

Quantitative Input Usage Static Analysis

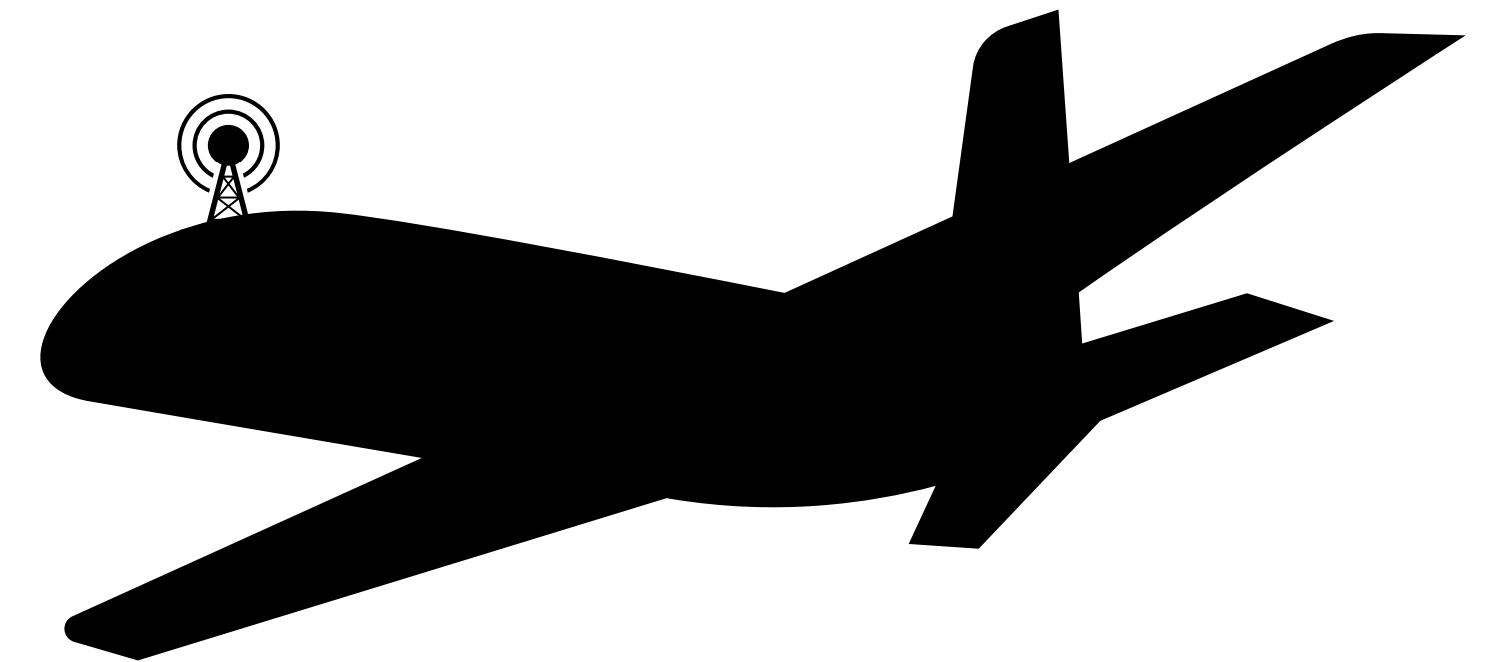
inria



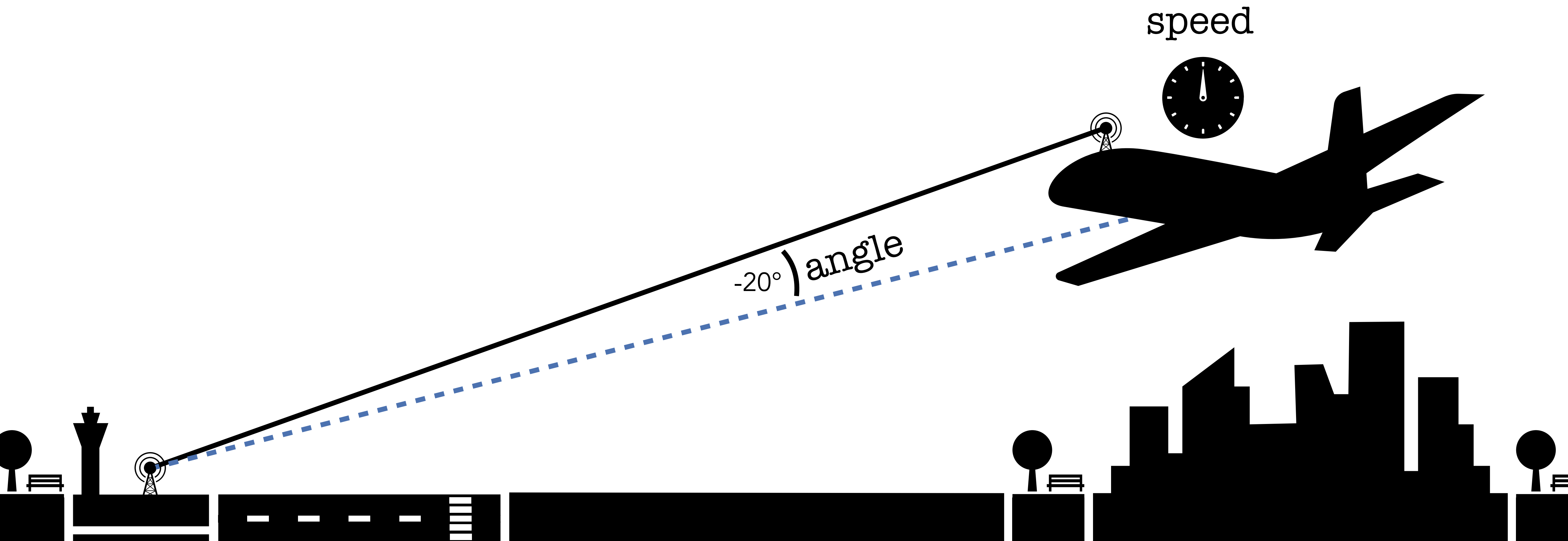
Denis Mazzucato, Marco Campion, and Caterina Urban

4 June 2024

Aircraft Landing Alarm System 🚨

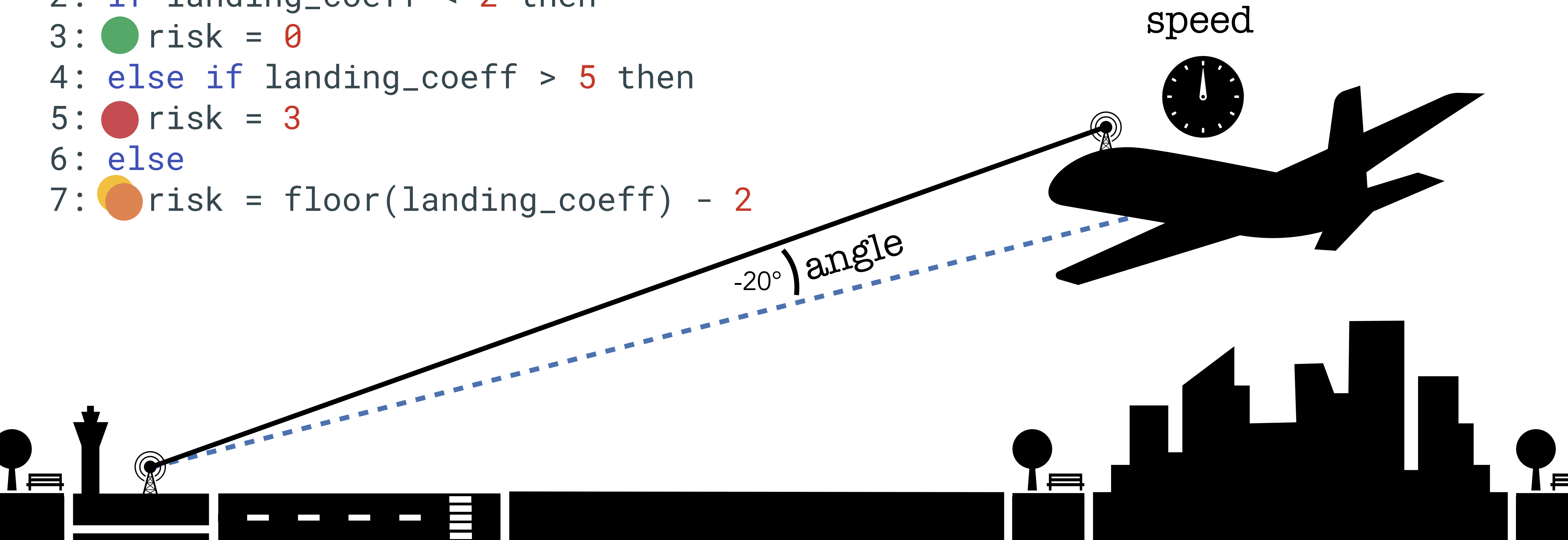


Aircraft Landing Alarm System 🚨



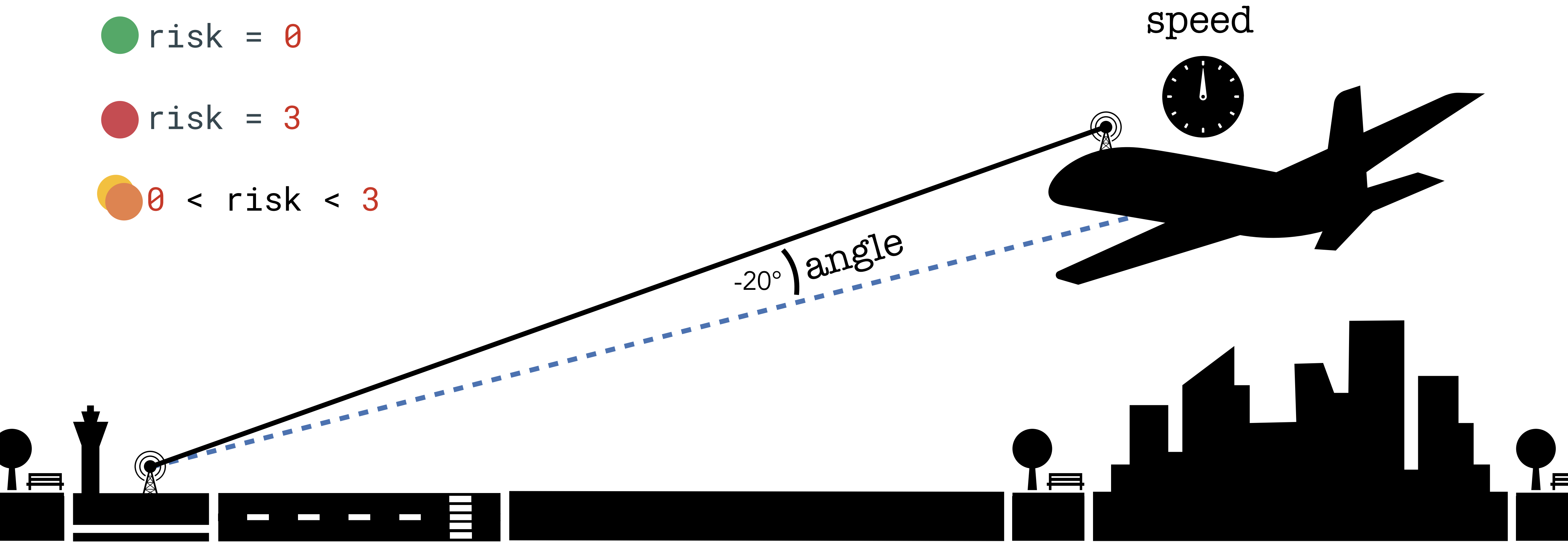
Aircraft Landing Alarm System 🚨

```
1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3:   ● risk = 0
4: else if landing_coeff > 5 then
5:   ● risk = 3
6: else
7:   ● risk = floor(landing_coeff) - 2
```

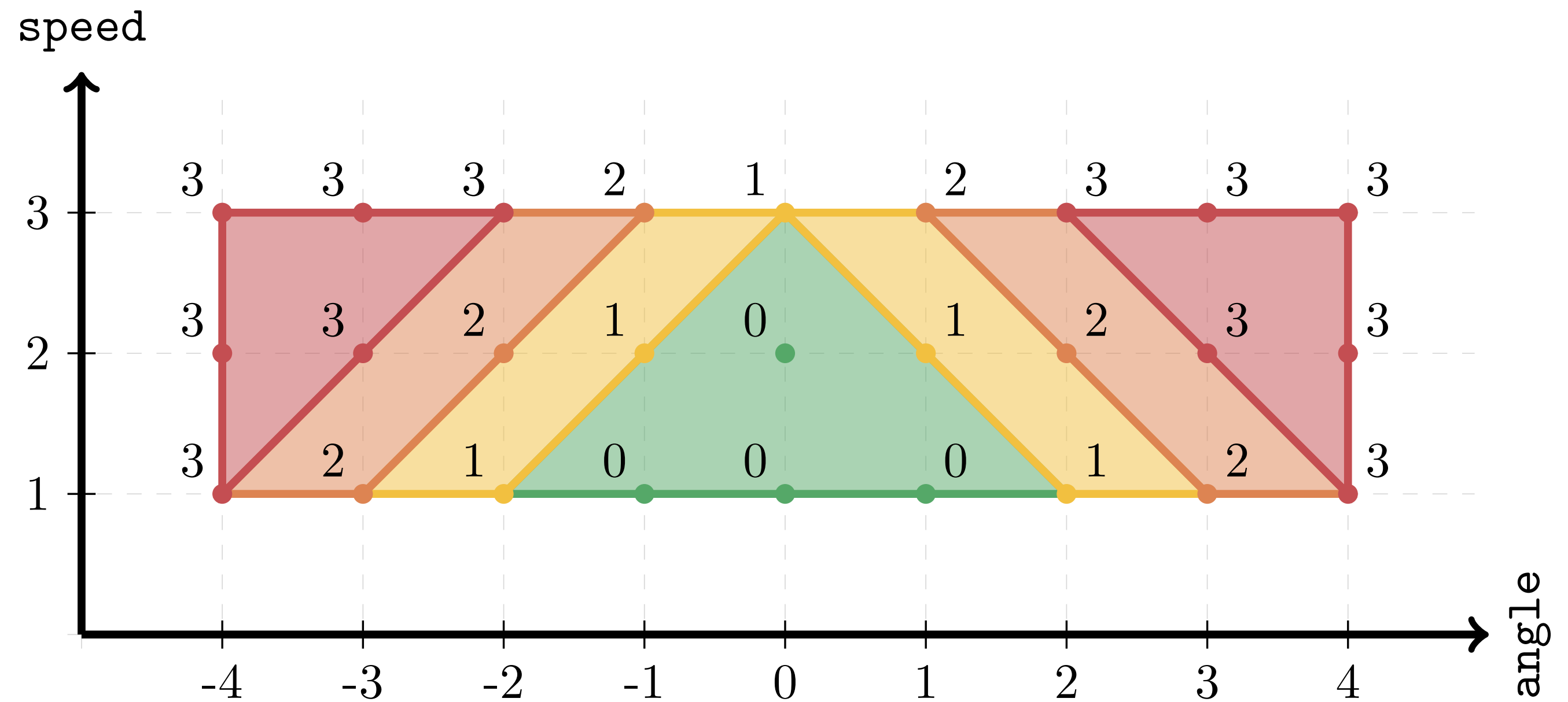


Aircraft Landing Alarm System 🚨

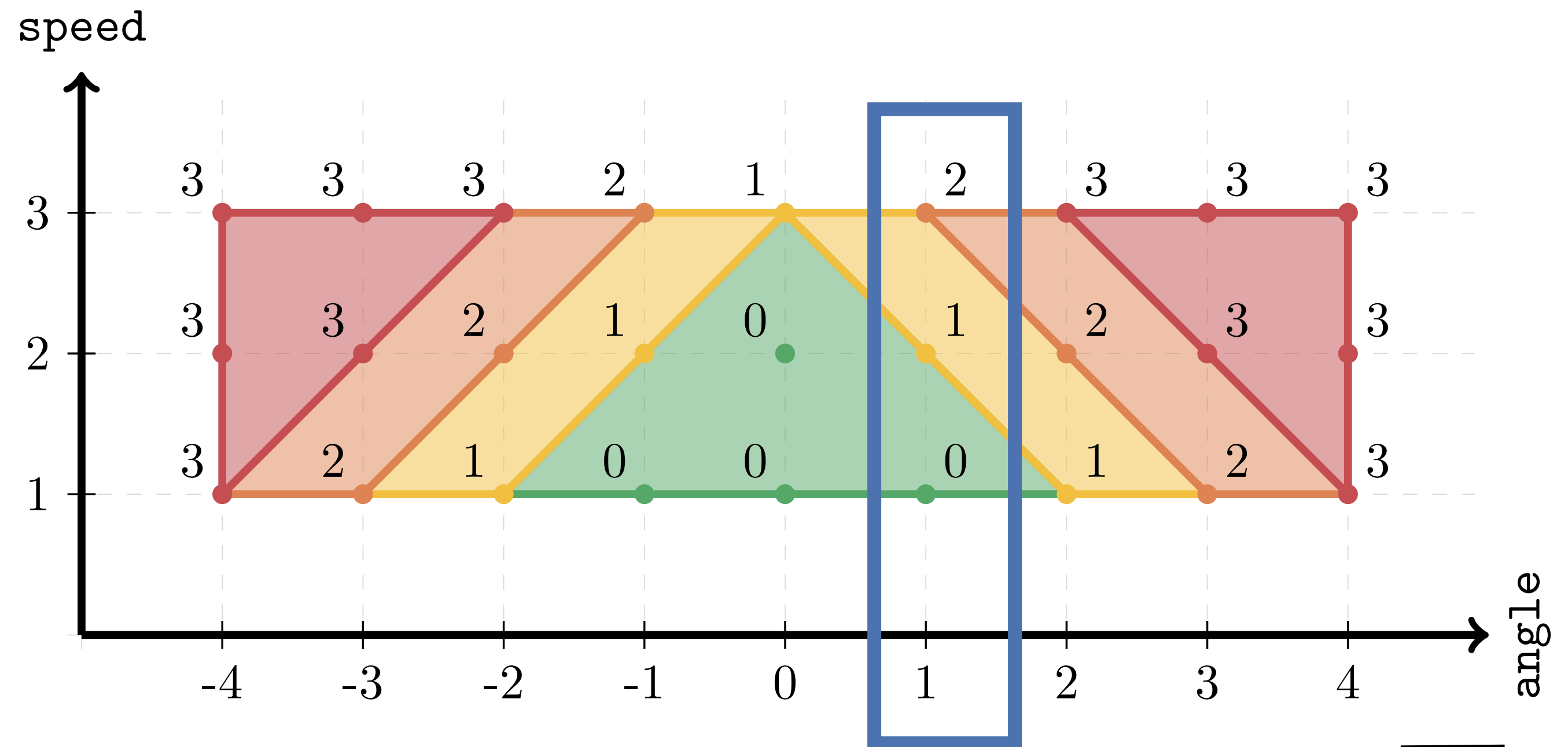
- risk = 0
- risk = 3
- 0 < risk < 3



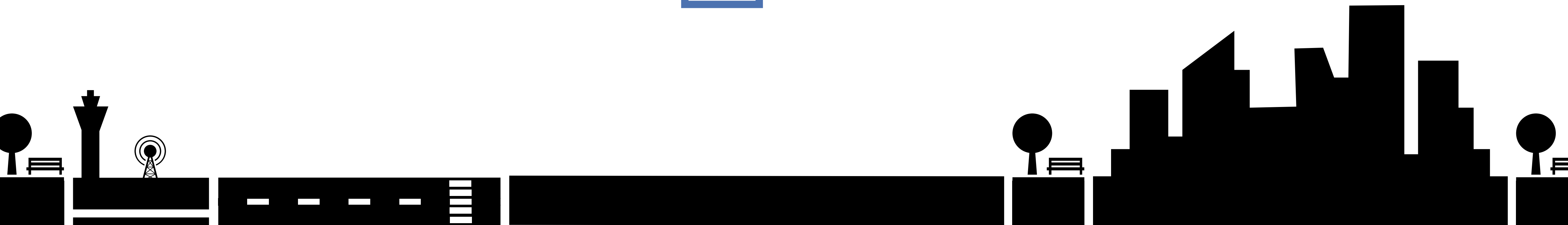
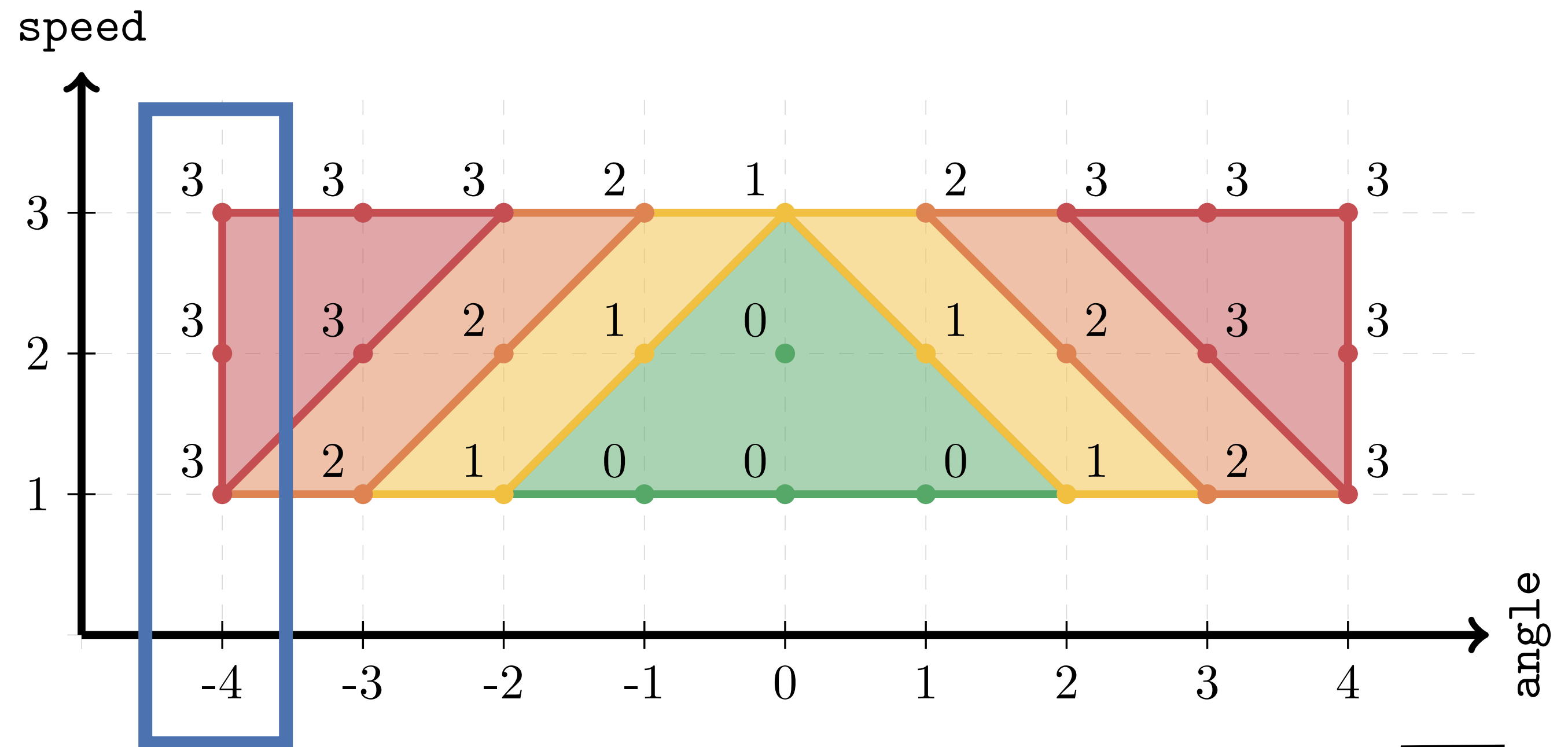
Aircraft Landing Alarm System 🚨



