\natural$	$\mathtt{Range}^\natural$
$\mathtt{angle}$	2	3	2	3	$\Longrightarrow$ 3
$\mathtt{speed}$			2		$\Longrightarrow$ 2

Goal: Quantify the impact of `speed` and `angle` on `risk`

	Range	Outcomes
`angle`	3	2
`speed`	2	3

Goal: Quantify the impact of `speed` and `angle` on `risk`

	RANGE	OUTCOMES	Range$^\natural$	Outcomes$^\natural$
angle	3	2	3	4
speed	2	3	2	3

Goal: Quantify the impact of speed and angle on `risk`

	IMPACT		Impact$^\natural$	
	RANGE	OUTCOMES	Range$^\natural$	Outcomes$^\natural$
angle	3	2	3	4
speed	2	3	2	3

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

(iii) $\mathtt{Impact}^\natural$ is a **sound implementation** of IMPACT

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

(iii) $\texttt{Impact}^\natural$ is a **sound implementation** of $\textsc{Impact}$

$B^\natural$

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

(iii) $\mathtt{Impact}^\natural$ is a **sound implementation** of IMPACT

$$\Lambda^\natural[\![P]\!]B^\natural$$

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

(iii) $\mathtt{Impact}^\natural$ is a **sound implementation** of IMPACT

$$\mathtt{Impact}^\natural_{\mathtt{i}}(\Lambda^\natural[\![P]\!]B^\natural)$$

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

(iii) $\mathtt{Impact}^\natural$ is a **sound implementation** of IMPACT

$$\mathtt{Impact}^\natural_{\mathtt{i}}(\Lambda^\natural[\![P]\!]B^\natural) \leq k$$

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^\natural$ **over-approximates** the input-output relations

(ii) The output buckets $B^\natural$ **include** the whole output space

(iii) $\texttt{Impact}^\natural$ is a **sound implementation** of IMPACT

$$\texttt{Impact}^\natural_{\texttt{i}}(\Lambda^\natural[\![P]\!]B^\natural) \leq k \quad \Longrightarrow$$

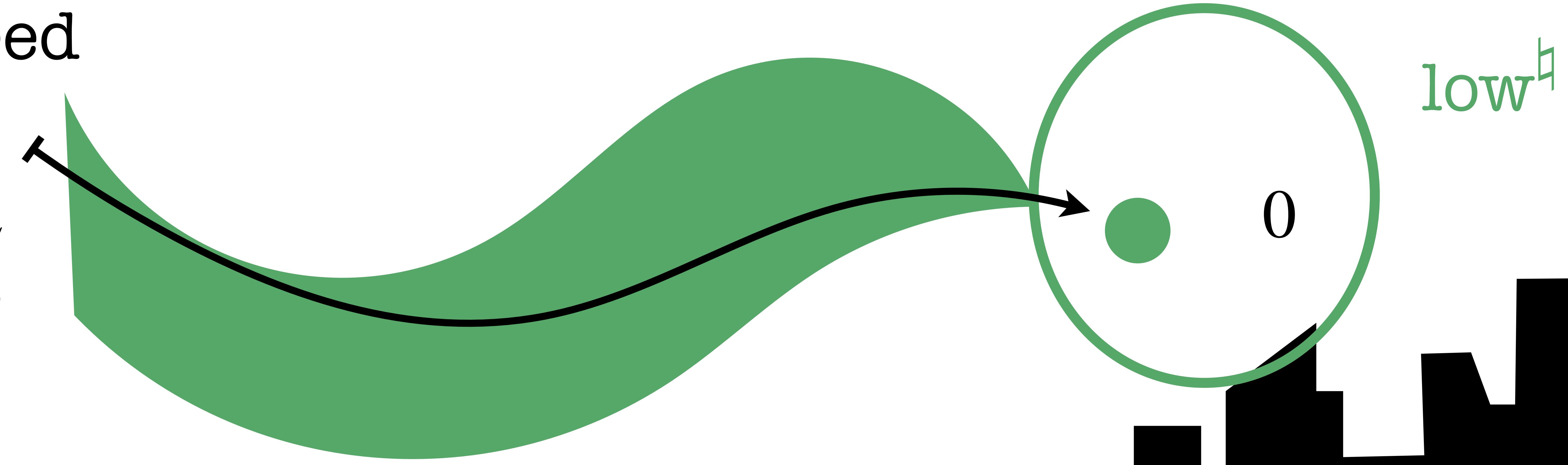
The variable $\texttt{i}$ on the program P
has an impact of **at most** k

Source of Imprecision

Abstraction of the Backward Analysis

$$\texttt{angle} = 1 \\ \wedge \\ 1 \leq \texttt{speed} \leq 3$$

angle	speed
1	1
1	2
1	3



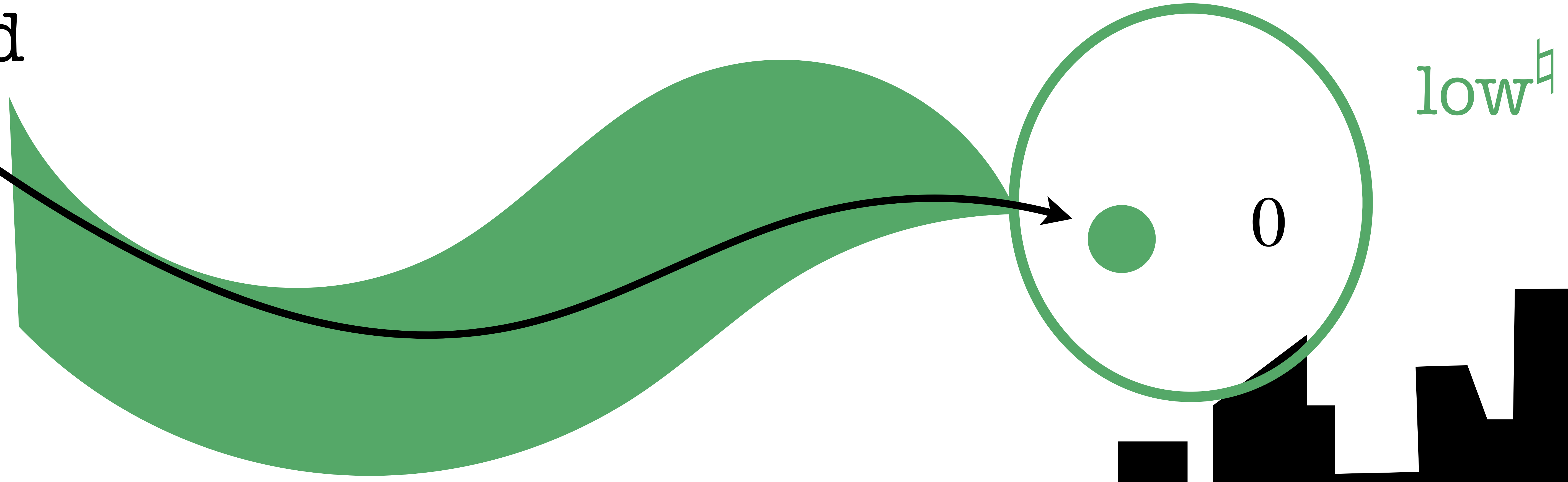
Source of Imprecision

Choice of the Output Buckets

Abstraction of the Backward Analysis

$$\text{angle} = 1 \;\wedge\; 1 \leq \text{speed} \leq 3$$

angle	speed
1	1
1	2
1	3



Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.

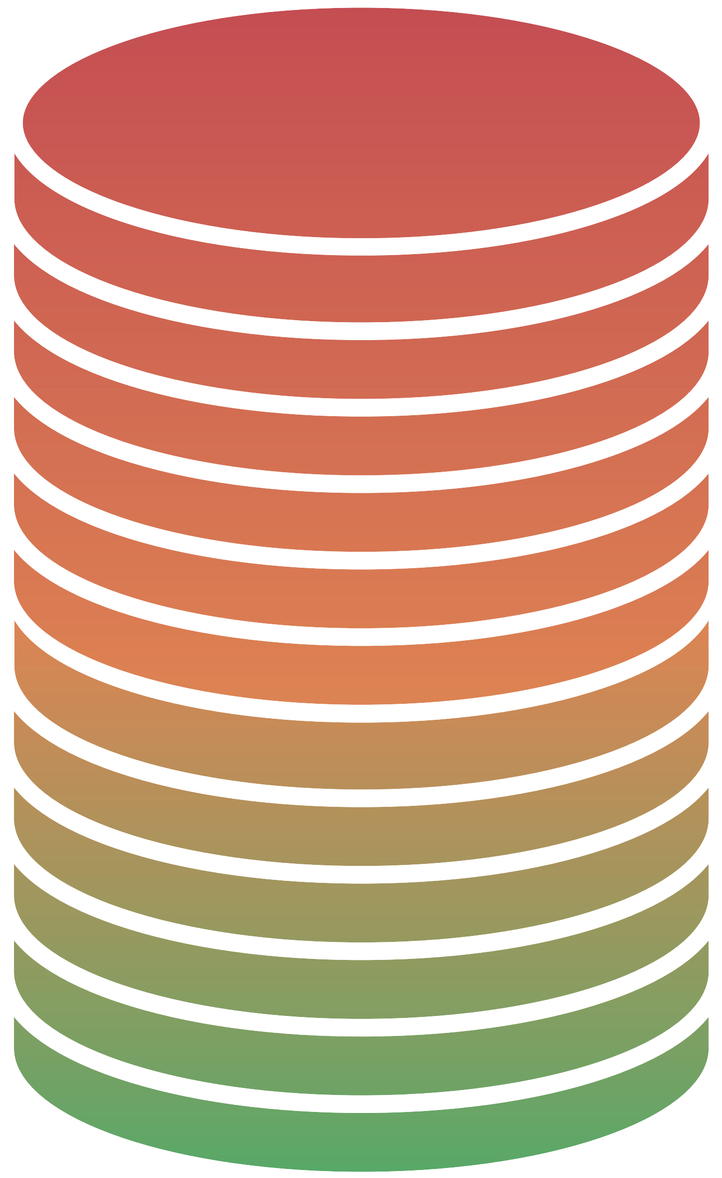
```
def mean_growth_rate_60_90(
        portugal1, portugal2, portugal3,
        norway1,
        uk1, uk2, uk3, uk4,
        usa1, usa2, usa3):
    portugal_avg = avg(portugal1, portugal2, portugal3)
    norway_avg = avg(norway1)
    uk_avg = avg(uk1, uk2, uk3, uk4)
    usa_avg = avg(usa1, usa2, usa3)
    return avg(portugal_avg, norway_avg, uk_avg, usa_avg)
```

