Nasa Formal Methods (NFM) 2024, Moffett Field, California (USA)

Quantitative Input Usage Static Analysis

Denis Mazzucato, Marco Campion, and Caterina Urban

4 June 2024

Aircraft Landing Alarm System 🚨

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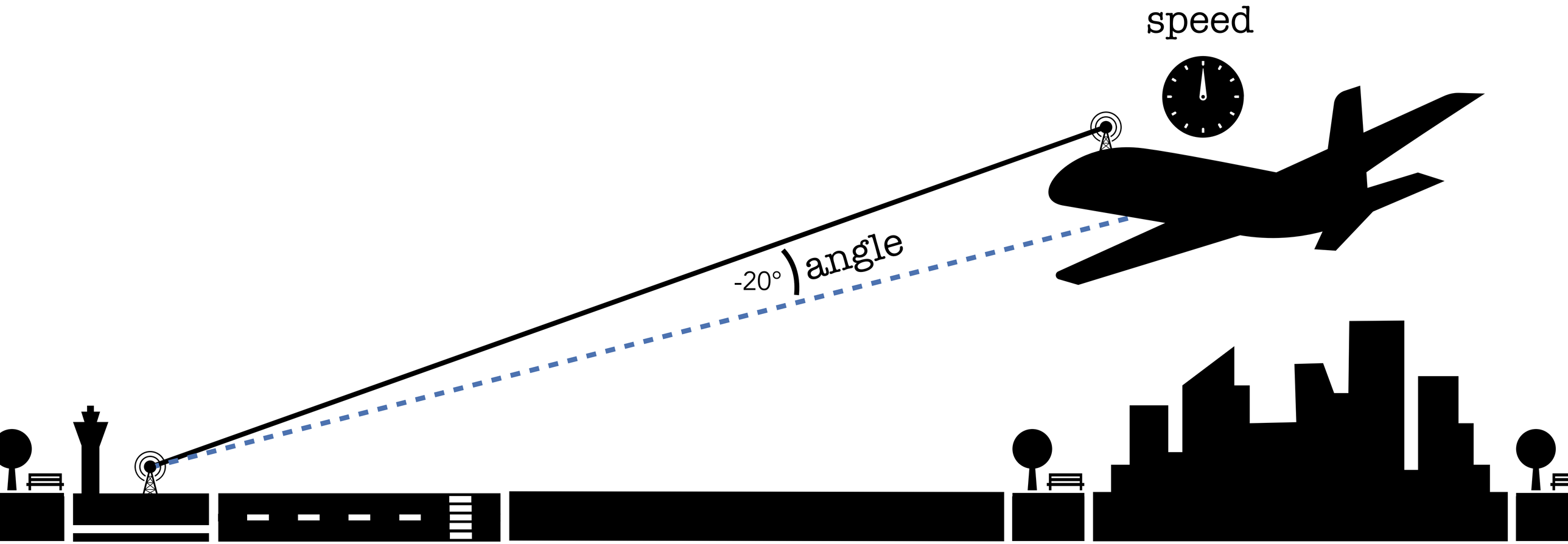

Denis Mazzucato et al. *Quantitative Input Usage Static Analysis*, NFM 2024

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Aircraft Landing Alarm System 🚨

```
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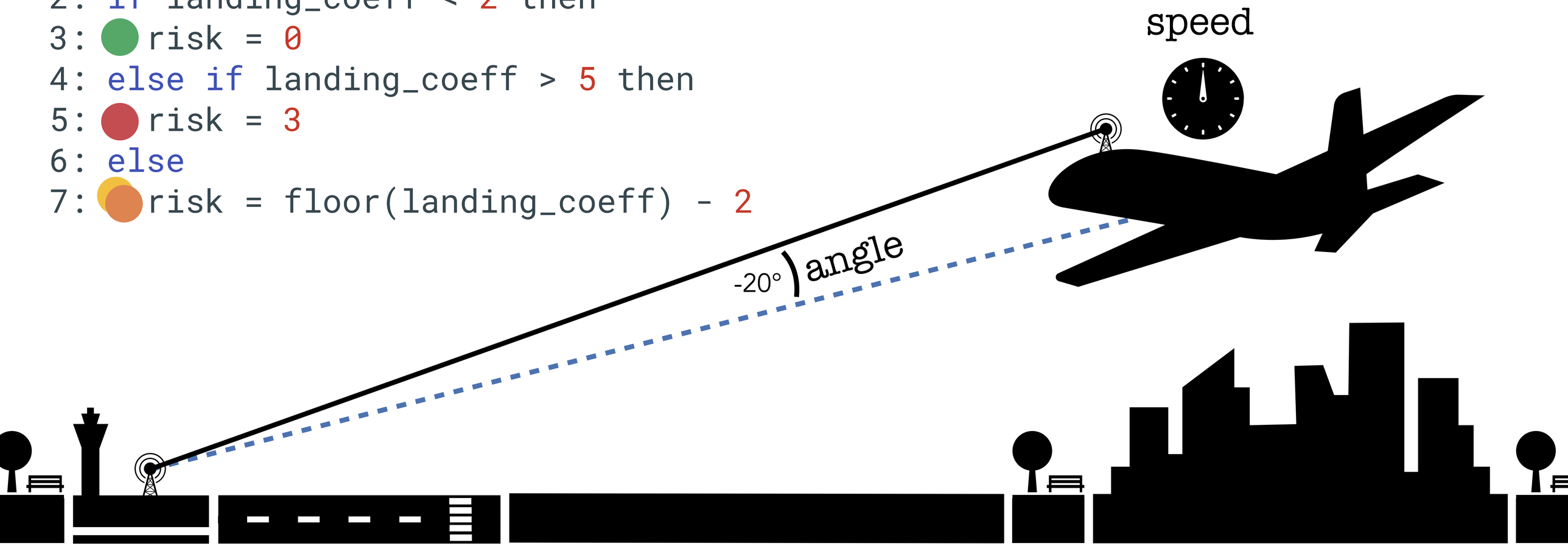

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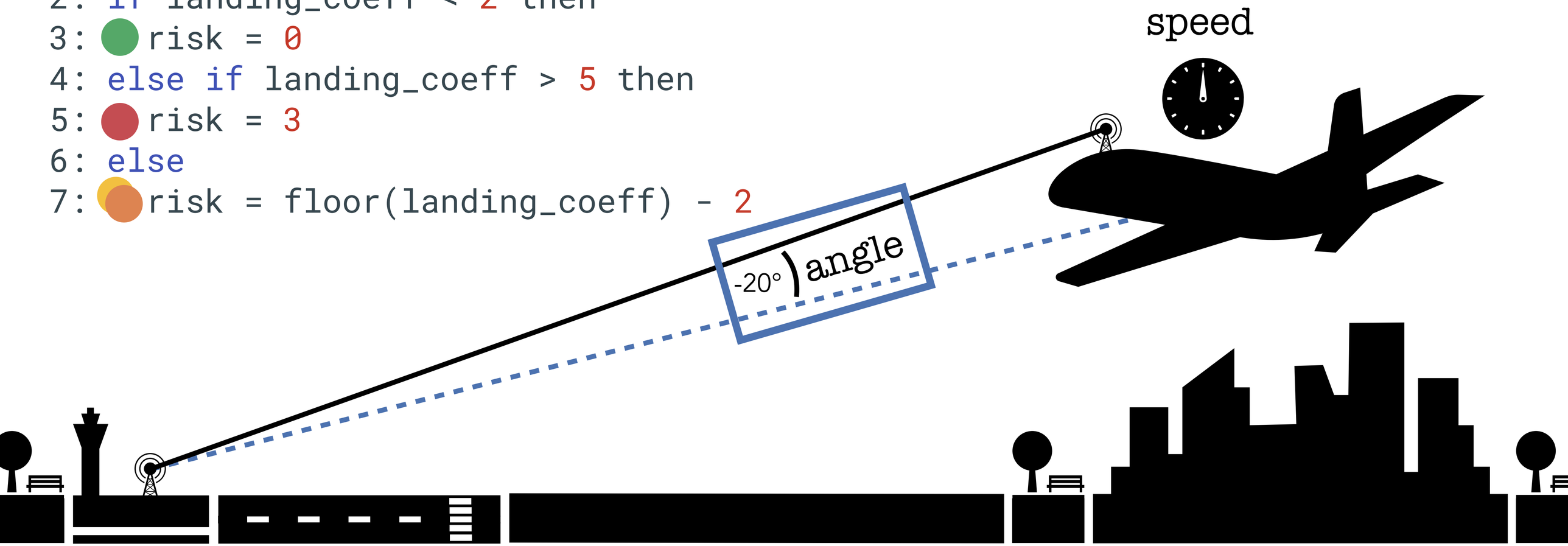

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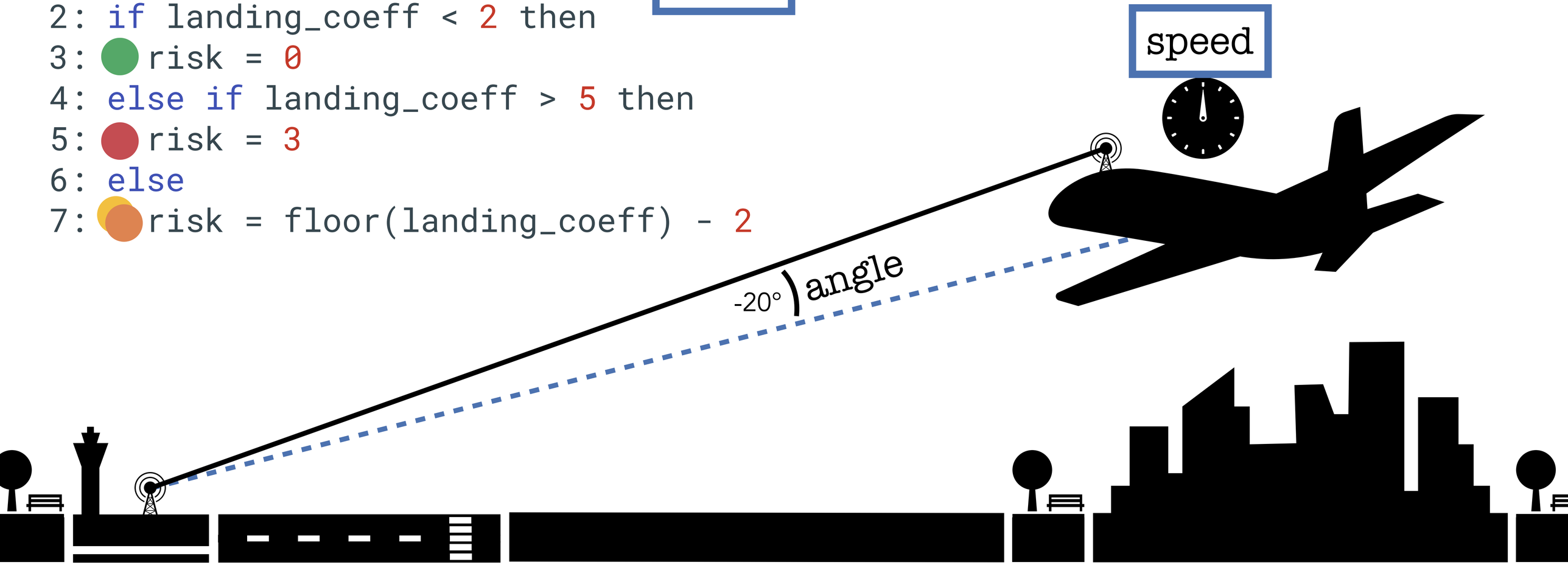

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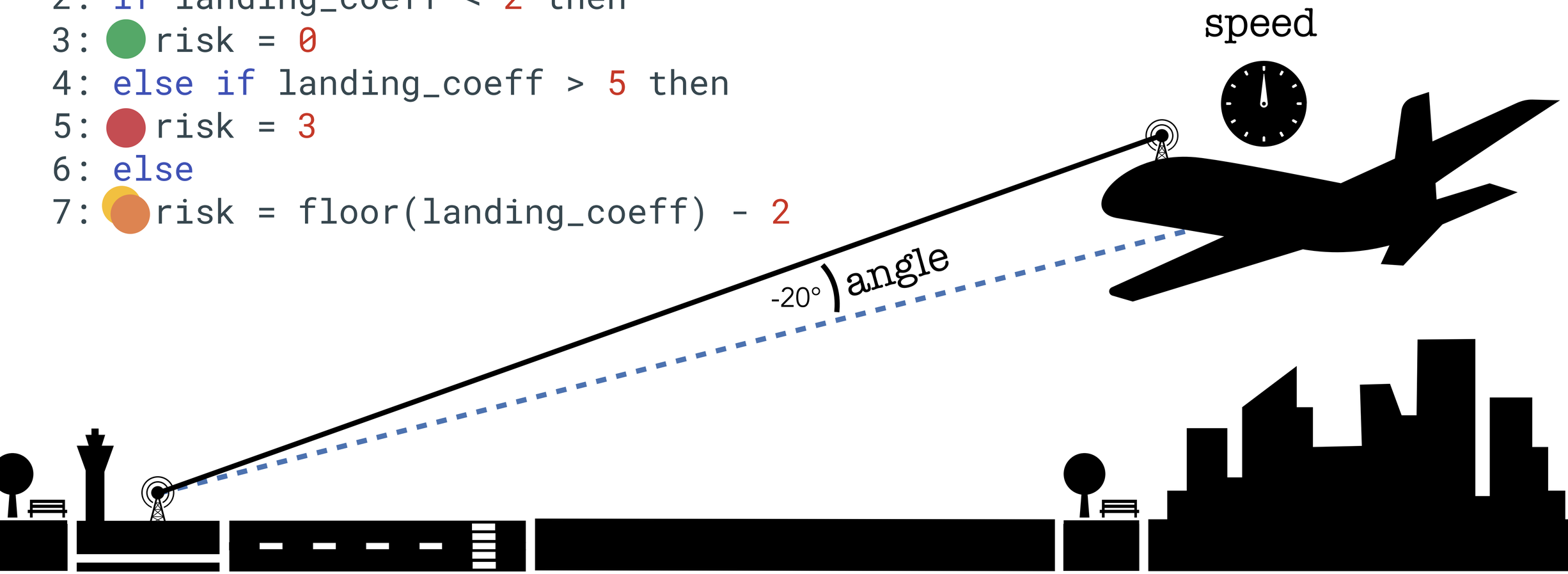

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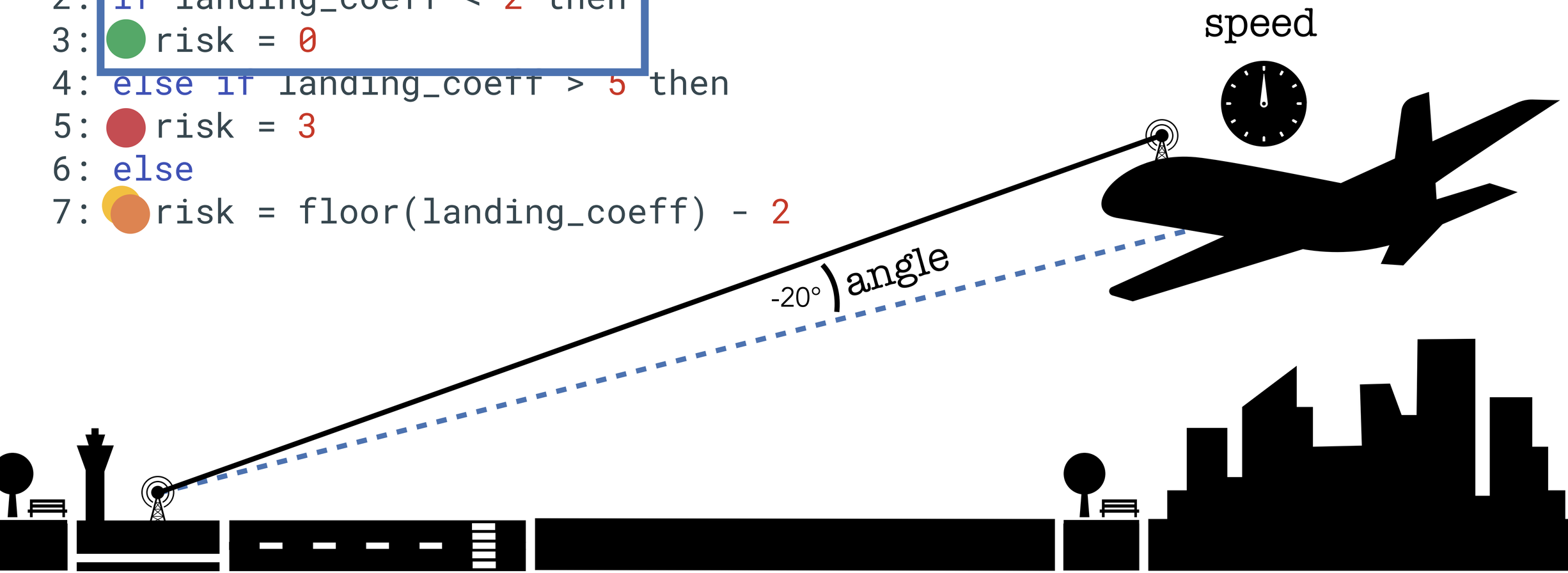

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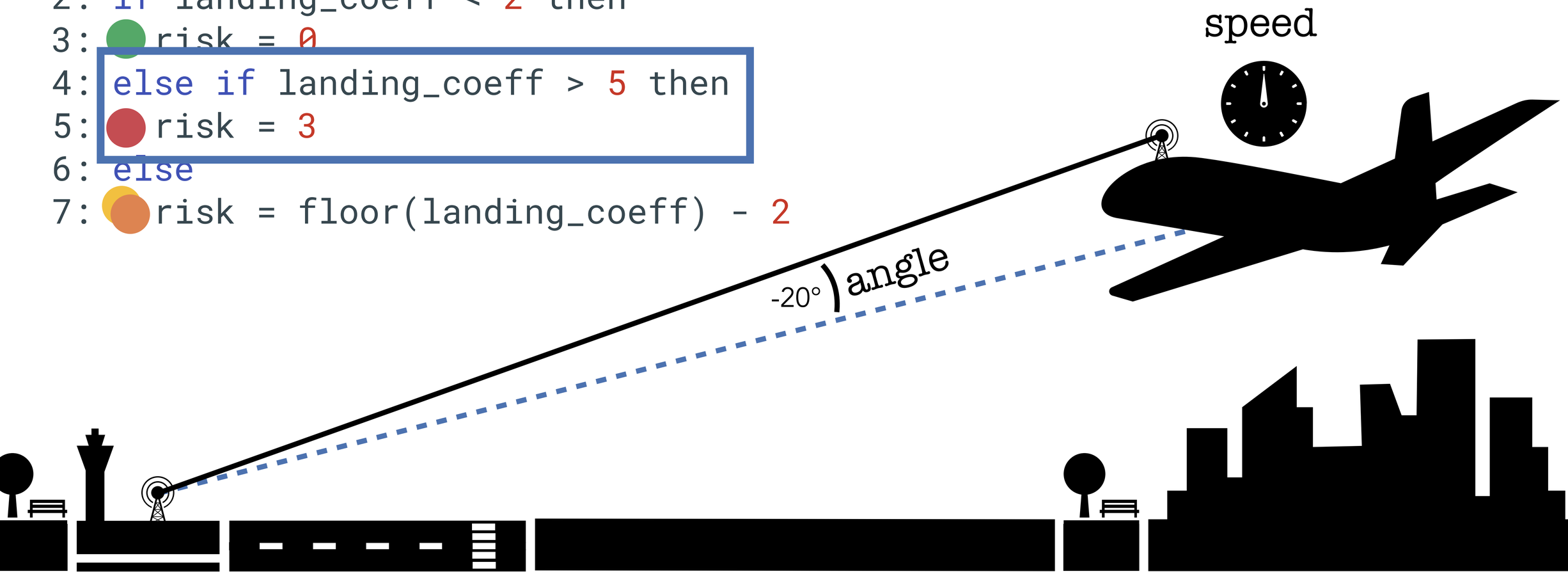