Aircraft Landing Alarm System 🚨

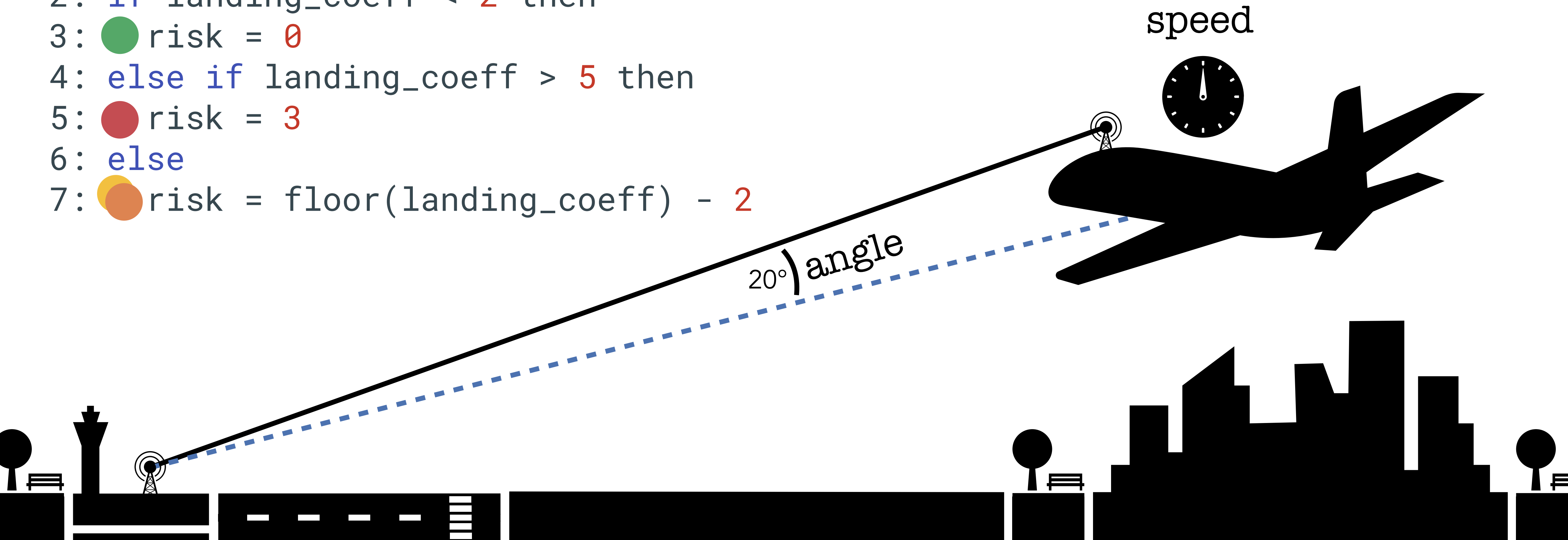


Aircraft Landing Alarm System 🚨

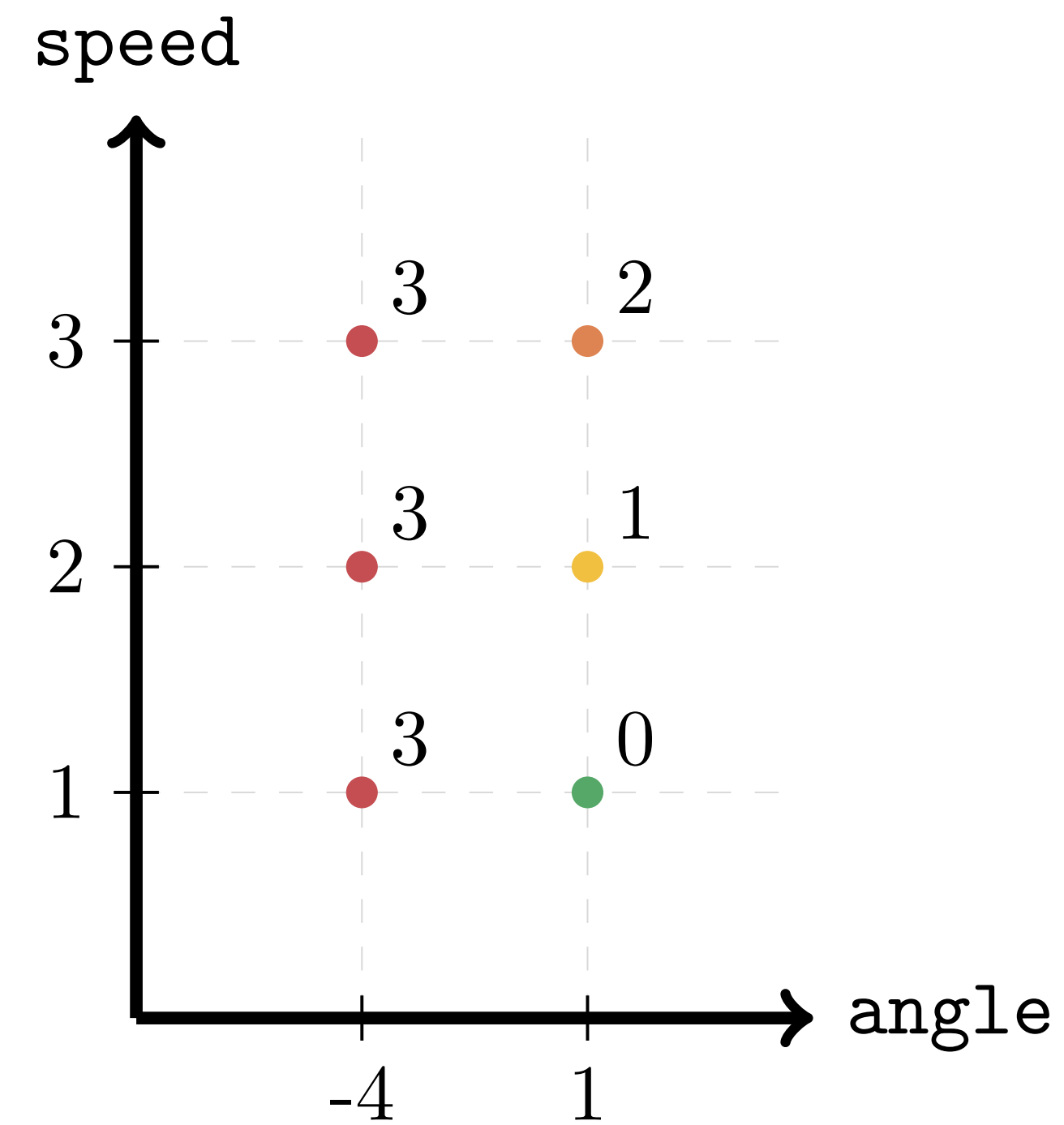


Goal: Quantify the impact of speed and angle on risk

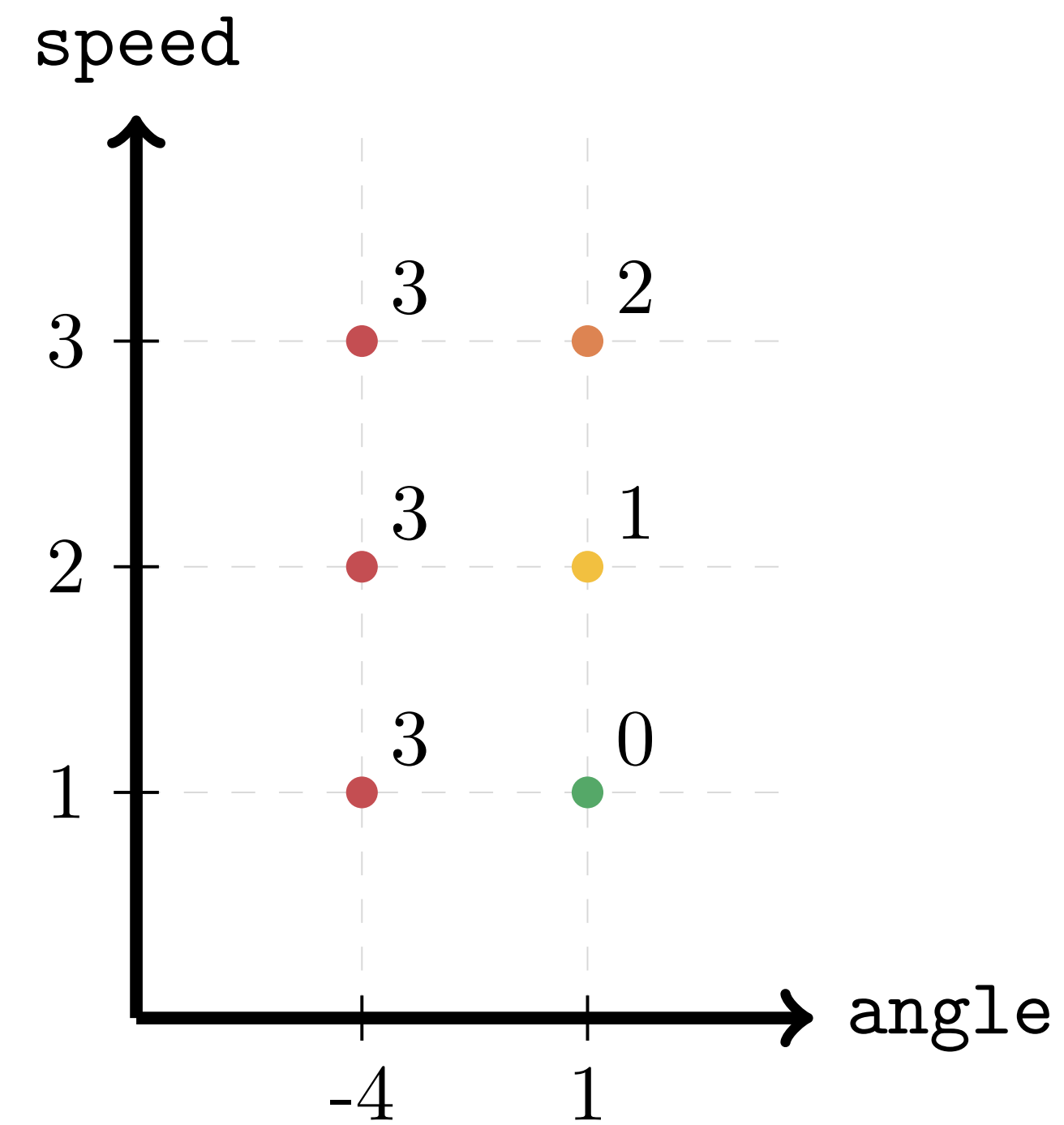
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```



Goal: Quantify the impact of speed and angle on risk



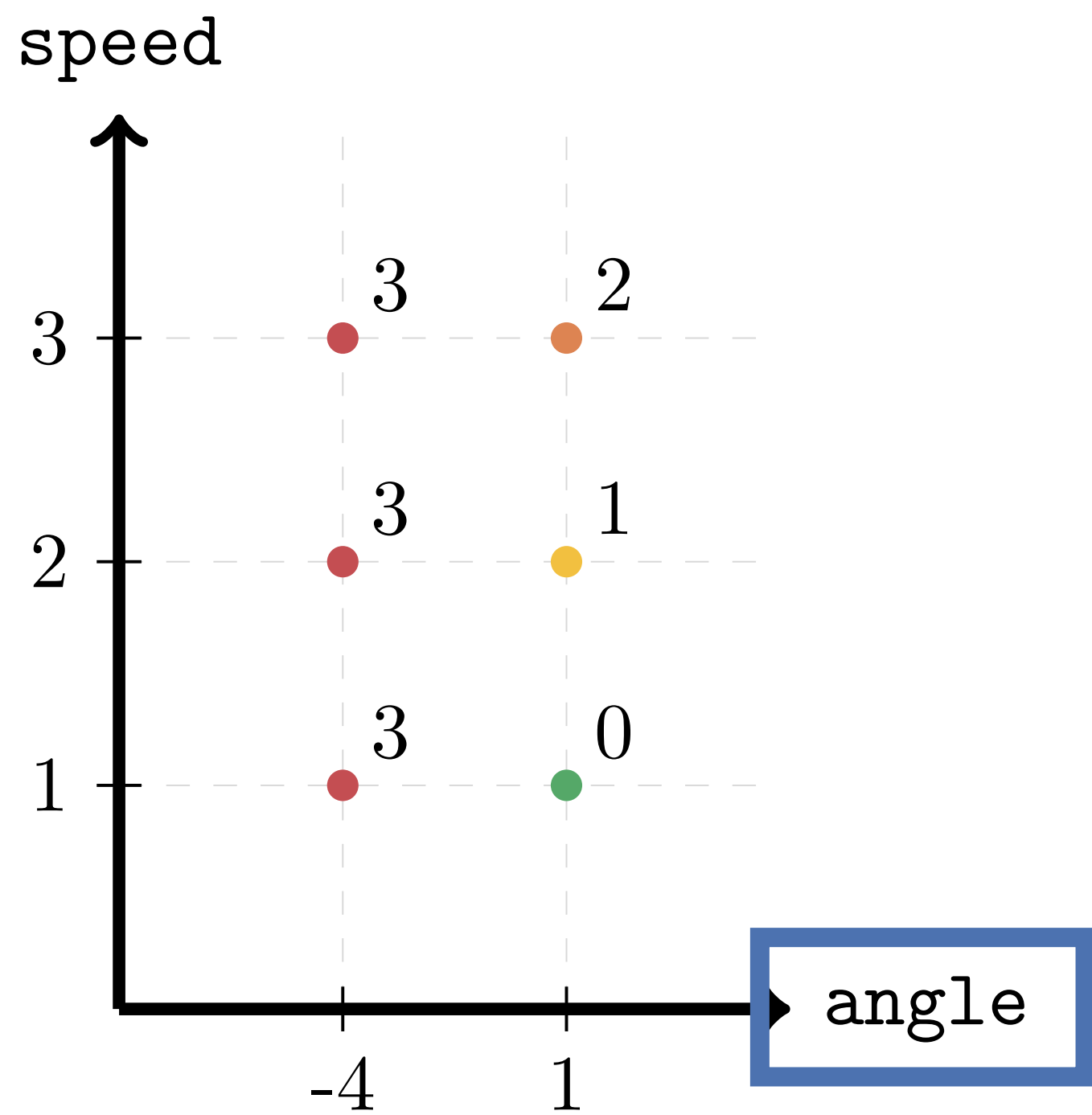
Goal: Quantify the impact of speed and angle on risk