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.

```
def mean_growth_rate_60_90(
      portugal1, portugal2, portugal3,
      norway1,
      uk1, uk2, uk3, uk4,
      usa1, usa2, usa3):
    portugal_avg = avg(portugal1, portugal2, portugal3)
    norway_avg = avg(norway1)
    uk_avg = avg(uk1, uk2, uk3, uk4)
    usa_avg = avg(usa1, usa2, usa3)
    return avg(portugal_avg, norway_avg, uk_avg, usa_avg)
```

$\mathrm{avg}^\natural = 20\,\%$

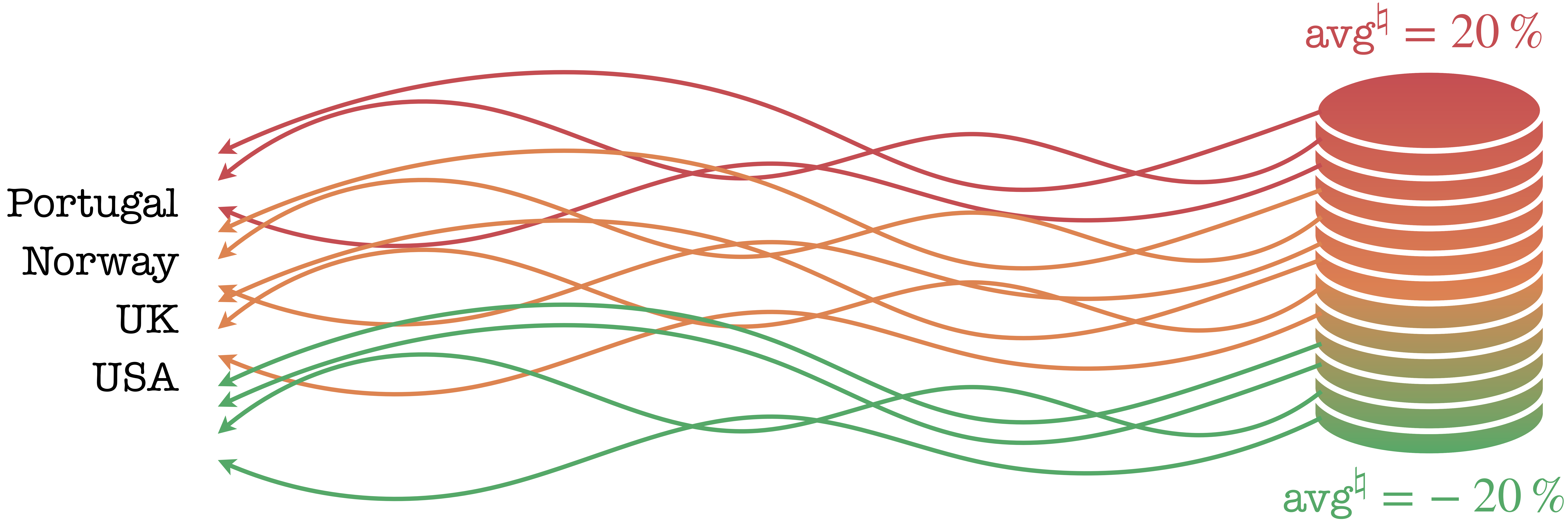


$\mathrm{avg}^\natural = -20\,\%$

41 Output Buckets

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.



41 Output Buckets

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
American Economic Review 2010.

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.

```
def mean_growth_rate_60_90(
    portugal1, portugal2, portugal3,
    norway1,
    uk1, uk2, uk3, uk4,
    usa1, usa2, usa3):
  portugal_avg = avg(portugal1, portugal2, portugal3)
  norway_avg = avg(norway1)
  uk_avg = avg(uk1, uk2, uk3, uk4)
  usa_avg = avg(usa1, usa2, usa3)
  return avg(portugal_avg, norway_avg, uk_avg, usa_avg)
```

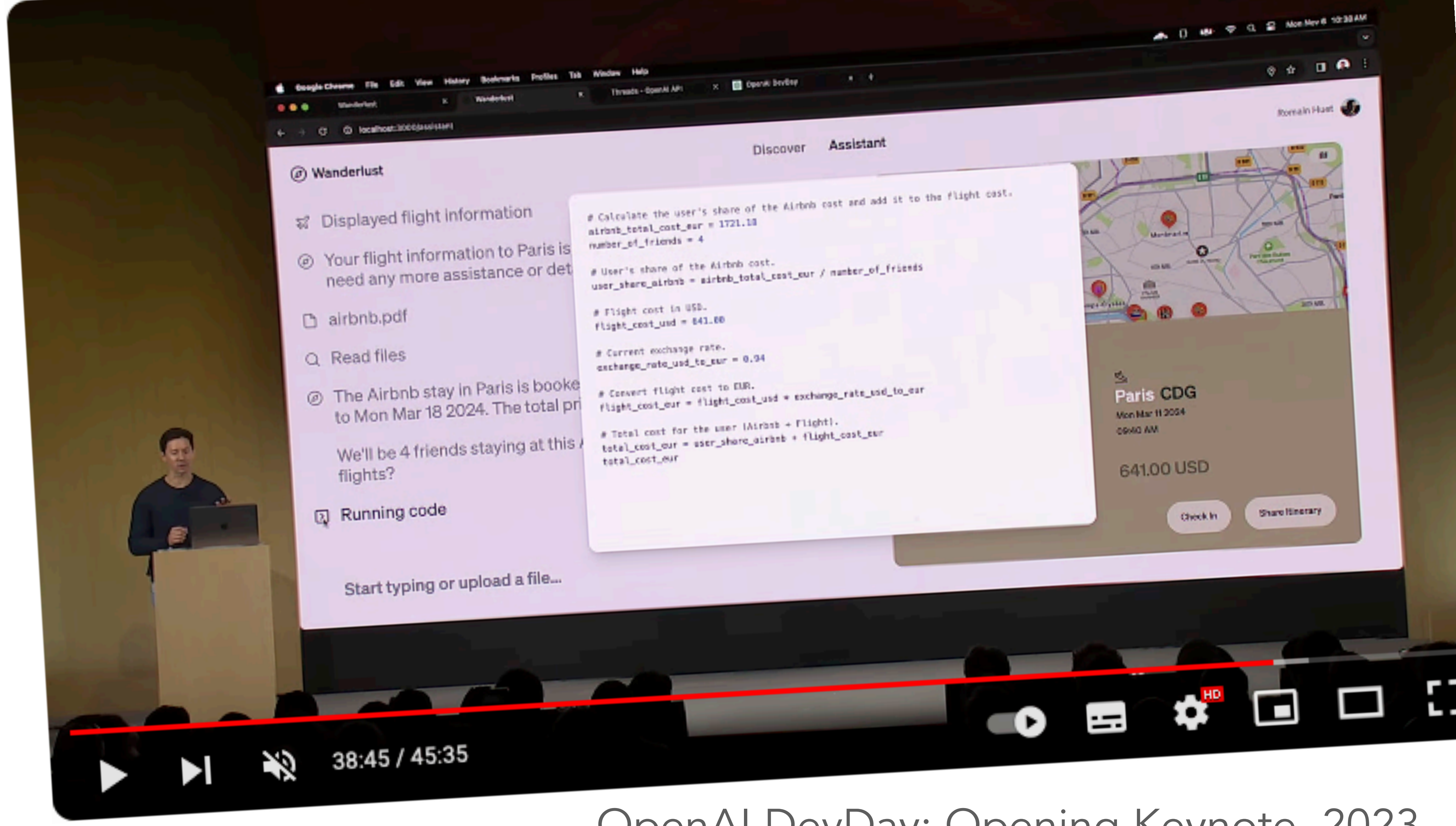
Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
American Economic Review 2010.

```
def mean_growth_rate_60_90(
        portugal1, portugal2, portugal3,
        norway1,
        uk1, uk2, uk3, uk4,
        usa1, usa2, usa3):
    portugal_avg = avg(portugal1, portugal2, portugal3)
    norway_avg = avg(norway1)
    uk_avg = avg(uk1, uk2, uk3, uk4)
    usa_avg = avg(usa1, usa2, usa3)
    return avg(portugal_avg, norway_avg, uk_avg, usa_avg)
```

Use Case: GPT-4 Turbo

```python
def share_division(
        airbnb_total_cost_eur,
        flight_cost_usd,
        number_of_friends):
    share_airbnb = airbnb_total_cost_eur / number_of_friends
    usd_to_eur = 0.92
    flight_cost_eur = flight_cost_usd * usd_to_eur
    total_cost_eur = share_airbnb + flight_cost_eur
    return total_cost_eur
```



OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

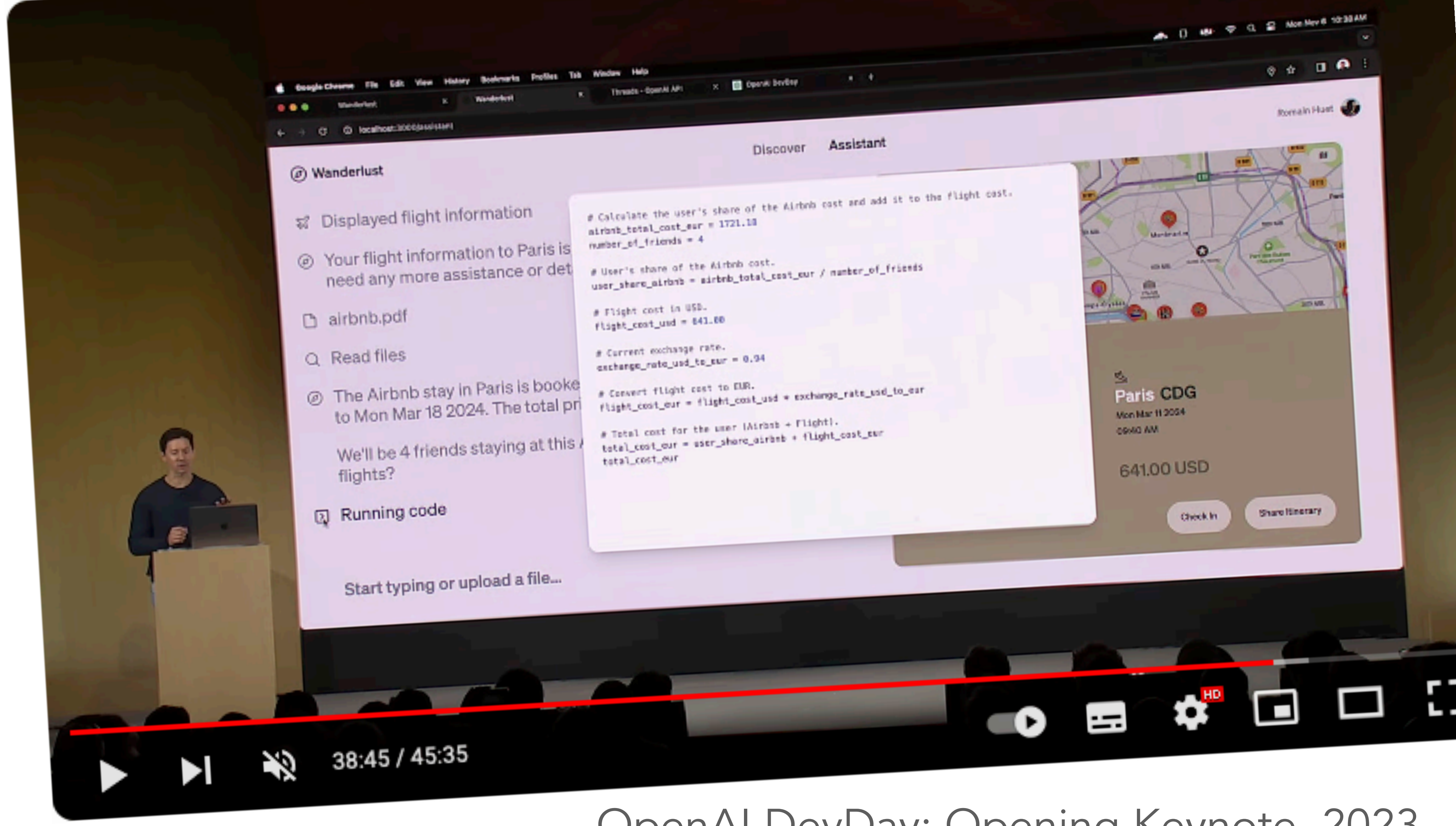
Use Case: GPT-4 Turbo

```
def share_division(
```