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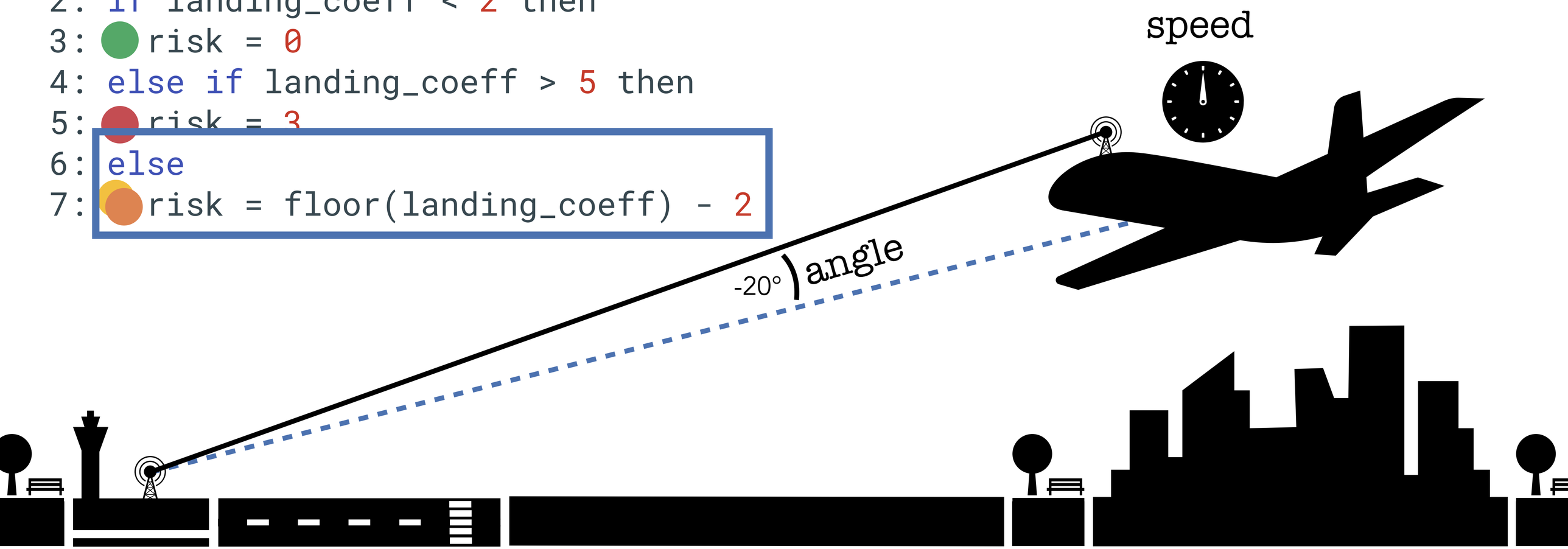

Aircraft Landing Alarm System 🚨

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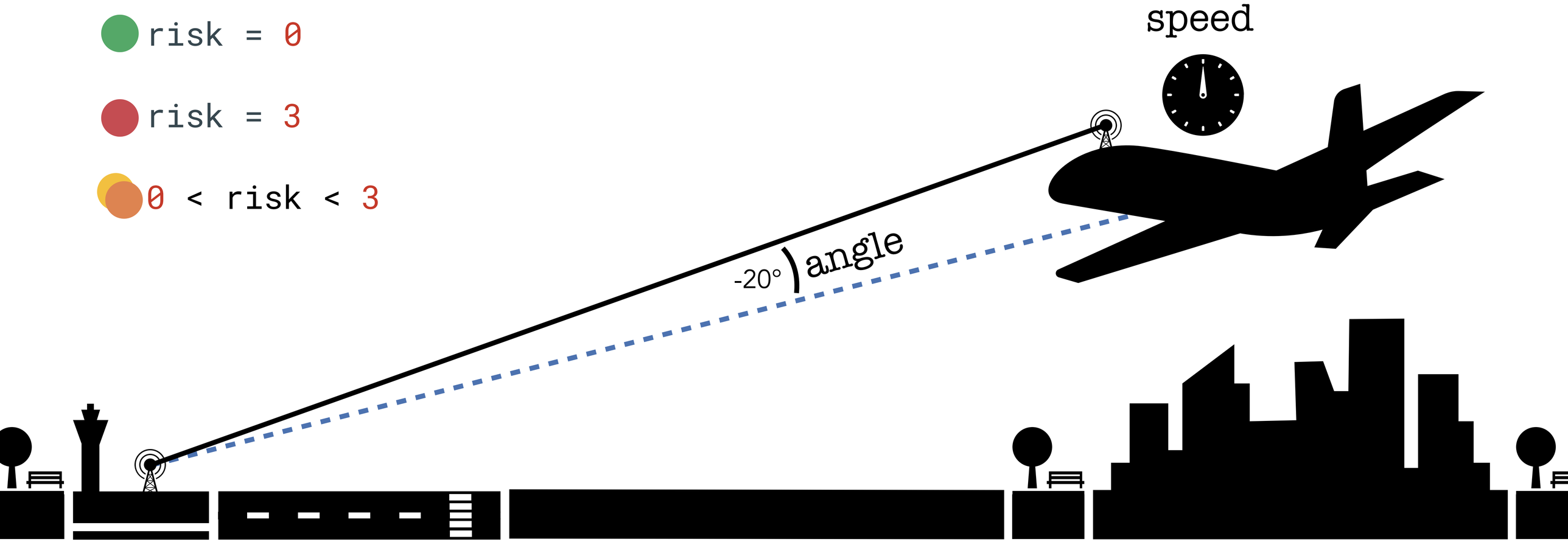

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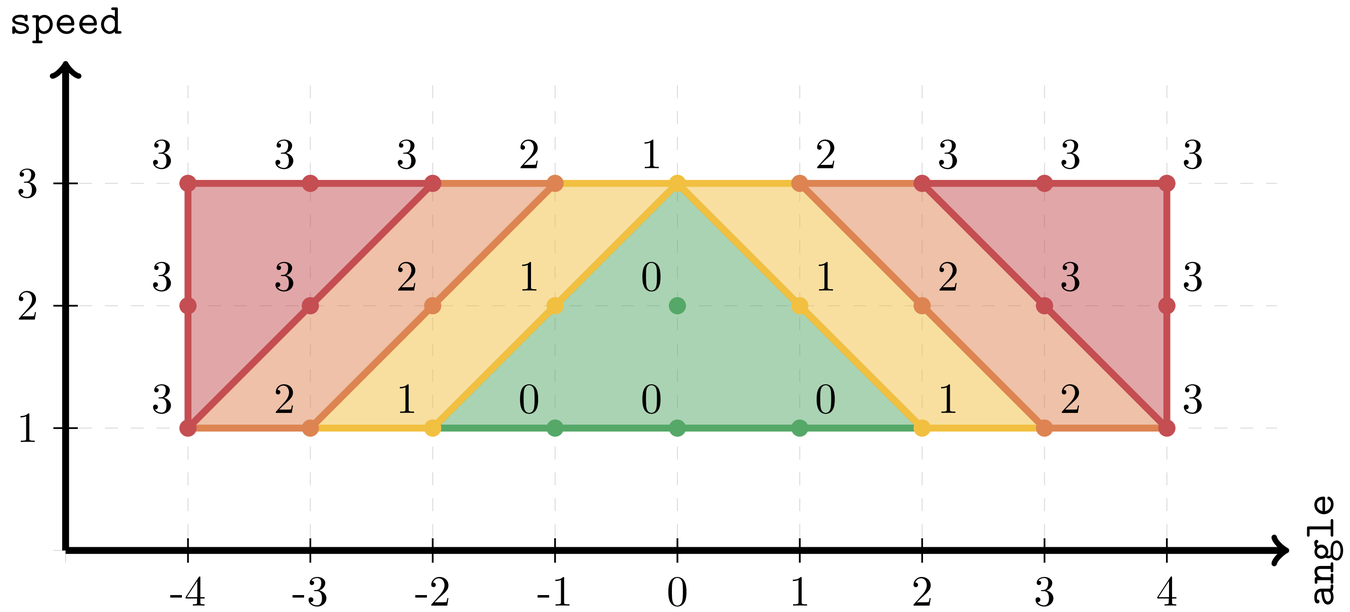

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Aircraft Landing Alarm System 🚨

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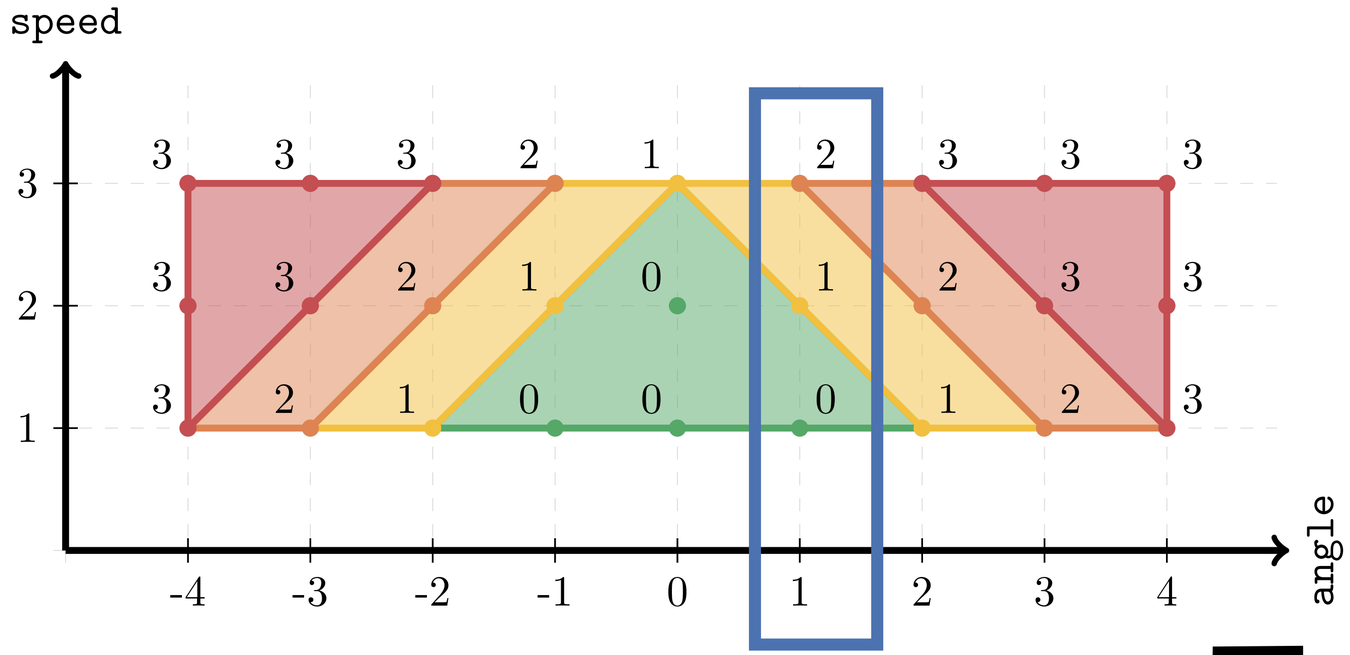

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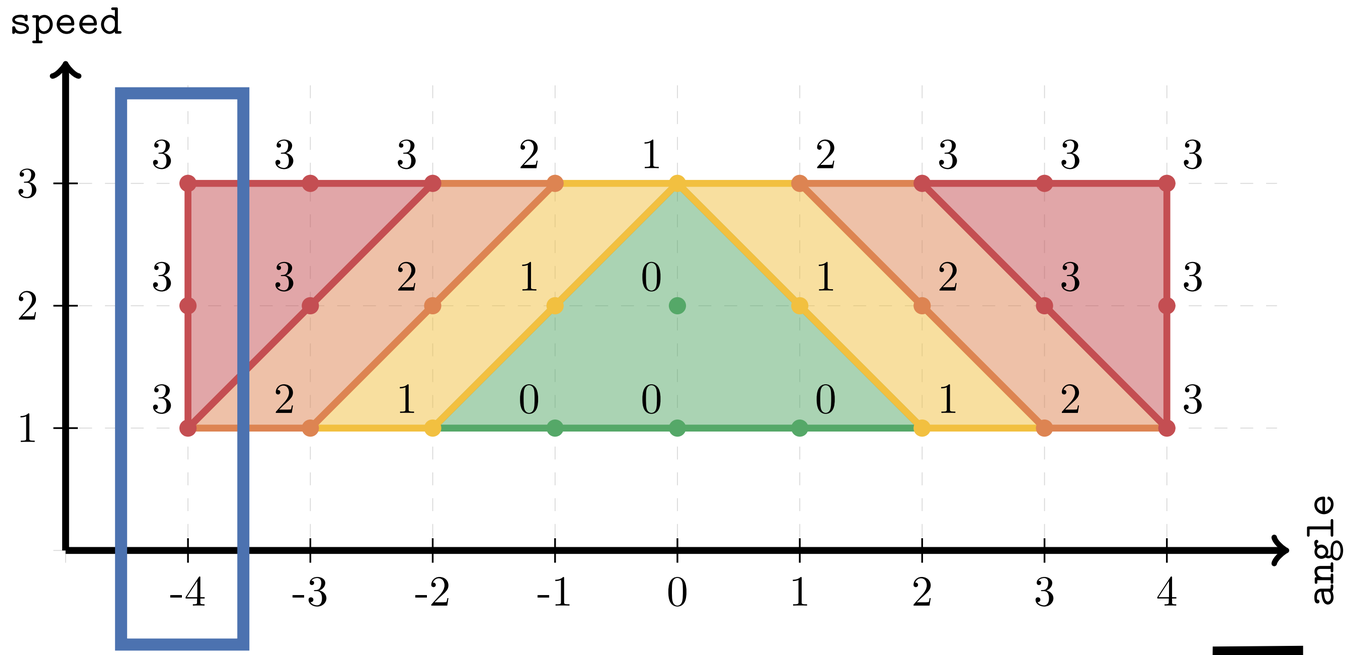

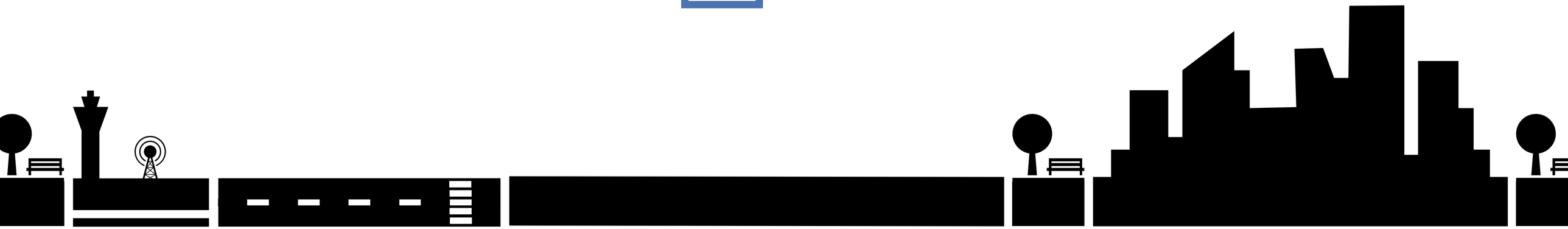

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Goal: Quantify the impact of speed and angle on risk

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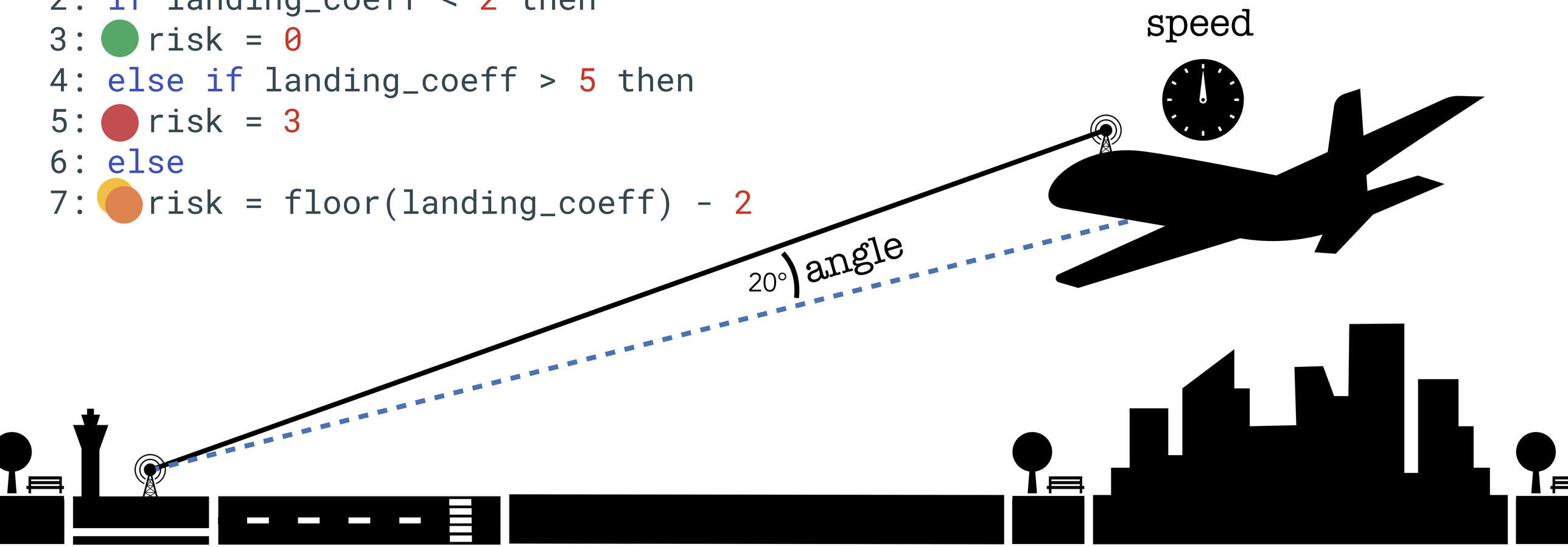

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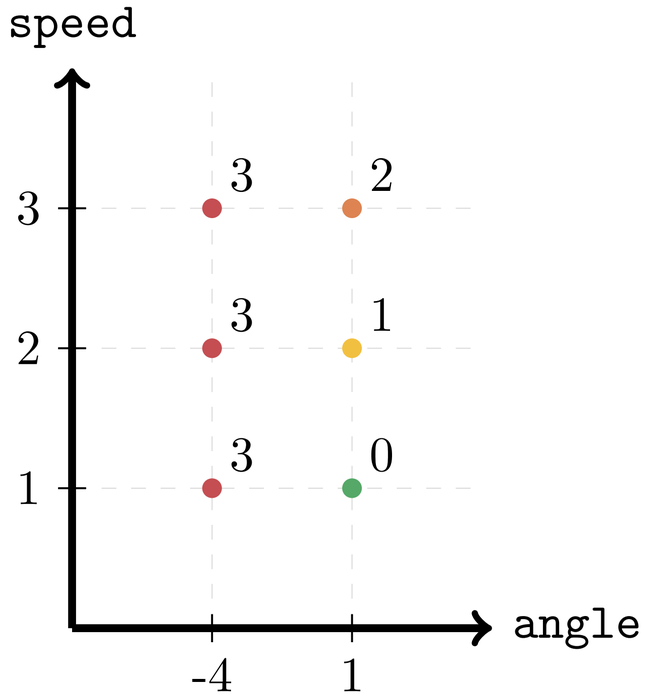
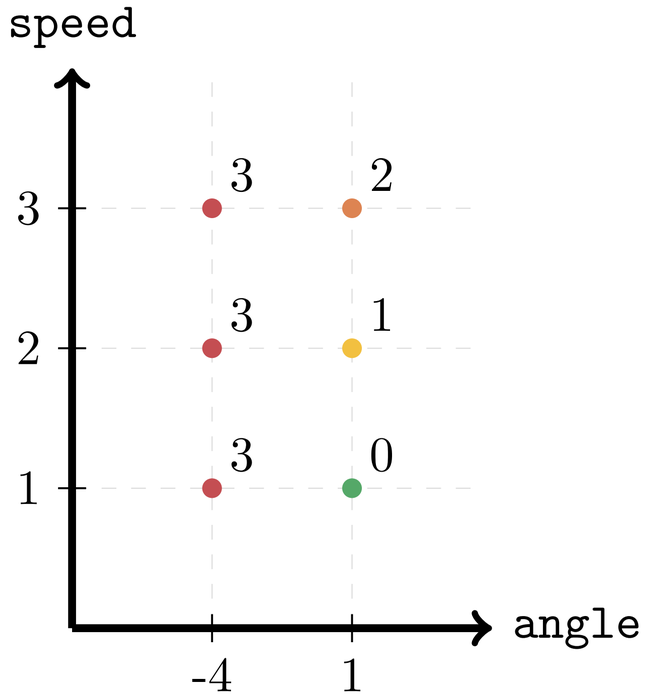


Goal: Quantify the impact of `speed` and `angle` on `risk`