Number of reachable outcomes



Goal: Quantify the impact of speed and angle on risk



Number of reachable outcomes

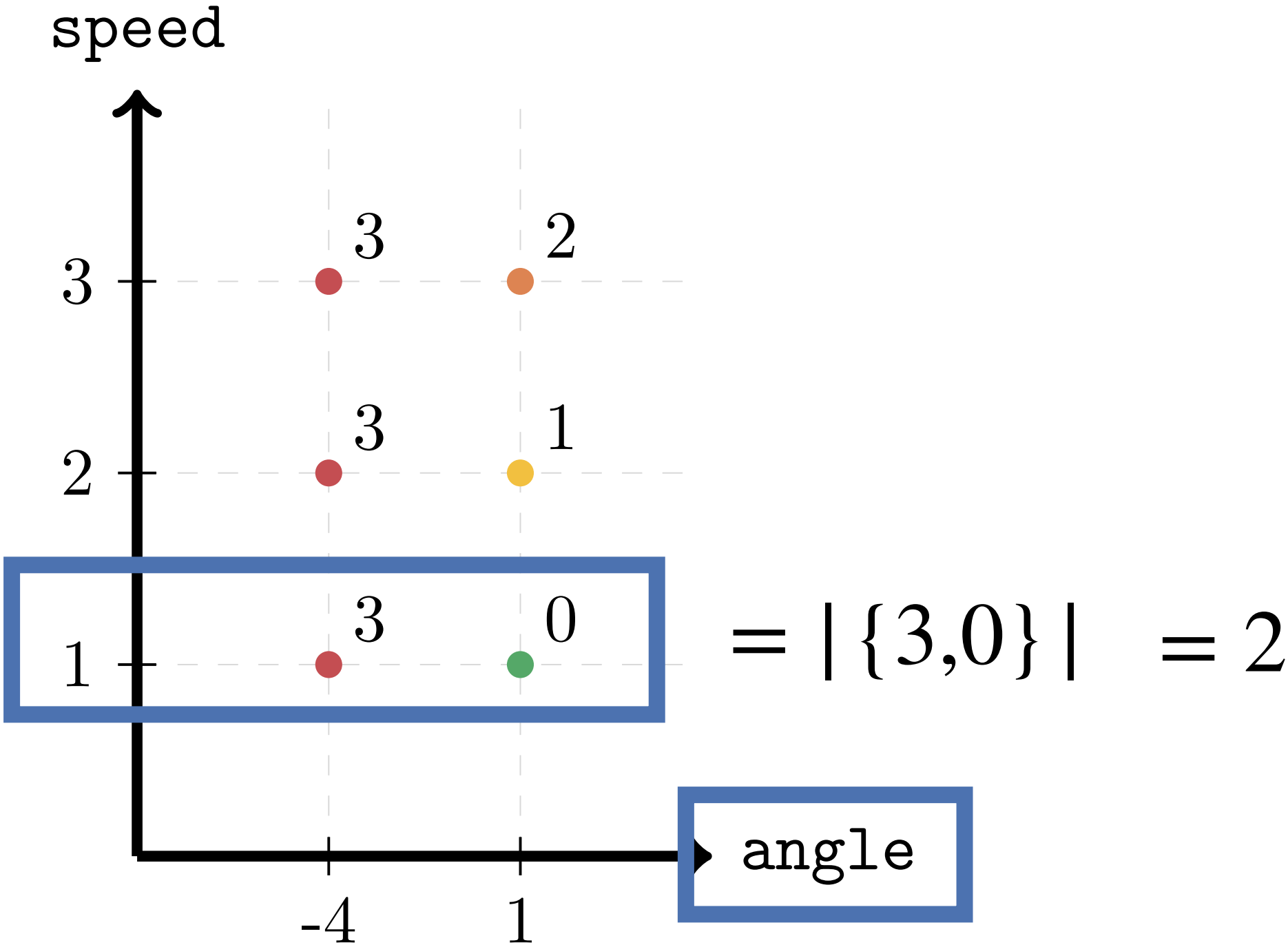
OUTCOMES



Goal: Quantify the impact of speed and angle on risk

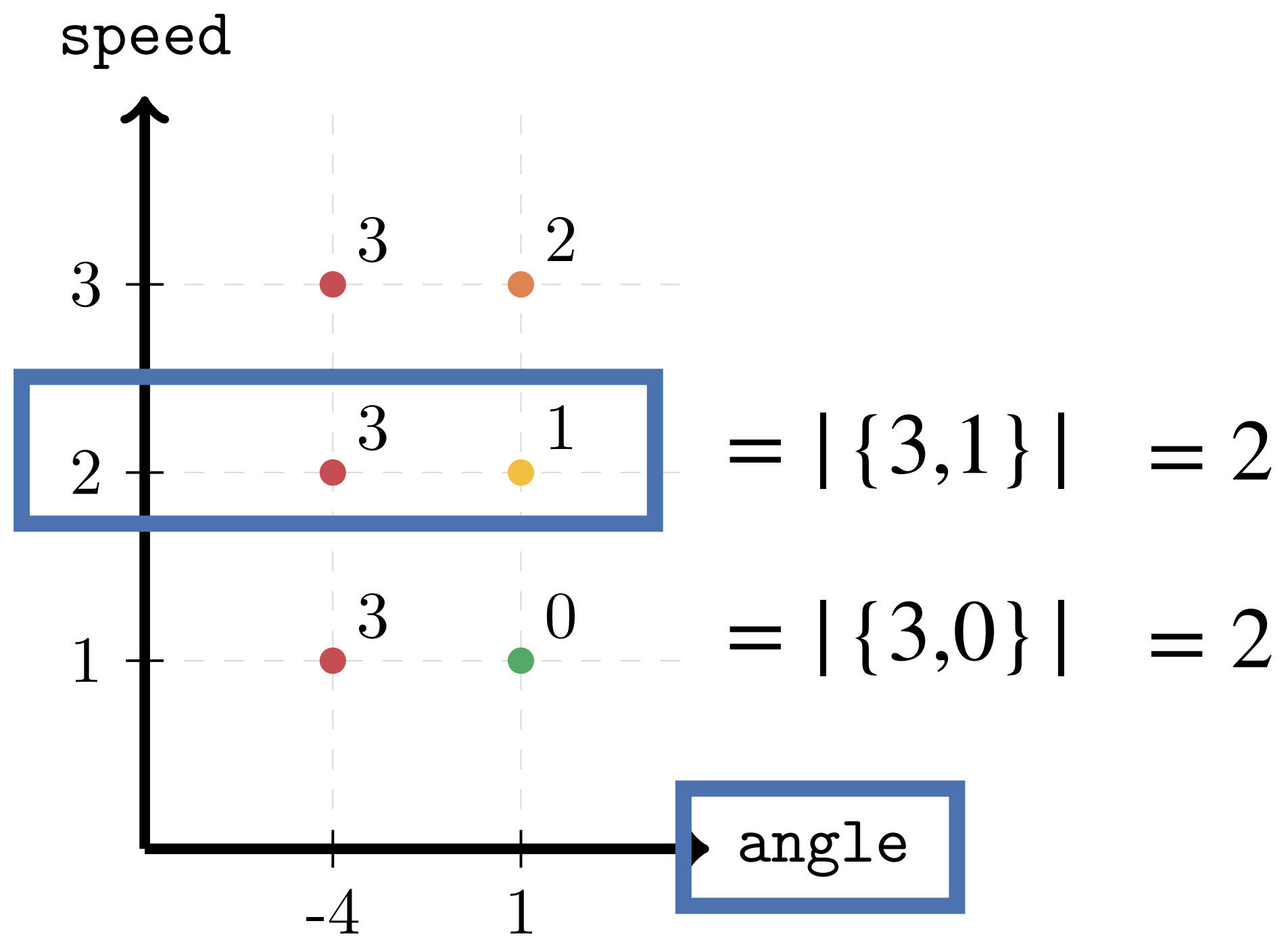
Number of reachable outcomes

OUTCOMES



Goal: Quantify the impact of speed and angle on risk

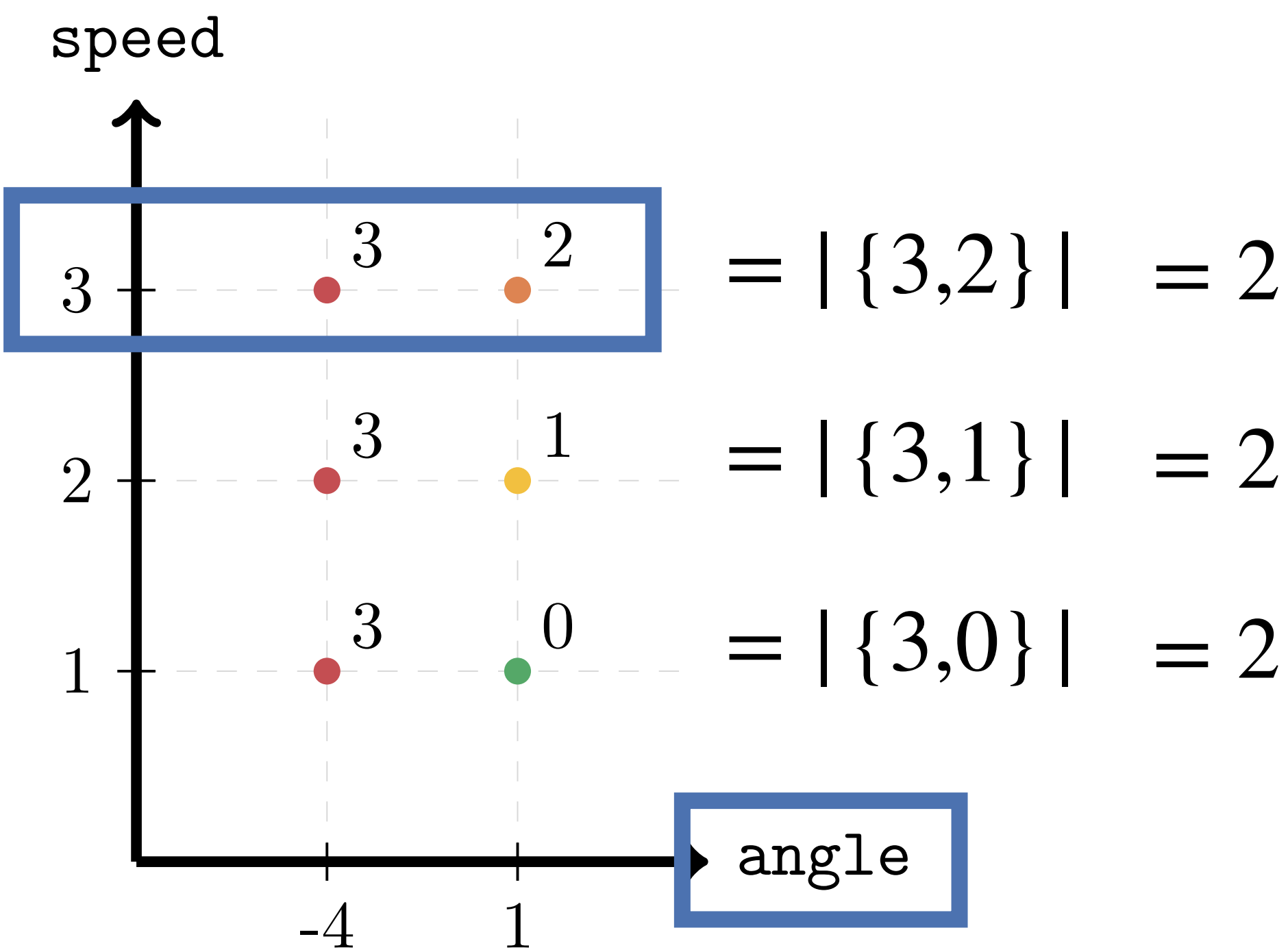
Number of reachable outcomes



OUTCOMES



Goal: Quantify the impact of speed and angle on risk

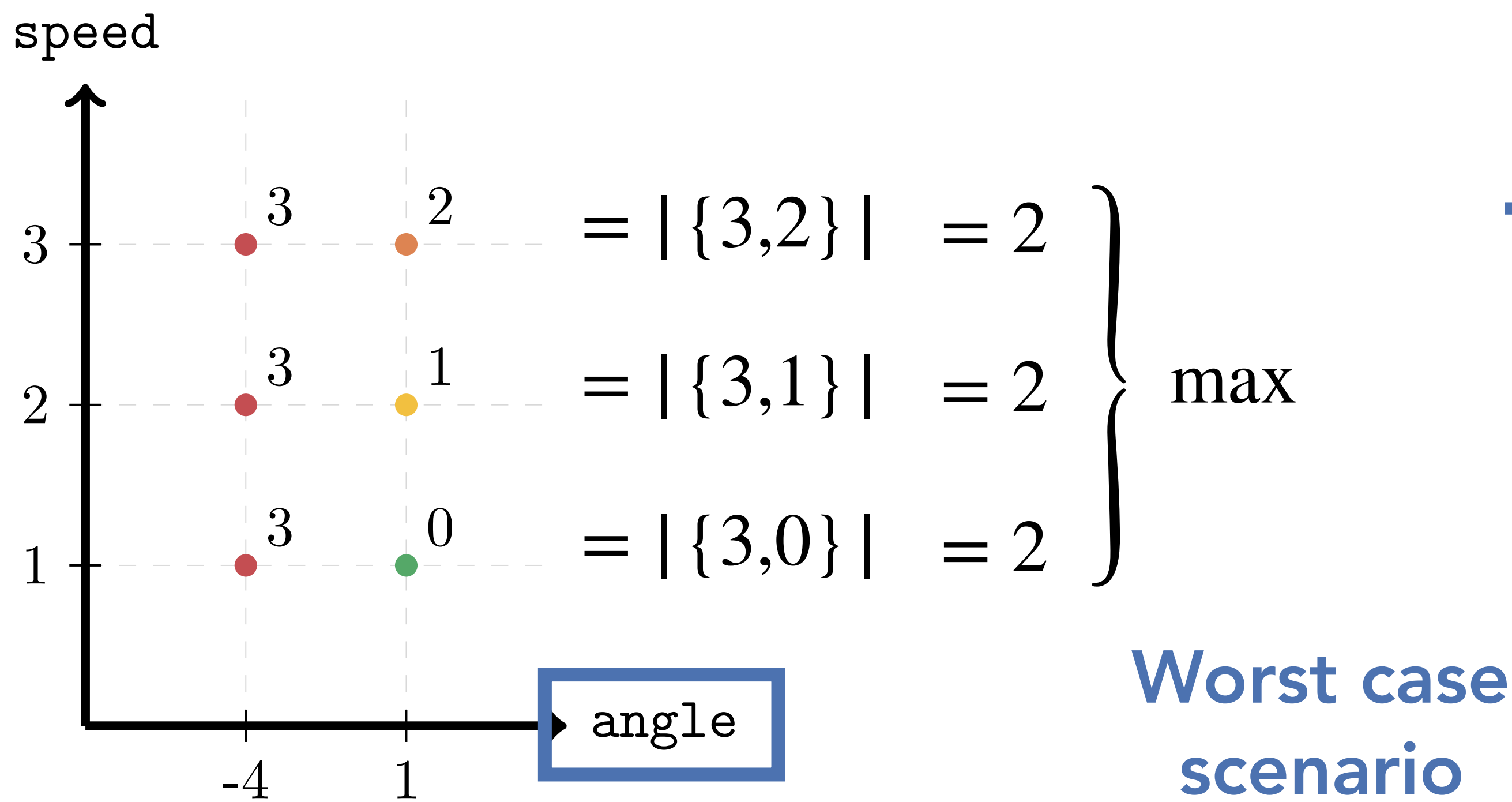


Number of reachable outcomes

OUTCOMES



Goal: Quantify the impact of speed and angle on risk



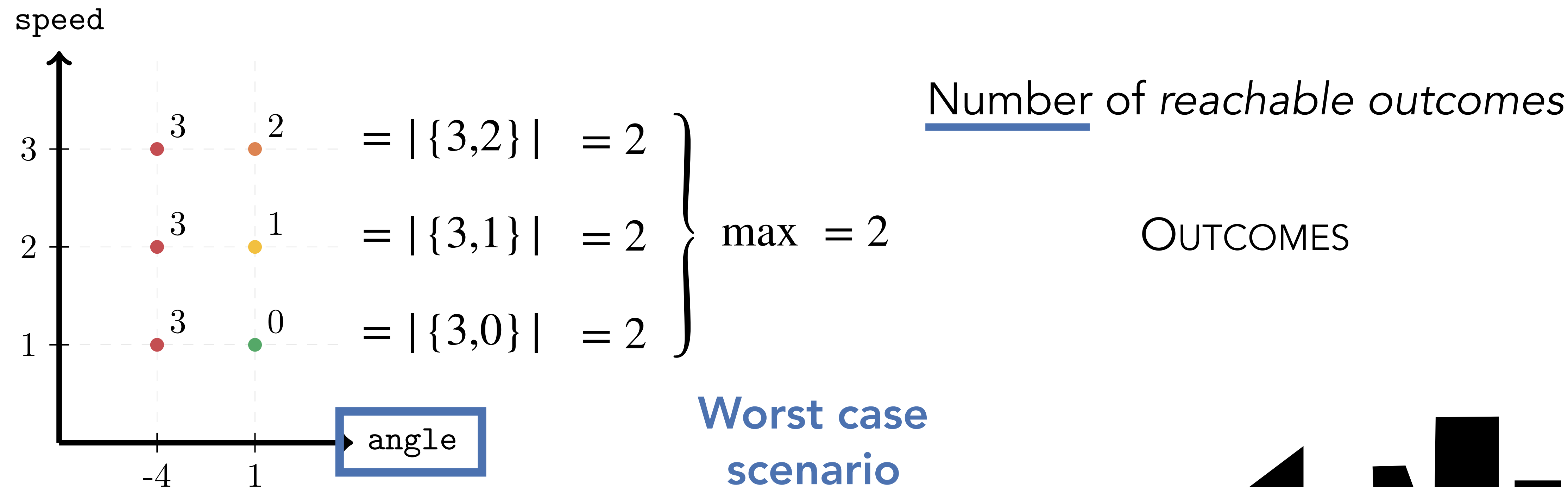
Number of reachable outcomes

OUTCOMES

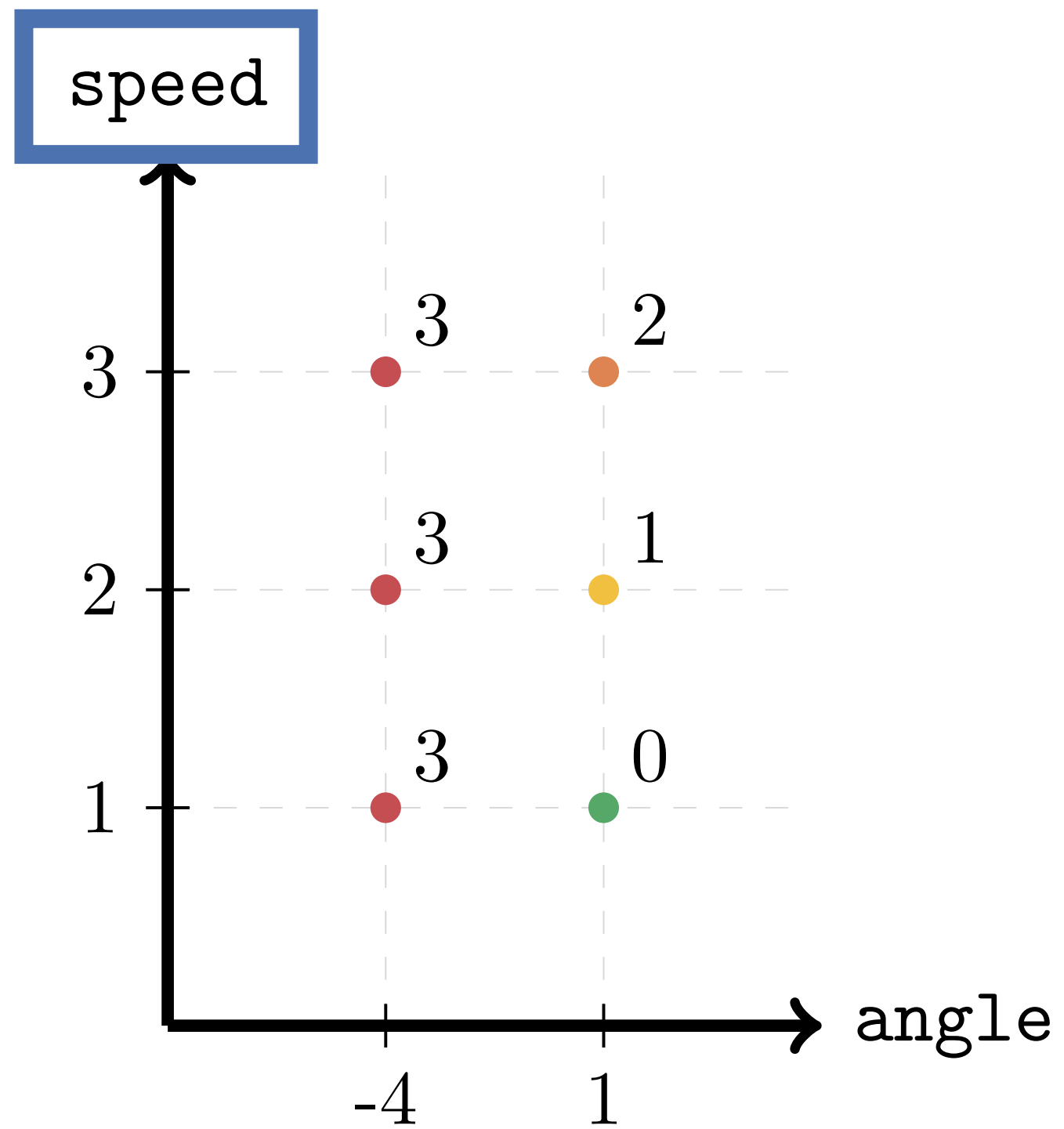
Worst case
scenario



Goal: Quantify the impact of speed and angle on risk



Goal: Quantify the impact of speed and angle on risk

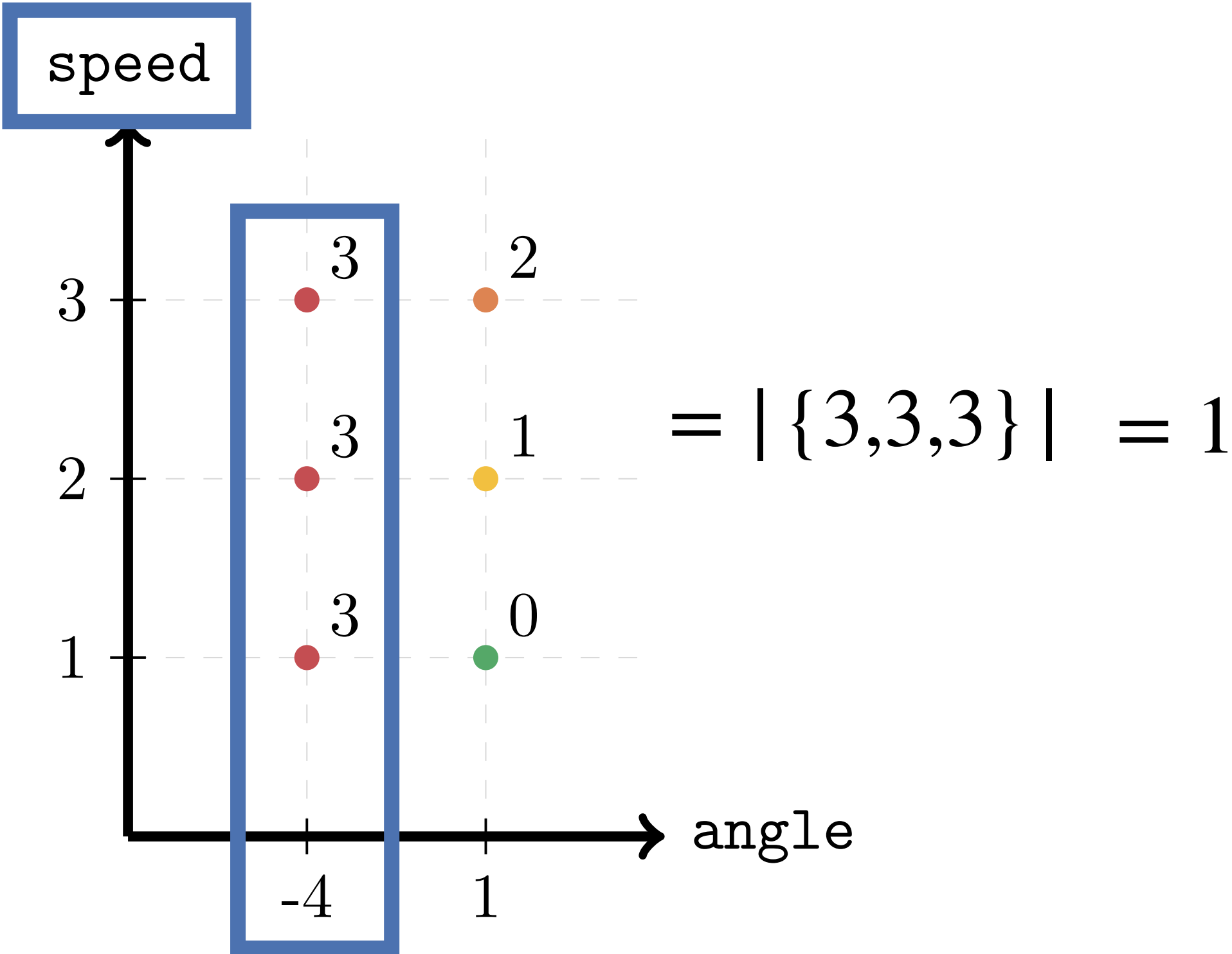


Number of reachable outcomes

OUTCOMES



Goal: Quantify the impact of speed and angle on risk

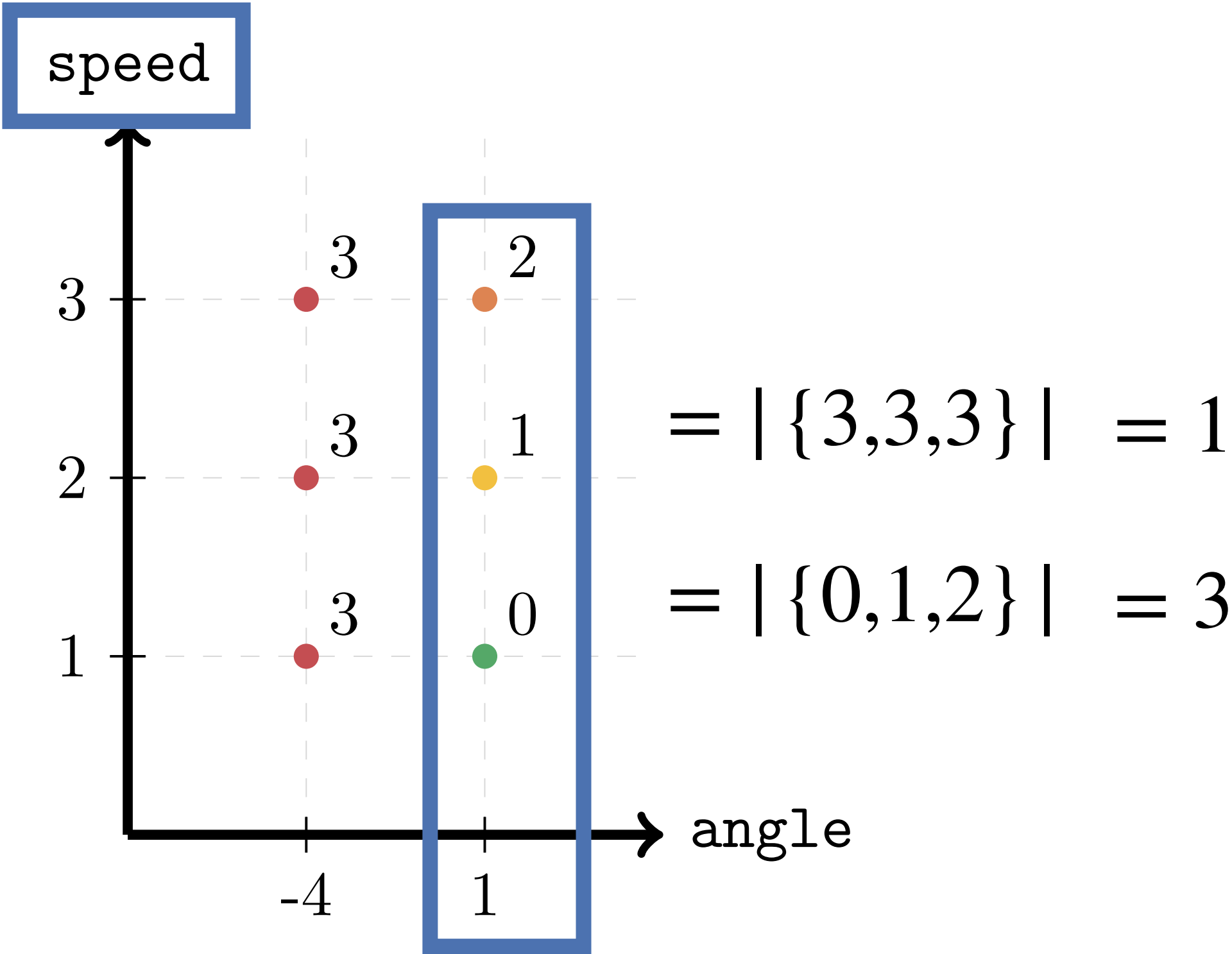


Number of reachable outcomes

OUTCOMES



Goal: Quantify the impact of speed and angle on risk

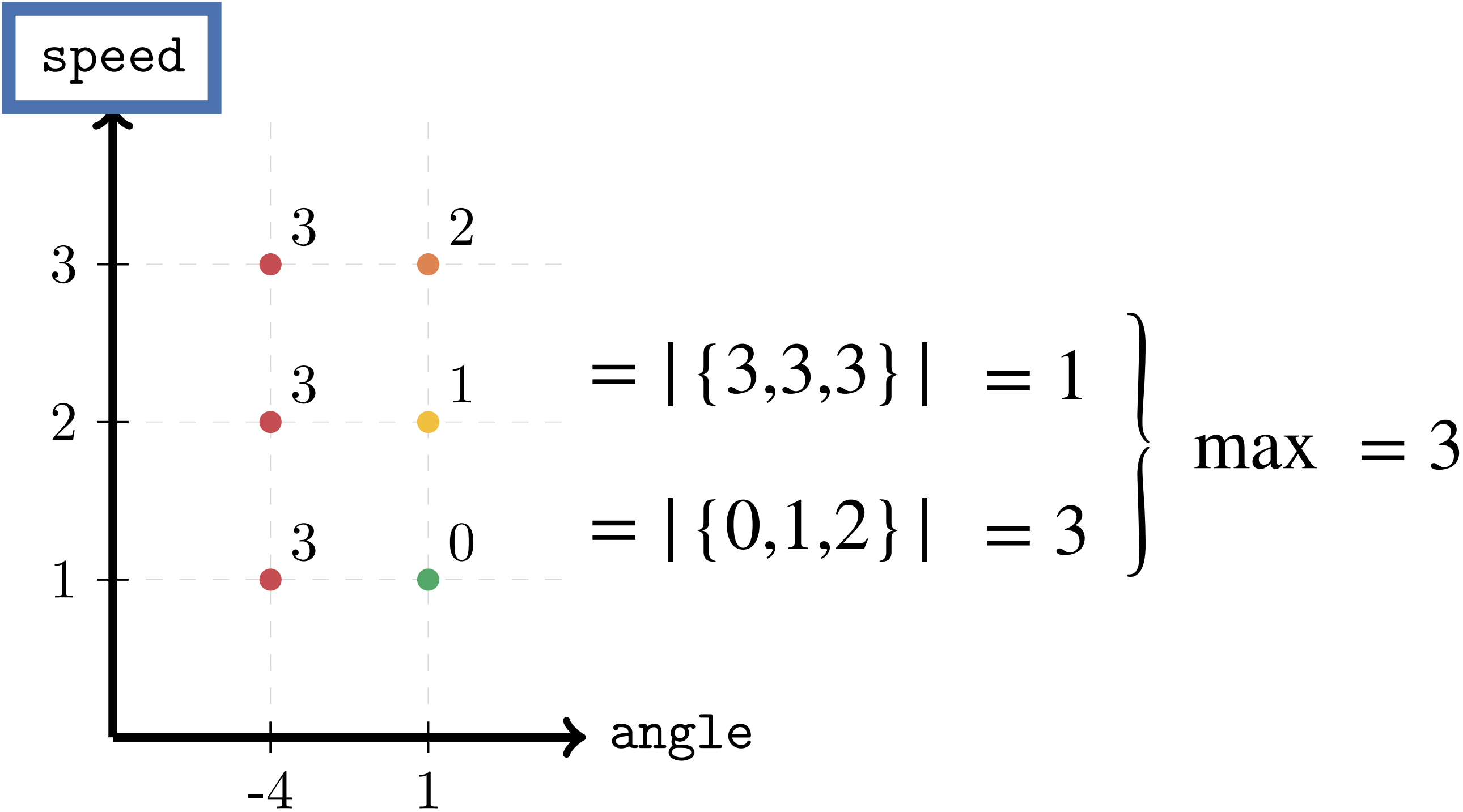


Number of reachable outcomes

OUTCOMES



Goal: Quantify the impact of speed and angle on risk



Number of reachable outcomes

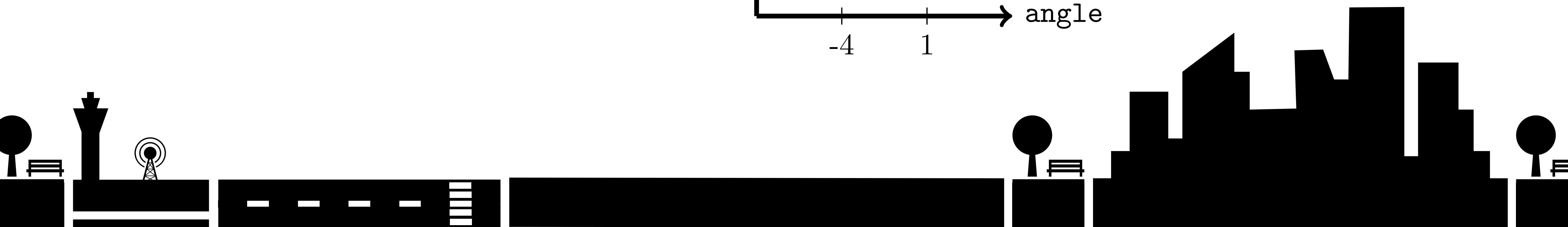
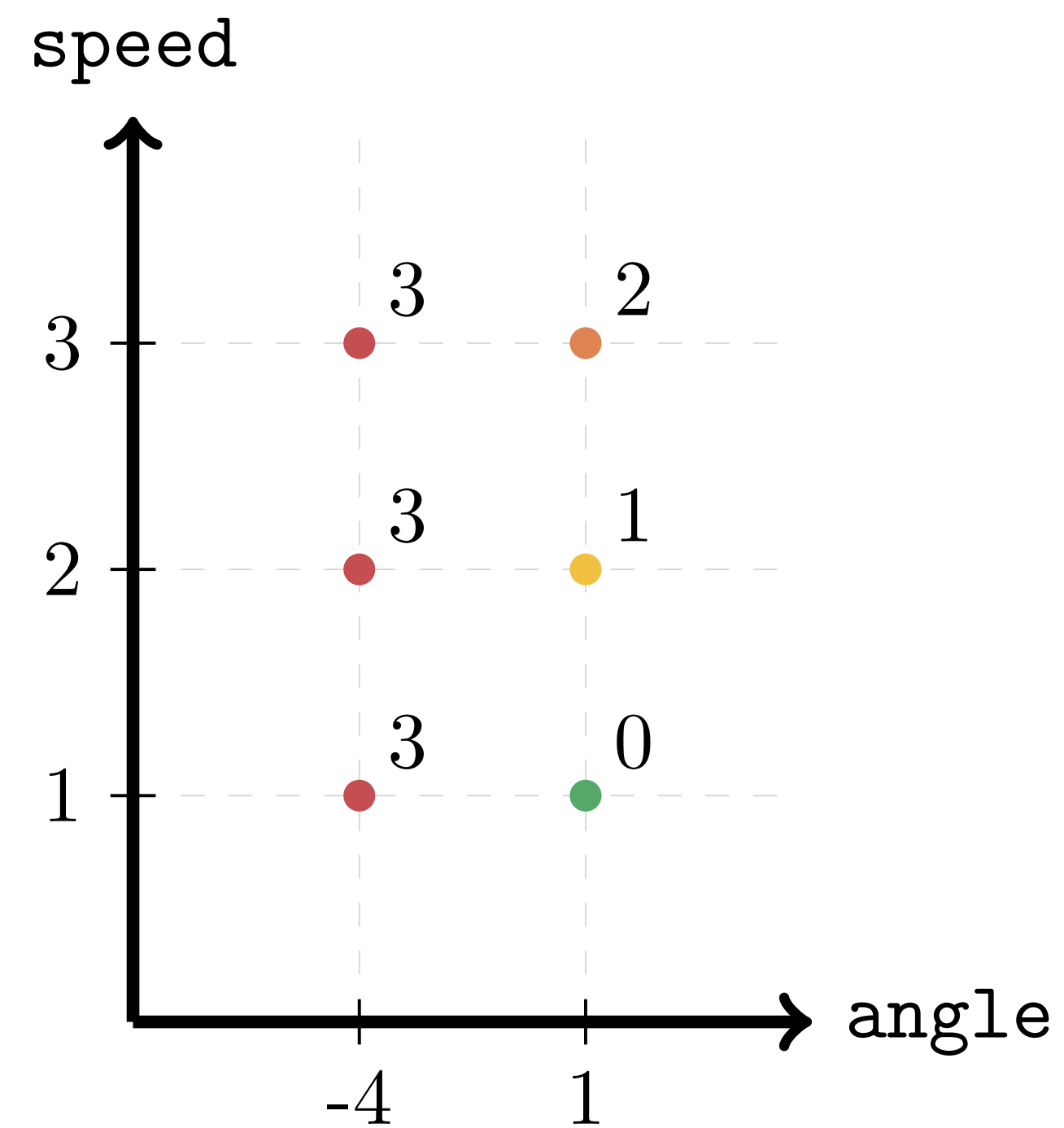
OUTCOMES



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

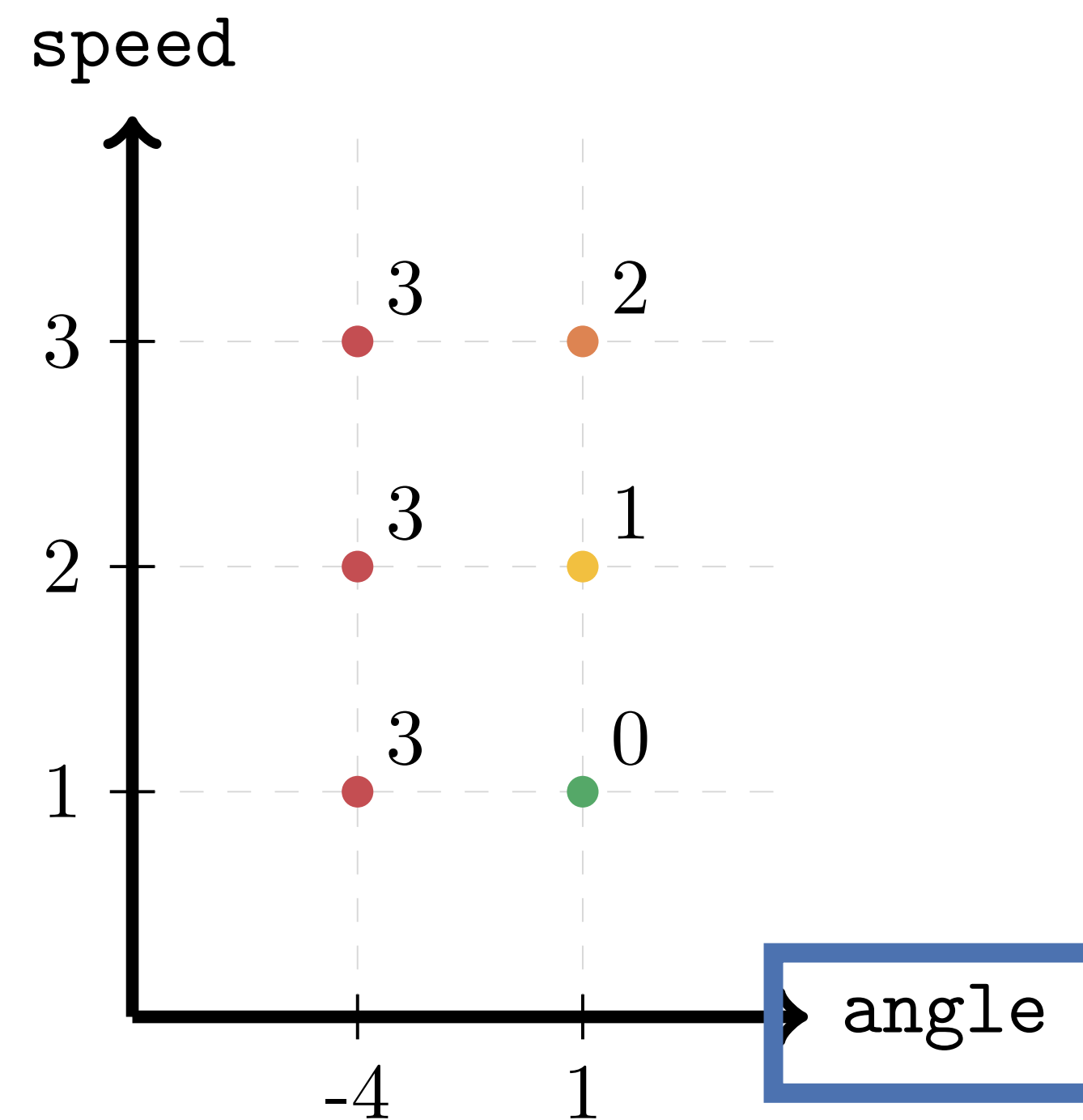
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

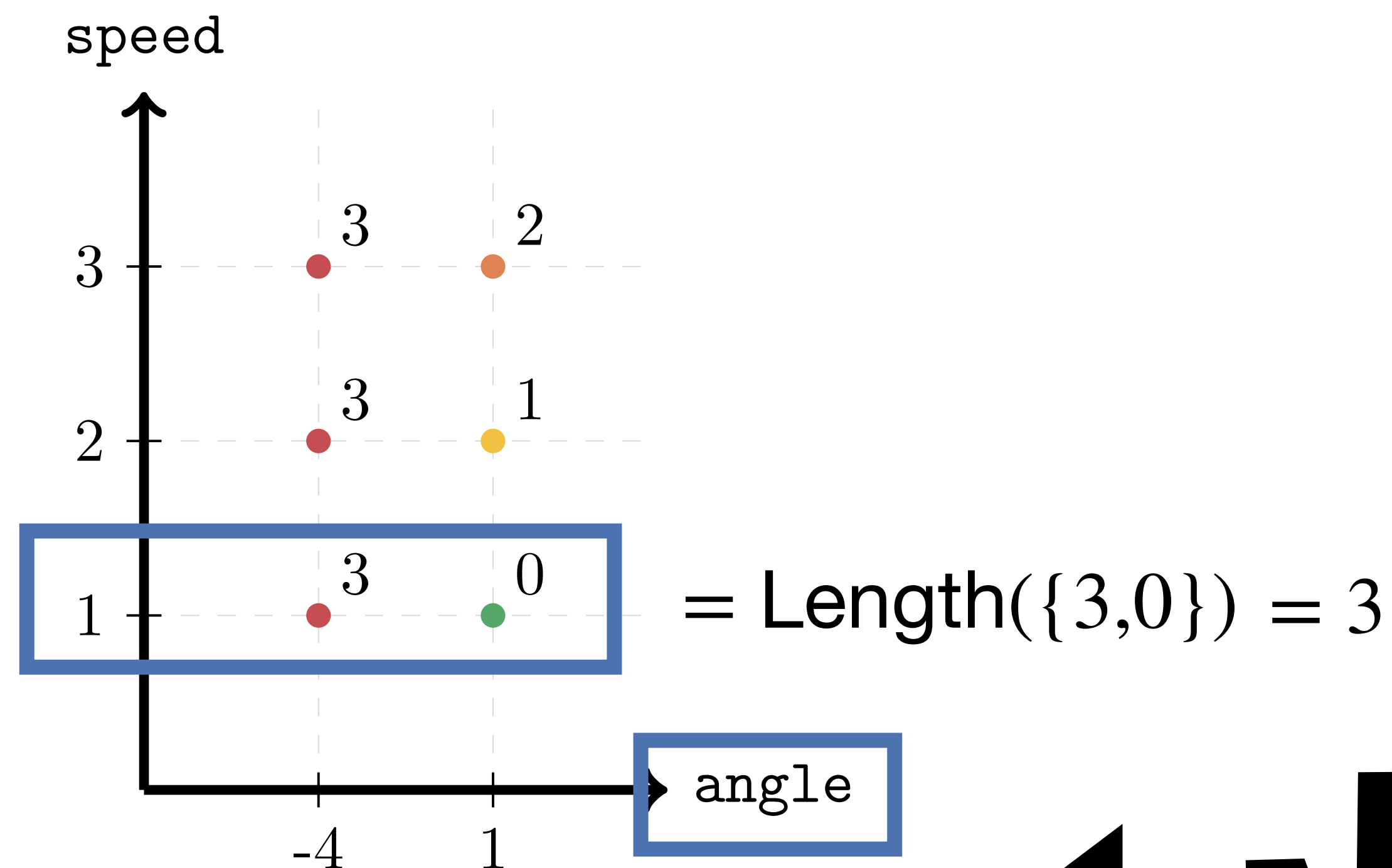
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of *reachable* outcomes

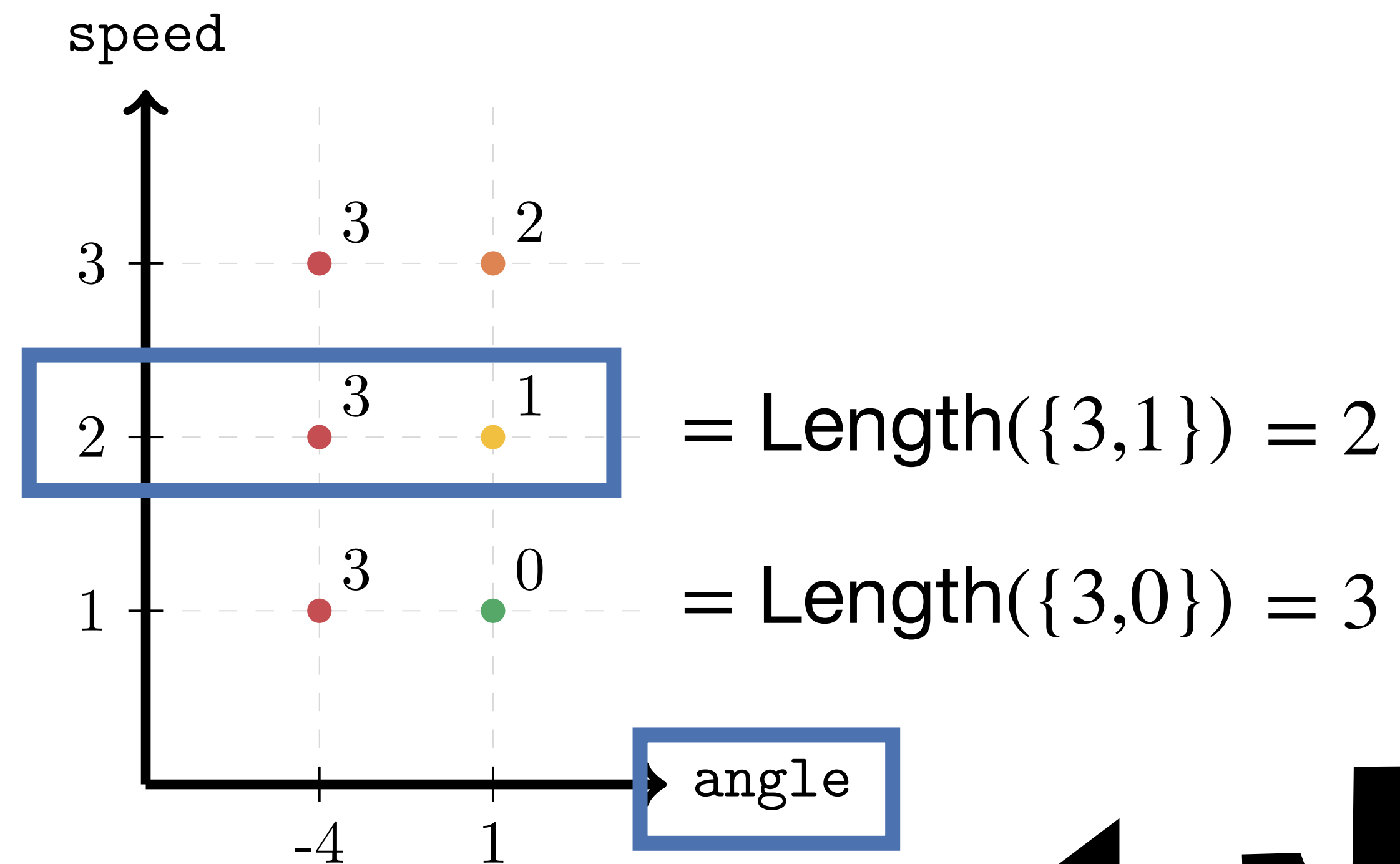
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