$500 \leq \texttt{airbnb\_total\_cost\_eur} \leq 2000$

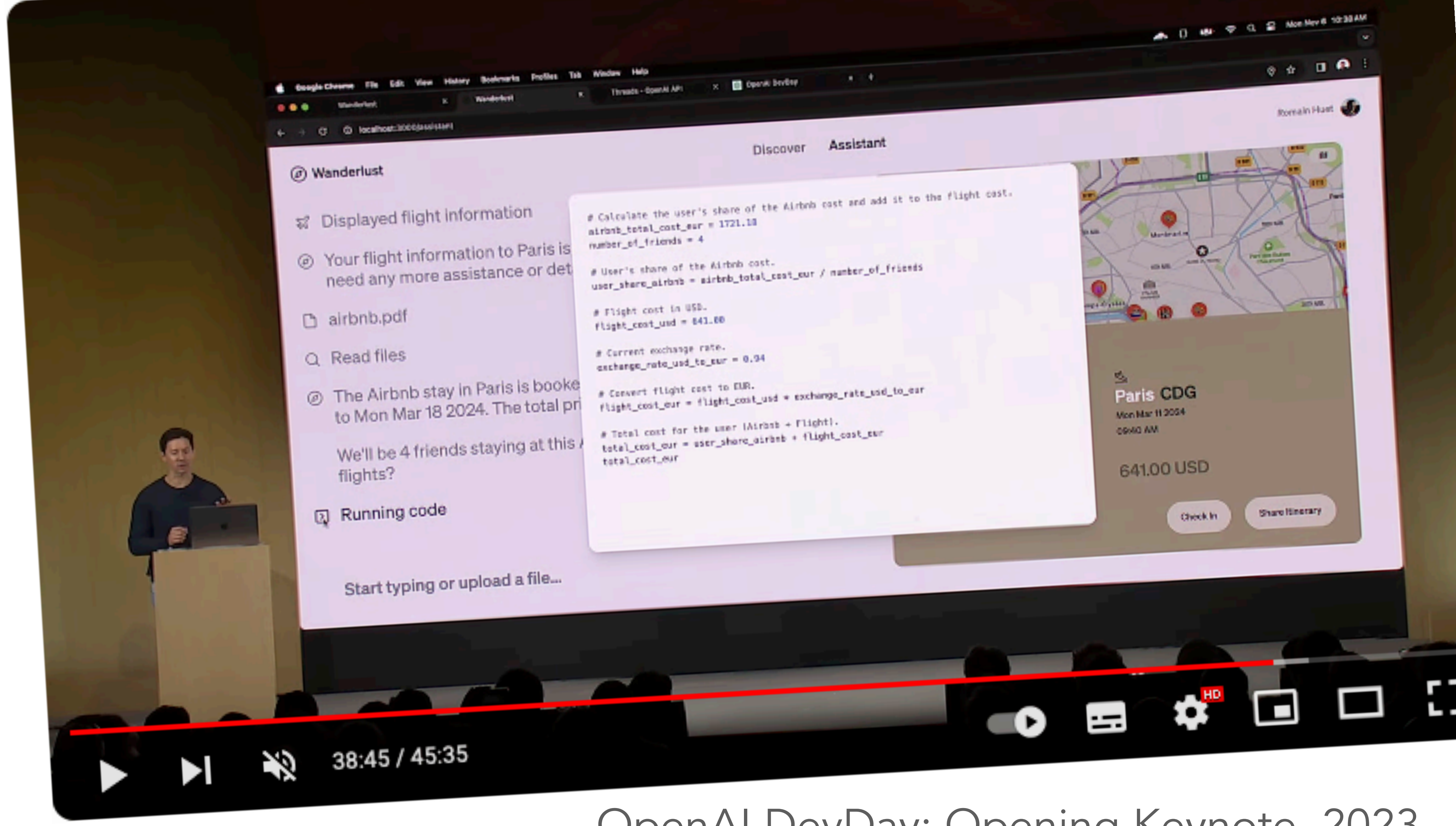
$50 \leq \texttt{flight\_cost\_usd} \leq 1000$

$2 \leq \texttt{number\_of\_friends} \leq 10$



OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

Use Case: GPT-4 Turbo

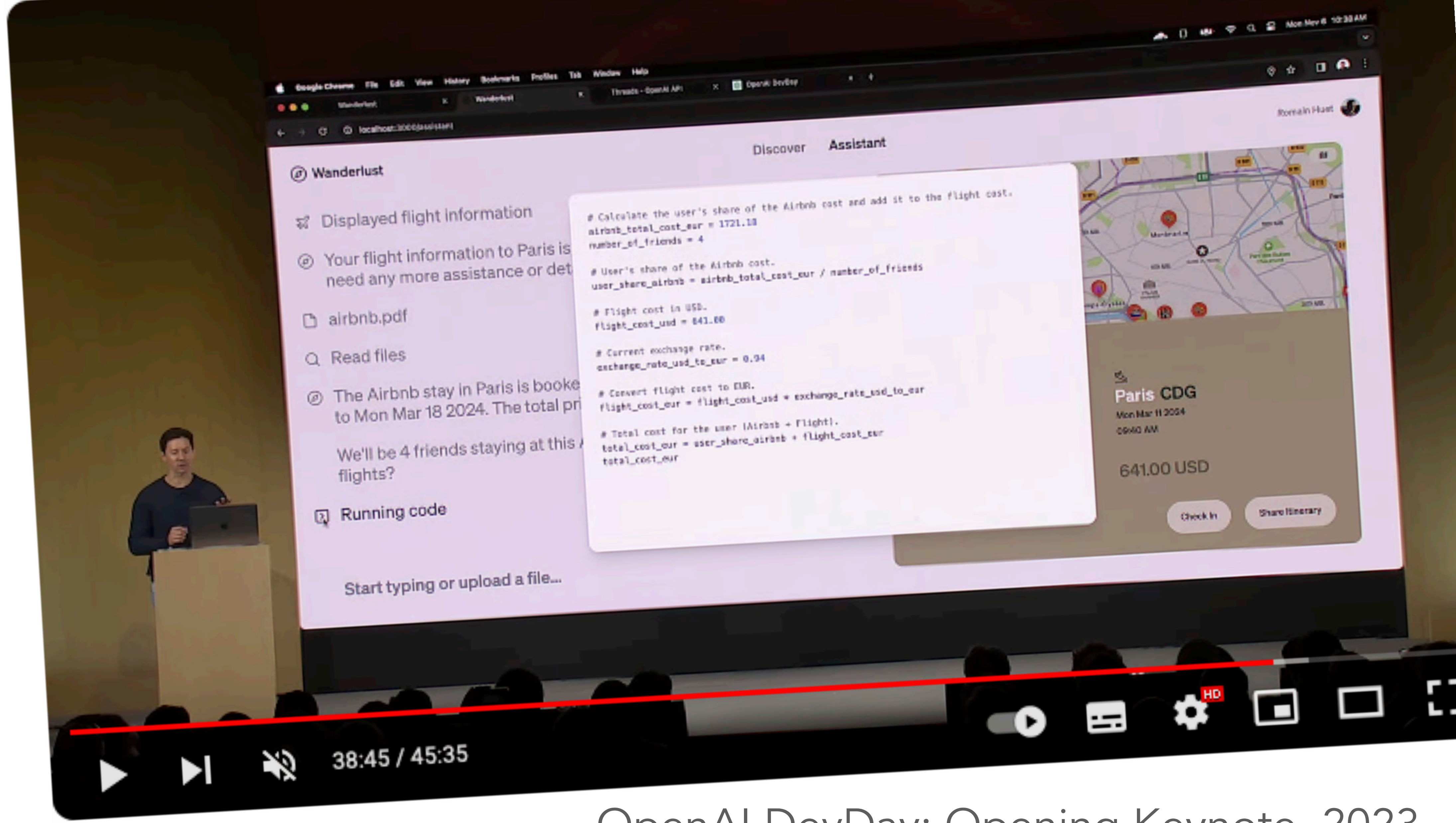


OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

```
def share_division(
500 ≤ airbnb_total_cost_eur ≤ 2000
 50 ≤ flight_cost_usd ≤ 1000
  2 ≤ number_of_friends ≤ 10
   share_airbnb = airbnb_total_cost_eur / number_of_friends
   usd_to_eur = 0.92
   flight_cost_eur = flight_cost_usd * usd_to_eur
   total_cost_eur = share_airbnb + flight_cost_eur
   return total_cost_eur
```

Denis Mazzucato et al. *Quantitative Input Usage Static Analysis*, NFM 2024

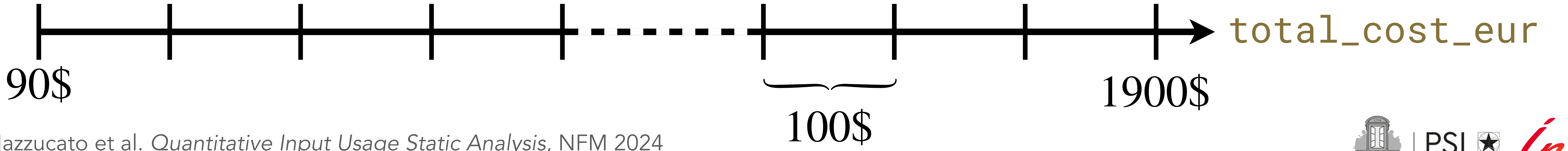
Use Case: GPT-4 Turbo



OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

```
def share_division(
500 ≤ airbnb_total_cost_eur ≤ 2000
 50 ≤ flight_cost_usd ≤ 1000
  2 ≤ number_of_friends ≤ 10
  share_airbnb = airbnb_total_cost_eur / number_of_friends
  usd_to_eur = 0.92
  flight_cost_eur = flight_cost_usd * usd_to_eur
  total_cost_eur = share_airbnb + flight_cost_eur
  return total_cost_eur
```