Number of *reachable outcomes*

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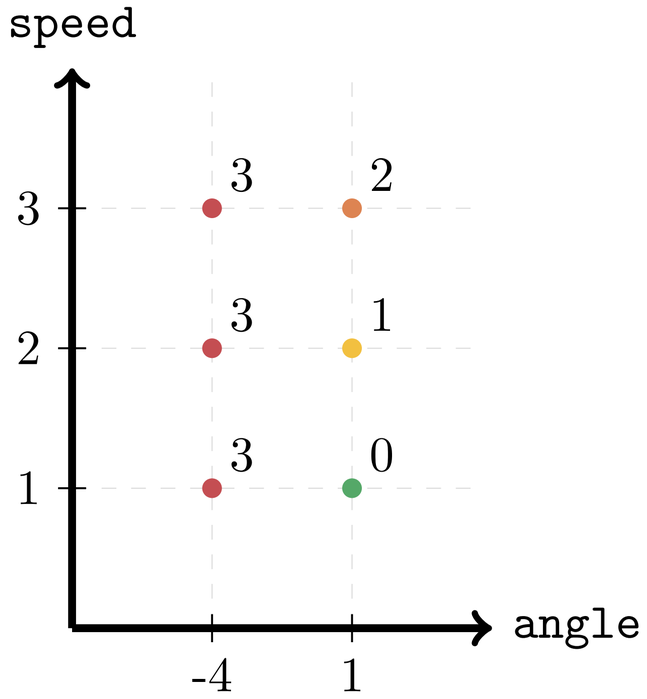
Goal: Quantify the impact of `speed` and `angle` on `risk`



Number of *reachable outcomes*

OUTCOMES

ENS | PSL Inria

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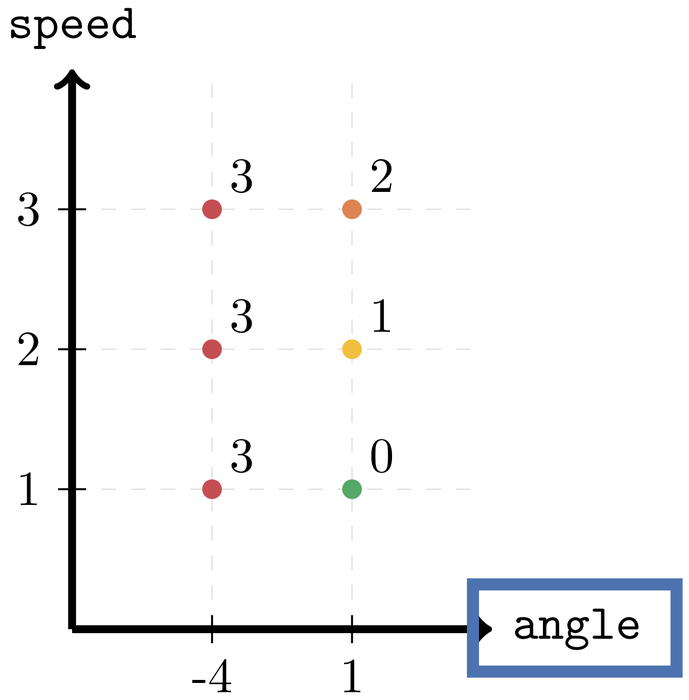

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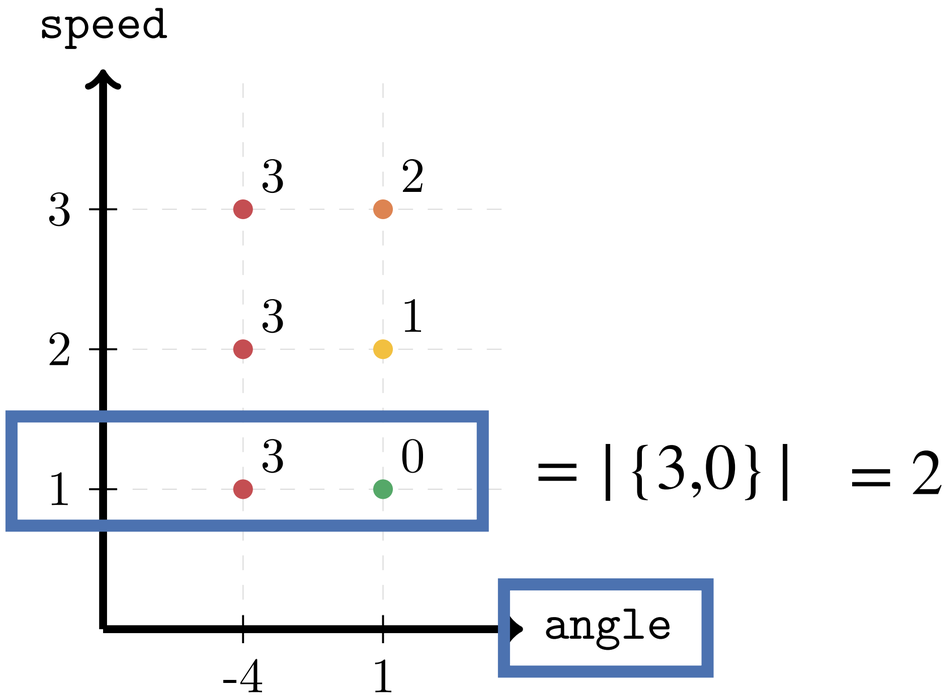
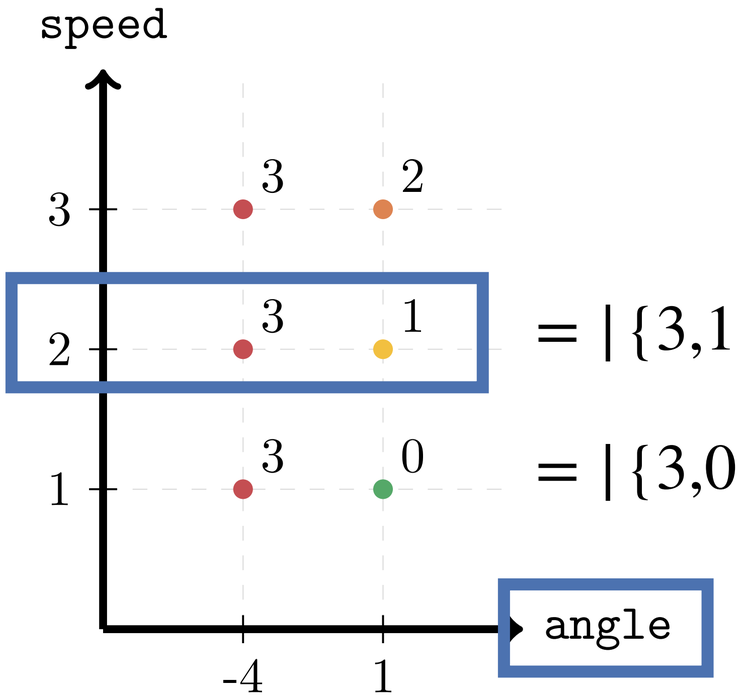


$= |\{3,0\}| \quad = 2$

Number of *reachable outcomes*

OUTCOMES

Goal: Quantify the impact of `speed` and `angle` on `risk`



$= |\{3,1\}| = 2$

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Number of *reachable outcomes*

OUTCOMES

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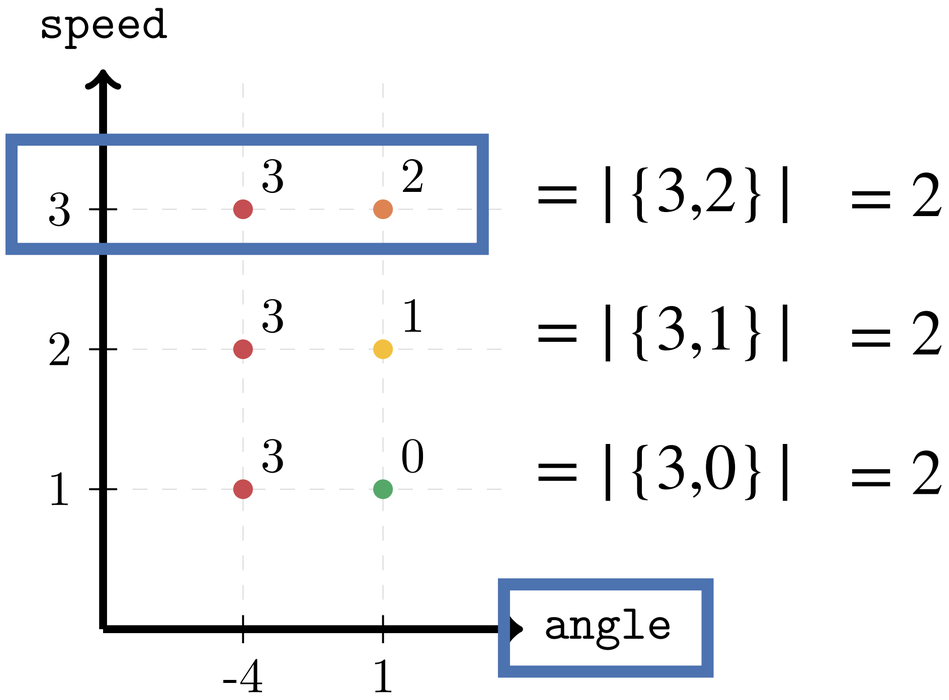

$= |\{3,2\}| = 2$

$= |\{3,1\}| = 2$

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Number of *reachable outcomes*

OUTCOMES

ENS | PSL Inria