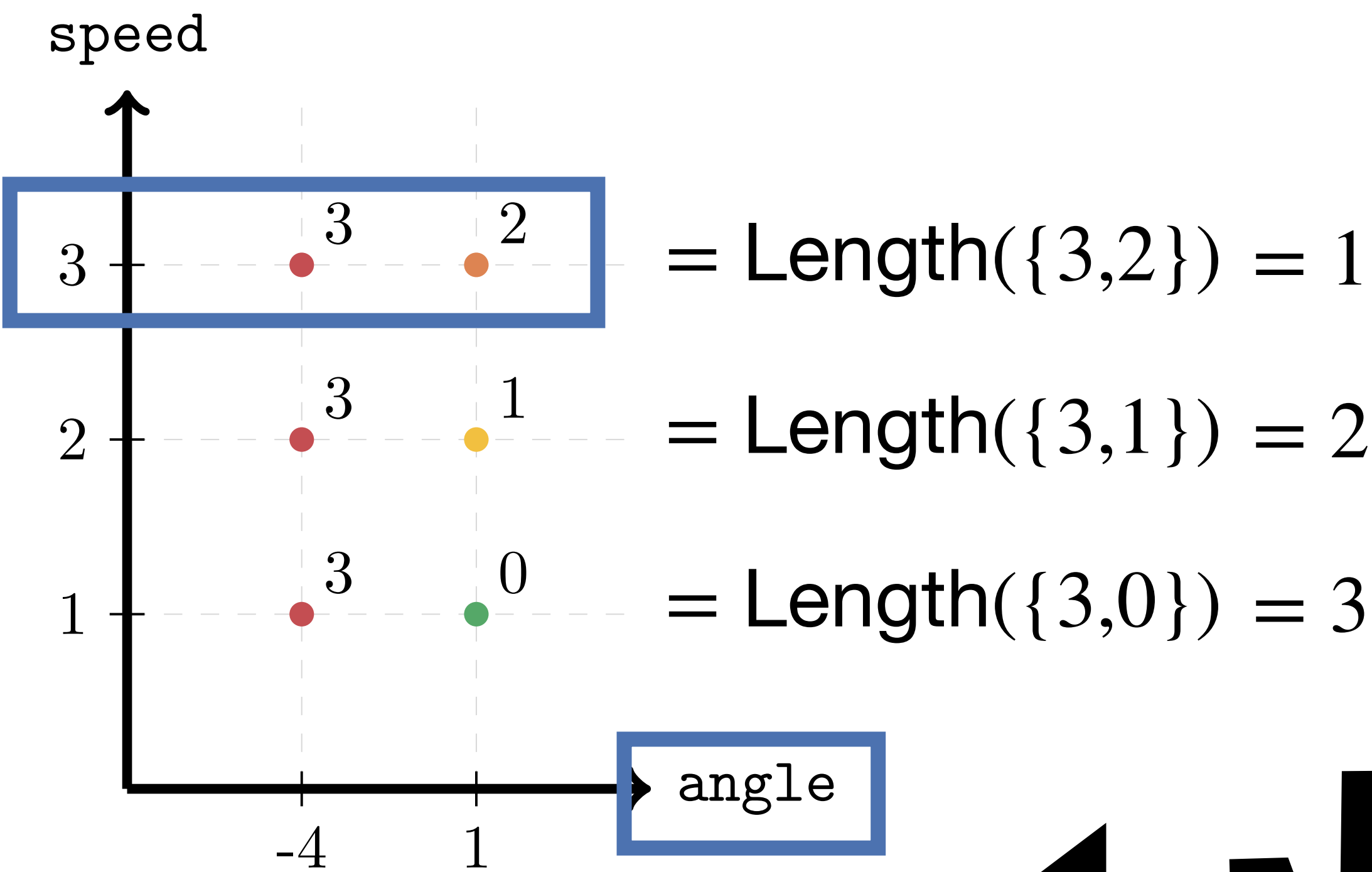
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

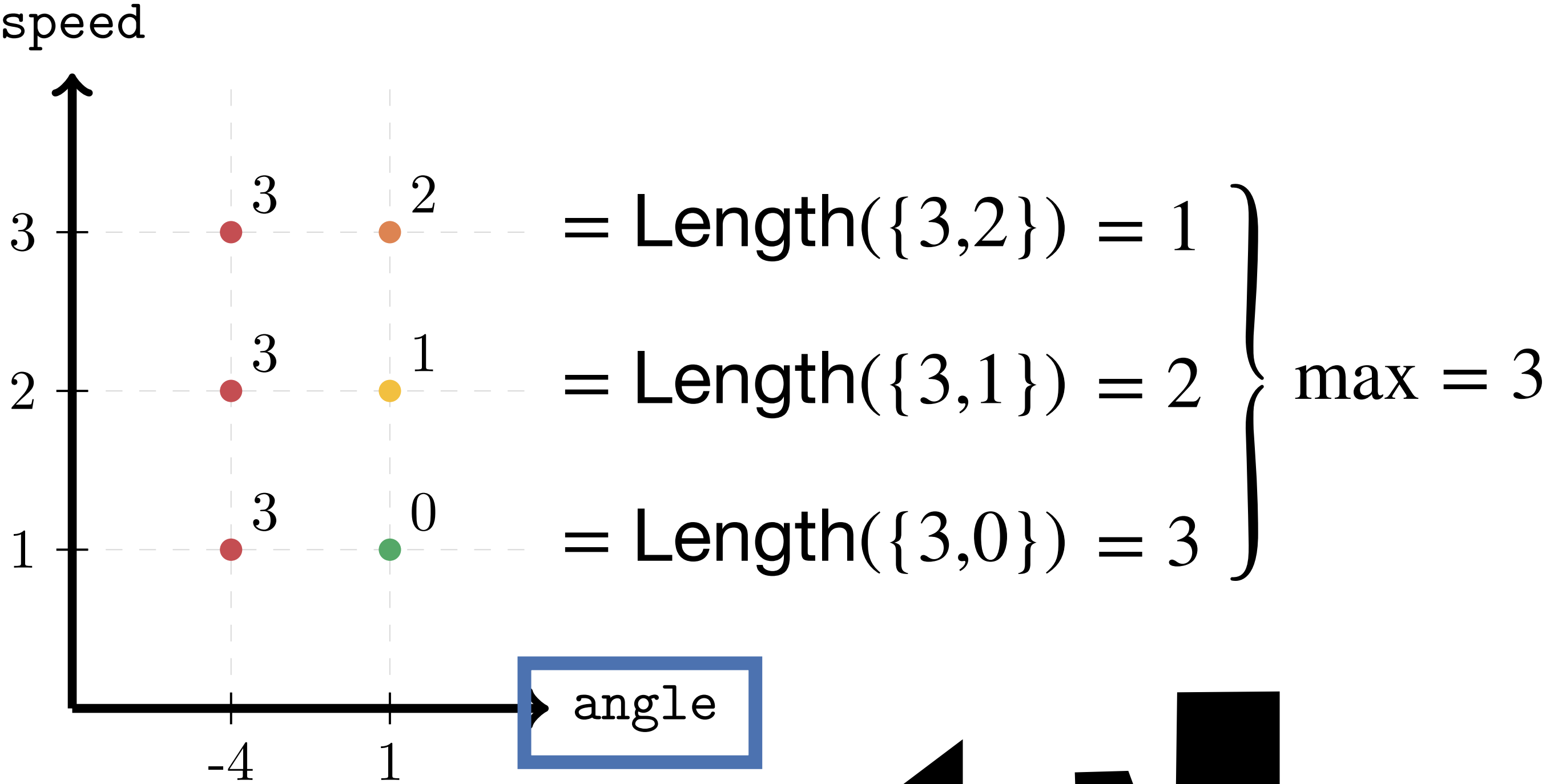
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

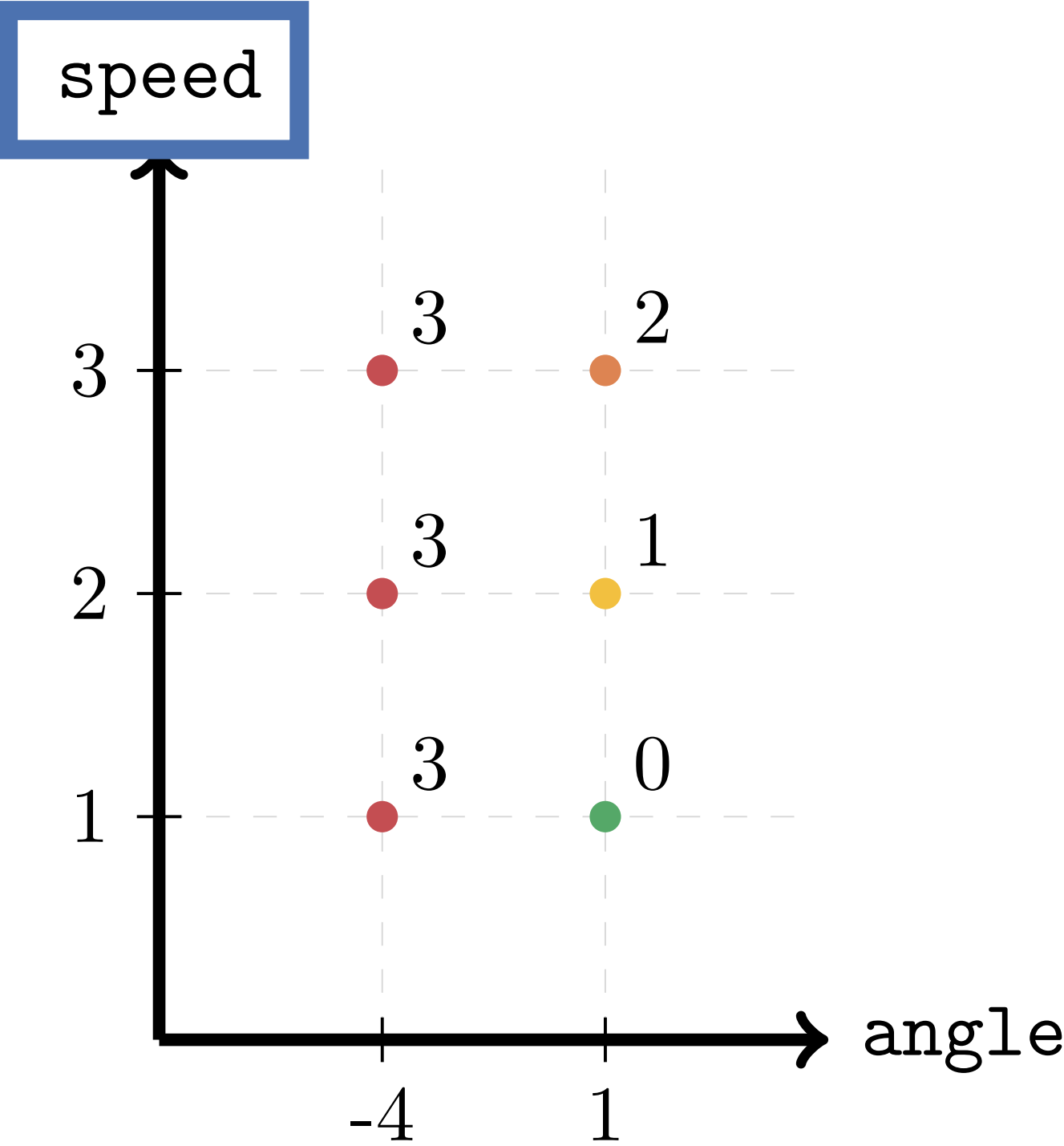
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

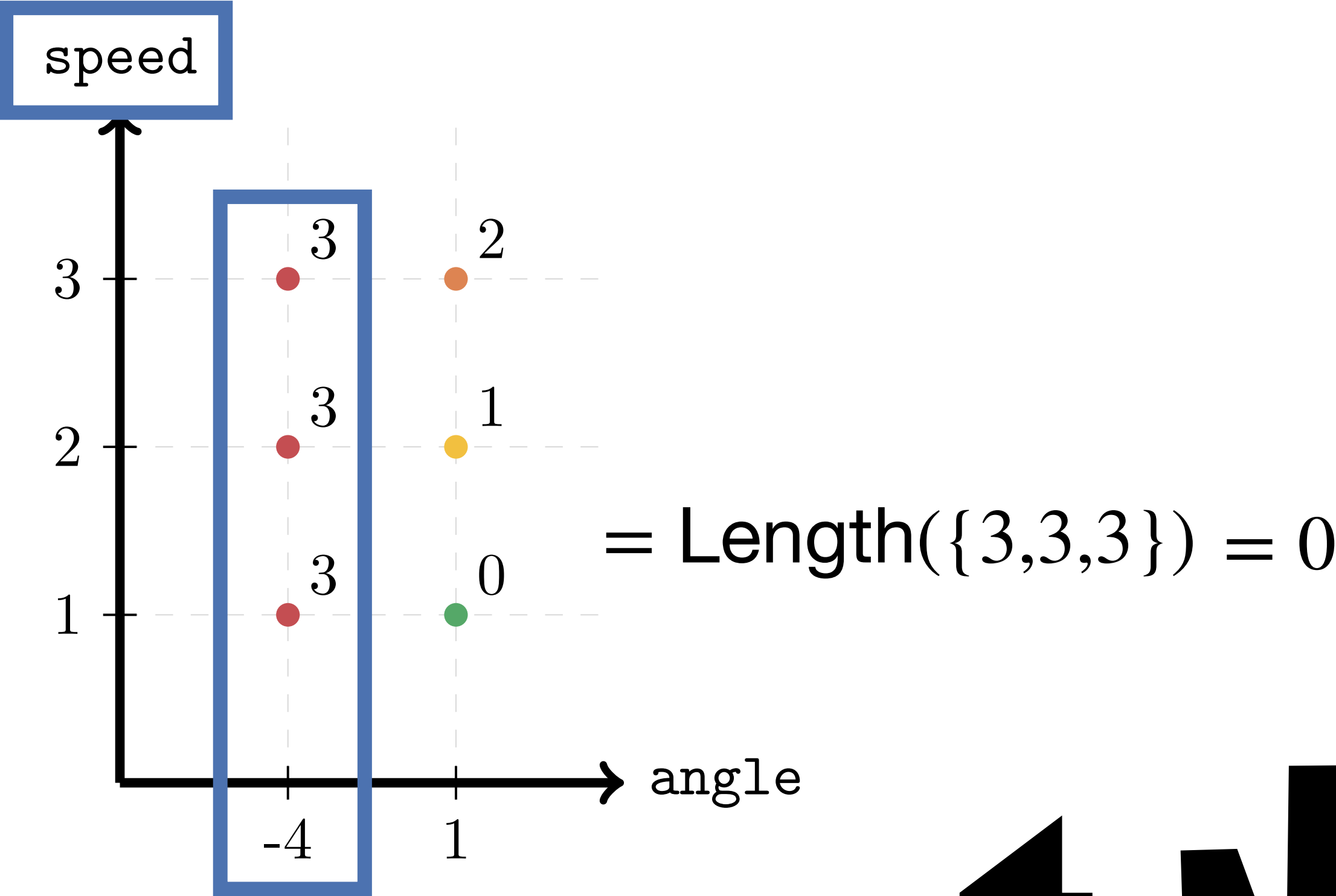
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

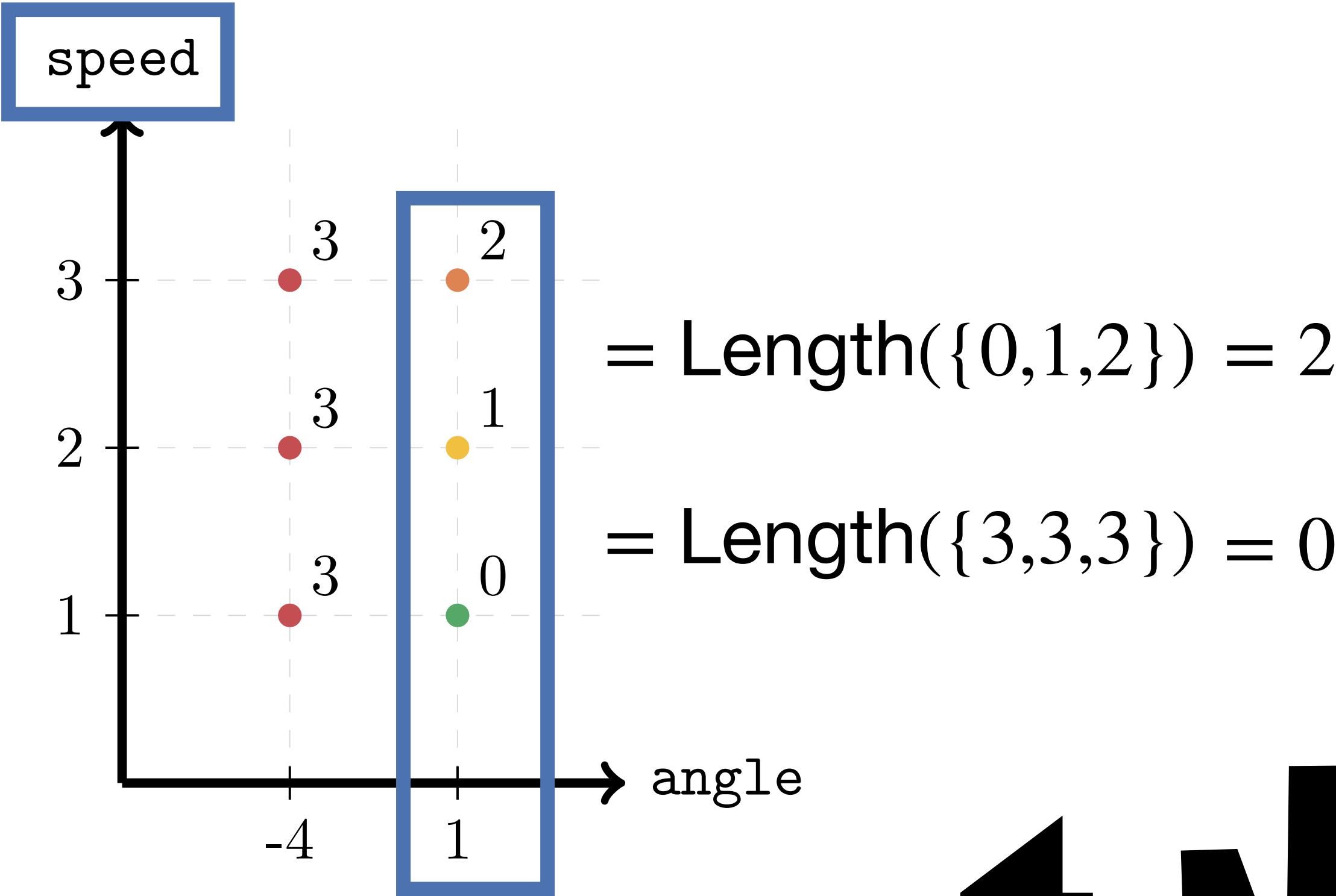
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

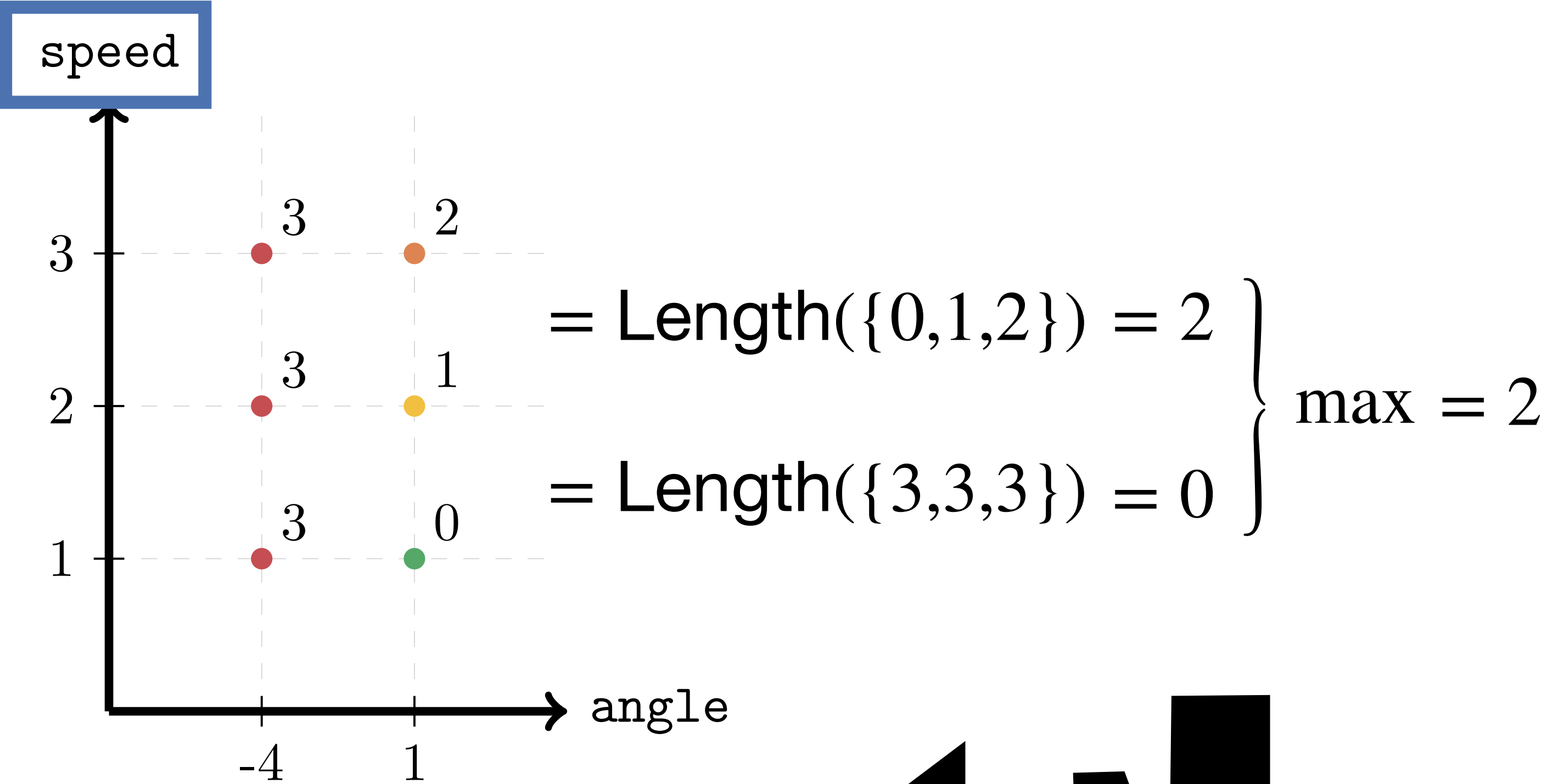
RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

RANGE



Goal: Quantify the impact of speed and angle on risk

Distance of reachable outcomes

Number of reachable outcomes

	RANGE	OUTCOMES
angle	3	2
speed	2	3



Automatic & Sound Static Analysis by Abstract Interpretation

	RANGE	OUTCOMES
angle	3	2
speed	2	3

Automatic & Sound Static Analysis by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

Automatic & Sound Static Analysis by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

**Smallest k permitted
by the abstraction!**

Automatic & Sound Static Analysis by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

1. Output Buckets

**Smallest k permitted
by the abstraction!**

Automatic & Sound Static Analysis

by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

**Smallest k permitted
by the abstraction!**

1. Output Buckets
2. Backward Abstract Analysis

Automatic & Sound Static Analysis

by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

**Smallest k permitted
by the abstraction!**

1. Output Buckets
2. Backward Abstract Analysis
3. Abstract Implementations of RANGE and OUTCOMES

Automatic & Sound Static Analysis

by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

**Smallest k permitted
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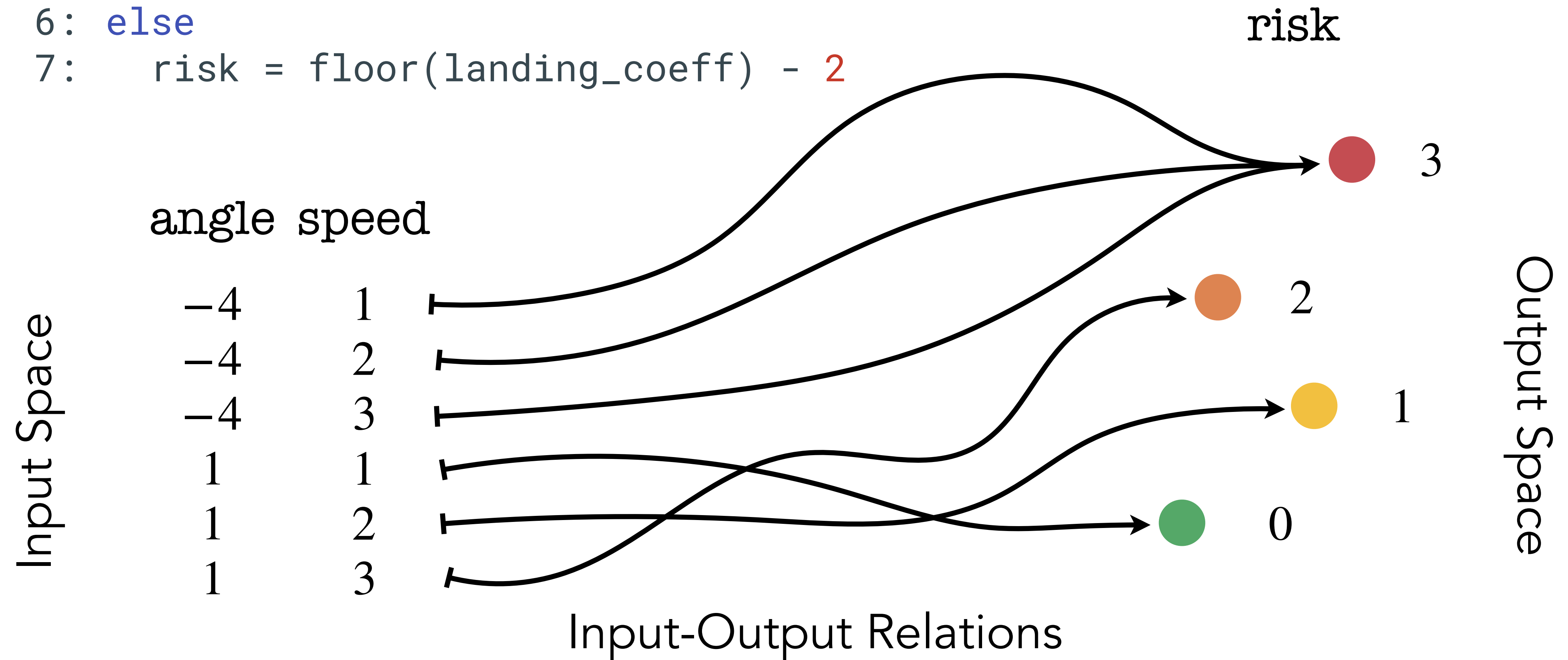
1. Output Buckets
2. Backward Abstract Analysis
3. Abstract Implementations of RANGE and OUTCOMES

RANGE[‡] and OUTCOMES[‡]