20 Output Buckets

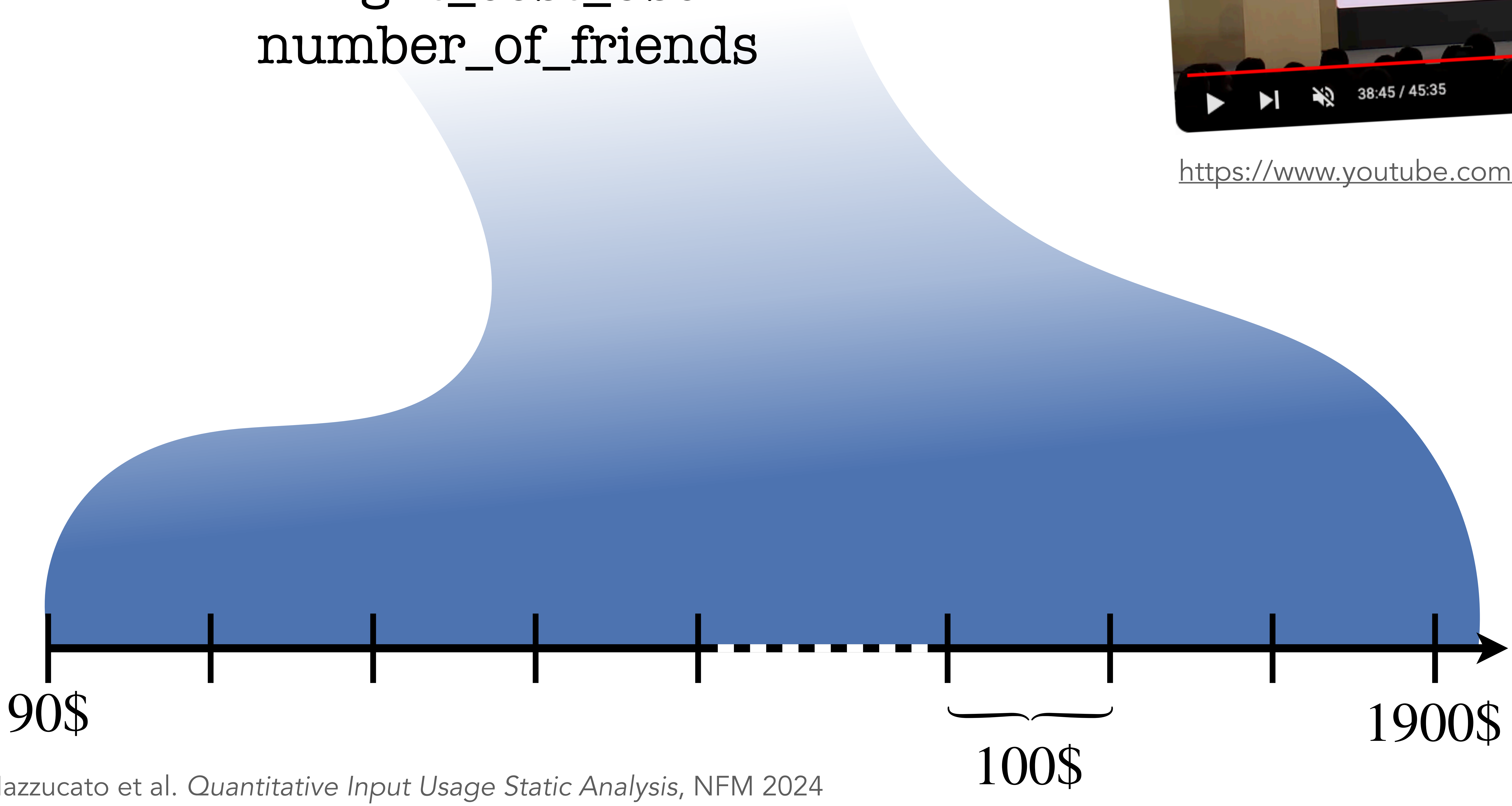


Denis Mazzucato et al. *Quantitative Input Usage Static Analysis*, NFM 2024

ENS | PSL Inria

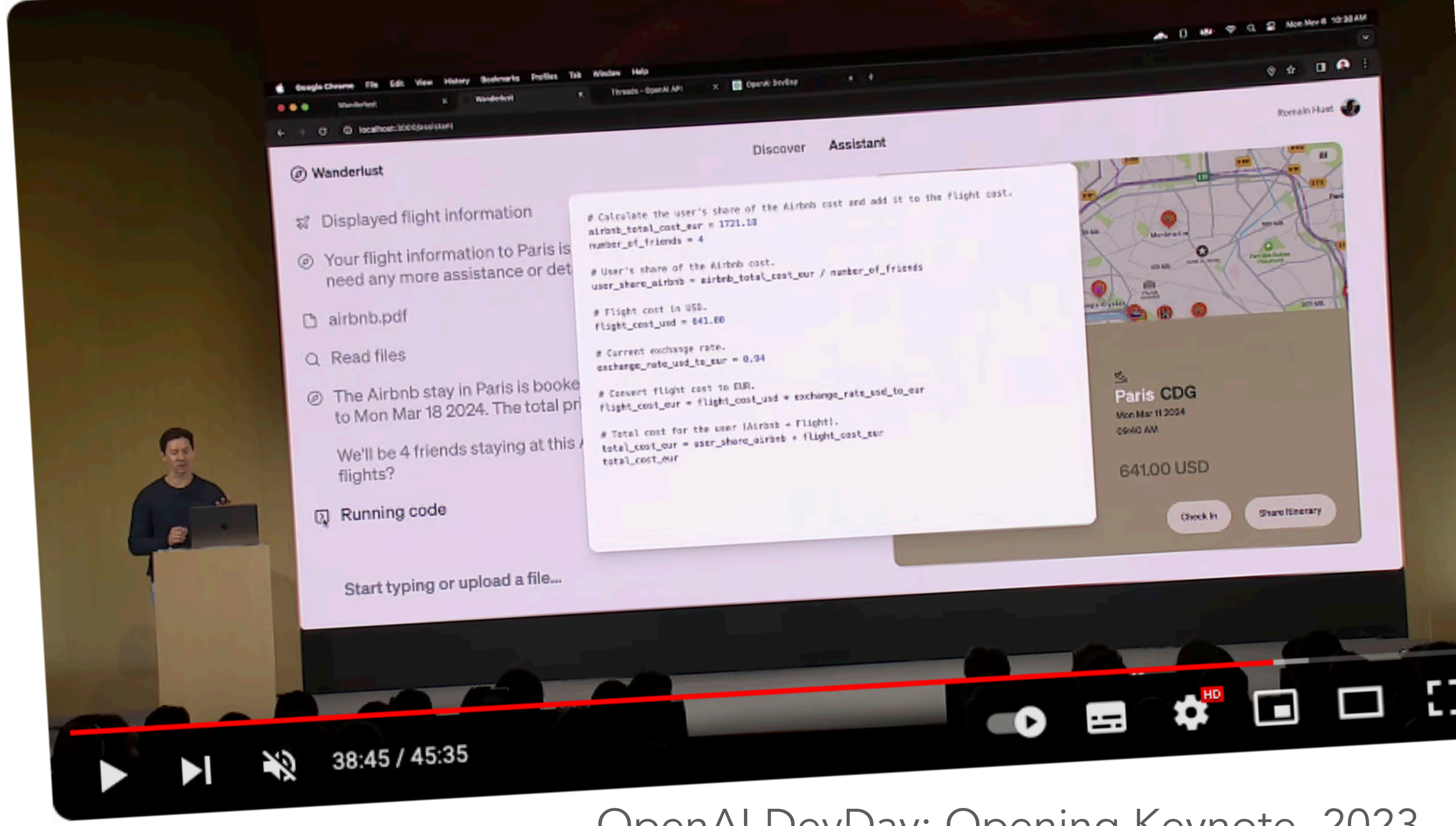
Use Case: GPT-4 Turbo

airbnb_total_cost_eur
flight_cost_usd
number_of_friends