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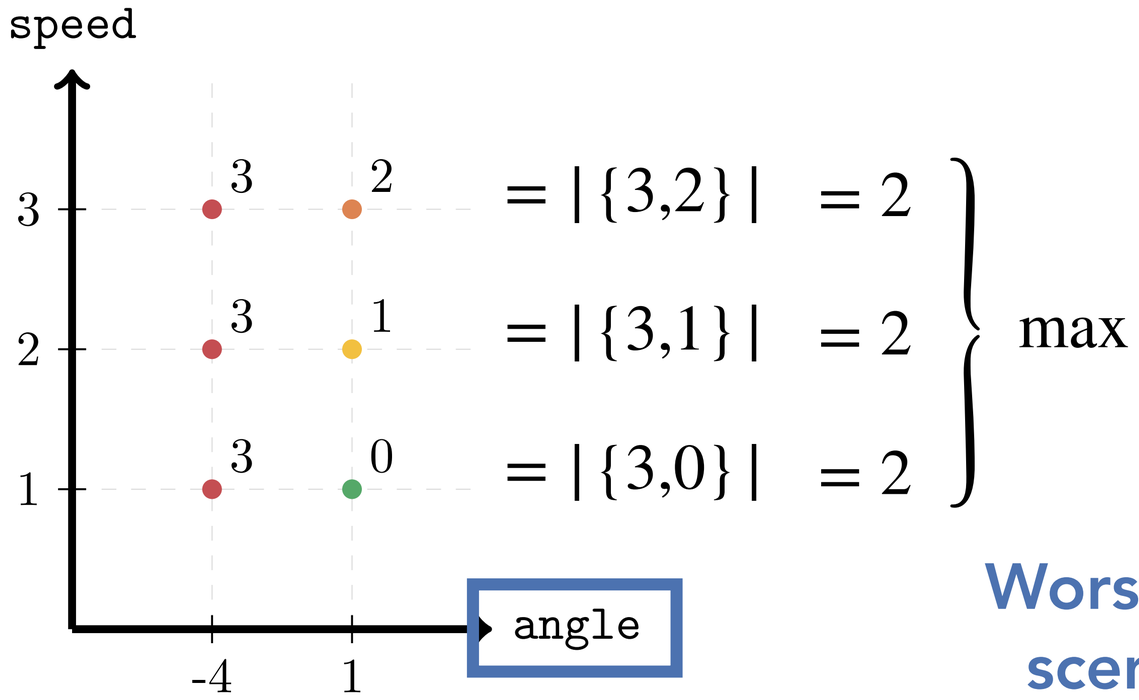
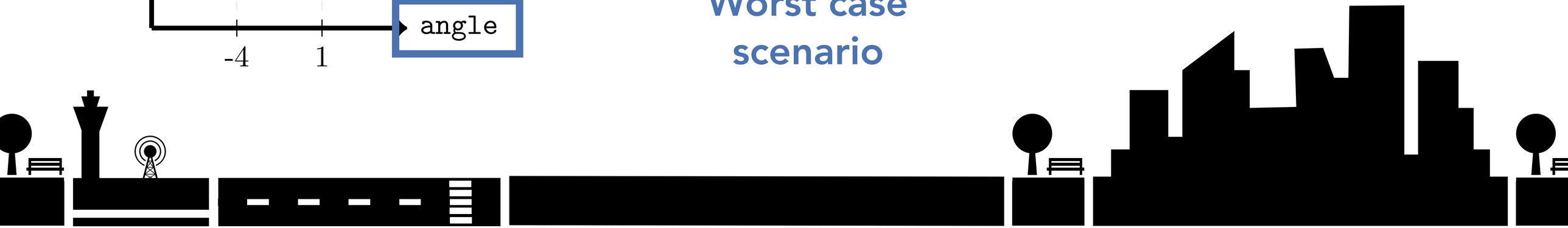


$$\left.\begin{array}{lll} = |\{3,2\}| & = 2 \\ = |\{3,1\}| & = 2 \\ = |\{3,0\}| & = 2 \end{array}\right\} \max$$

Number of *reachable outcomes*

OUTCOMES

Worst case scenario



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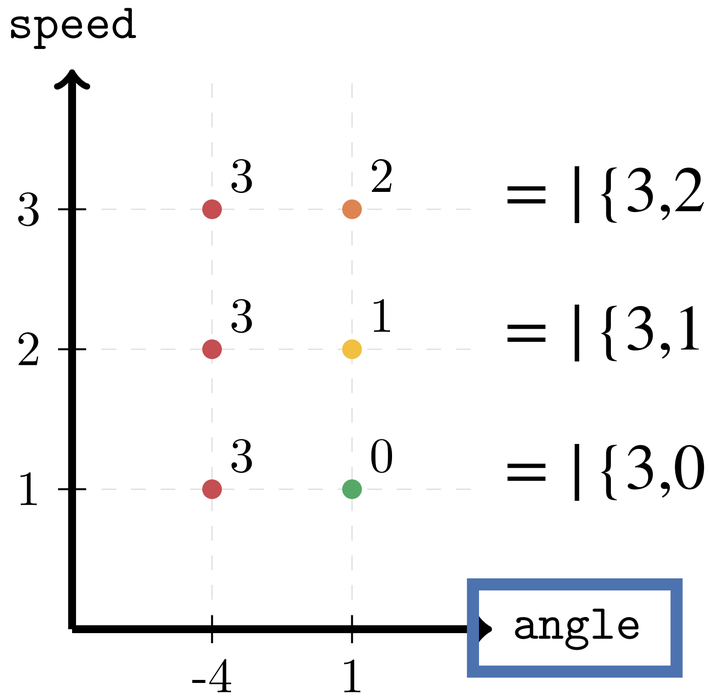

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OUTCOMES

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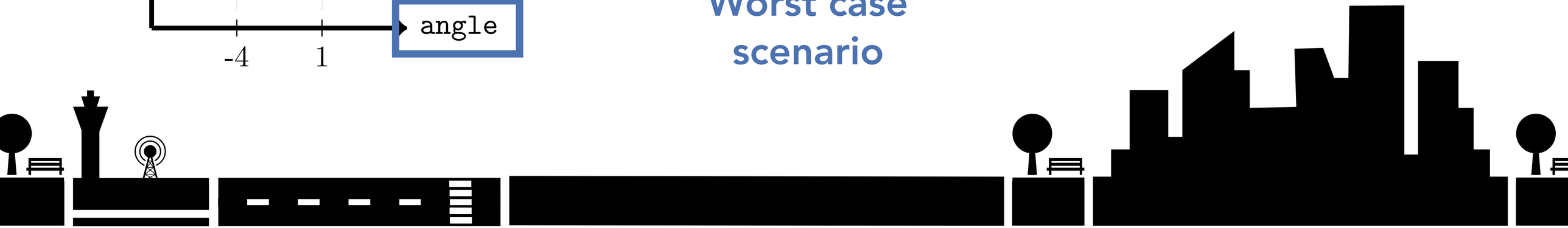

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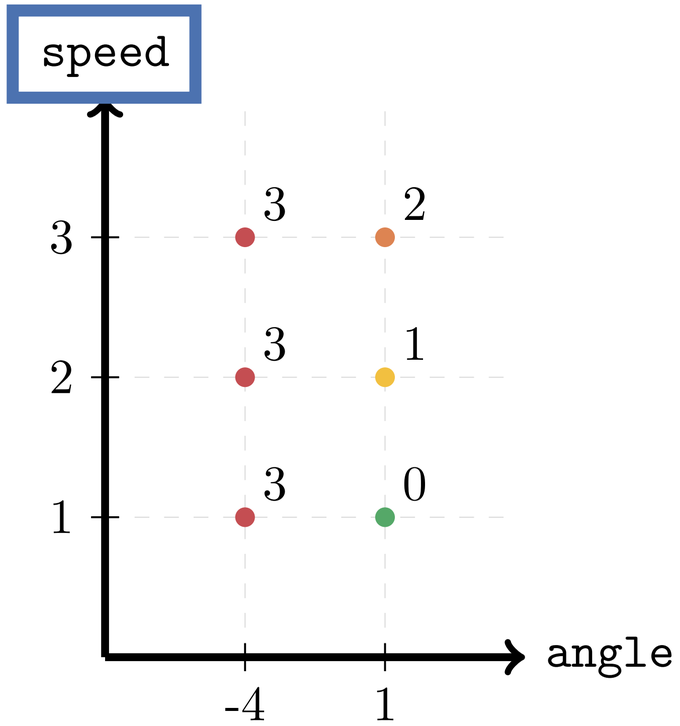

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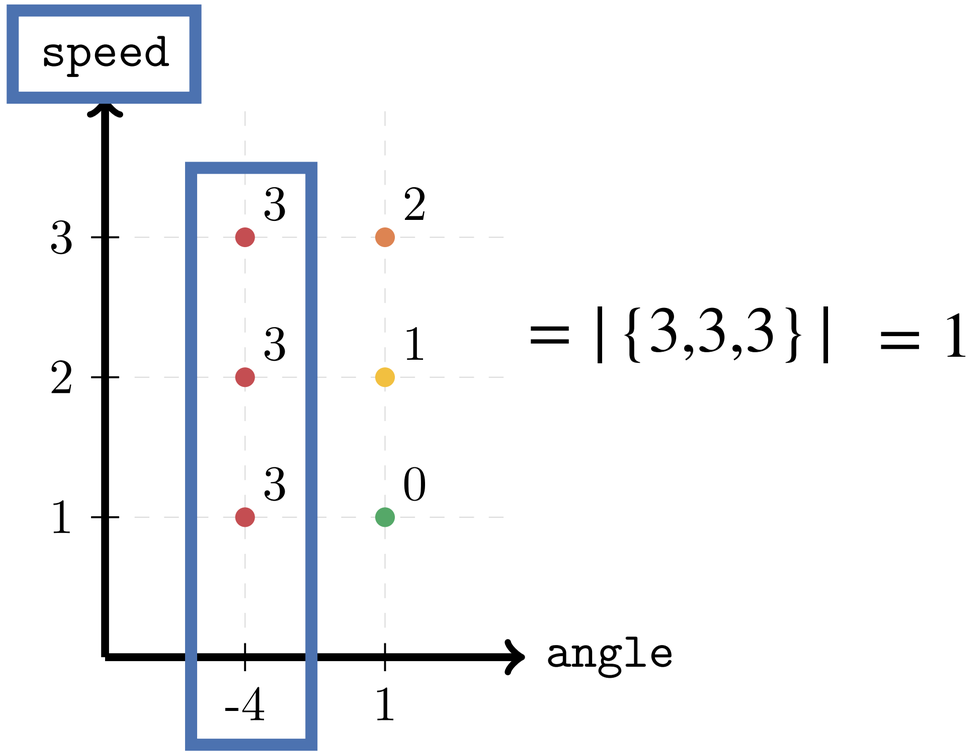

$= |\{3,3,3\}| = 1$

Number of *reachable outcomes*

OUTCOMES

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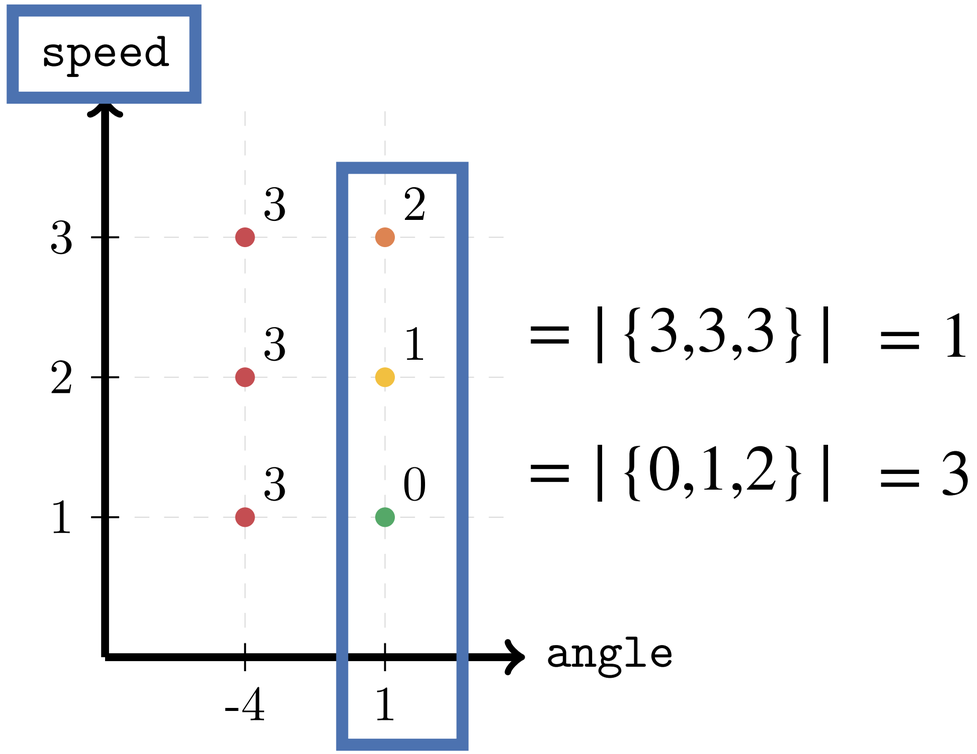

$= |\{3,3,3\}| = 1$

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Number of *reachable outcomes*

OUTCOMES

ENS | PSL Inria