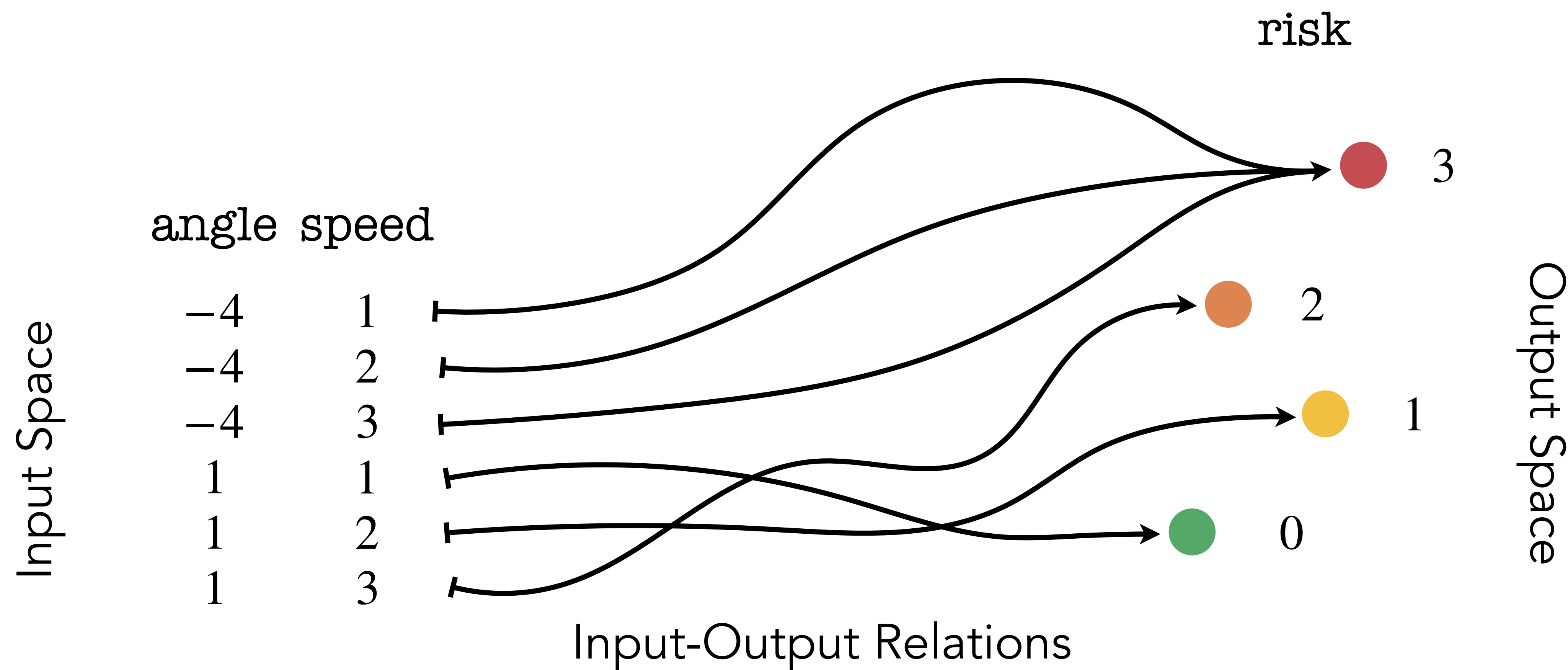
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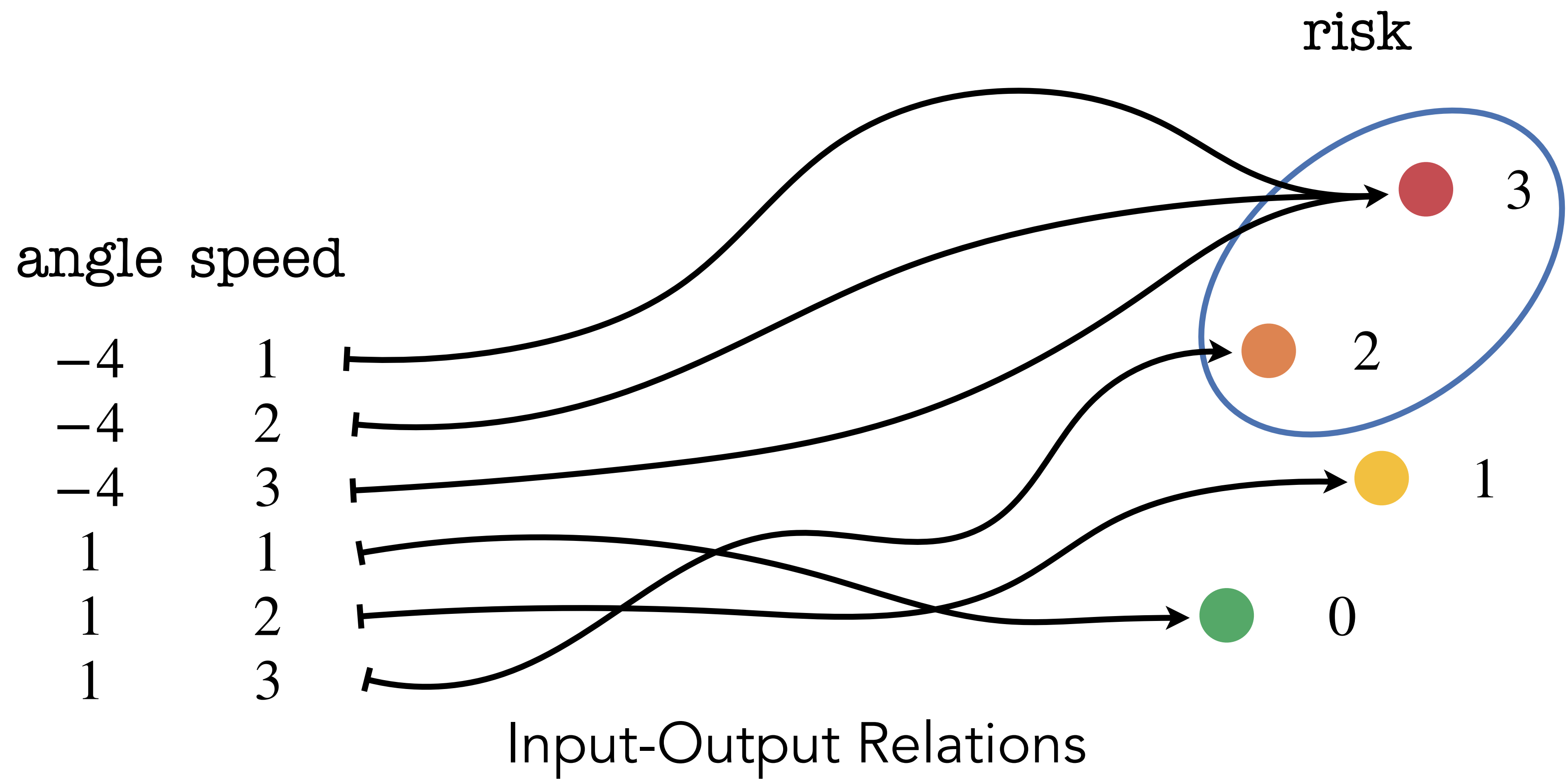
```



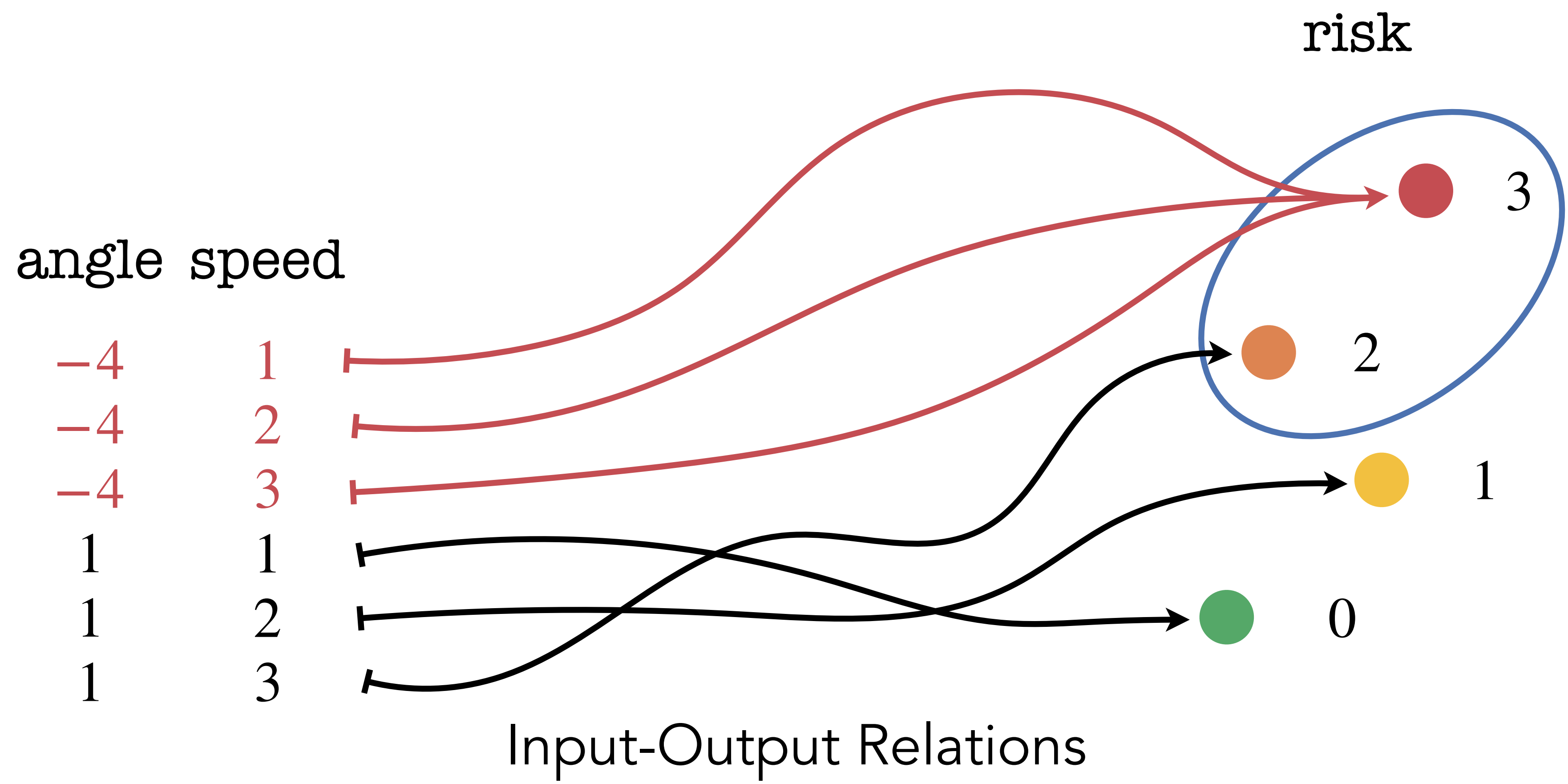
Example: OUTCOMES for the variable `angle`



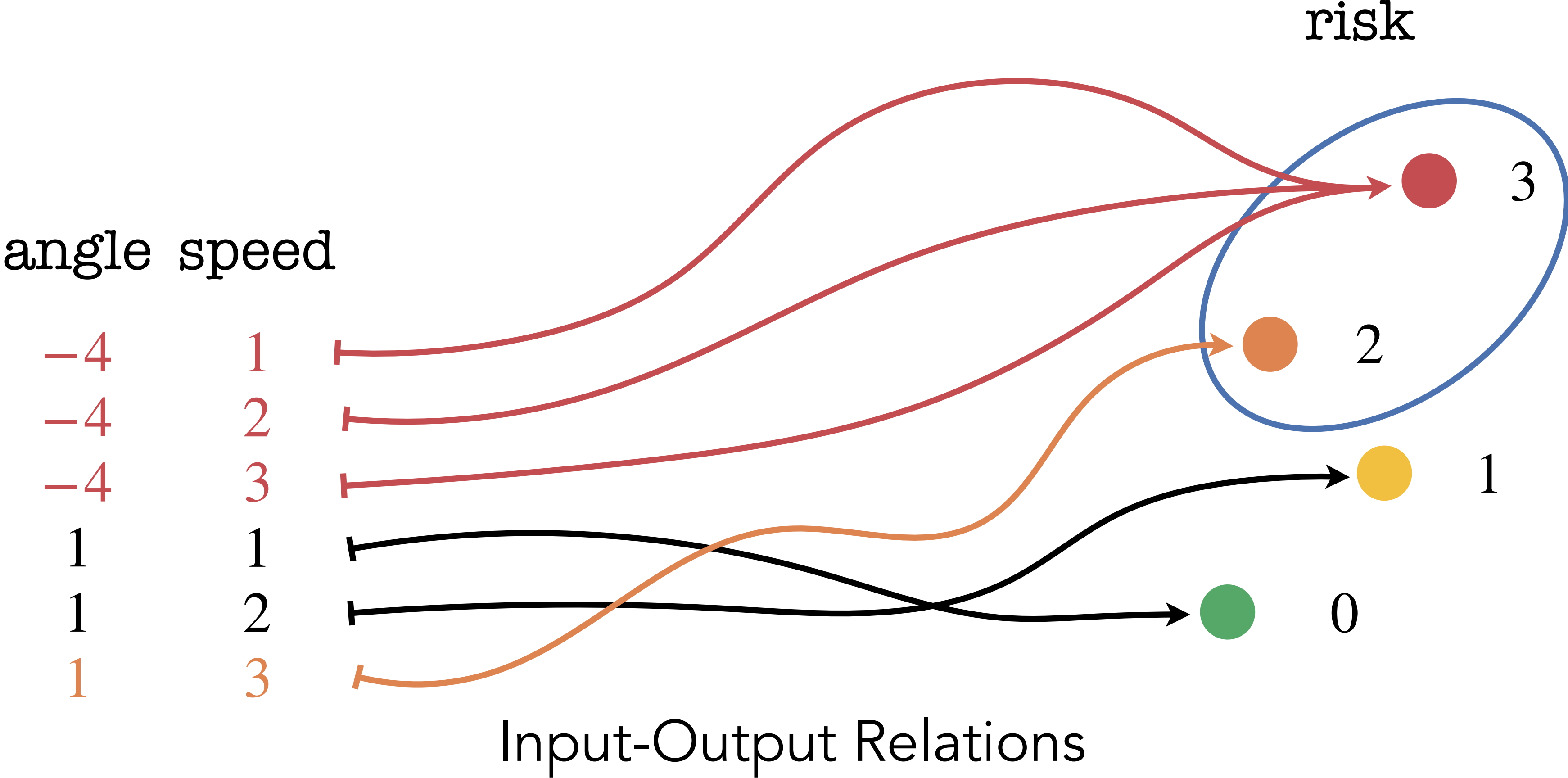
Example: OUTCOMES for the variable `angle`



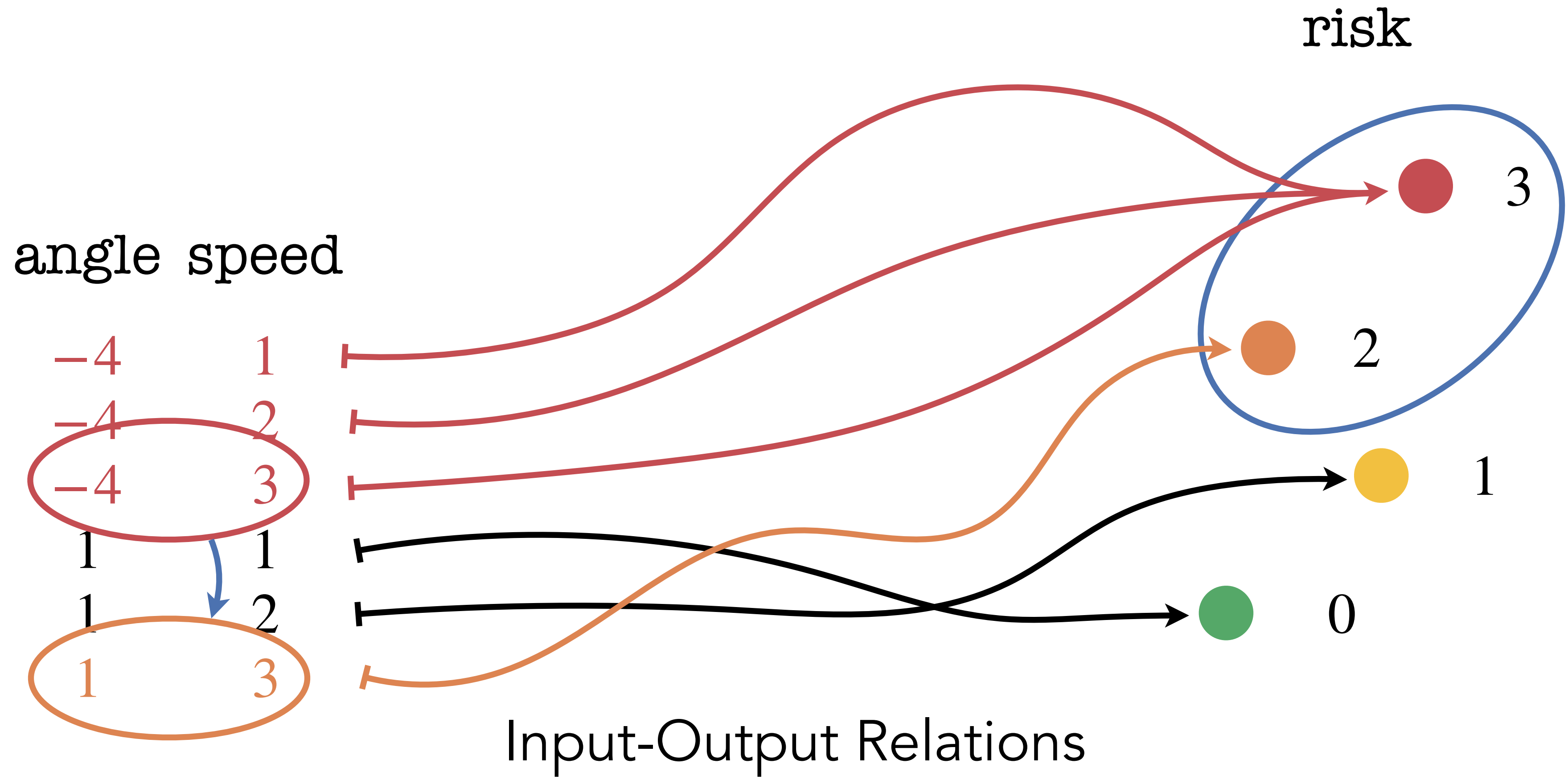
Example: OUTCOMES for the variable `angle`



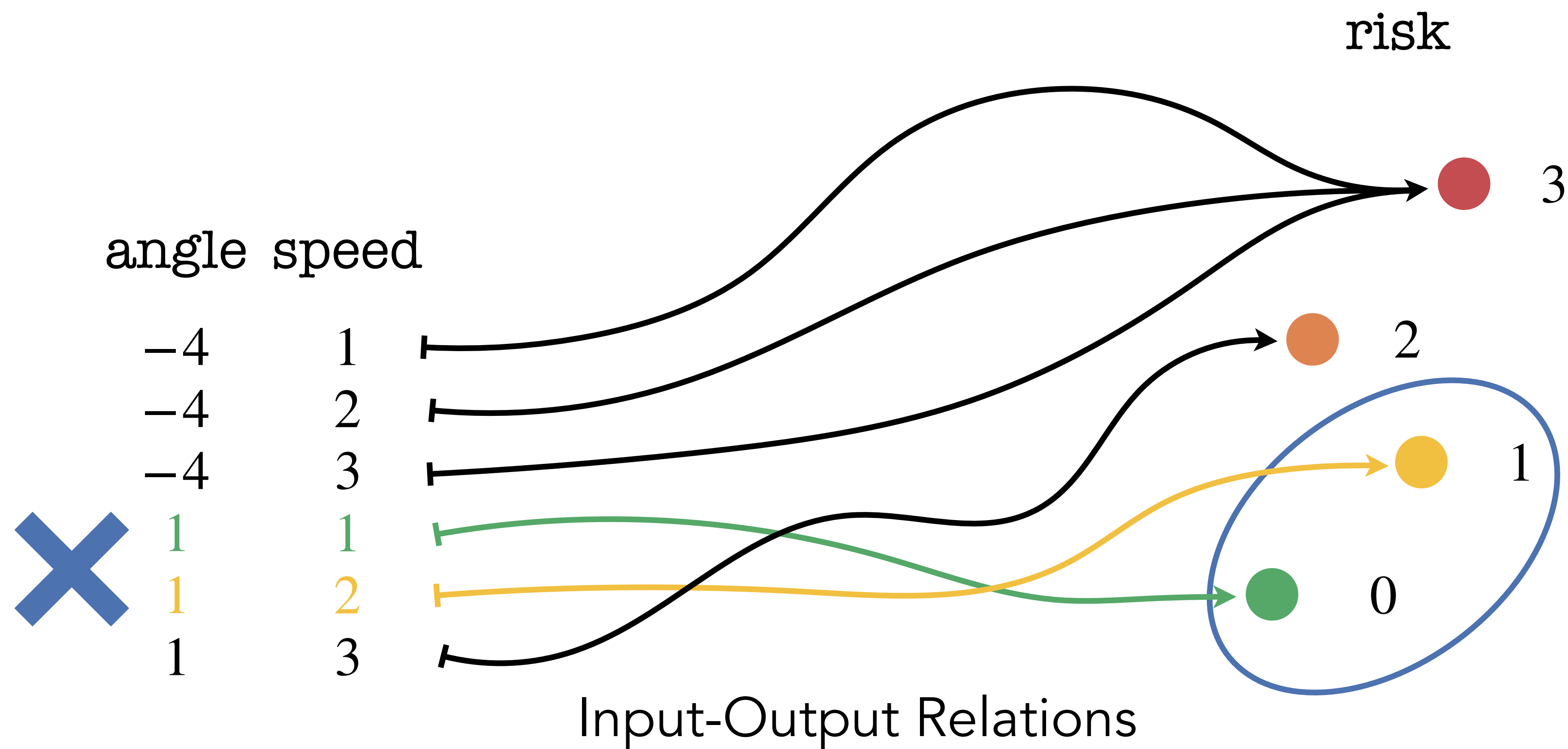
Example: OUTCOMES for the variable `angle`



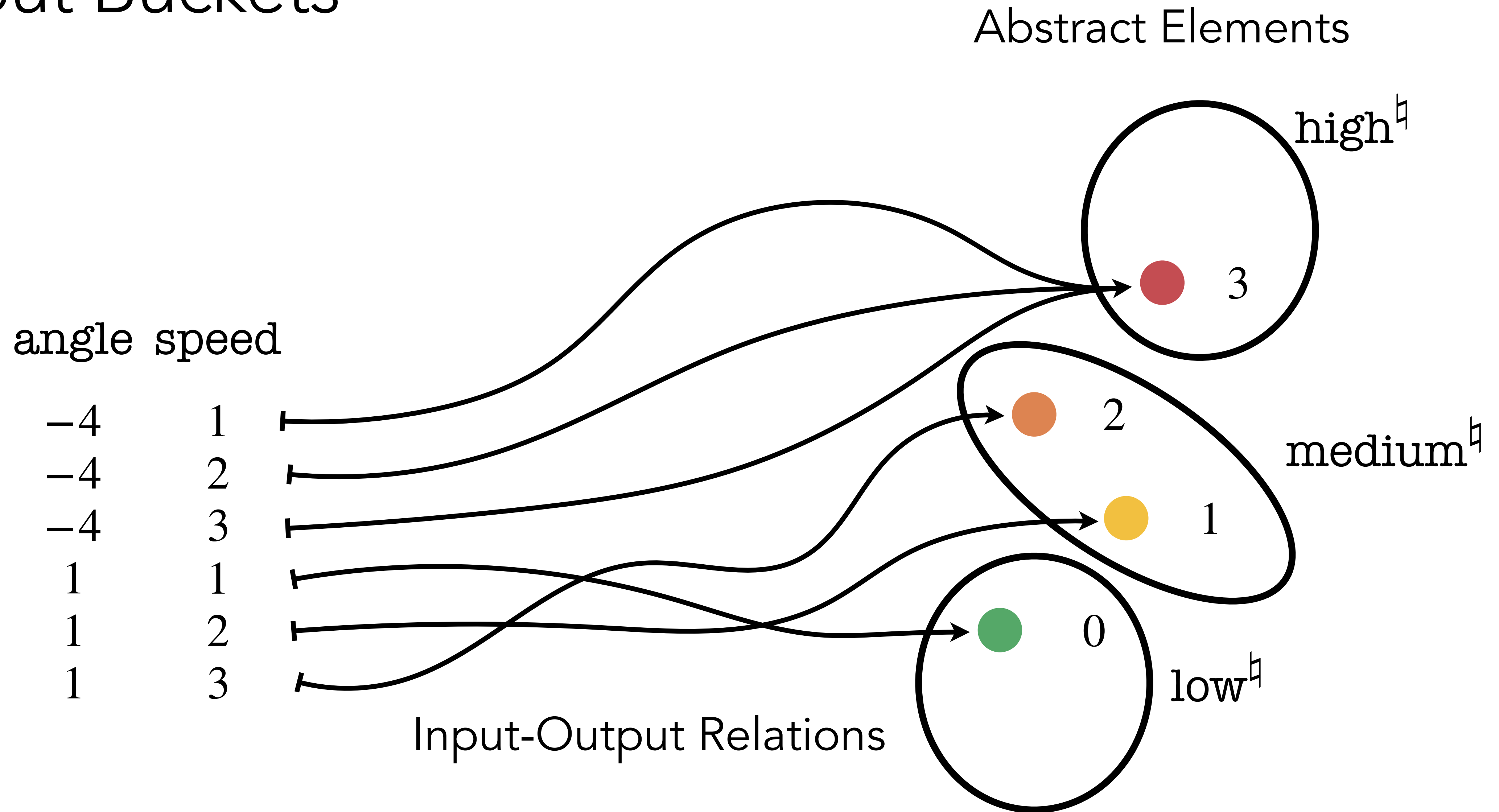
Example: OUTCOMES for the variable `angle`



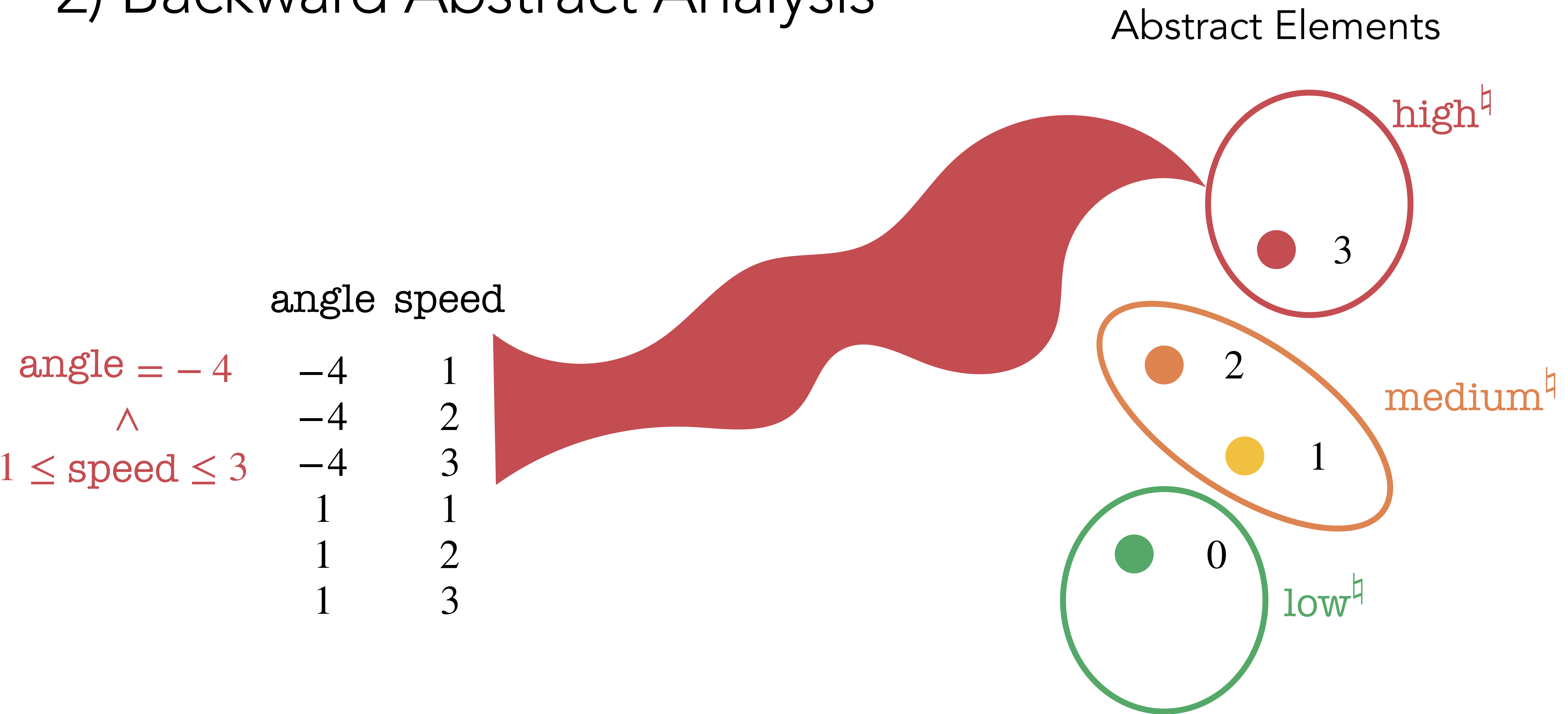
Example: OUTCOMES for the variable angle



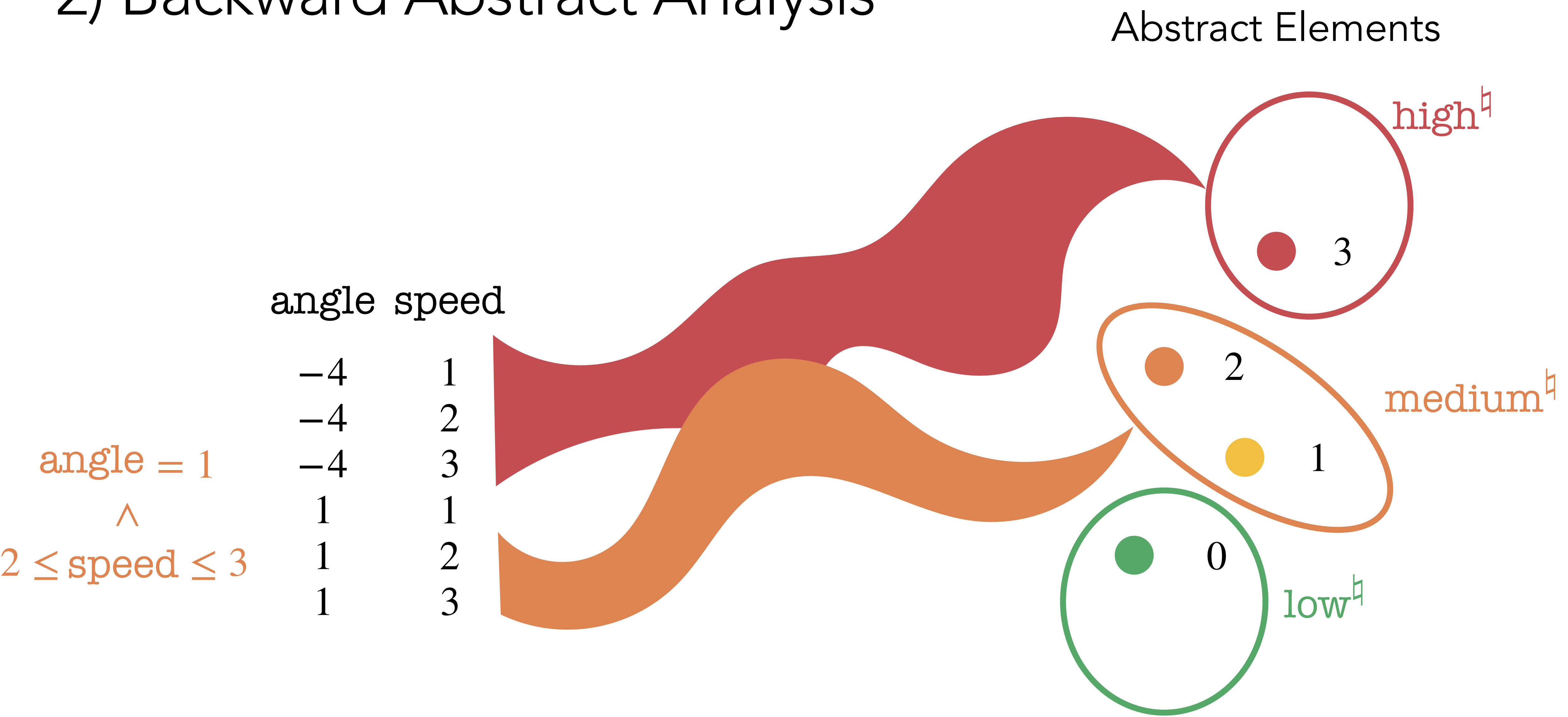
1) Output Buckets



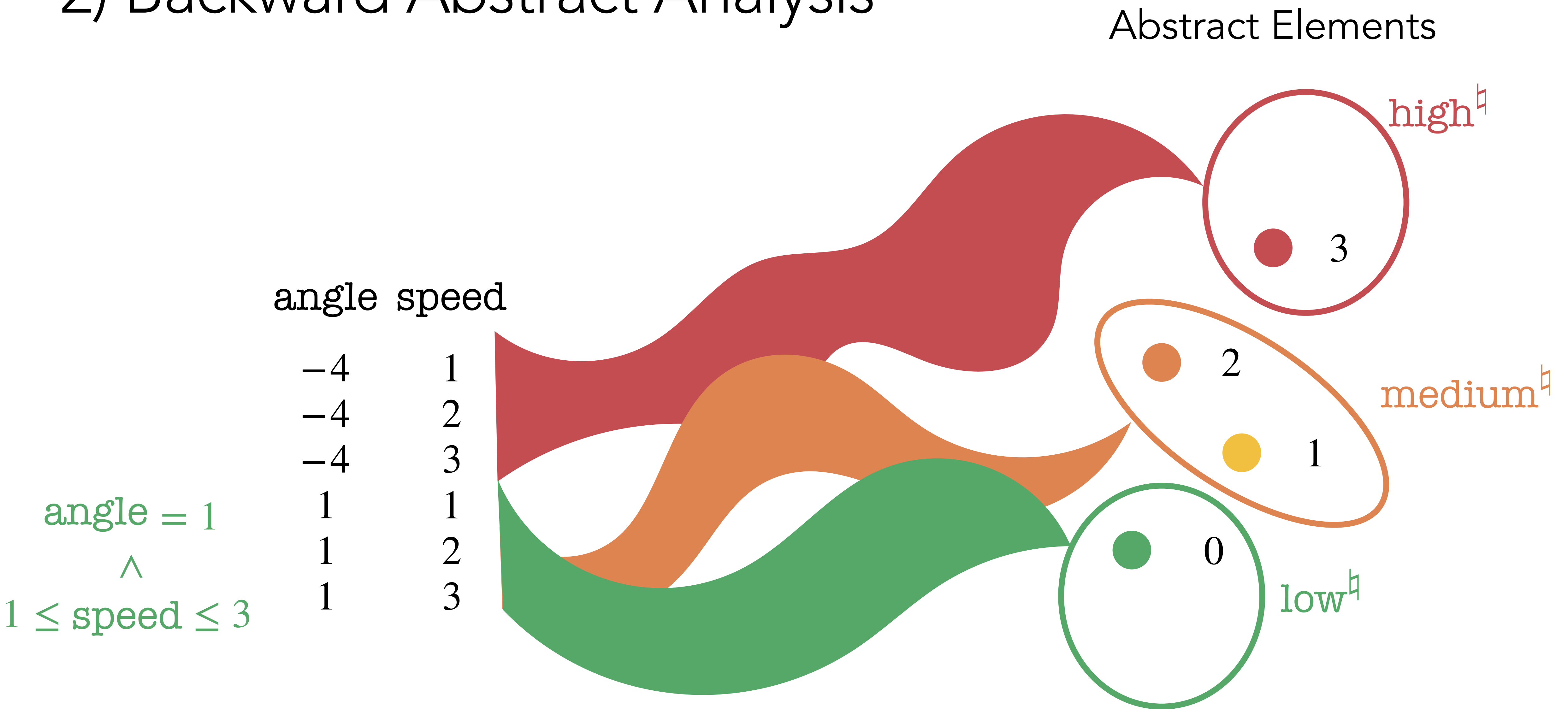
2) Backward Abstract Analysis



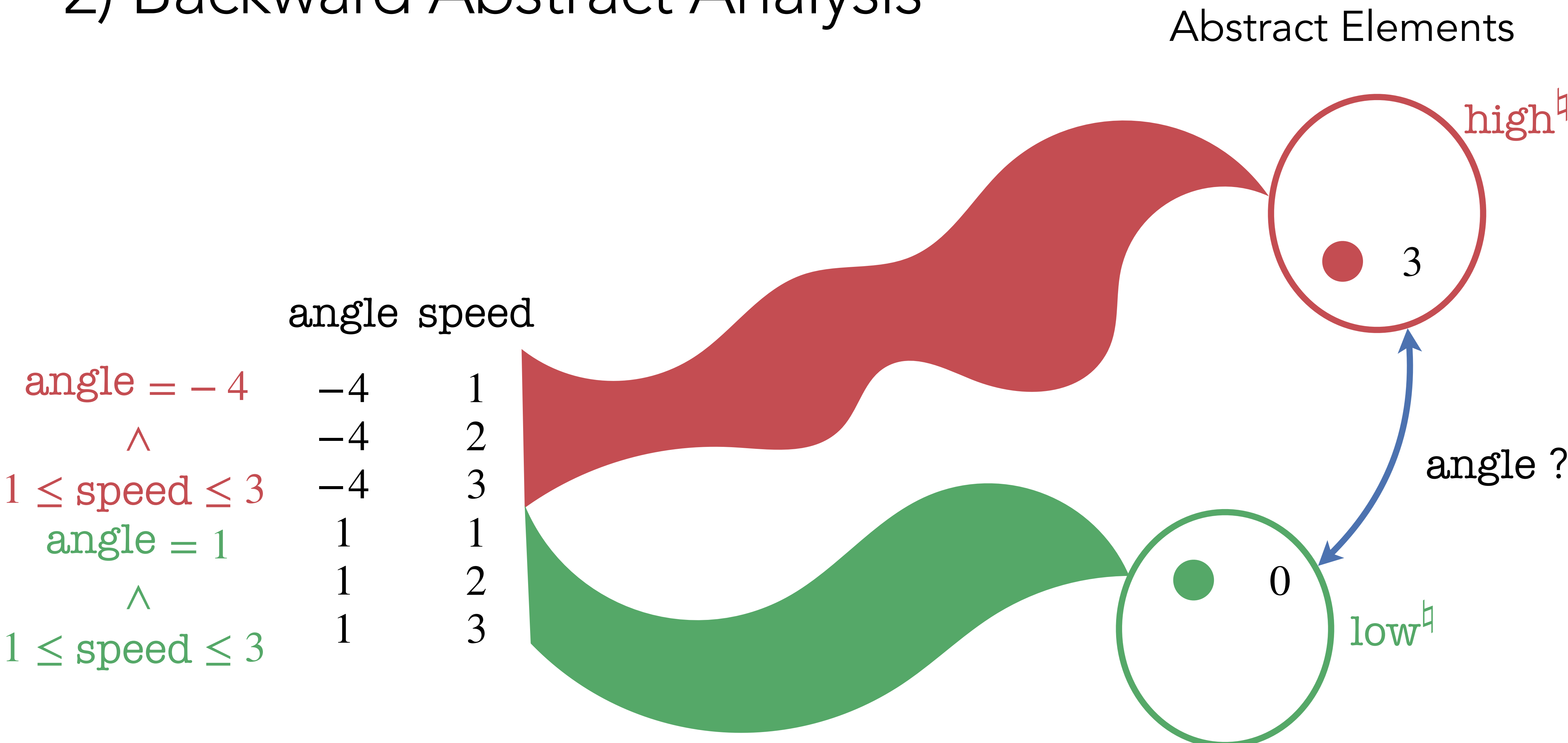
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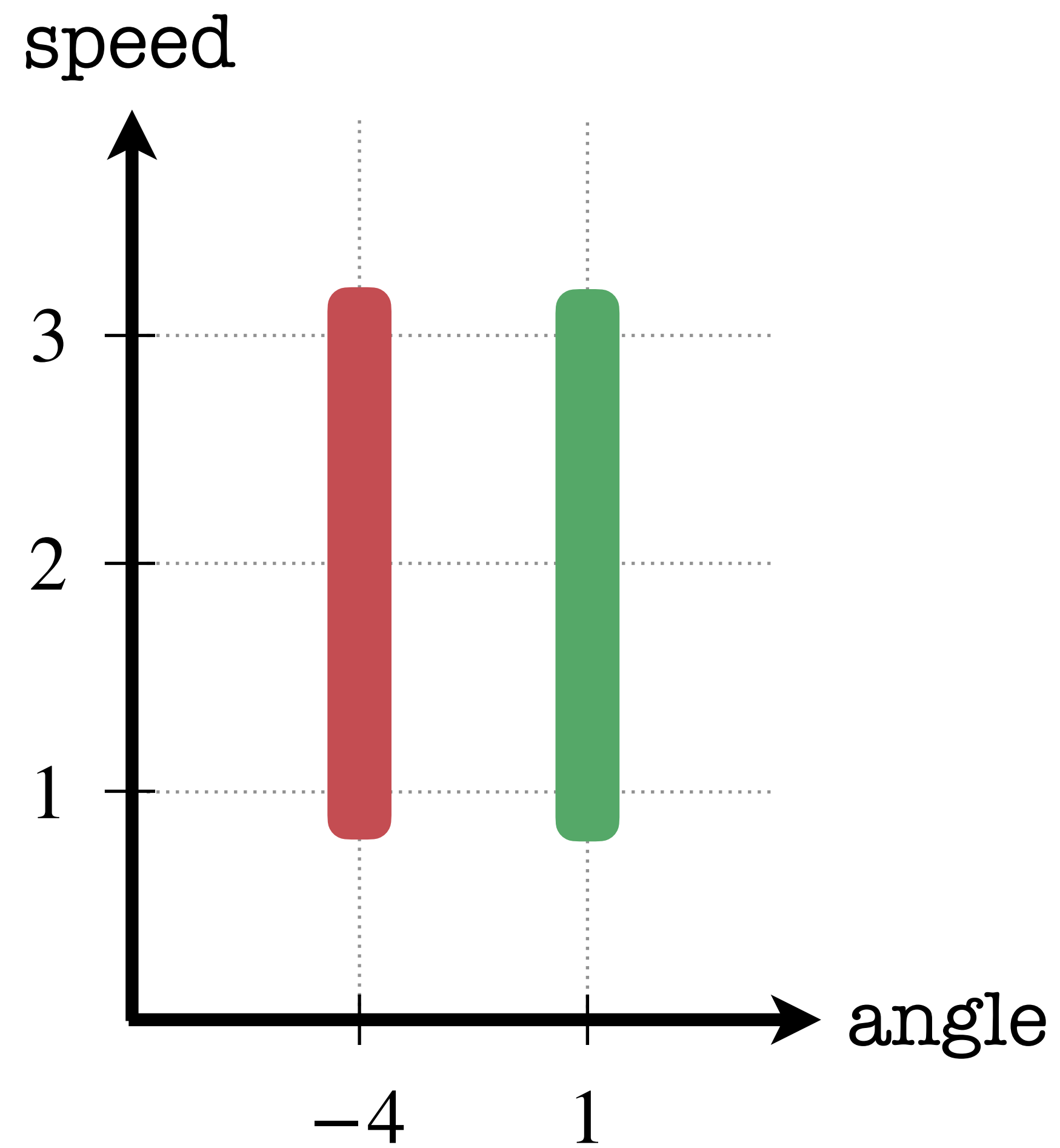


2) Backward Abstract Analysis



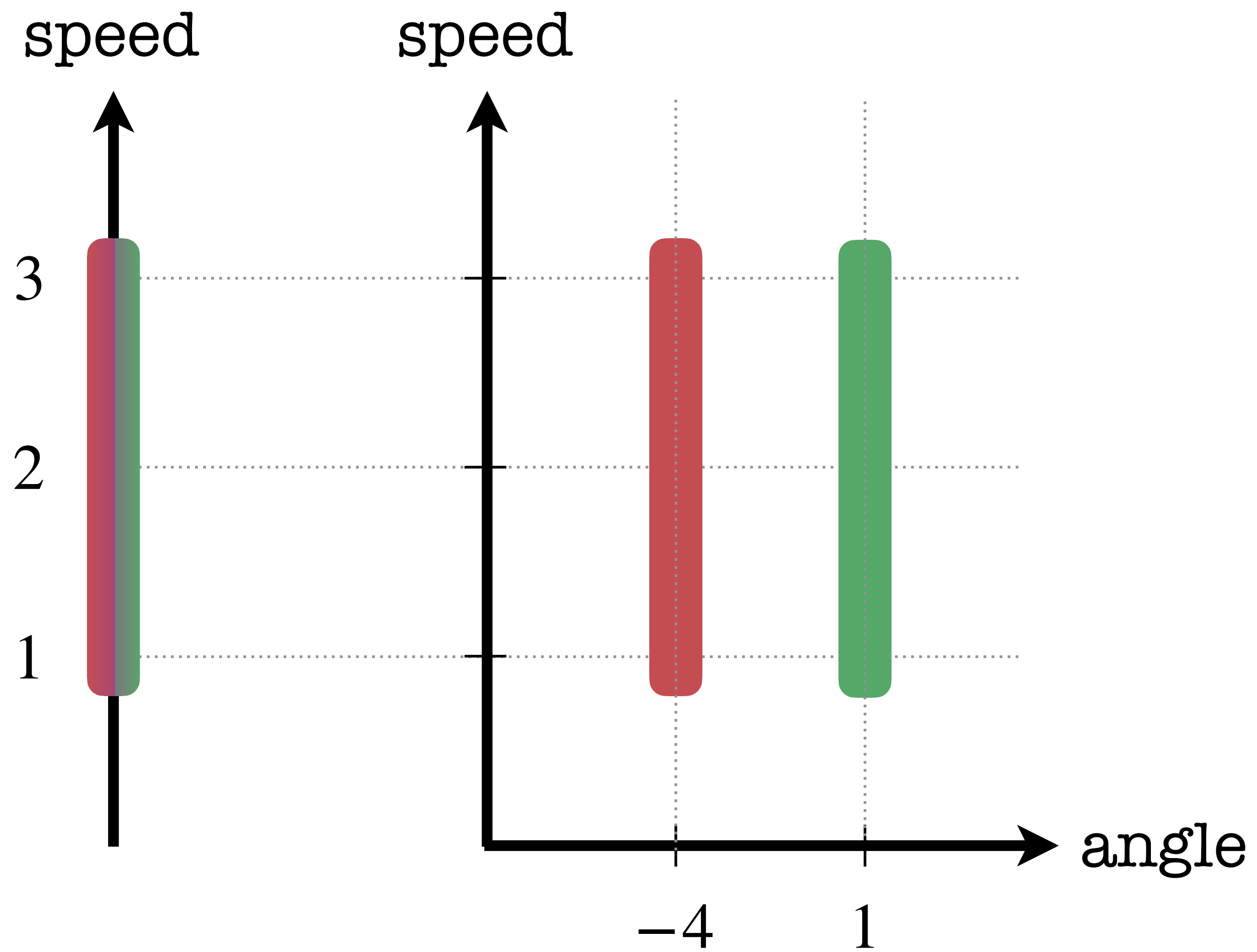
2) Backward Abstract Analysis





angle = -4
 \wedge
 $1 \leq \text{speed} \leq 3$

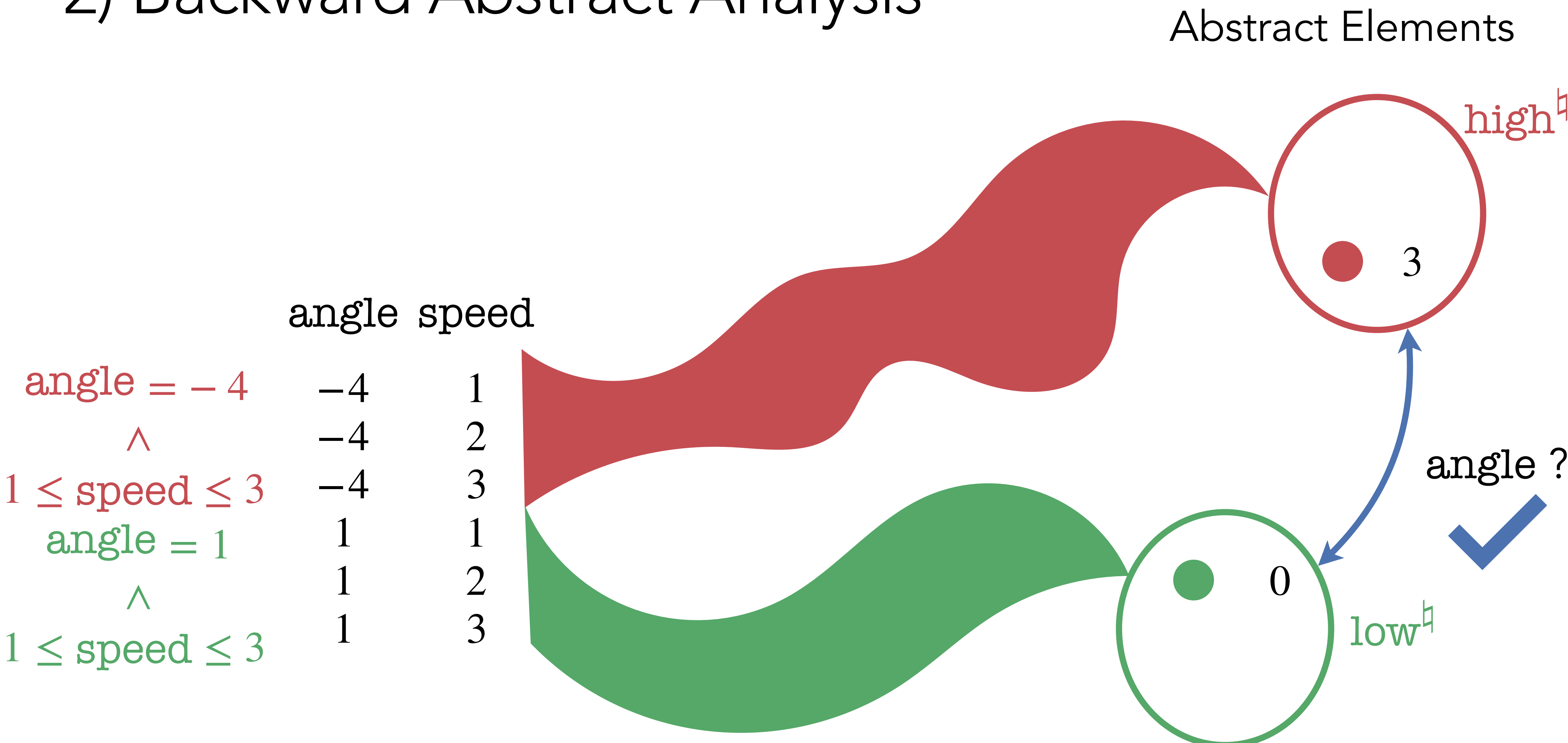
angle = 1
 \wedge
 $1 \leq \text{speed} \leq 3$



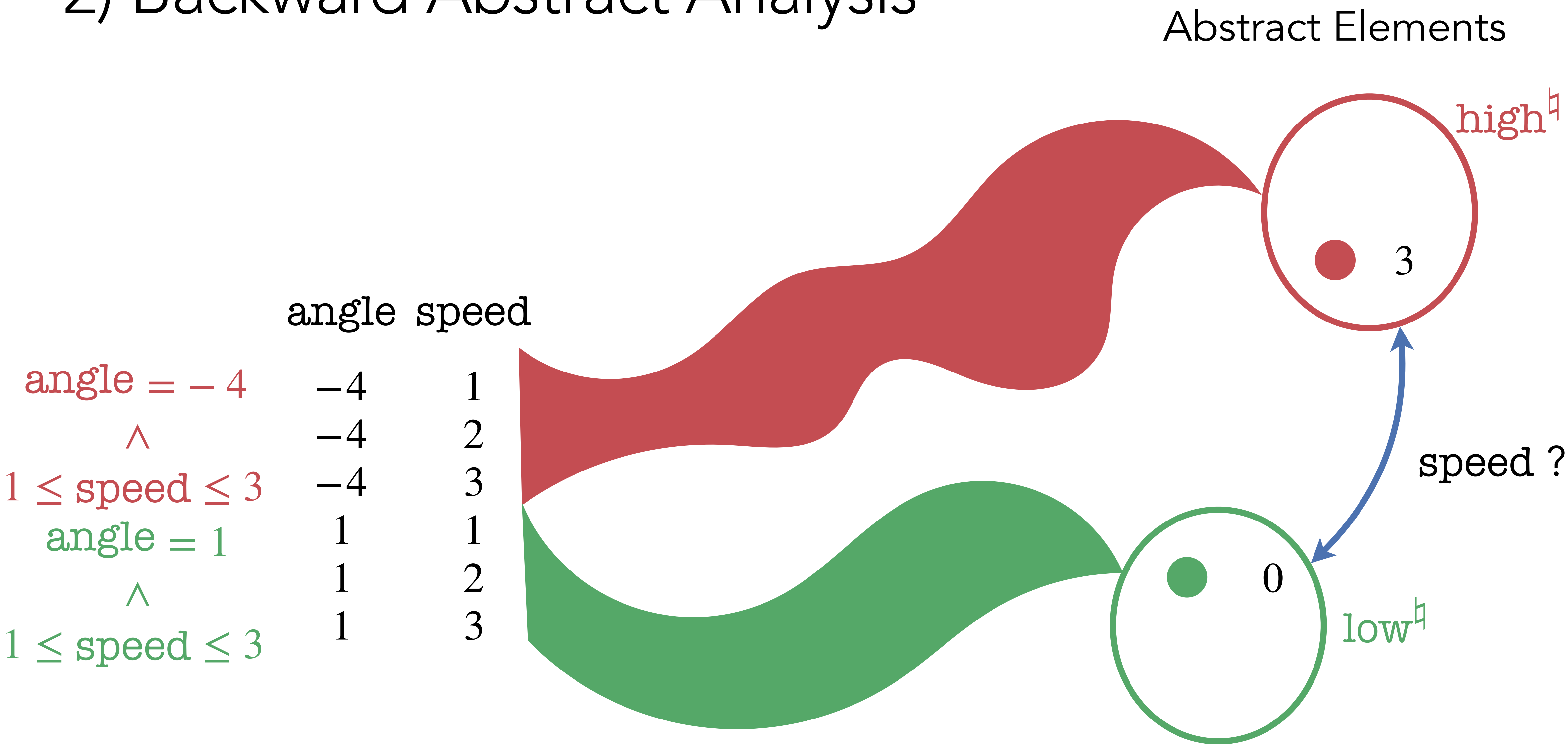
angle = -4
 \wedge
 $1 \leq \text{speed} \leq 3$

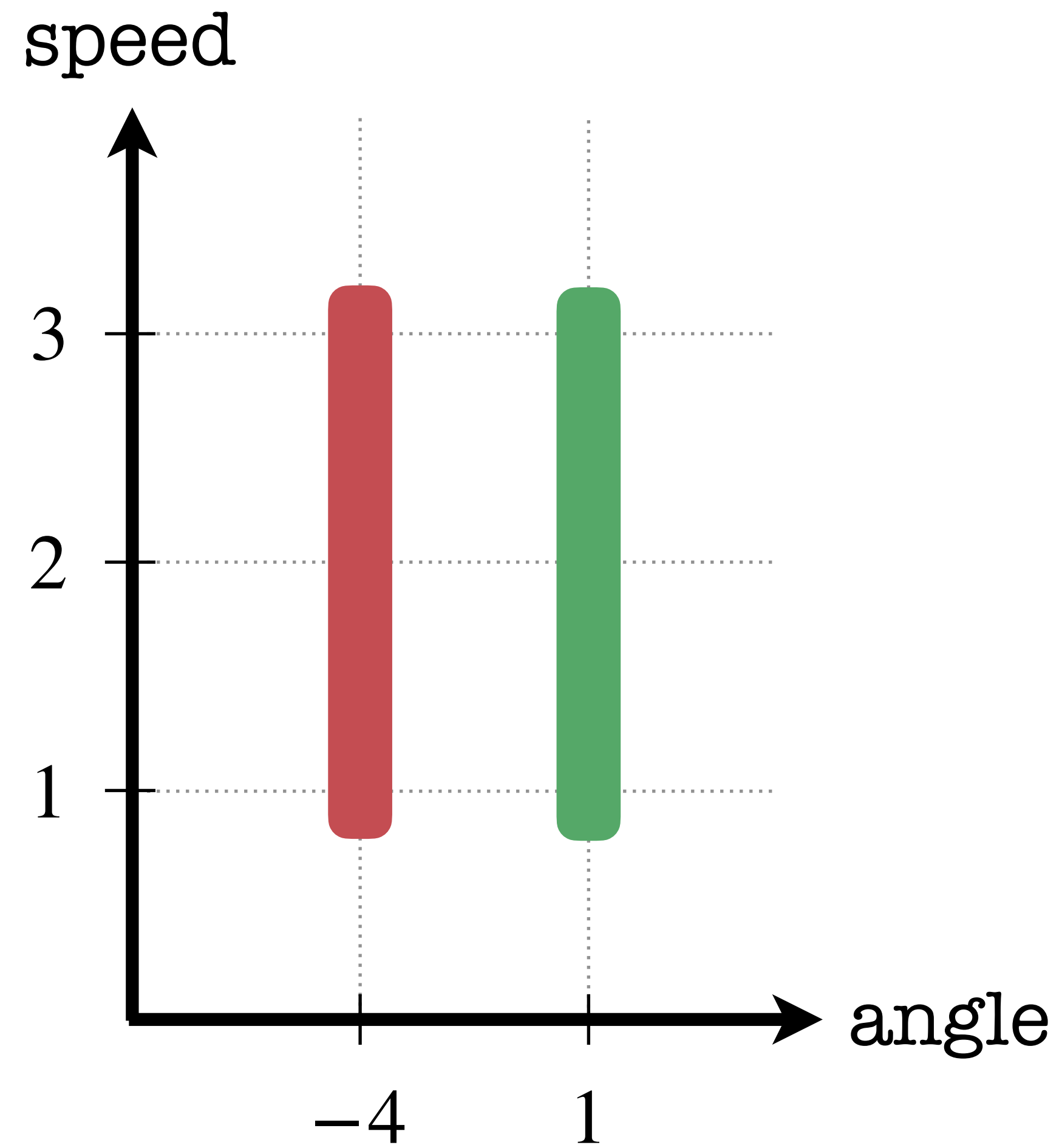
angle = 1
 \wedge
 $1 \leq \text{speed} \leq 3$

2) Backward Abstract Analysis



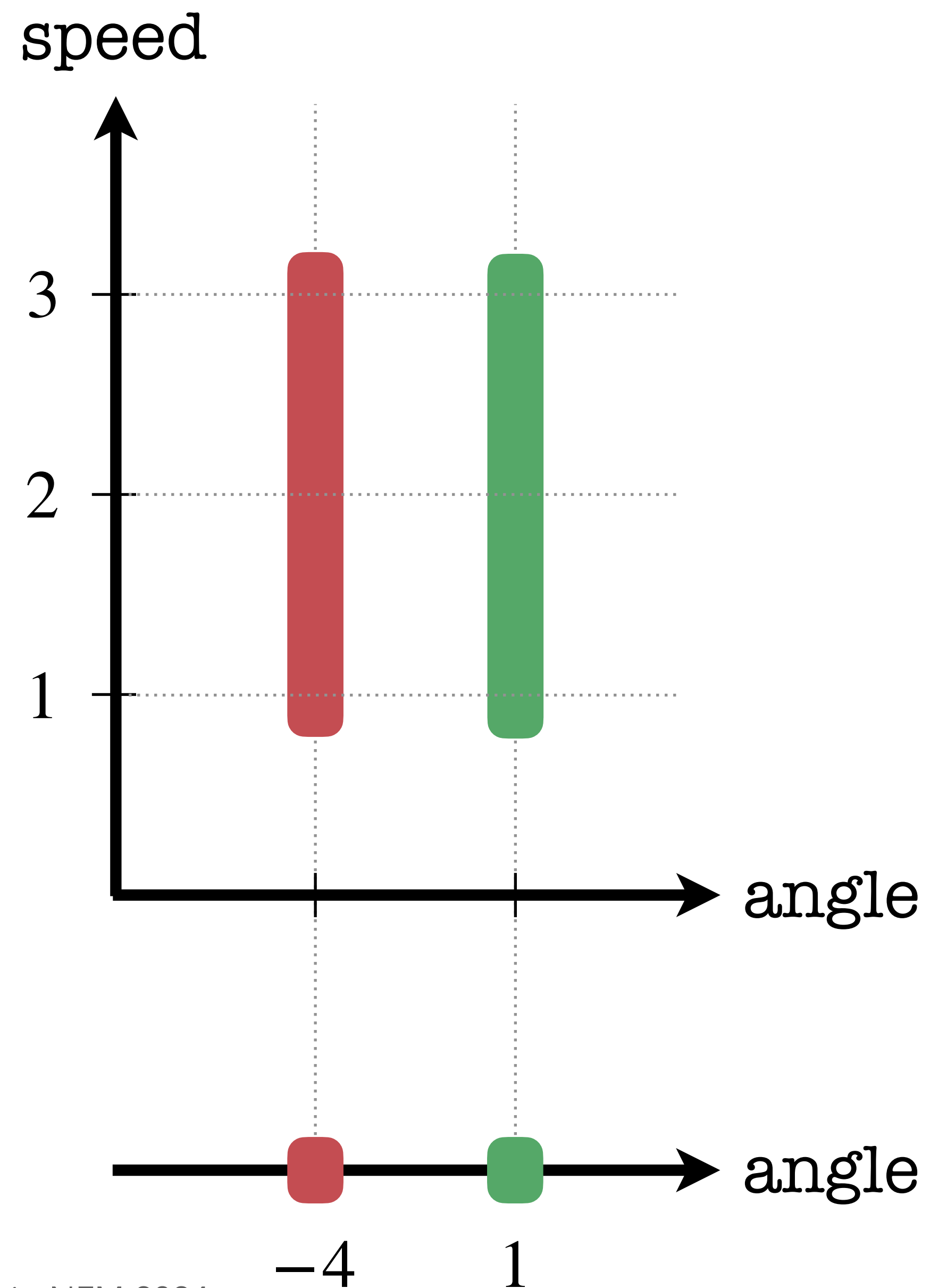
2) Backward Abstract Analysis





angle = -4
 \wedge
 $1 \leq \text{speed} \leq 3$

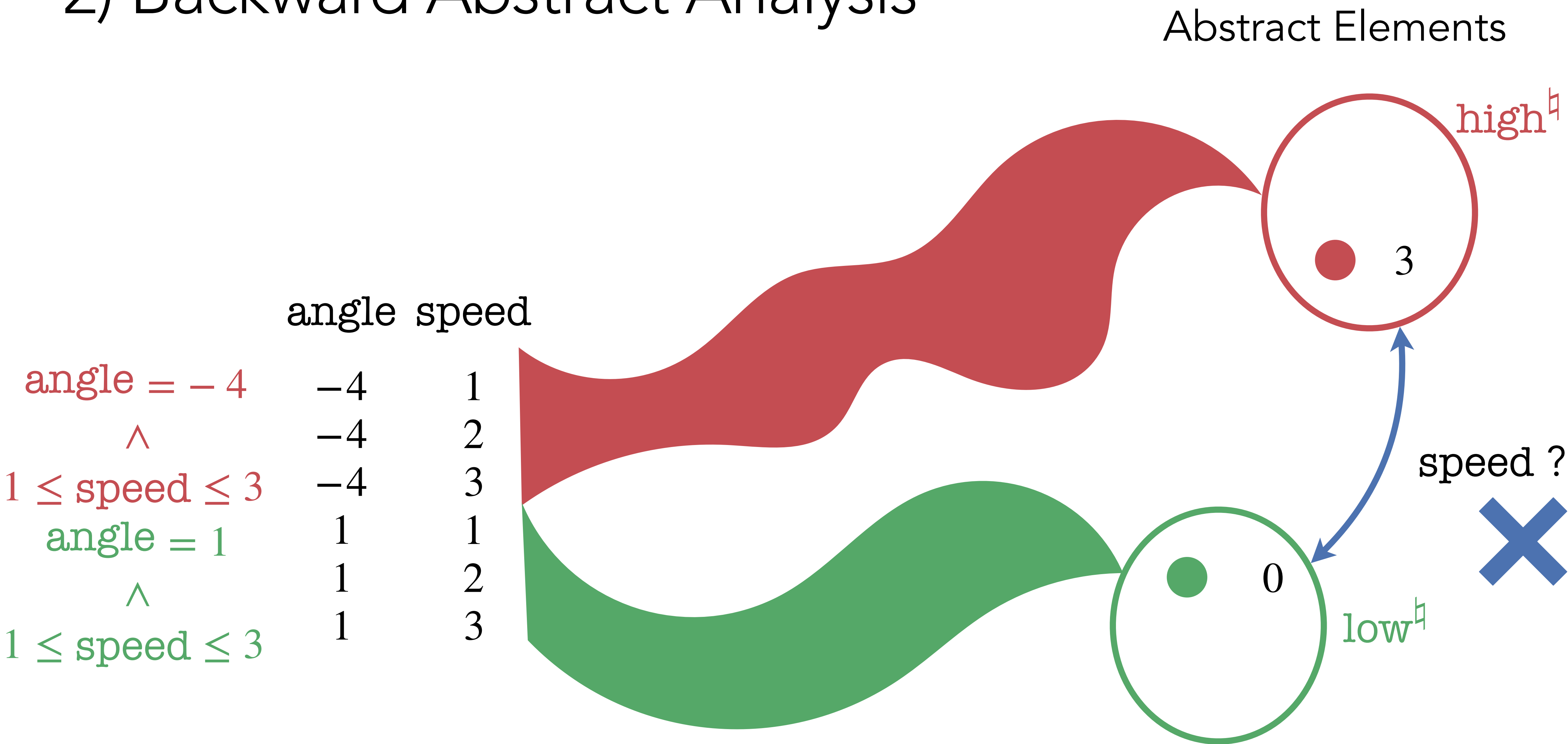
angle = 1
 \wedge
 $1 \leq \text{speed} \leq 3$



angle = -4
 \wedge
 $1 \leq \text{speed} \leq 3$

angle = 1
 \wedge
 $1 \leq \text{speed} \leq 3$

2) Backward Abstract Analysis



3) Abstract Implementation of Impact Definitions

Combinations	<div>high[⚡] medium[⚡]</div>	<div>high[⚡] low[⚡]</div>	<div>low[⚡] medium[⚡]</div>	<div>high[⚡] medium[⚡] low[⚡]</div>
angle	✓	✓	✓	✓
speed			✓	

Abstract Impact Outcomes^h

Combinations

high^h
medium^h

high^h
low^h

0, 3

low^h
medium^h

high^h
medium^h
low^h

angle



speed



Abstract Impact Outcomes^h

Combinations

high^h
medium^h

high^h
low^h

2

low^h
medium^h

high^h
medium^h
low^h

angle



2



speed



Abstract Impact Outcomes^h

Combinations					
	<div>high^h medium^h</div>	<div>high^h low^h</div>	<div>low^h medium^h</div>	<div>high^h medium^h low^h</div>	
angle	1,2, 3	2	✓	✓	
speed			✓		

Abstract Impact Outcomes^h

Combinations					
		<div>high^h medium^h</div>	<div>high^h low^h</div>	<div>low^h medium^h</div>	<div>high^h medium^h low^h</div>
angle	3	2	✓	✓	
speed			✓		

Abstract Impact Outcomes[⚡]

Combinations	<div>high[⚡] medium[⚡]</div>	<div>high[⚡] low[⚡]</div>	<div>low[⚡] medium[⚡]</div>	<div>high[⚡] medium[⚡] low[⚡]</div>	
angle	3	2	3	4	⇒ 4
speed			3		⇒ 3

Abstract Impact RANGE[‡]

Combinations

high[‡]
medium[‡]

high[‡]
low[‡]

low[‡]
medium[‡]

high[‡]
medium[‡]
low[‡]

angle

2

3

2

3

⇒ 3

speed

2

⇒ 2

Goal: Quantify the impact of speed and angle on risk

	RANGE	OUTCOMES
angle	3	2
speed	2	3



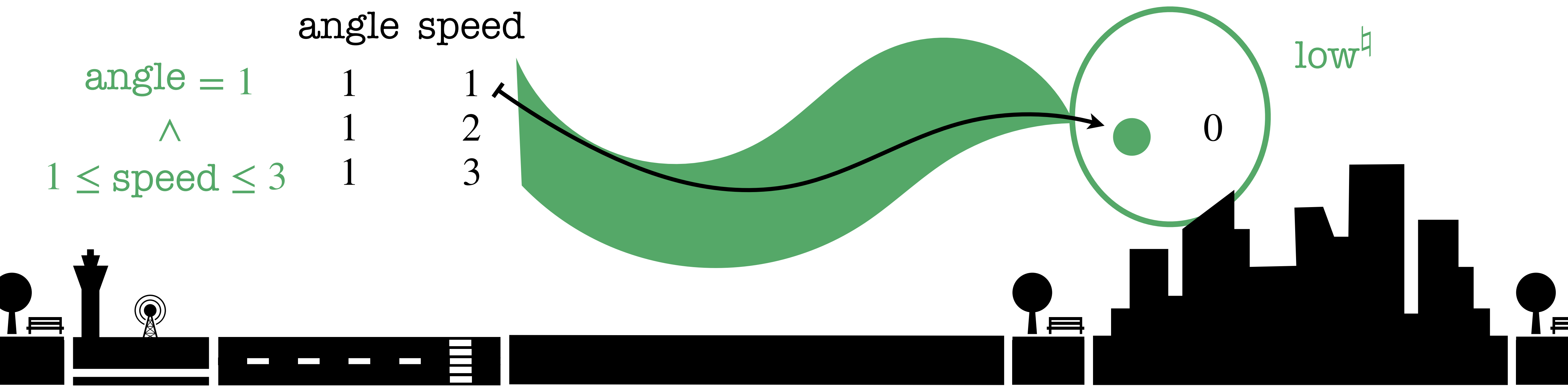
Goal: Quantify the impact of speed and angle on risk

	RANGE	OUTCOMES	RANGE ^h	OUTCOMES ^h
angle	3	2	3	4
speed	2	3	2	3



Source of Imprecision

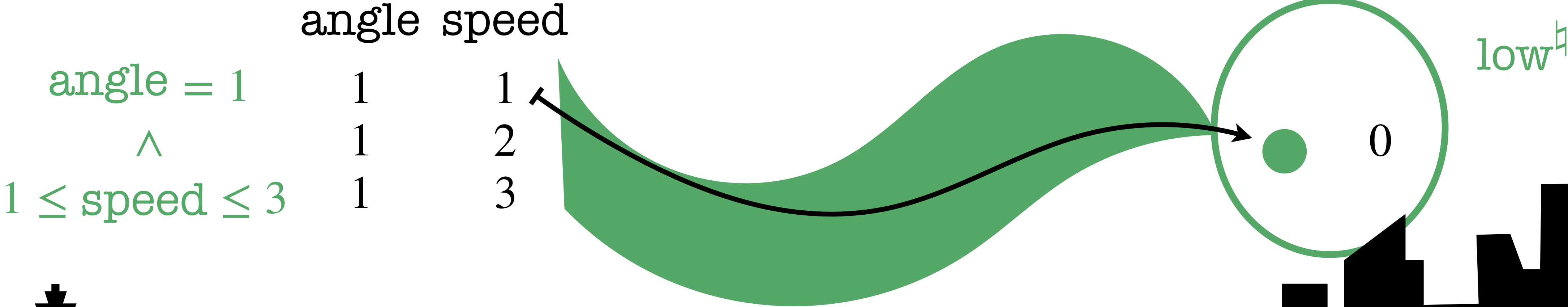
Abstraction of the Backward Analysis



Source of Imprecision

Abstraction of the
Backward Analysis

Choice of the
Output Buckets



Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
American Economic Review 2010.

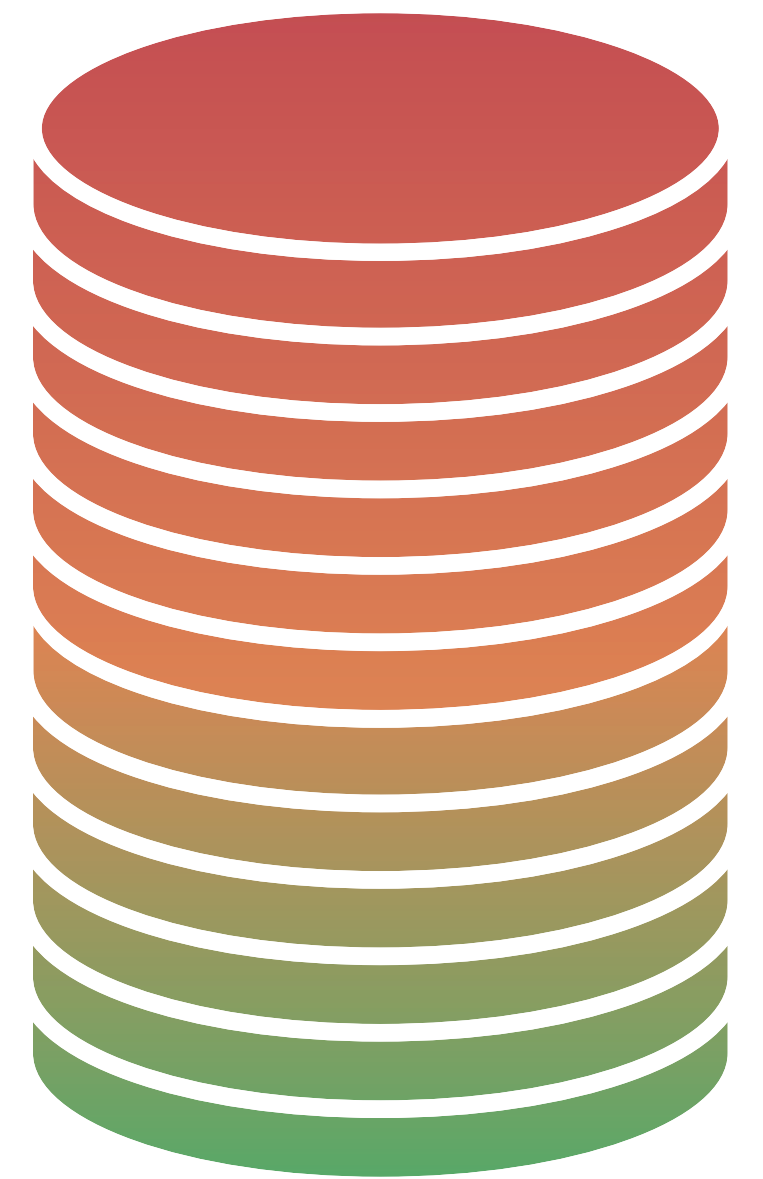
```
1: def mean_growth_rate_60_90(  
2:     portugal1, portugal2, portugal3,  
3:     norway1,  
4:     uk1, uk2, uk3, uk4,  
5:     usa1, usa2, usa3):  
6:     portugal_avg = (portugal1 + portugal2 + portugal3) / 3  
7:     norway_avg = norway1  
8:     uk_avg = (uk1 + uk2 + uk3 + uk4) / 4  
9:     usa_avg = (usa1 + usa2 + usa3) / 3  
10:    avg = (portugal_avg + norway_avg + uk_avg + usa_avg) / 4
```

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
American Economic Review 2010.