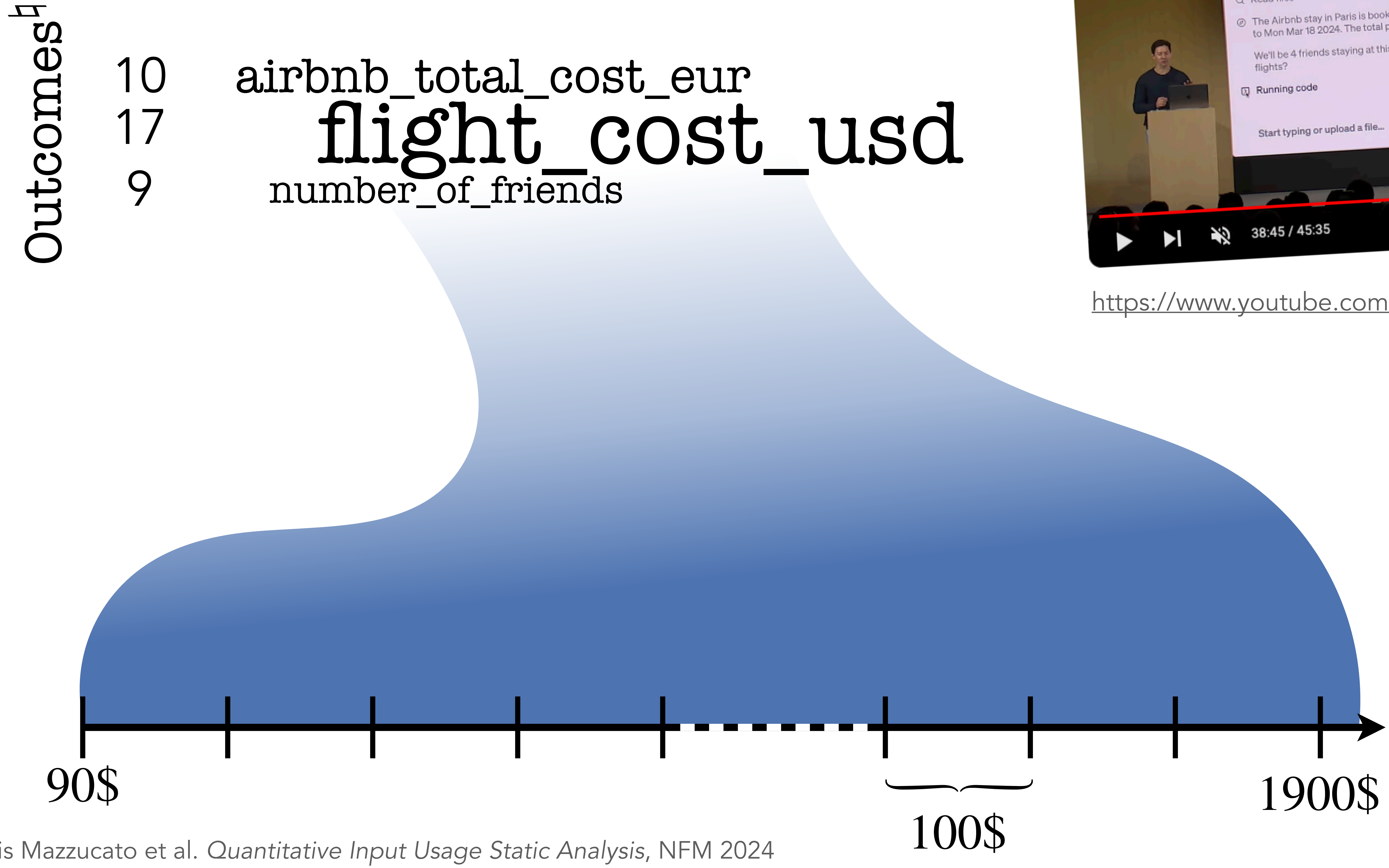
20 Output Buckets

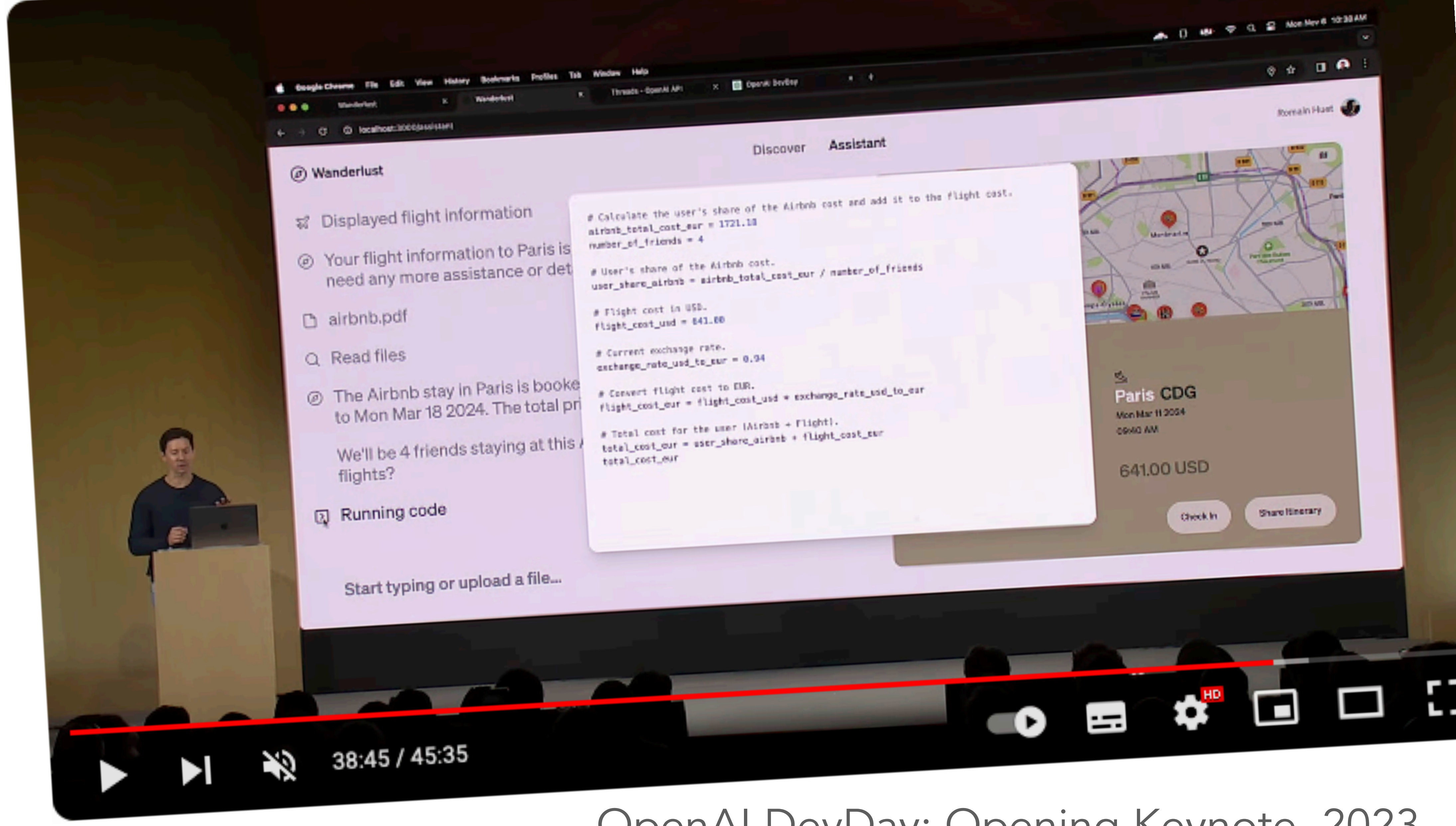
total_cost_eur



OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

Use Case: GPT-4 Turbo





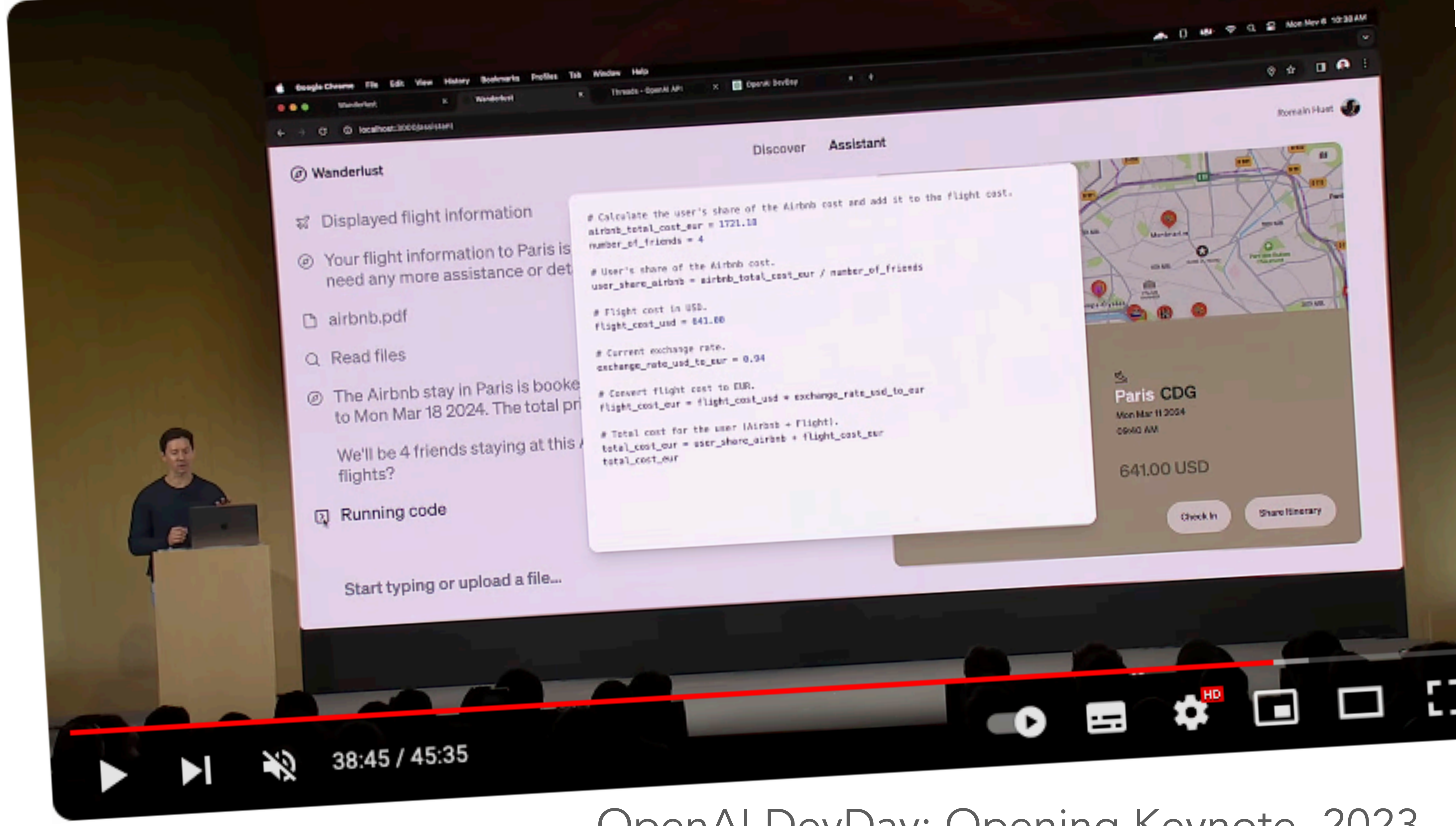
OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