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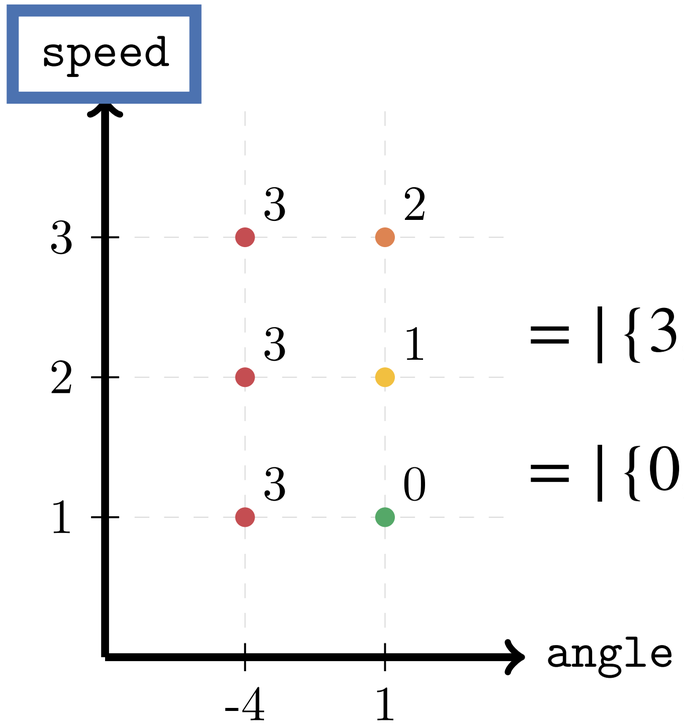

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Number of *reachable outcomes*

OUTCOMES

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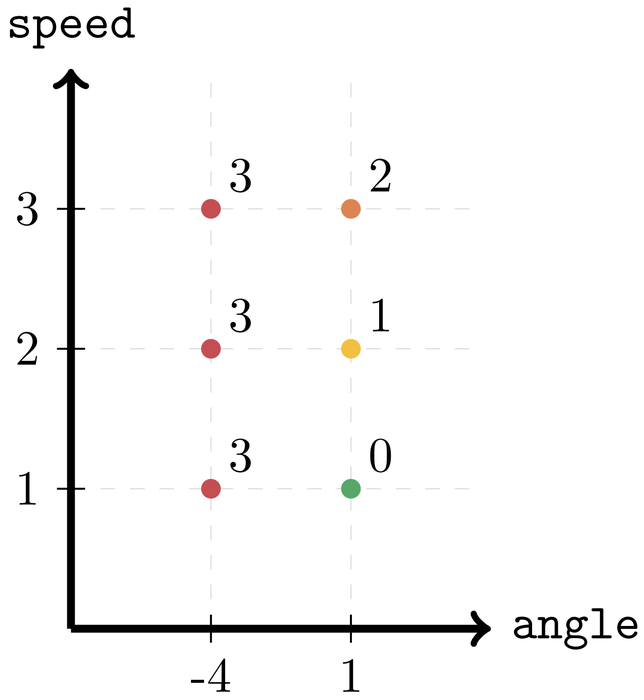
Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

RANGE



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Distance of *reachable outcomes*

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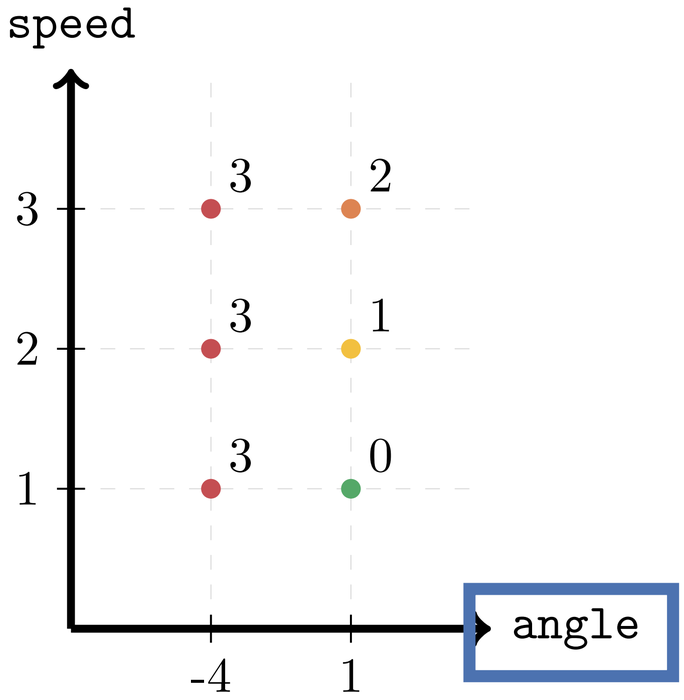

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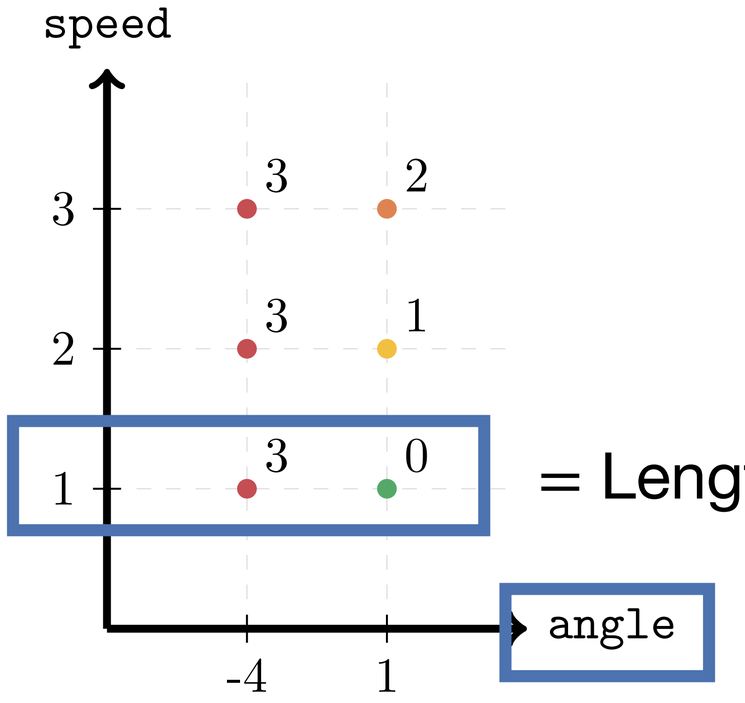
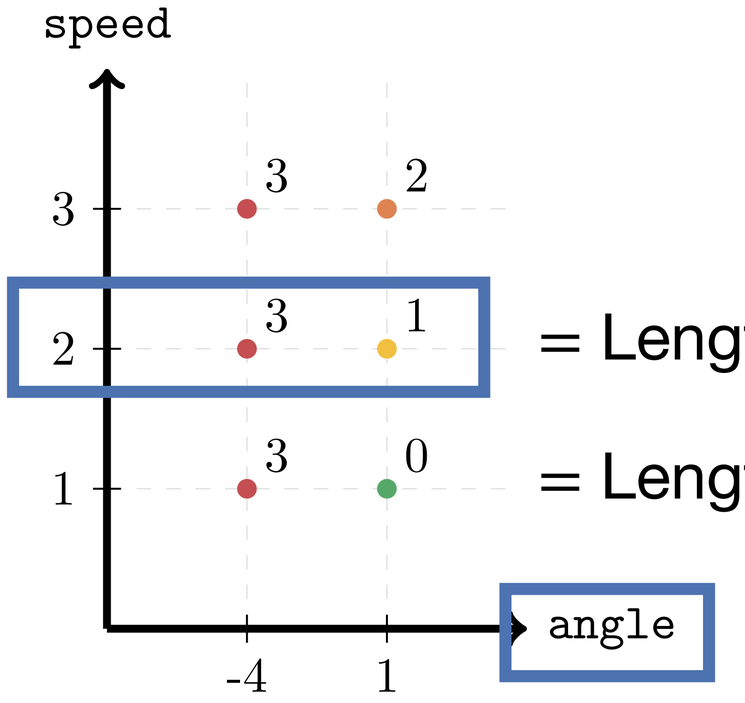


$= \text{Length}(\{3,0\}) = 3$

Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

RANGE



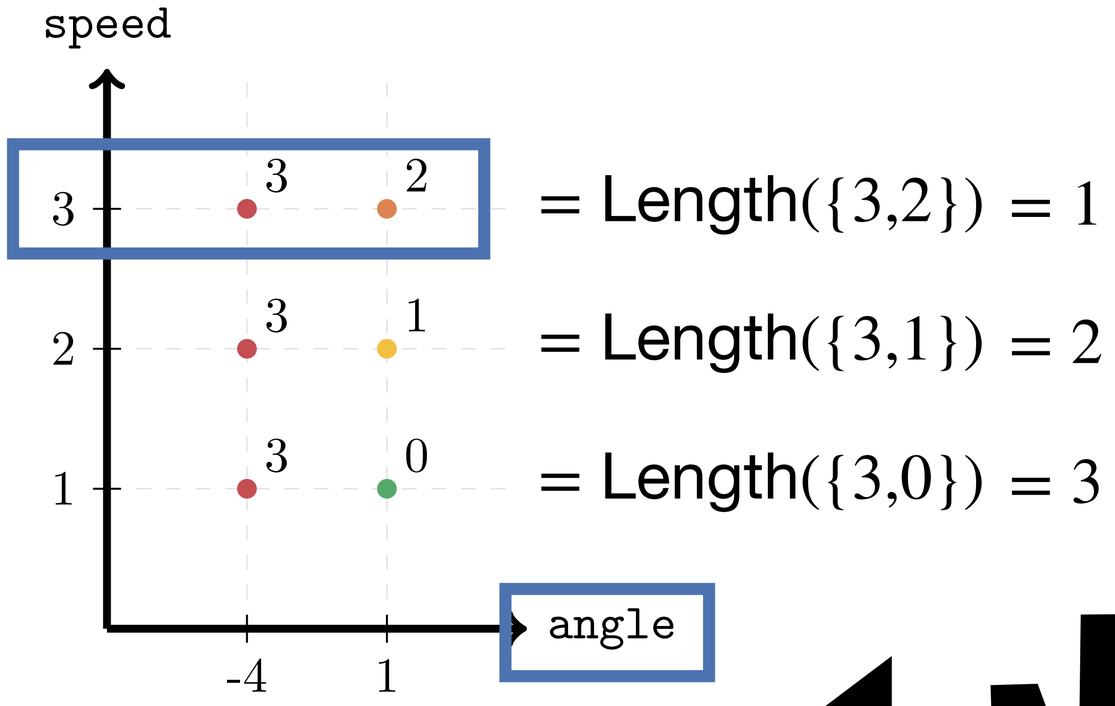
$= \text{Length}(\{3,1\}) = 2$

$= \text{Length}(\{3,0\}) = 3$

Goal: Quantify the impact of speed and angle on risk

Distance of *reachable outcomes*

RANGE



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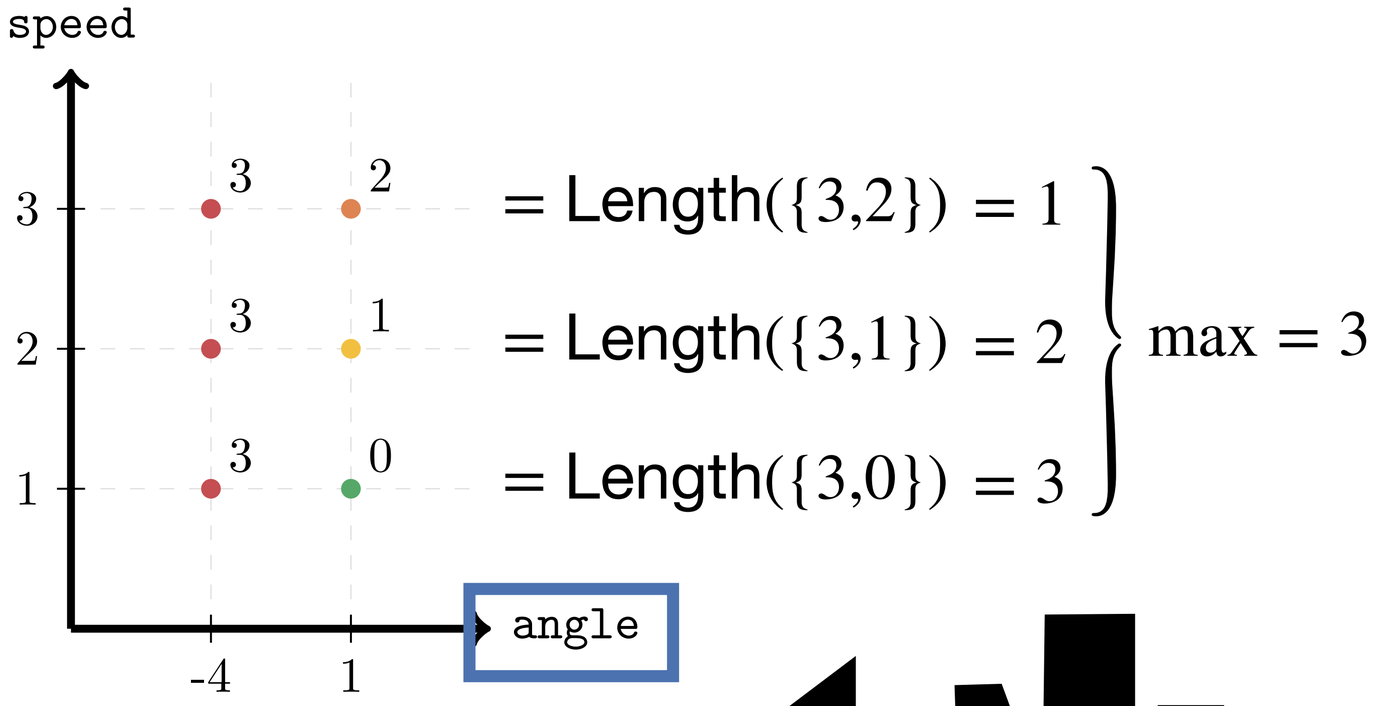