```
1: def mean_growth_rate_60_90(  
2:     portugal1, portugal2, portugal3,  
3:     norway1,  
4:     uk1, uk2, uk3, uk4,  
5:     usa1, usa2, usa3):  
6:     portugal_avg = (portugal1 + portugal2 + portugal3) / 3  
7:     norway_avg = norway1  
8:     uk_avg = (uk1 + uk2 + uk3 + uk4) / 4  
9:     usa_avg = (usa1 + usa2 + usa3) / 3  
10:    avg = (portugal_avg + norway_avg + uk_avg + usa_avg) / 4
```

$avg^{\uparrow} = 20\%$

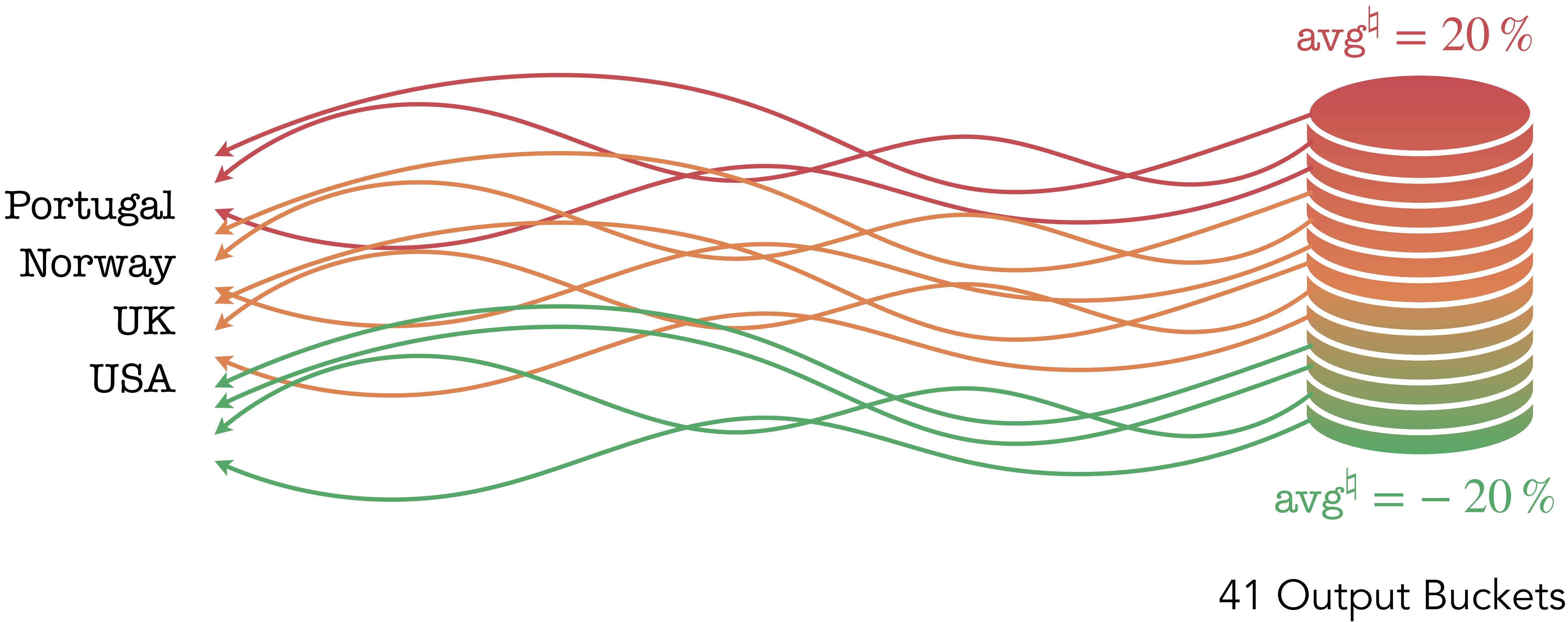


$avg^{\downarrow} = -20\%$

41 Output Buckets

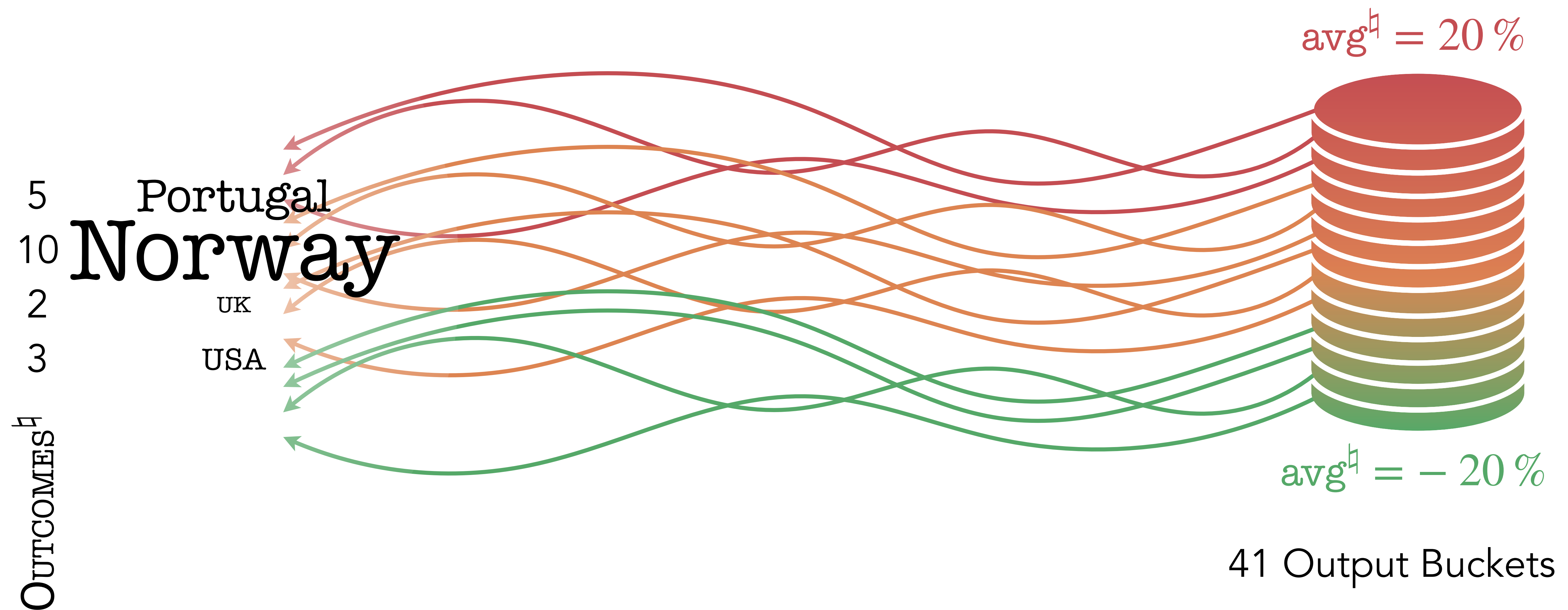
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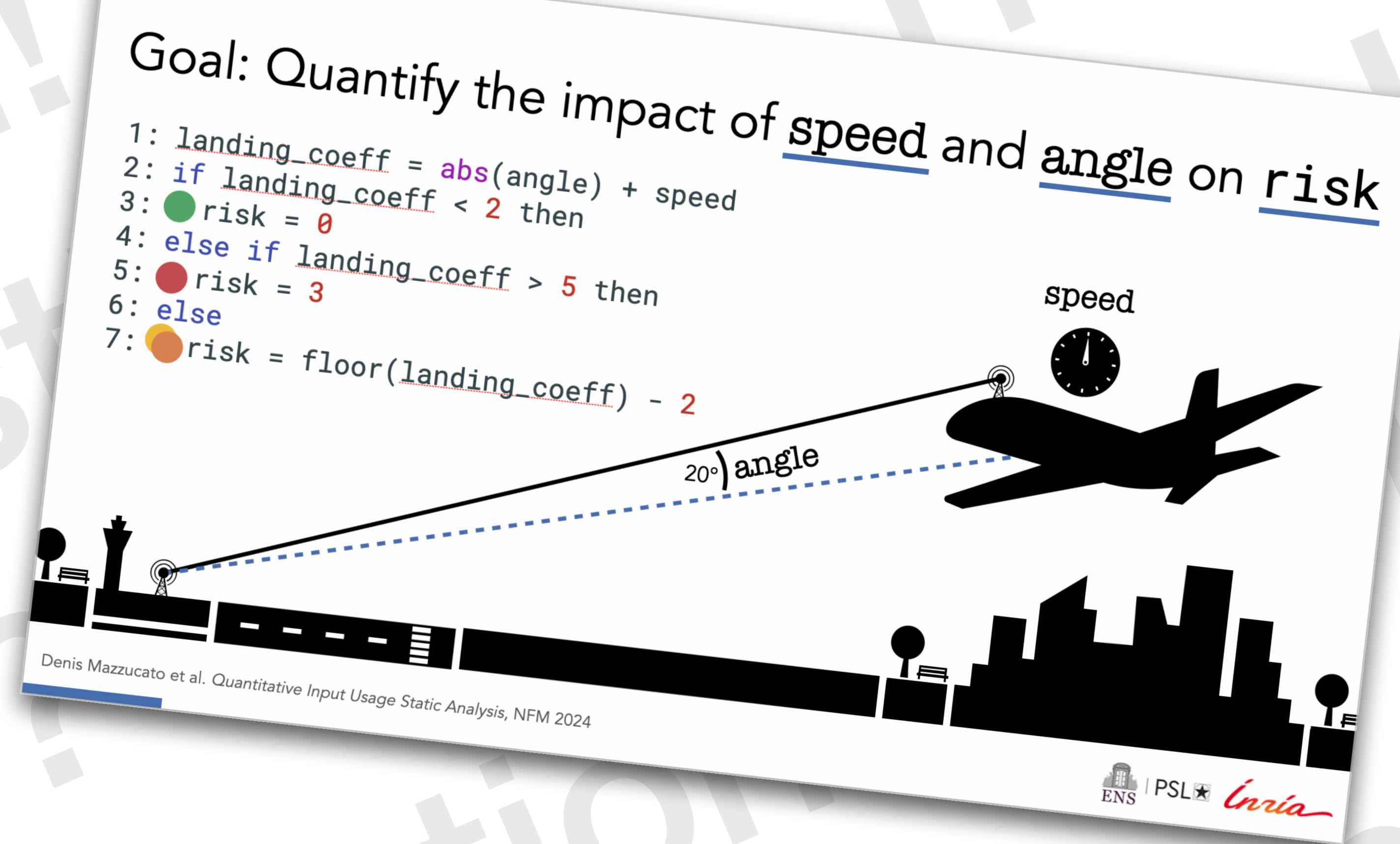
Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
American Economic Review 2010.