Denis Mazzucato et al. *Quantitative Input Usage Static Analysis*, NFM 2024

ENS | PSL Inria

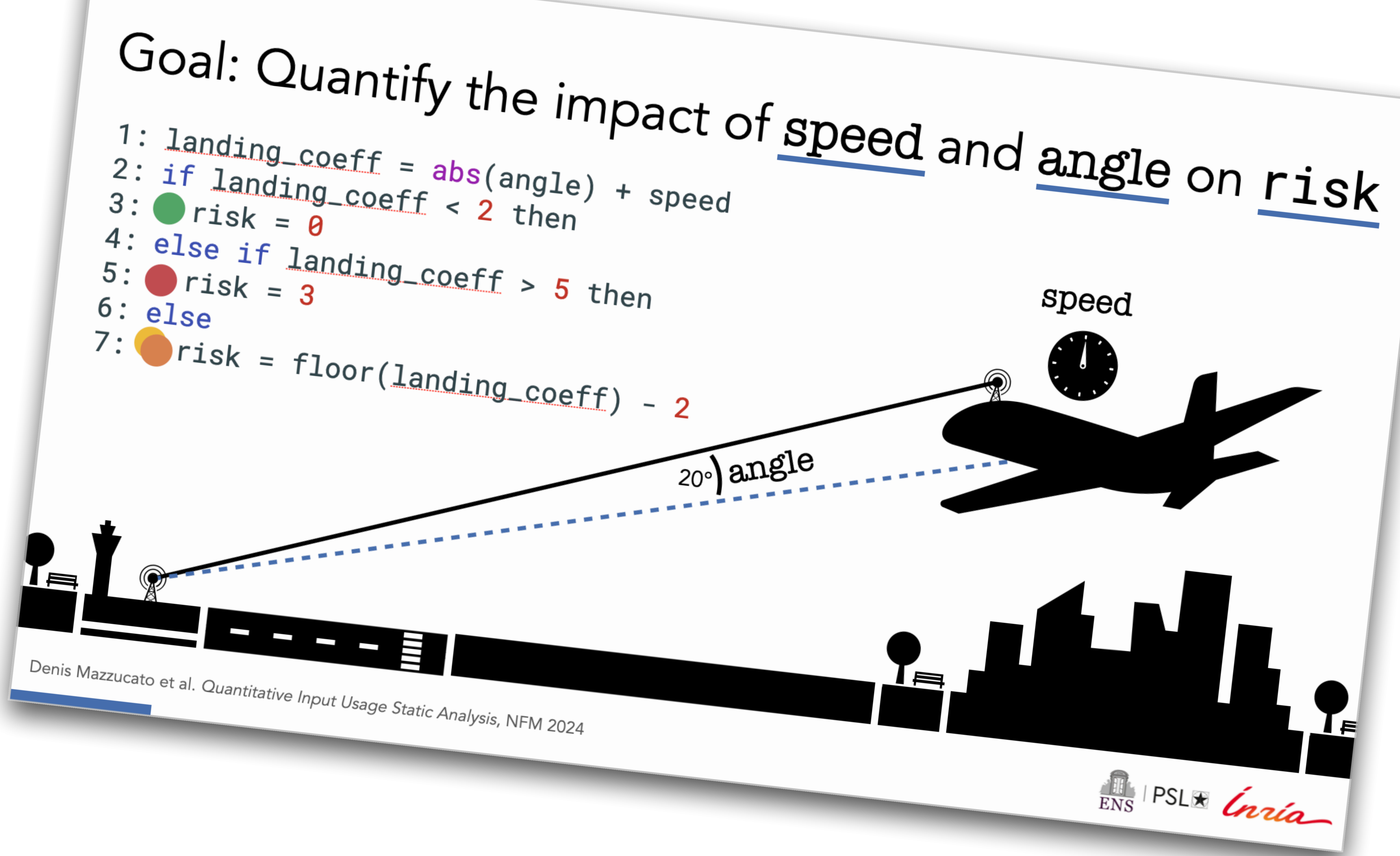
Use Case: GPT-4 Turbo

```python
def share_division(
        airbnb_total_cost_eur,
        flight_cost_usd,
        number_of_friends):
    share_airbnb = airbnb_total_cost_eur / number_of_friends
    usd_to_eur = 0.92
    flight_cost_eur = flight_cost_usd * usd_to_eur
    total_cost_eur = share_airbnb + flight_cost_eur
    return total_cost_eur
```



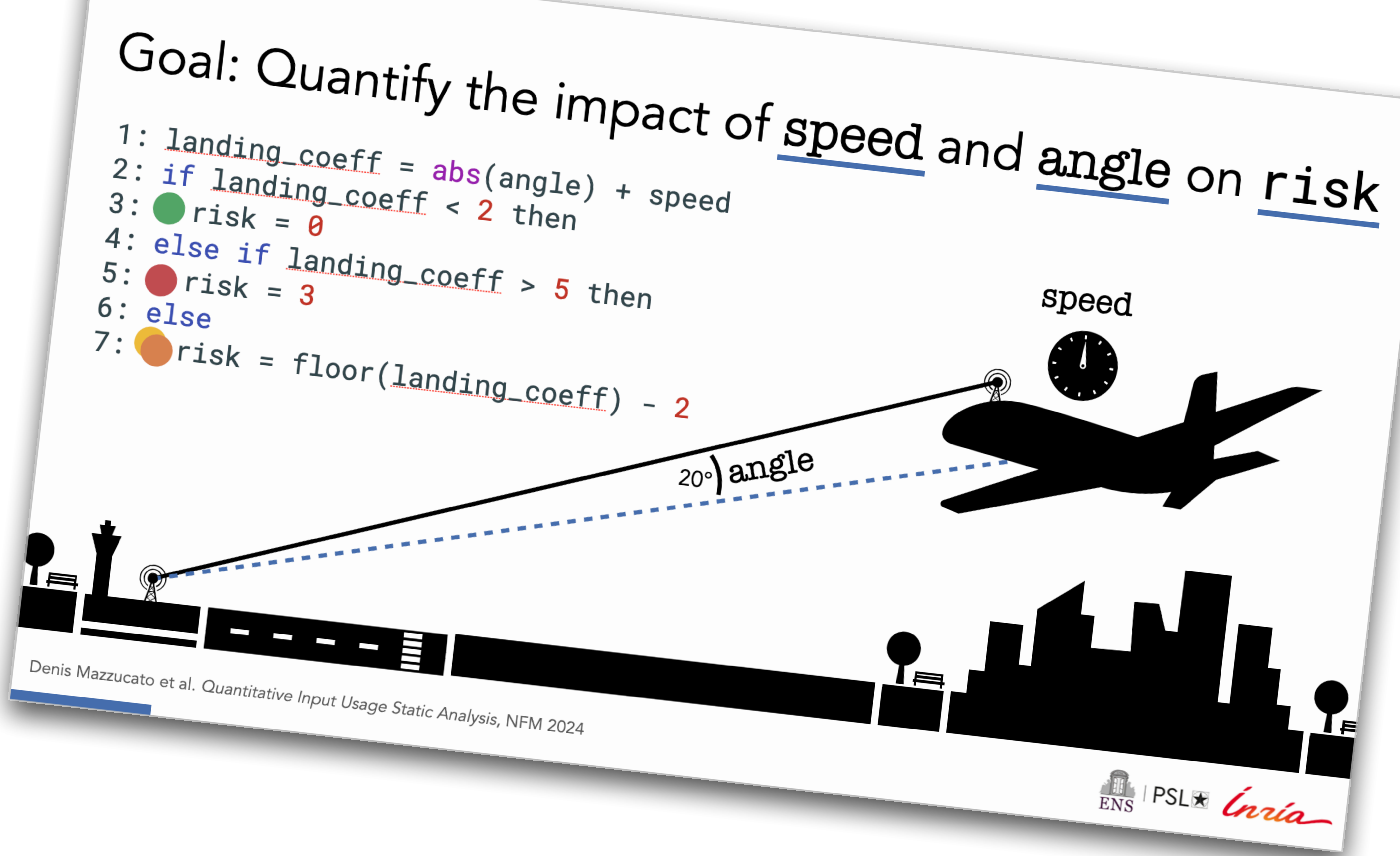
OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l

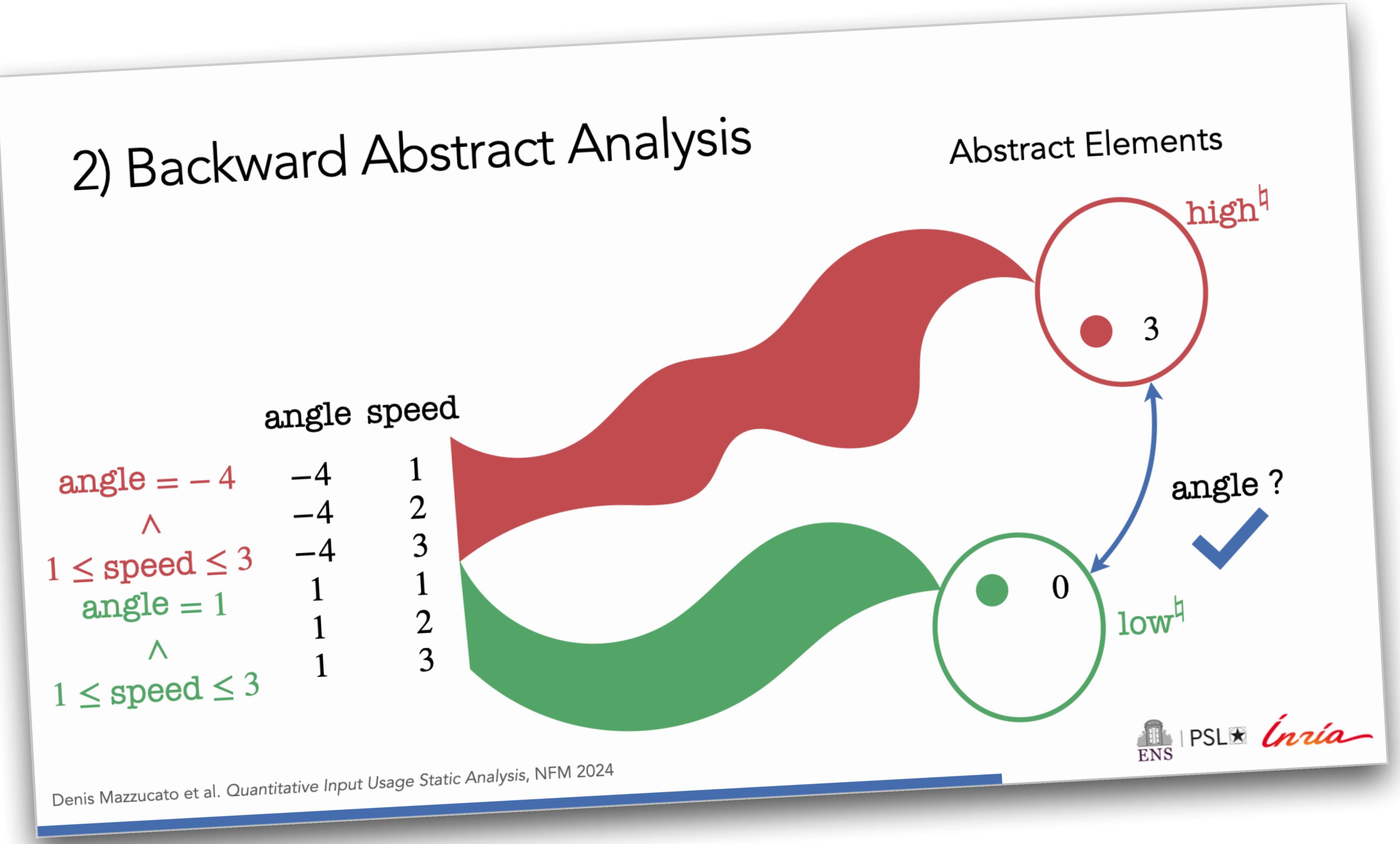
Conclusion



Goal: Quantify the impact of speed and angle on risk
1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3: risk = 0
4: else if landing_coeff > 5 then
5: risk = 3
6: else
7: risk = floor(landing_coeff) - 2
speed
20°
angle
Denis Mazzucato et al. Quantitative Input Usage Static Analysis, NFM 2024