Distance of *reachable outcomes*

RANGE



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Distance of *reachable outcomes*

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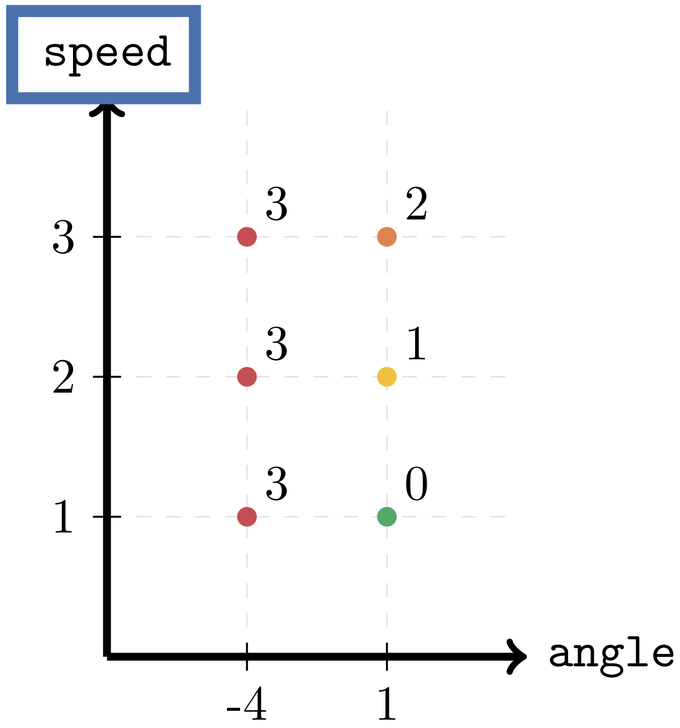

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Distance of *reachable outcomes*

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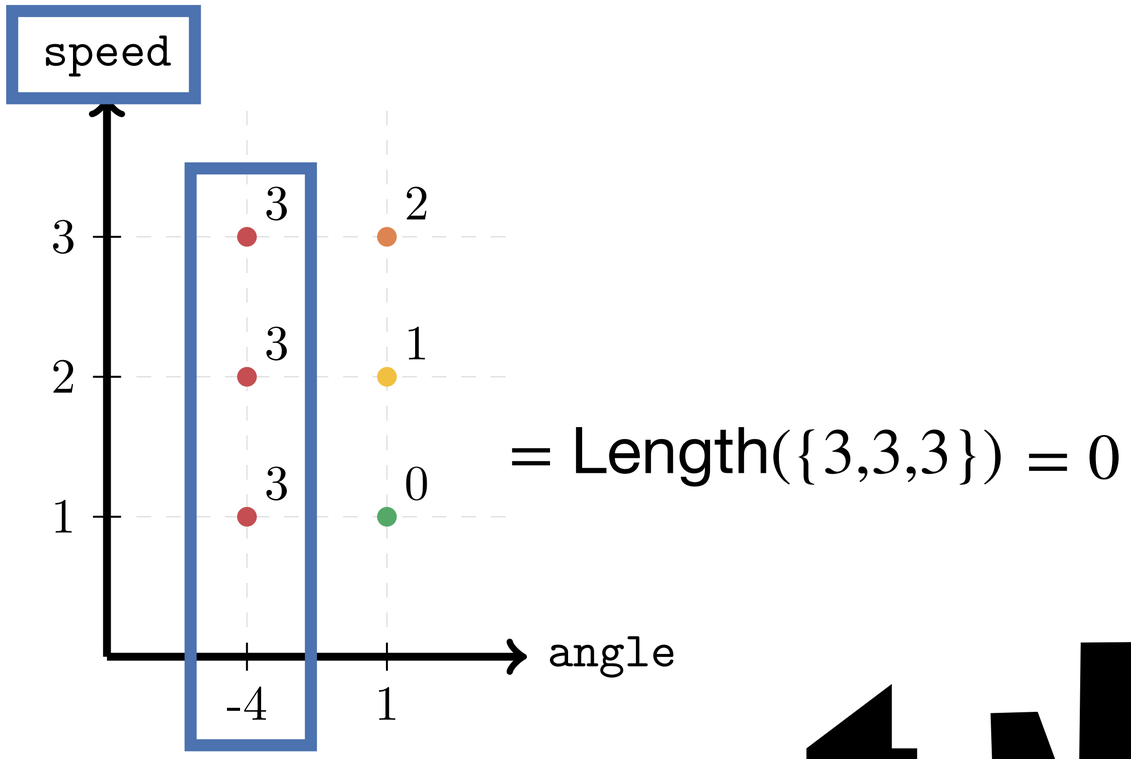
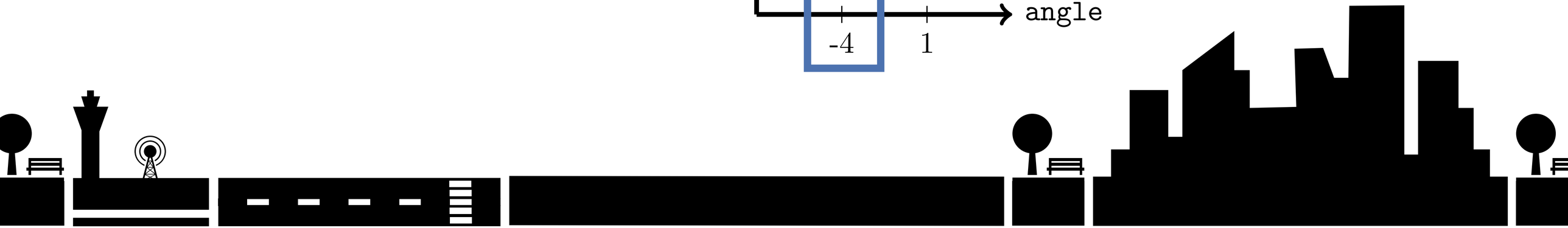


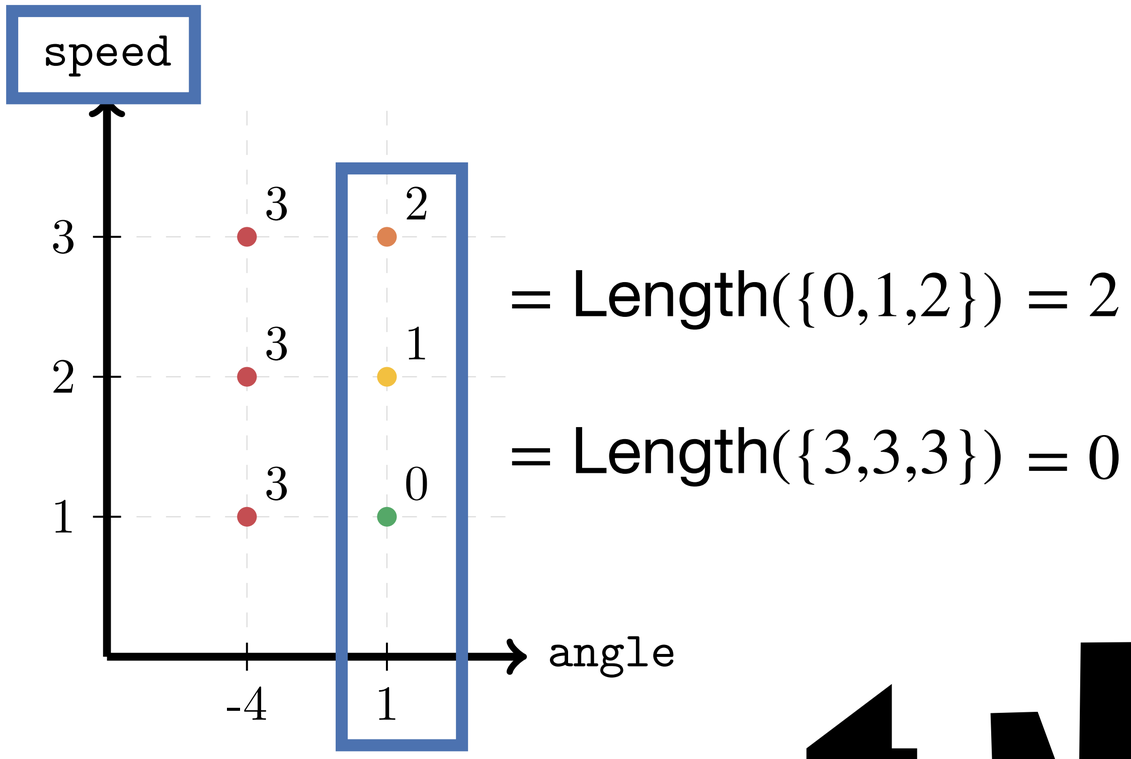
$= \text{Length}(\{3,3,3\}) = 0$



Goal: Quantify the impact of speed and angle on risk

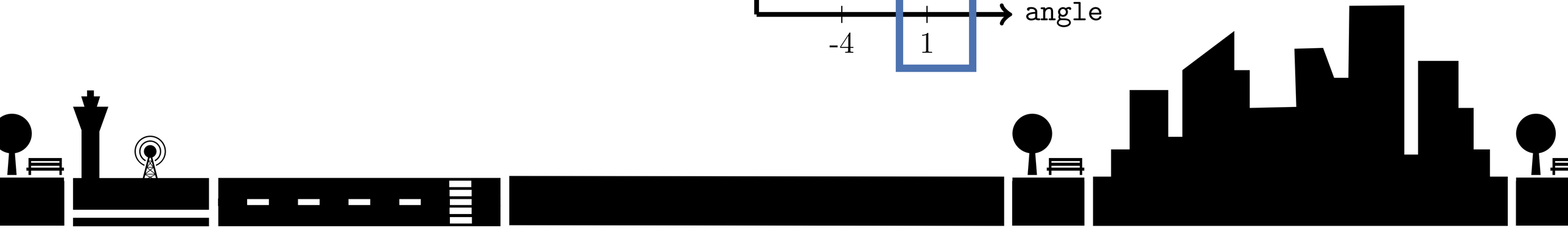
Distance of *reachable outcomes*

RANGE



$= \text{Length}(\{0,1,2\}) = 2$

$= \text{Length}(\{3,3,3\}) = 0$



Goal: Quantify the impact of speed and angle on risk

Distance of *reachable outcomes*

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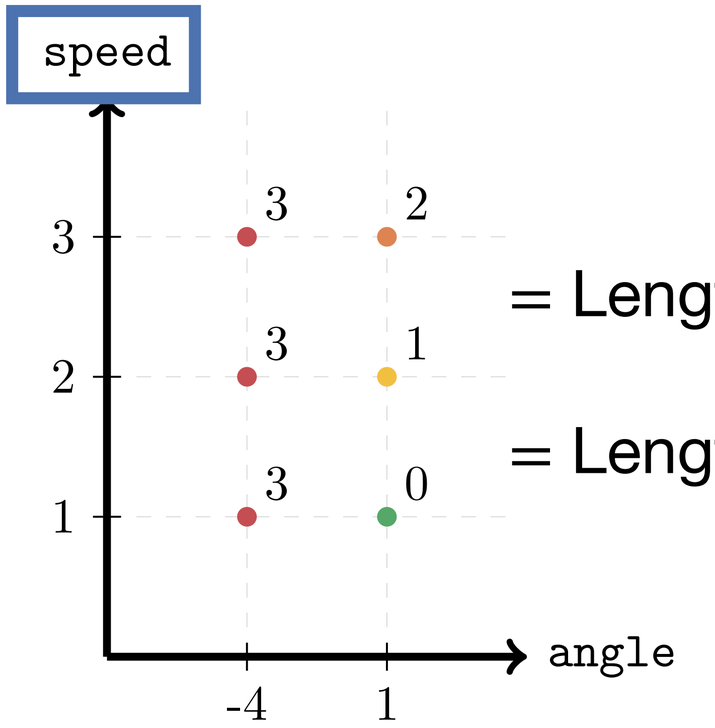
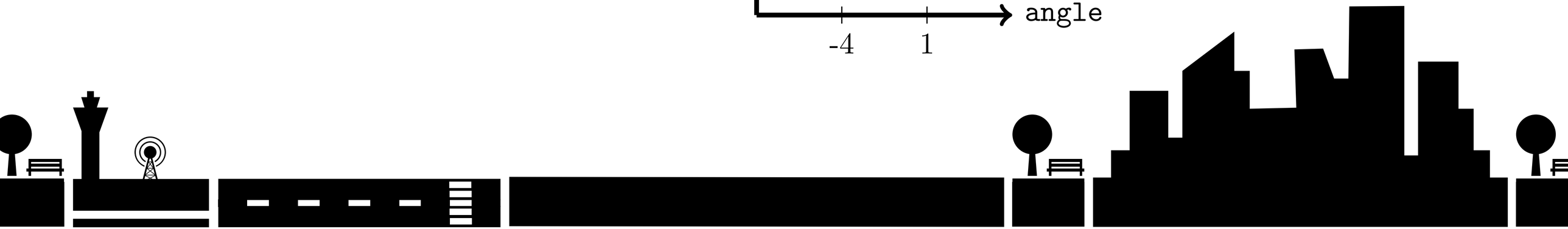