```
1: def mean_growth_rate_60_90(  
2:     portugal1, portugal2, portugal3,  
3:     norway1,  
4:     uk1, uk2, uk3, uk4,  
5:     usa1, usa2, usa3):  
6:     portugal_avg = (portugal1 + portugal2 + portugal3) / 3  
7:     norway_avg = norway1  
8:     uk_avg = (uk1 + uk2 + uk3 + uk4) / 4  
9:     usa_avg = (usa1 + usa2 + usa3) / 3  
10:    avg = (portugal_avg + norway_avg + uk_avg + usa_avg) / 4
```

Conclusion

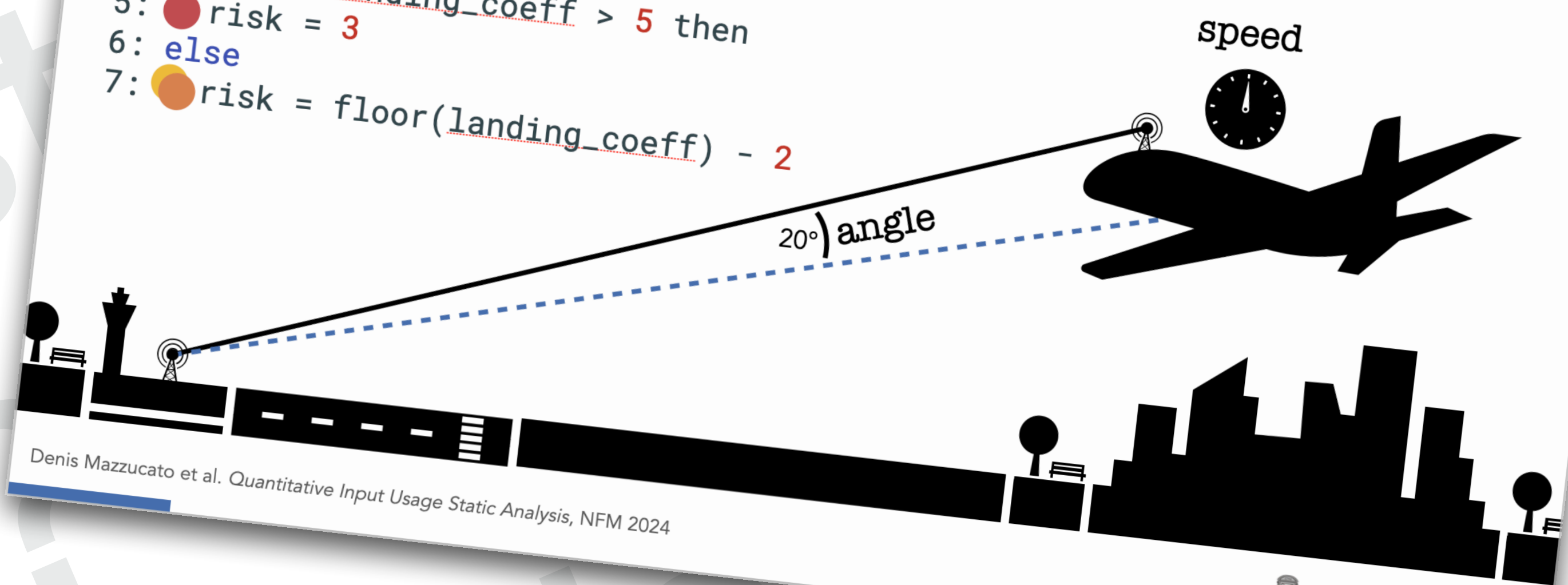
Conclusion



Conclusion

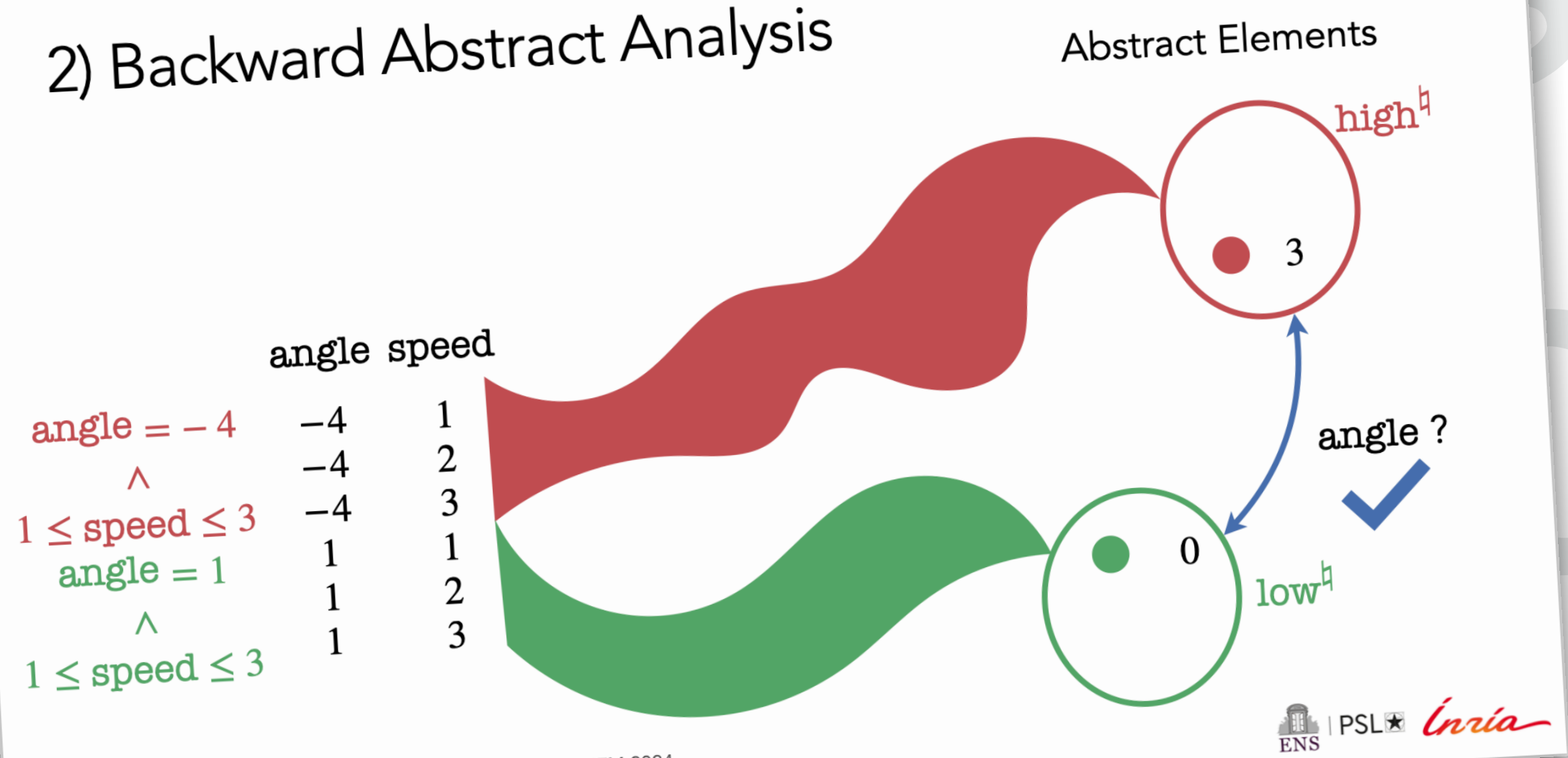
Goal: Quantify the impact of speed and angle on risk