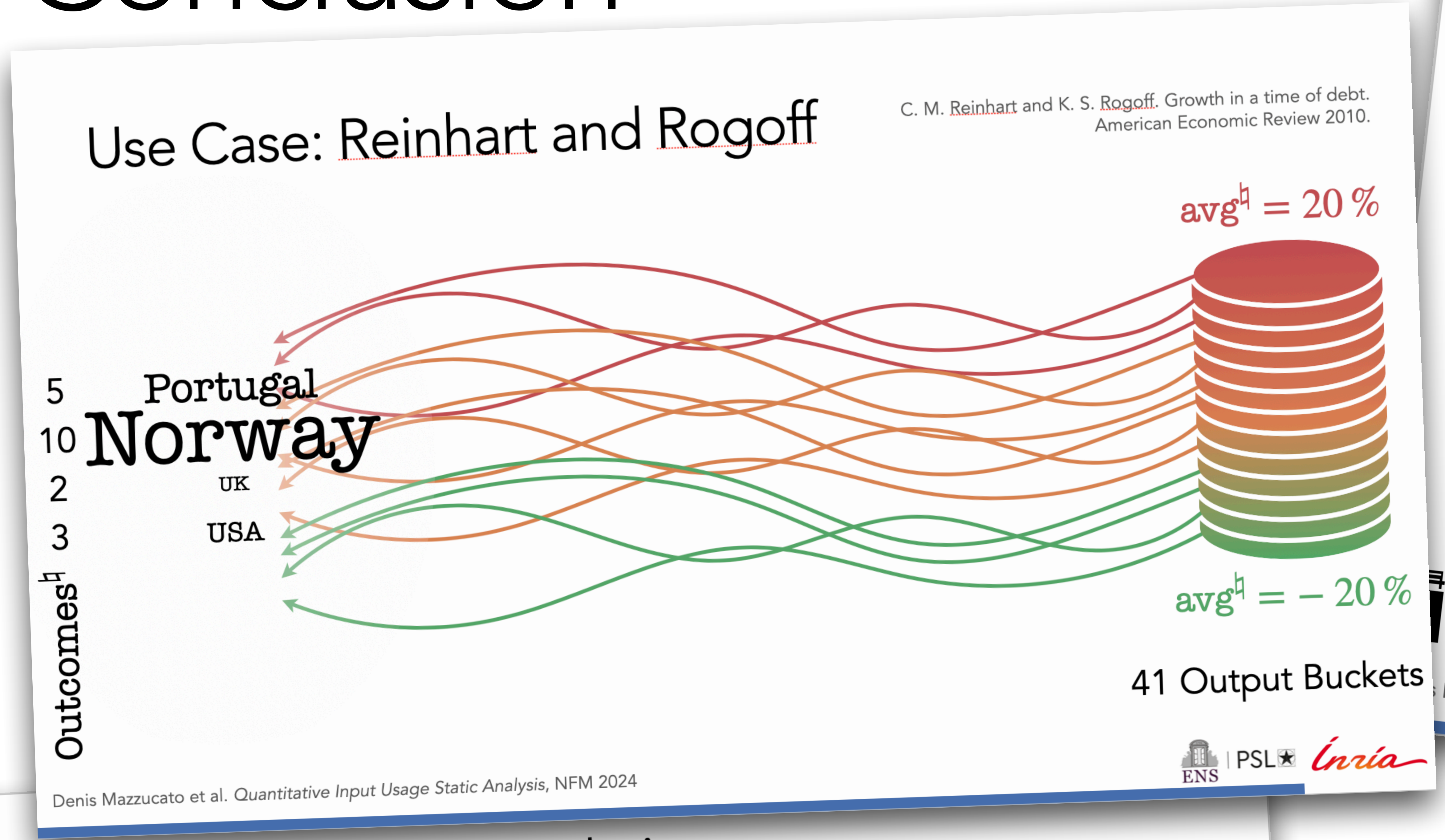
Conclusion

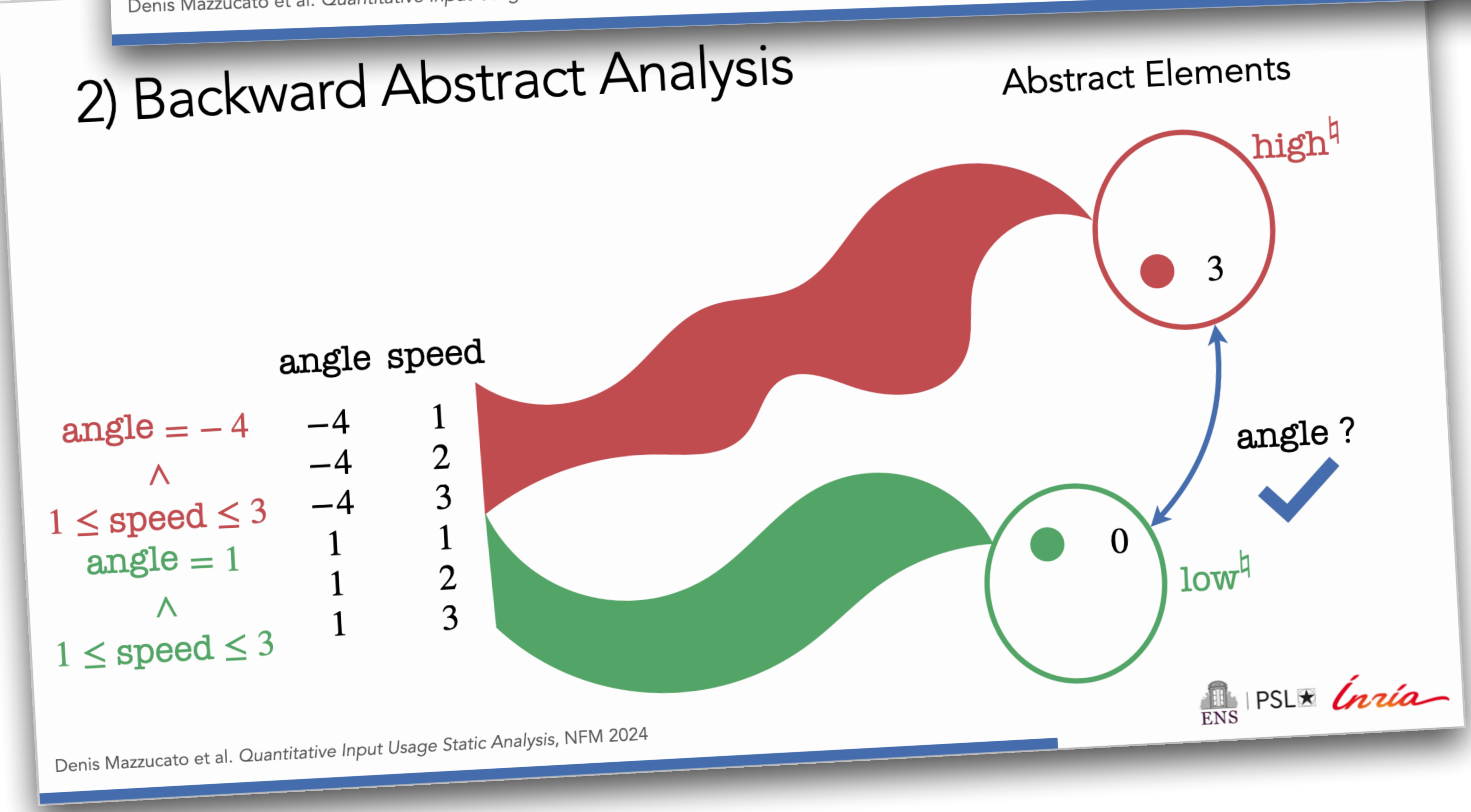


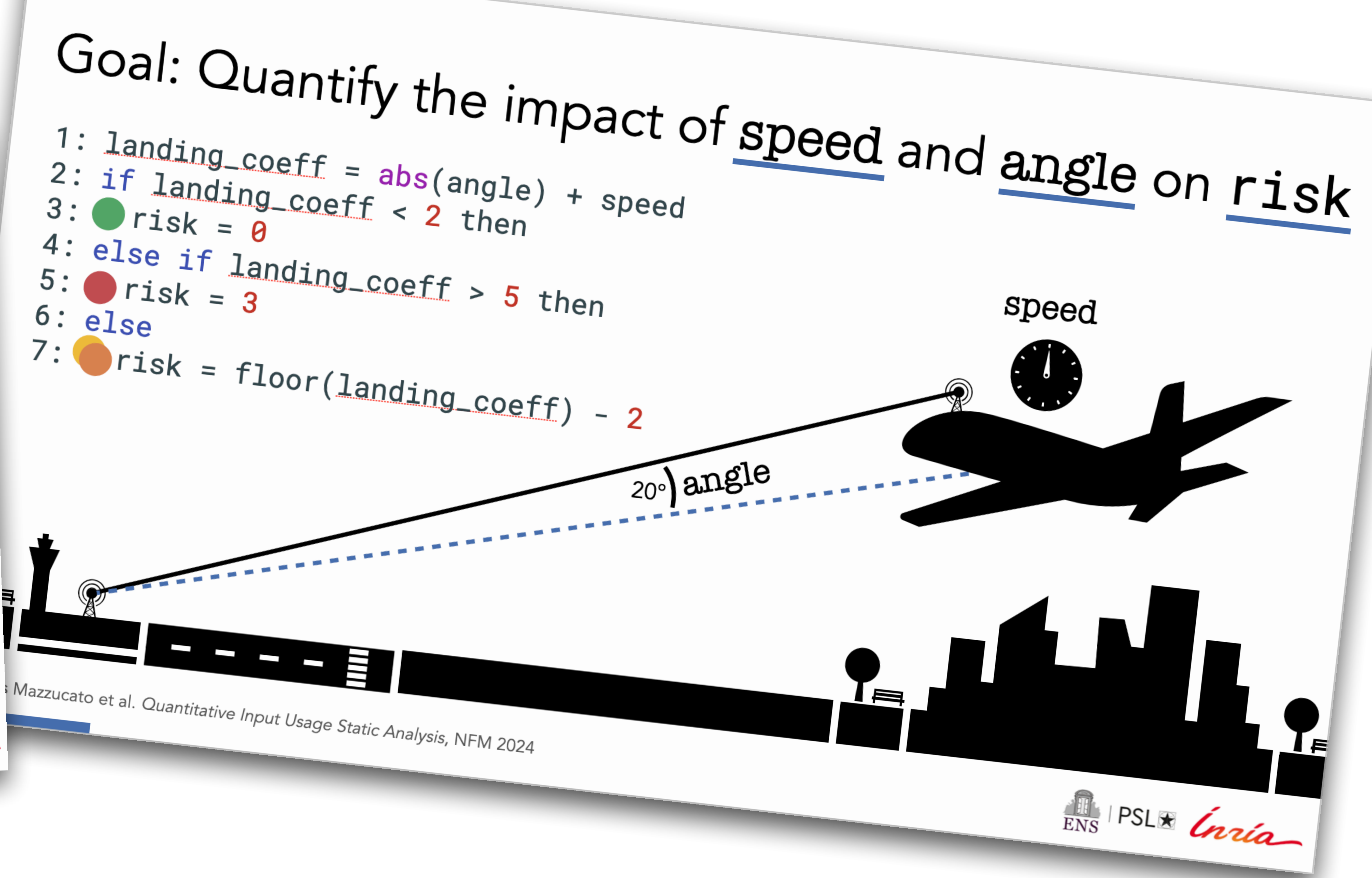


ENS | PSL★ Inria

Conclusion


Use Case: Reinhart and Rogoff
C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.
avg♮ = 20 %
5 Portugal
10 Norway
UK
2
3 USA
Outcomes♮
avg♮ = – 20 %
41 Output Buckets
Denis Mazzucato et al. Quantitative Input Usage Static Analysis, NFM 2024
ENS | PSL★ Inria