$$\left.\begin{array}{l} = \text{Length}(\{0,1,2\}) = 2 \\ = \text{Length}(\{3,3,3\}) = 0 \end{array}\right\} \max = 2$$



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Goal: Quantify the impact of `speed` and `angle` on `risk`

Distance of *reachable outcomes*

Number of *reachable outcomes*

	RANGE	OUTCOMES
angle	3	2
speed	2	3

Automatic & Sound Static Analysis
by Abstract Interpretation

	Range	Outcomes
angle	3	2
speed	2	3

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

	RANGE	OUTCOMES
angle	$3 \leq k$	$2 \leq k$
speed	$2 \leq k$	$3 \leq k$

Smallest k permitted by the abstraction!

Automatic & Sound Static Analysis
by Abstract Interpretation

Find k such that

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1. Output Buckets

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Range♮ and Outcomes♮

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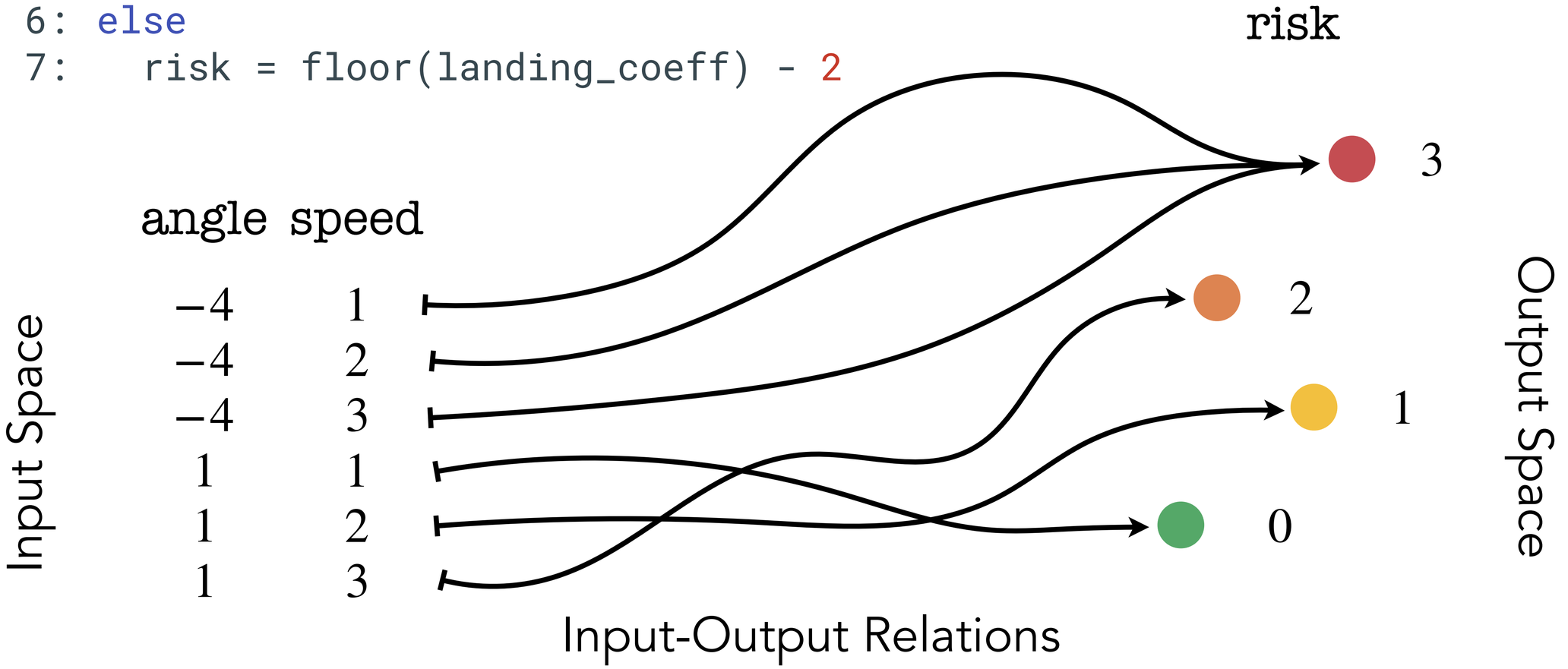
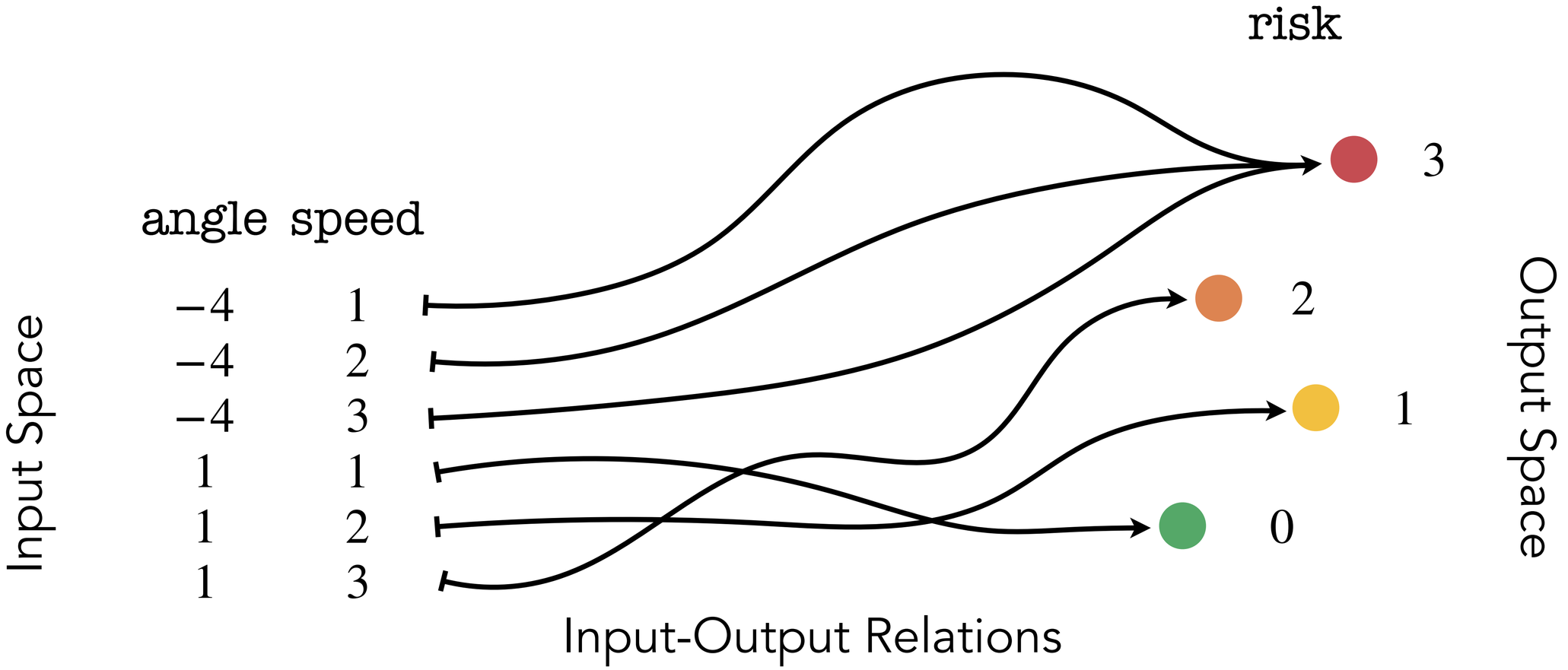
```
1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3:   risk = 0
4: else if landing_coeff > 5 then
5:   risk = 3