```
1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3:   risk = 0
4: else if landing_coeff > 5 then
5:   risk = 3
6: else
7:   risk = floor(landing_coeff) - 2
```



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2) Backward Abstract Analysis

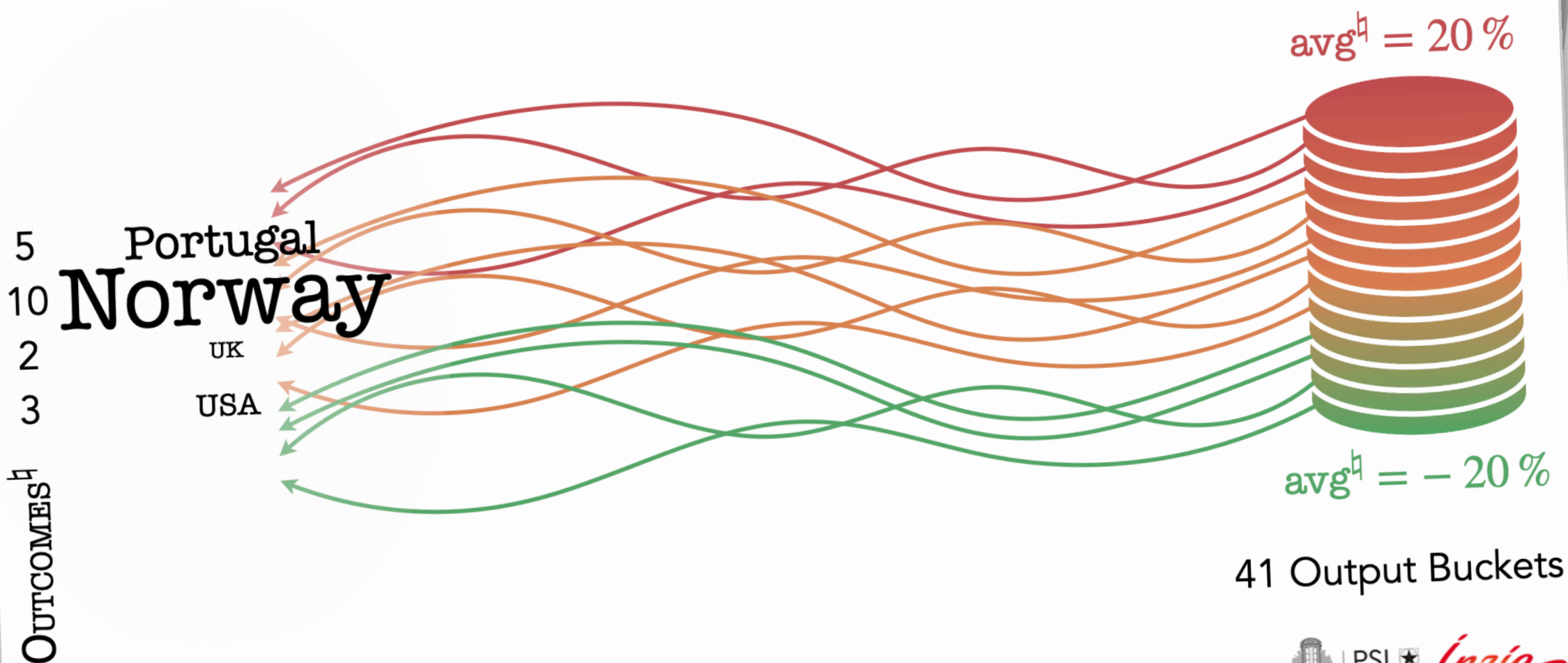


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Conclusion

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.

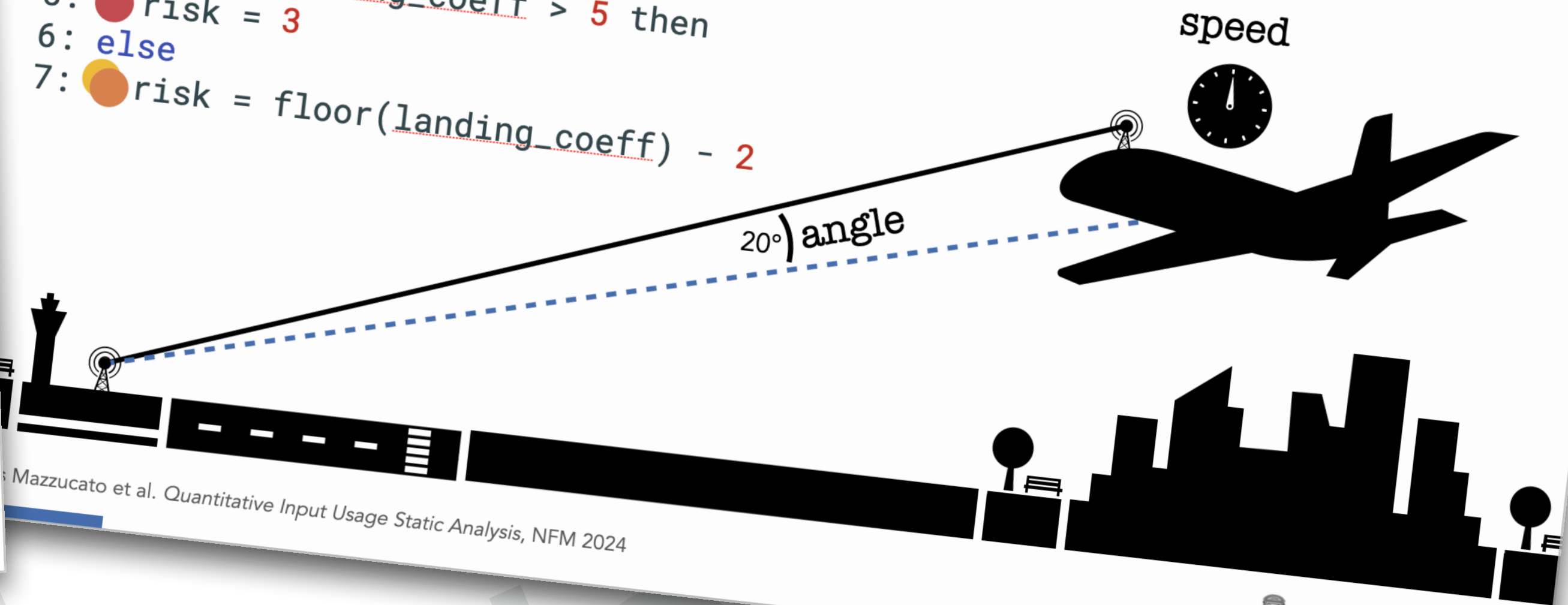


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ENS | PSL | Inria

Goal: Quantify the impact of speed and angle on risk

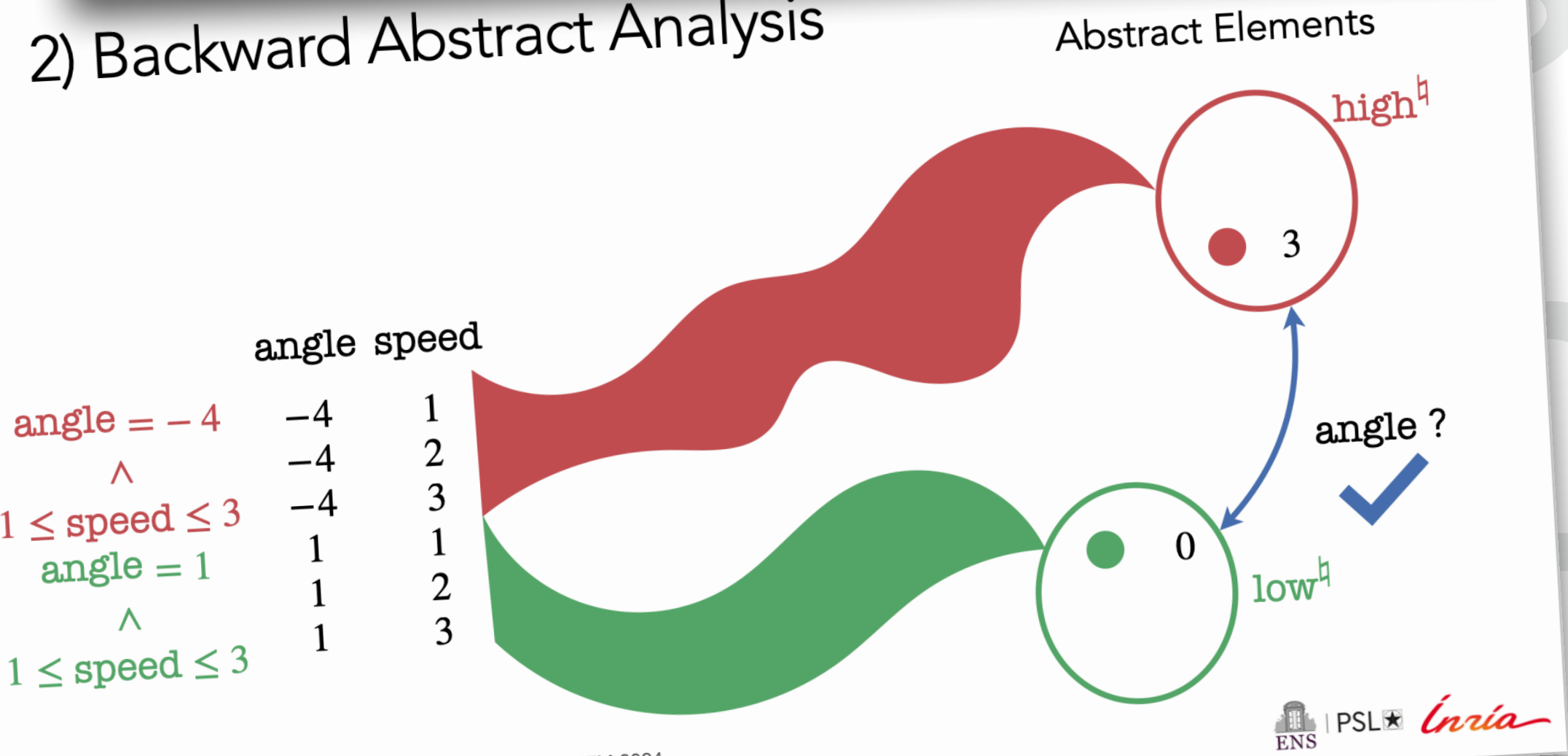
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7:   risk = floor(landing_coeff) - 2
```



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ENS | PSL | Inria

2) Backward Abstract Analysis



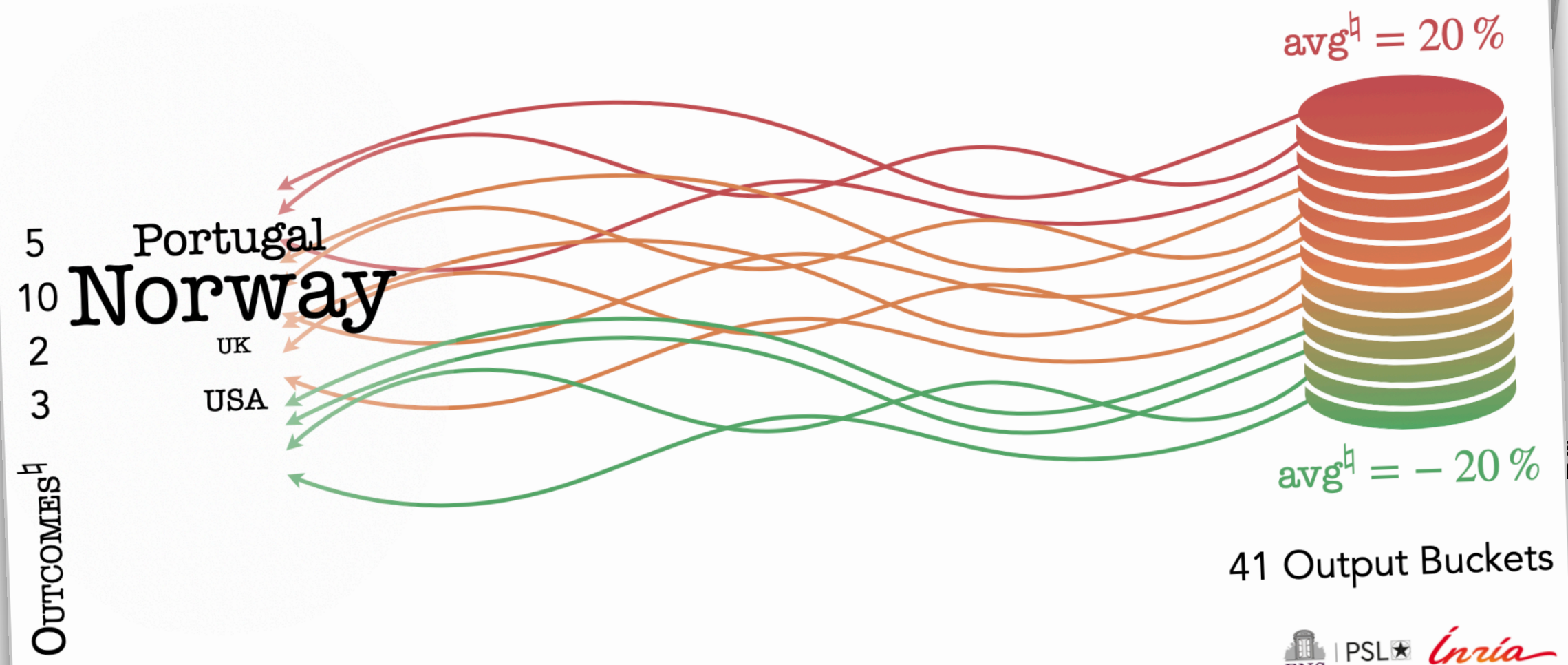
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ENS | PSL | Inria

Conclusion

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt. American Economic Review 2010.

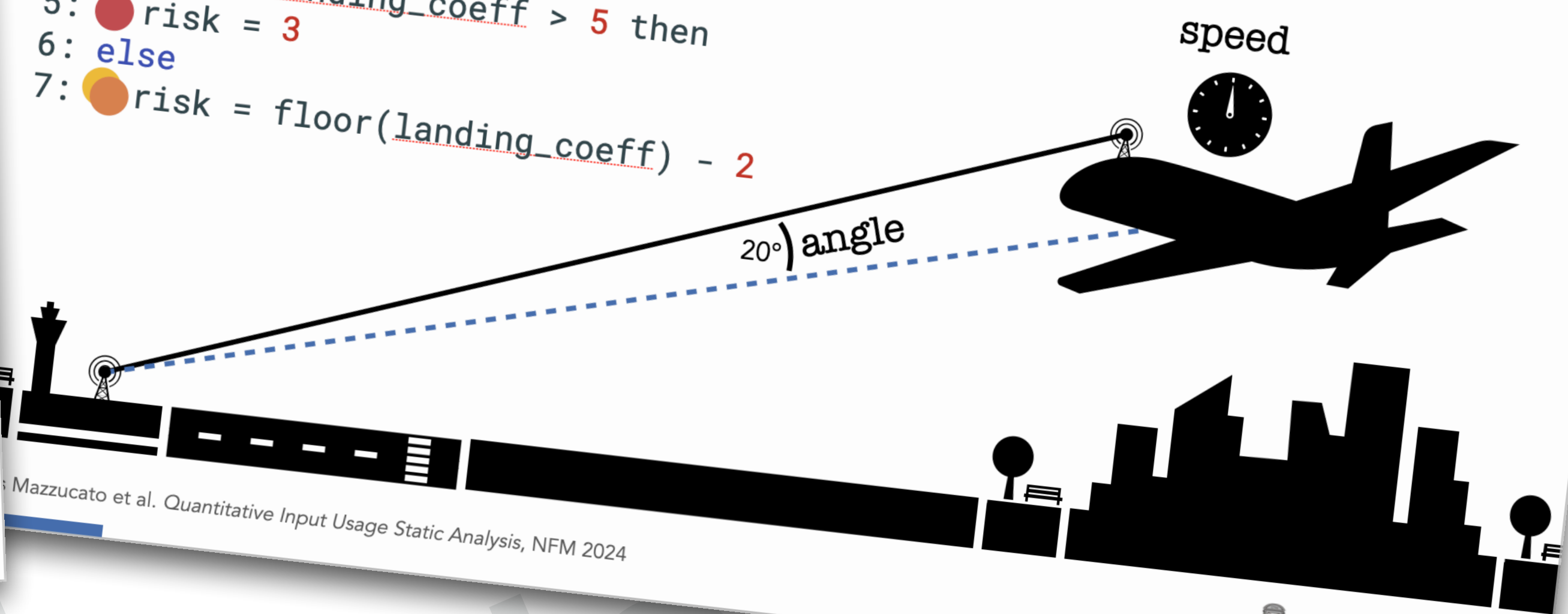


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Goal: Quantify the impact of speed and angle on risk

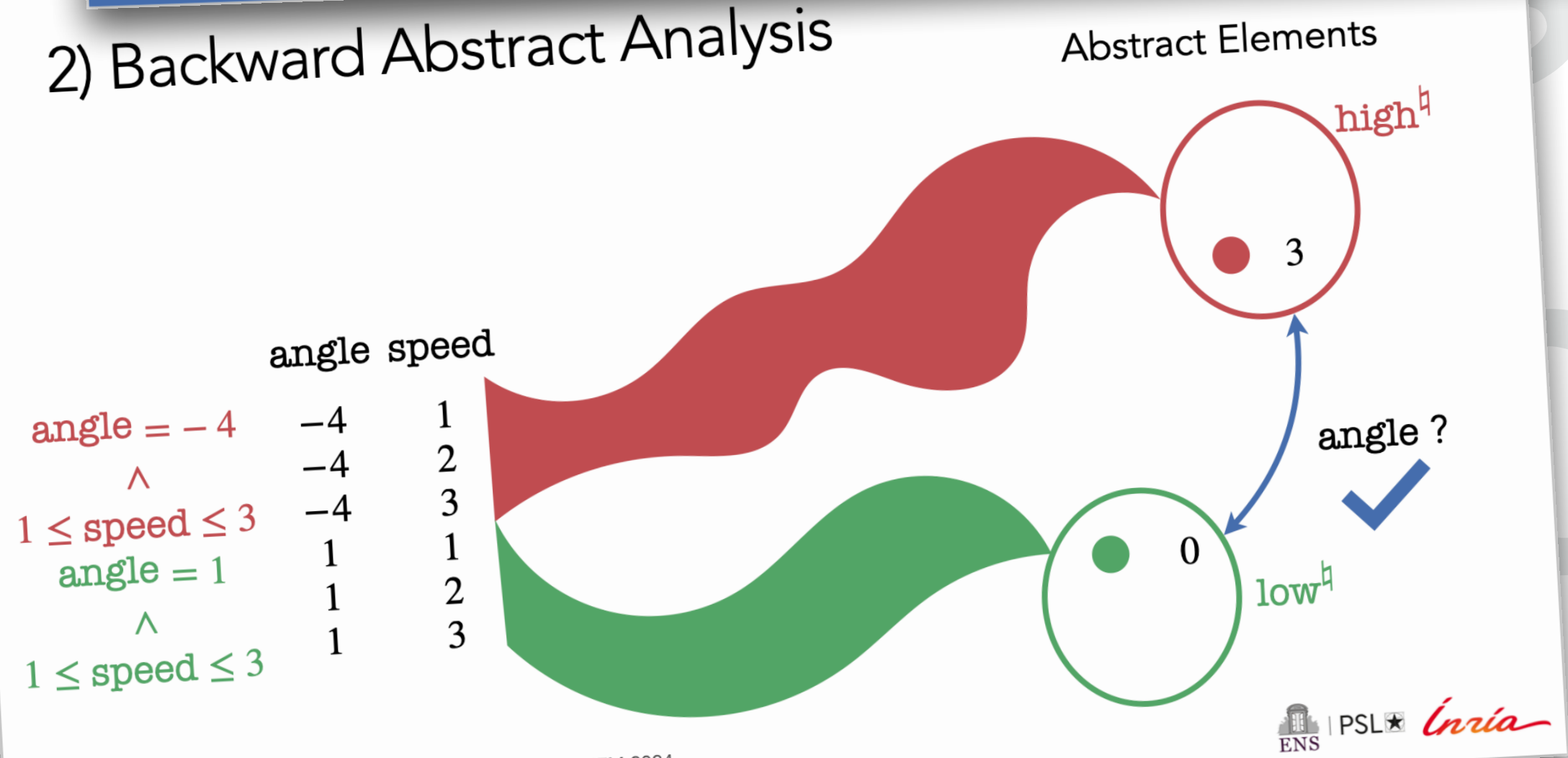
```
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5:   risk = 3
6: else
7:   risk = floor(landing_coeff) - 2
```



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2) Backward Abstract Analysis



	angle	speed
angle = -4	-4	1
^	-4	2
1 ≤ speed ≤ 3	-4	3
angle = 1	1	1
^	1	2
1 ≤ speed ≤ 3	1	3

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Spoiler (Submitted)

Quantify the Impact on Timing Behavior