2) Backward Abstract Analysis
Abstract Elements
high♮
3
angle speed
angle = – 4
∧
1 ≤ speed ≤ 3
angle = 1
∧
1 ≤ speed ≤ 3
–4 1
–4 2
–4 3
1 1
1 2
1 3
angle ?
0
low♮
Denis Mazzucato et al. Quantitative Input Usage Static Analysis, NFM 2024
ENS | PSL★ Inria


Goal: Quantify the impact of speed and angle on risk
1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3: risk = 0
4: else if landing_coeff > 5 then
5: risk = 3
6: else
7: risk = floor(landing_coeff) - 2
speed
20° angle
Mazzucato et al. Quantitative Input Usage Static Analysis, NFM 2024
ENS | PSL★ Inria

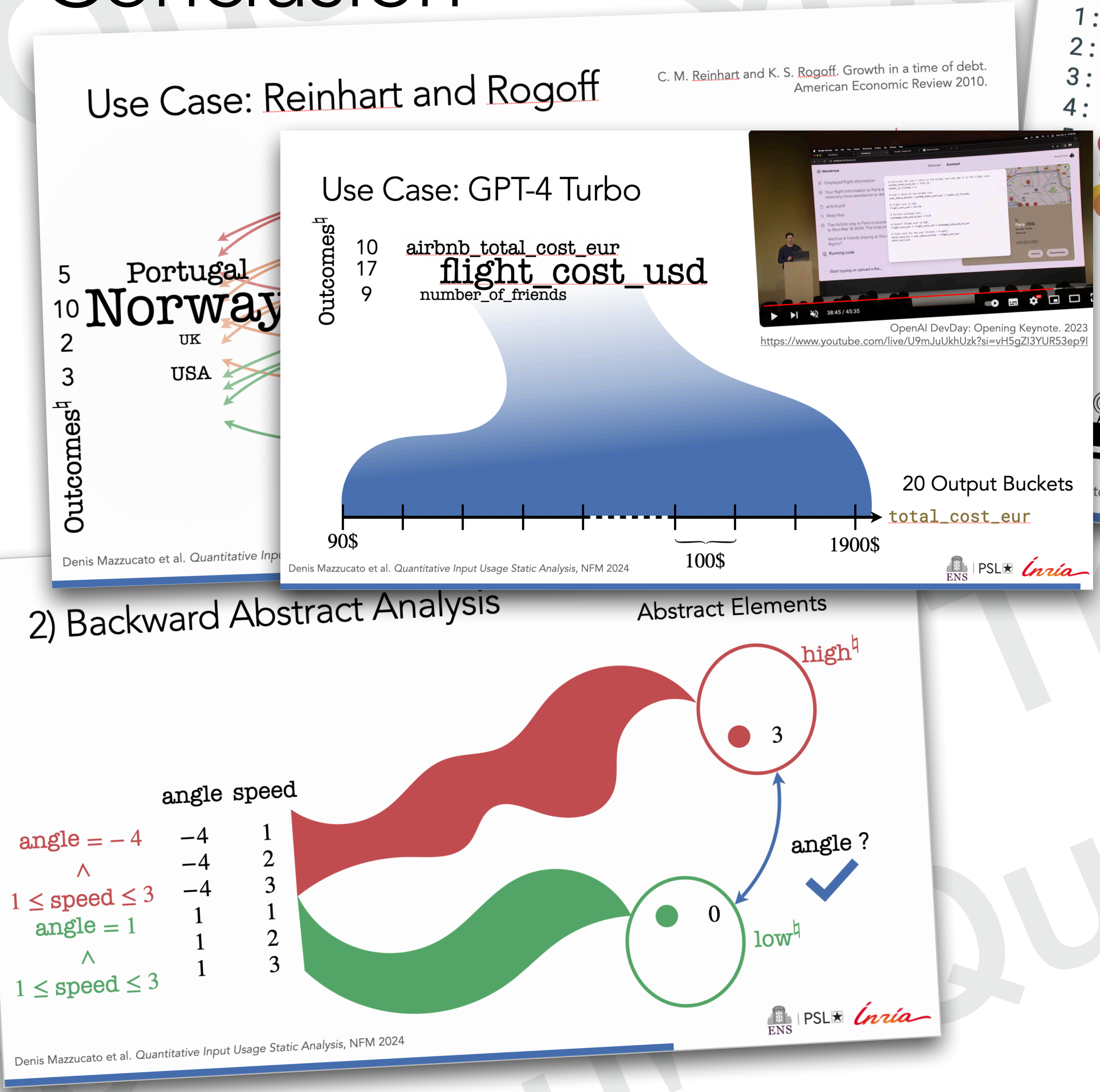
Conclusion

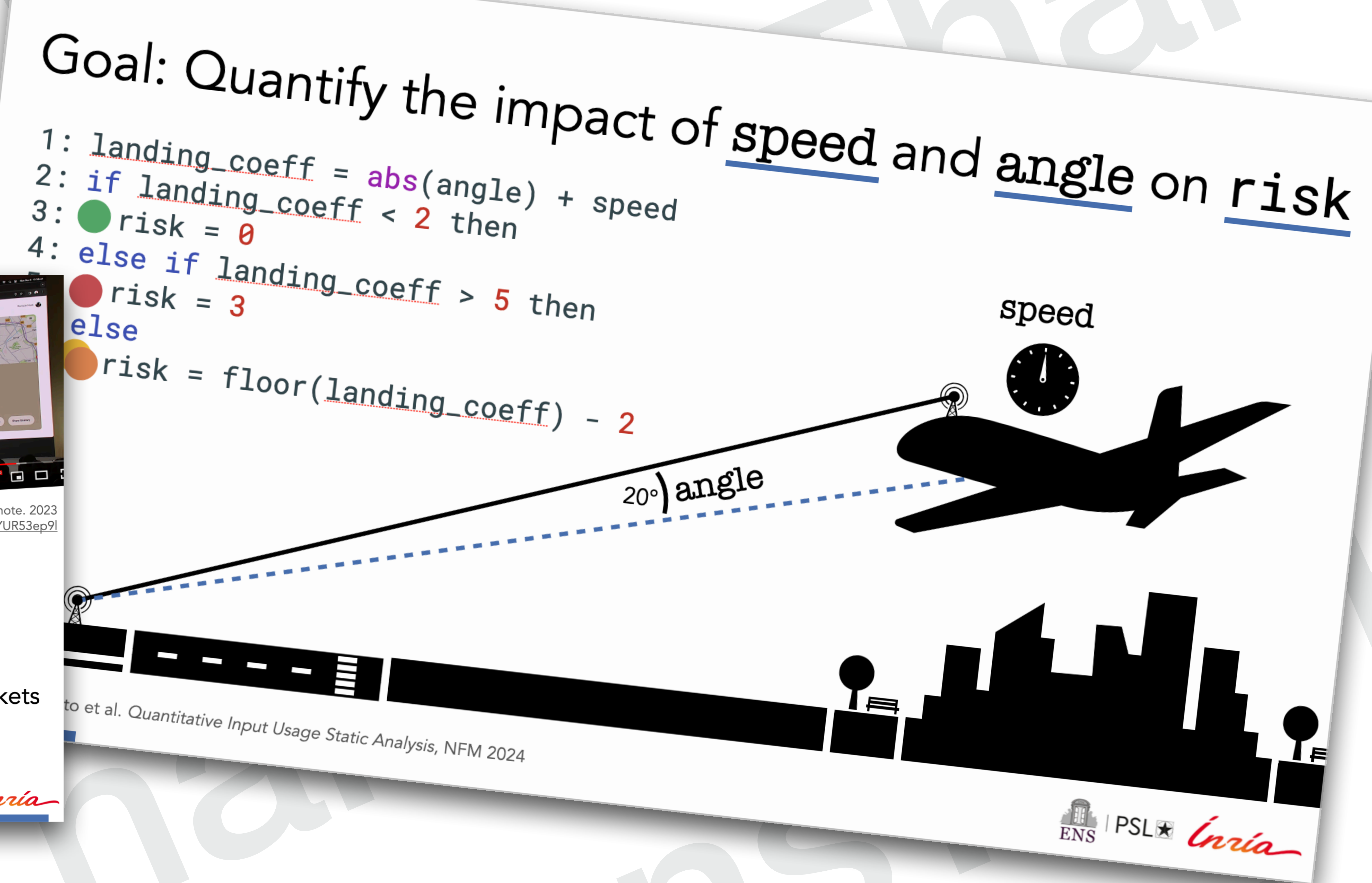
ENS | PSL Inria

Conclusion


Use Case: Reinhart and Rogoff
C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.
Portugal
Norway
UK
USA
5
10
2
3
Outcomes♮
Use Case: GPT-4 Turbo
Outcomes♮
10 airbnb_total_cost_eur
17 flight_cost_usd
9 number_of_friends
20 Output Buckets
total_cost_eur
90$
100$
1900$
OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9l
Goal: Quantify the impact of speed and angle on risk
1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3: risk = 0
4: else if landing_coeff > 5 then
risk = 3
else
risk = floor(landing_coeff) - 2
speed
20° angle
2) Backward Abstract Analysis
Abstract Elements
high♮
3
angle ?
0
low♮
angle speed
angle = −4
∧
1 ≤ speed ≤ 3
angle = 1
∧
1 ≤ speed ≤ 3
−4 1
−4 2
−4 3
1 1
1 2
1 3
Denis Mazzucato et al. Quantitative Input Usage Static Analysis, NFM 2024
Questions?
Thank You!

Conclusion





Future Work

Quantify the Impact on Timing Behavior