6: else
7:   risk = floor(landing_coeff) - 2
```



Intuition: OUTCOMES for the variable **angle**



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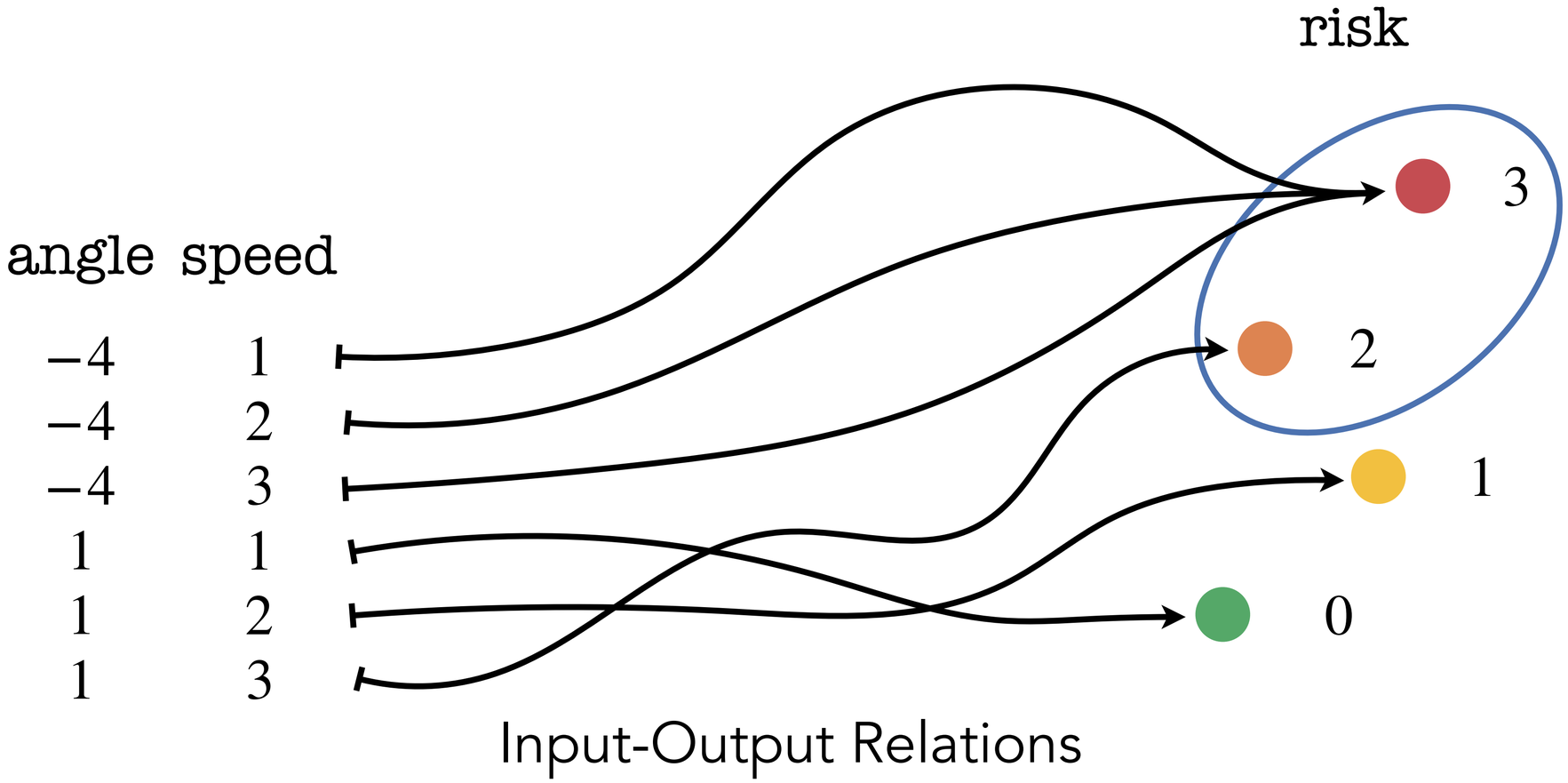

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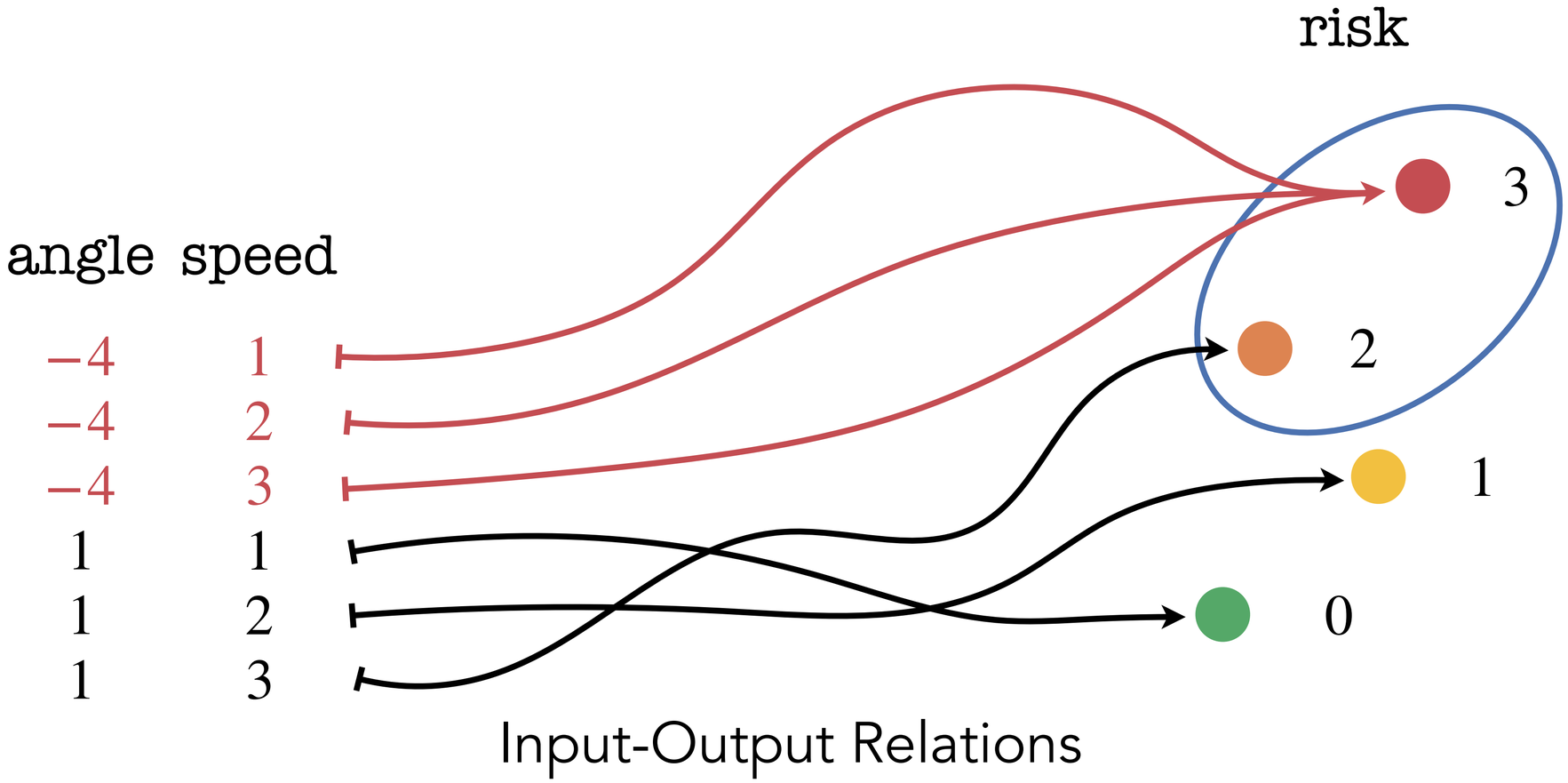

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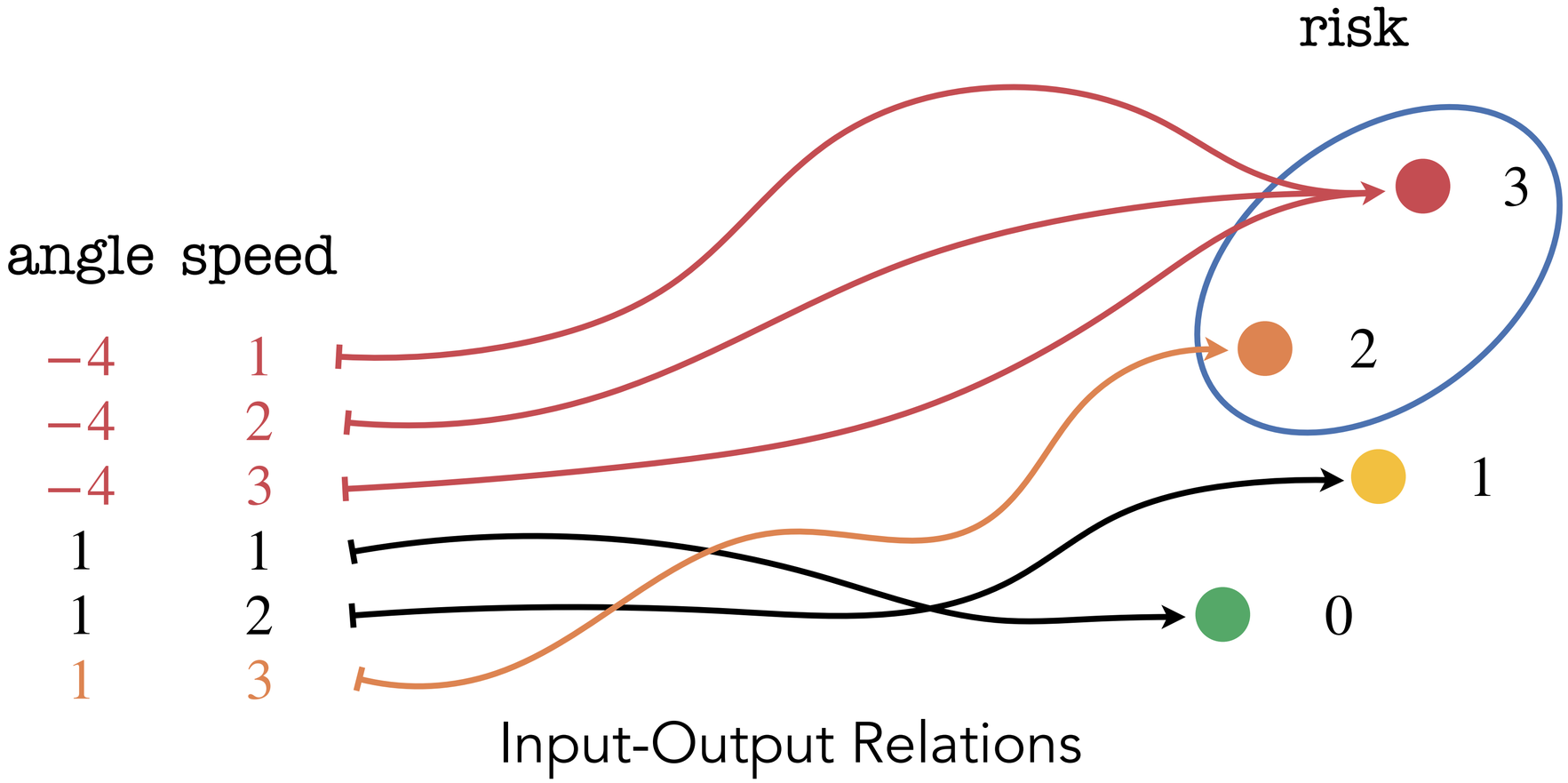

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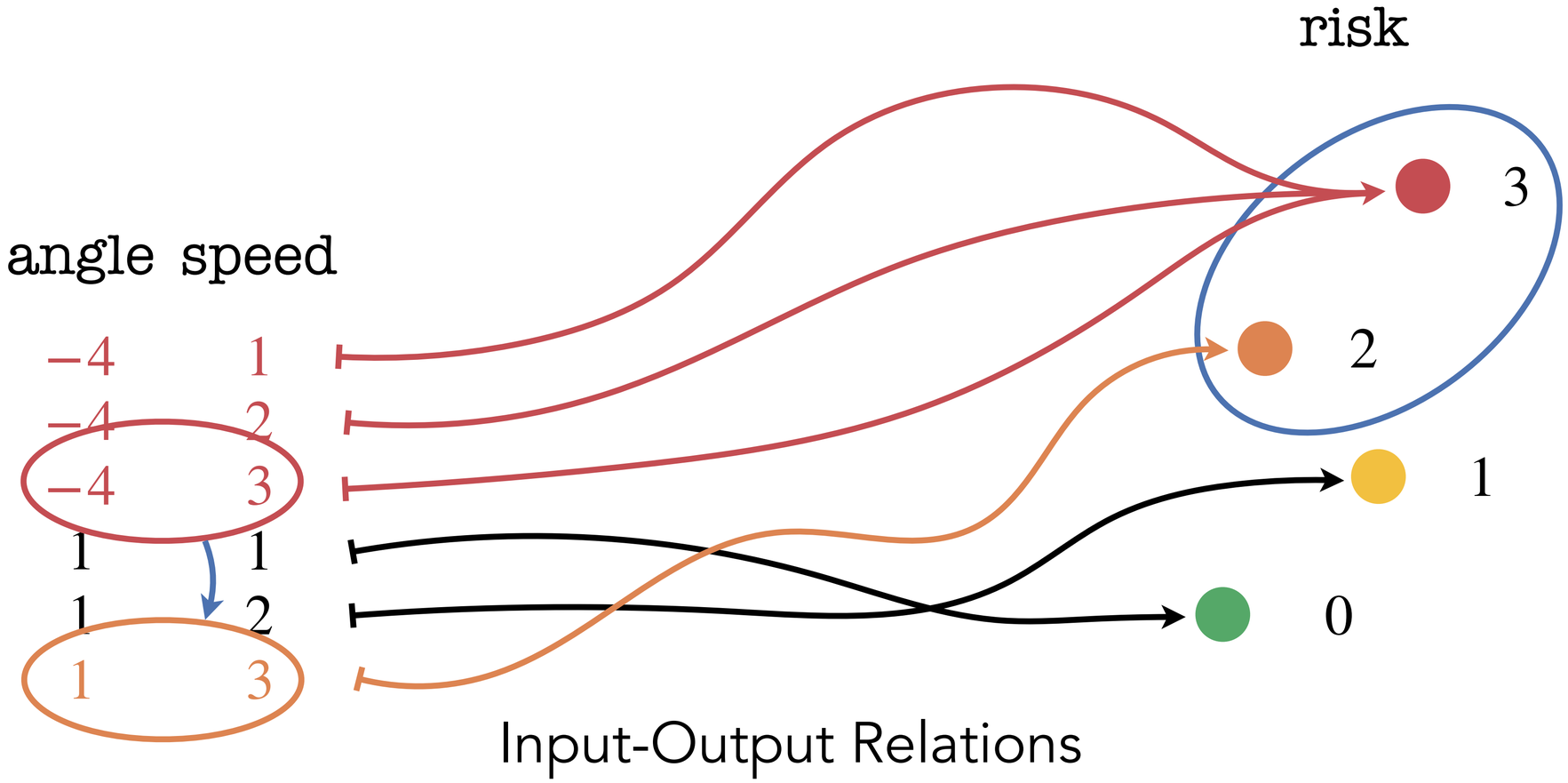

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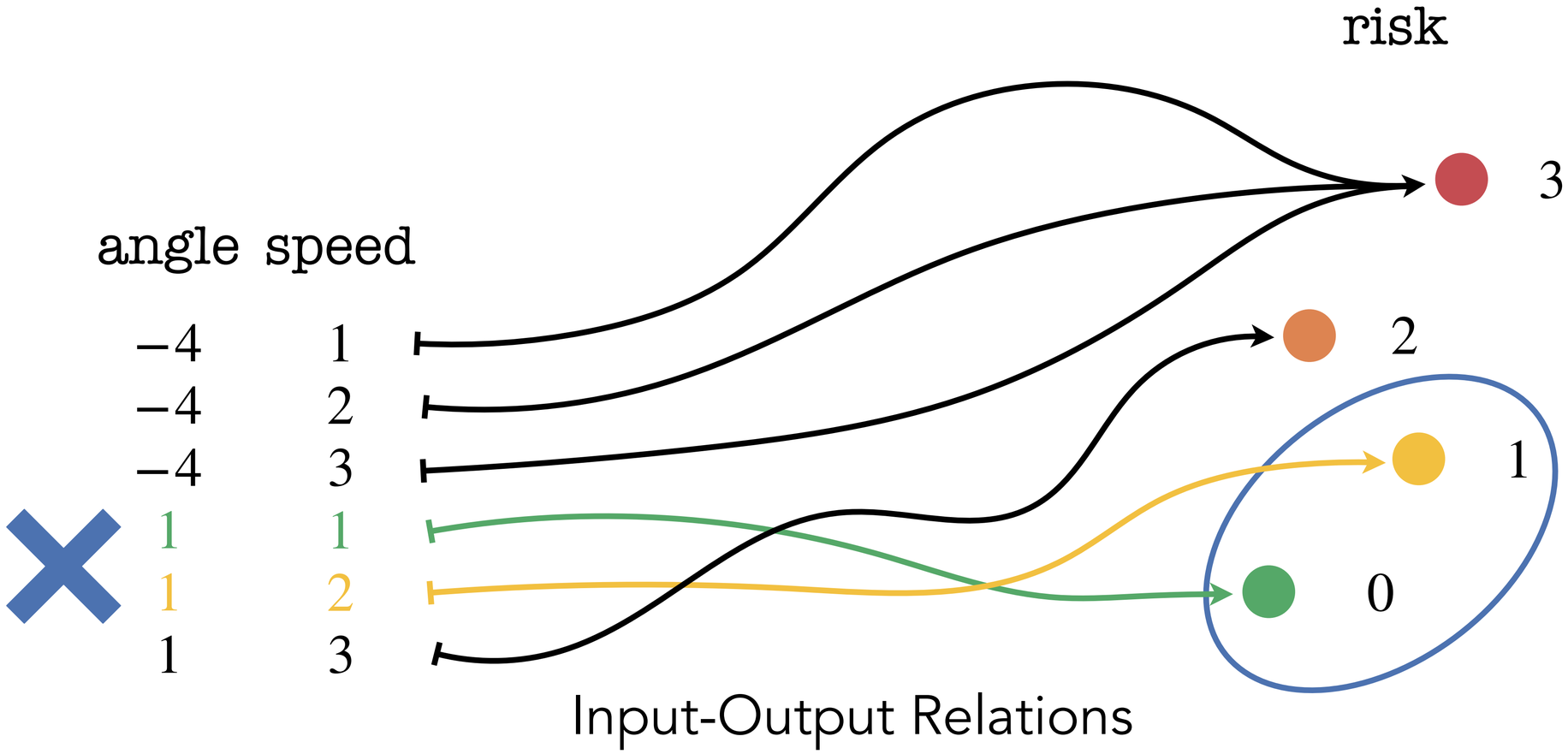
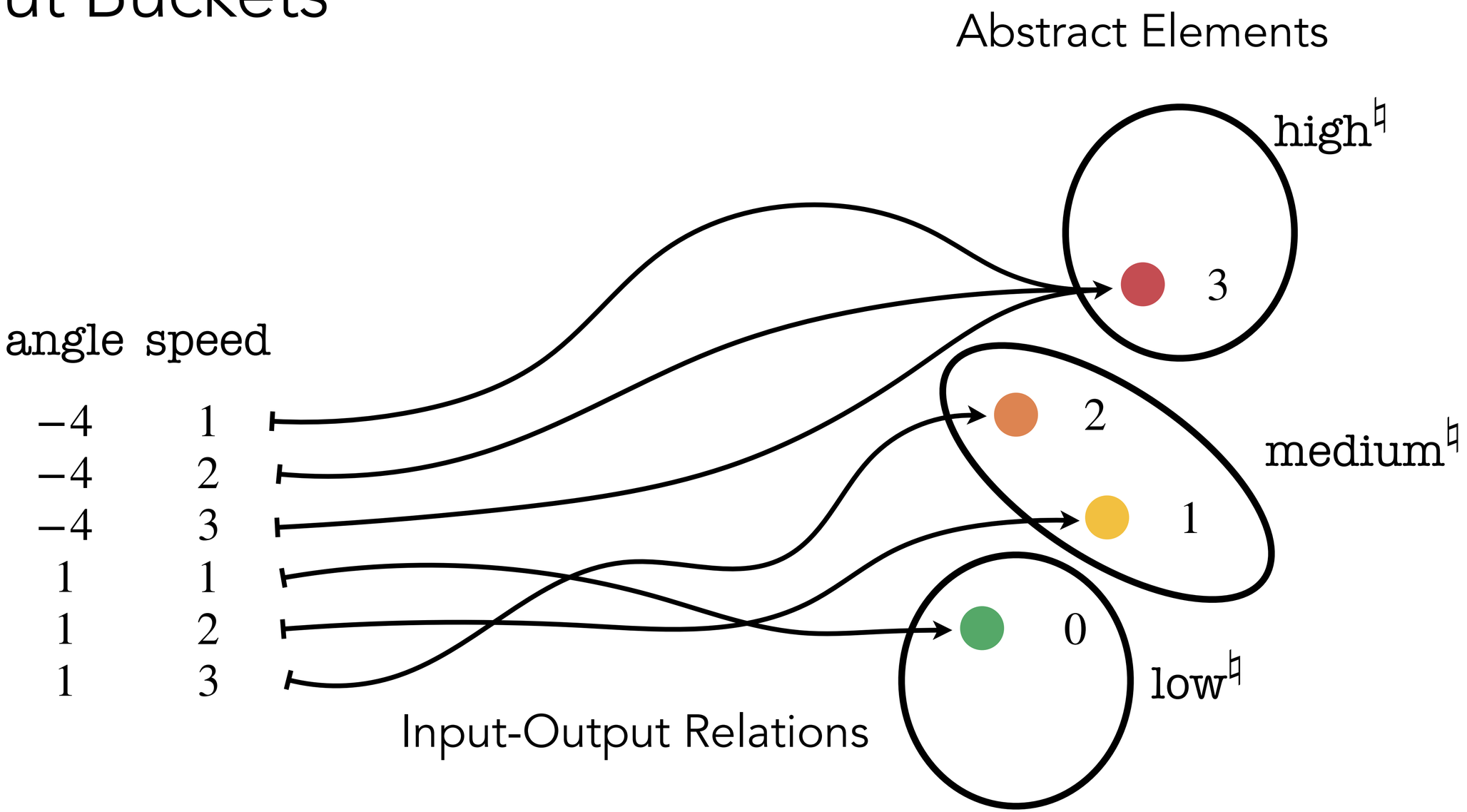
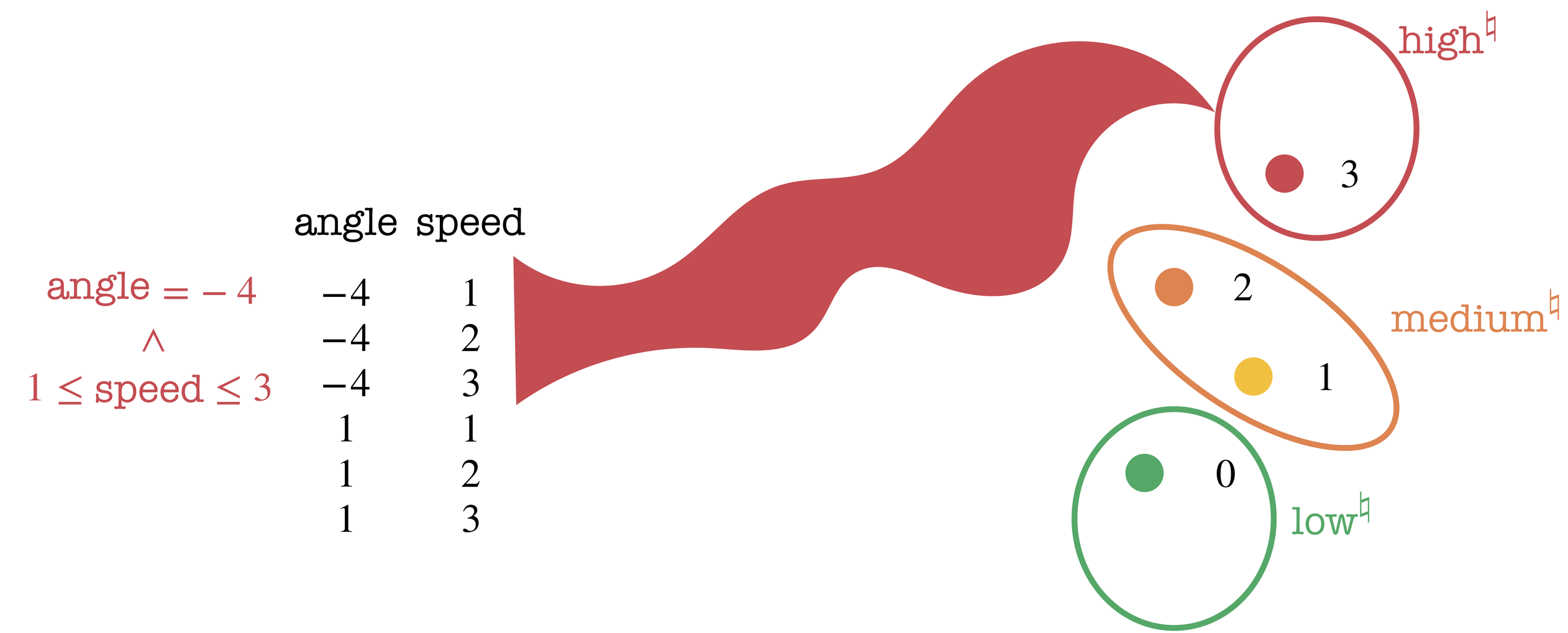


Input-Output Relations

1) Output Buckets



2) Backward Abstract Analysis



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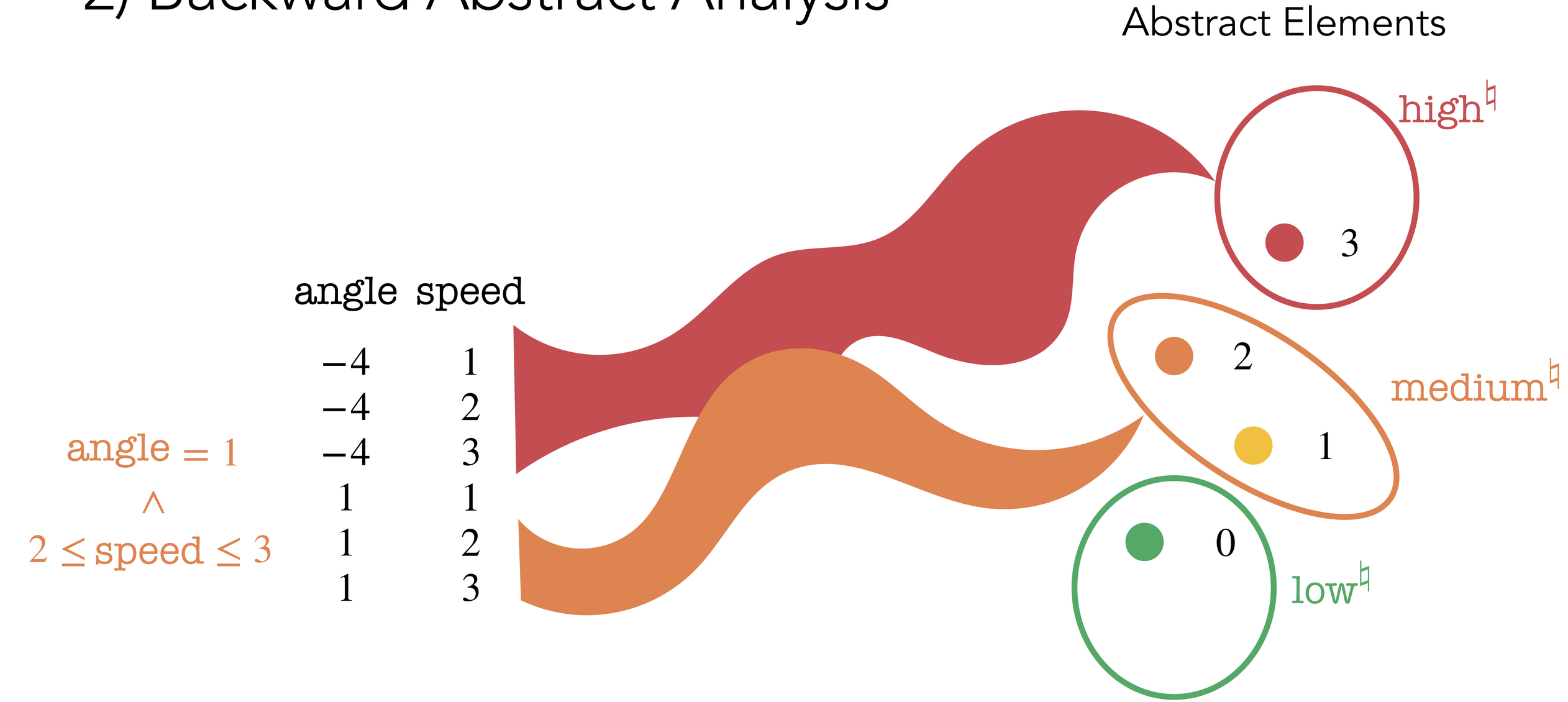

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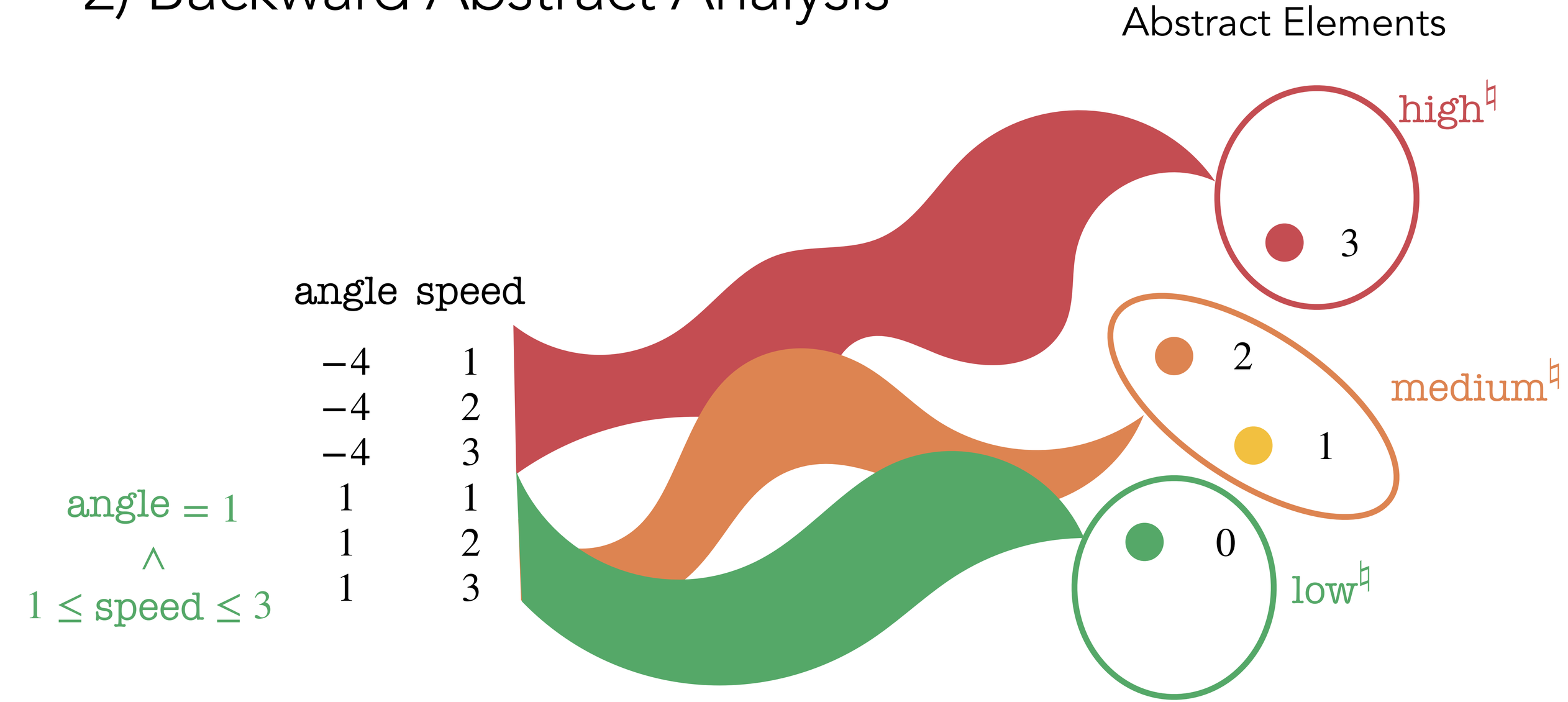

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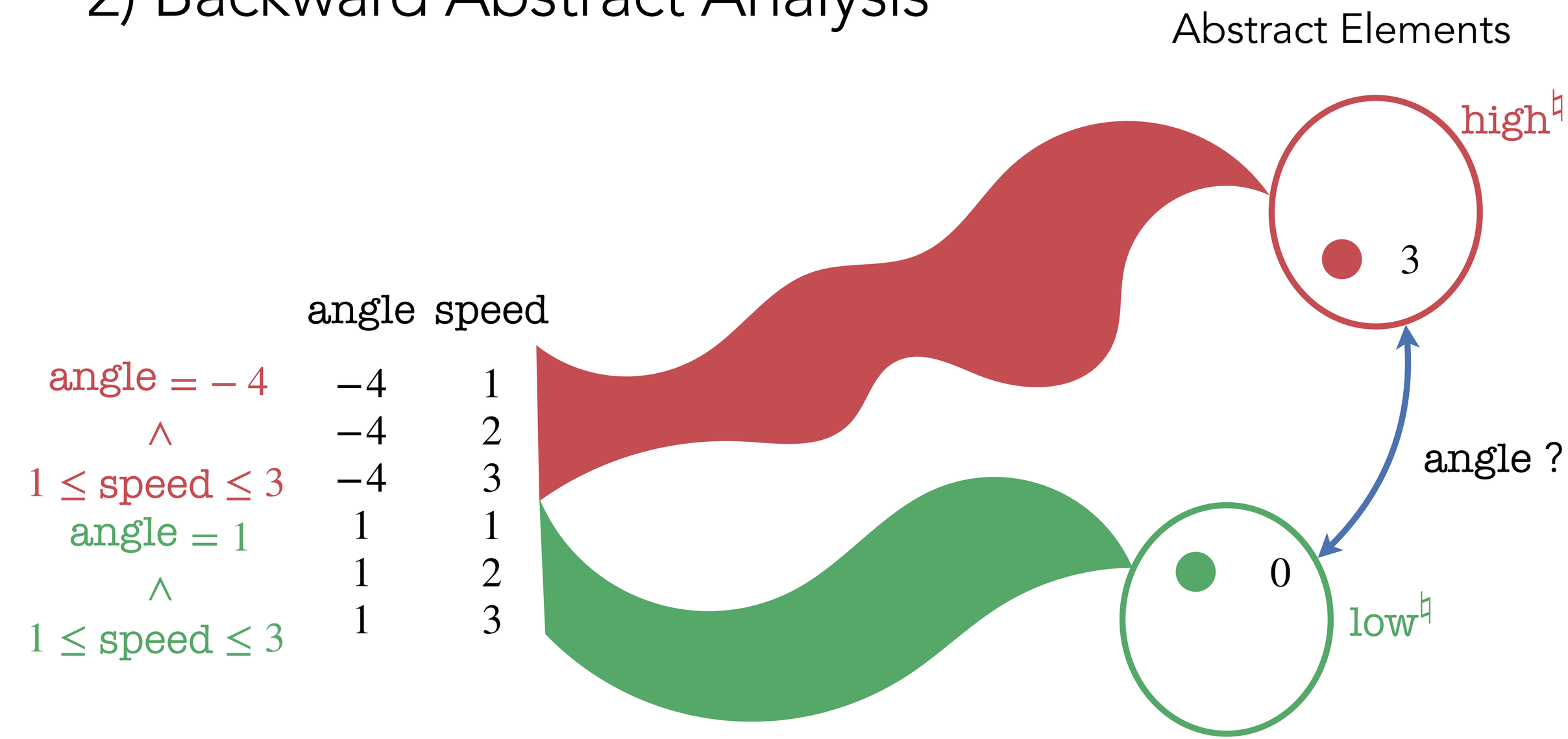
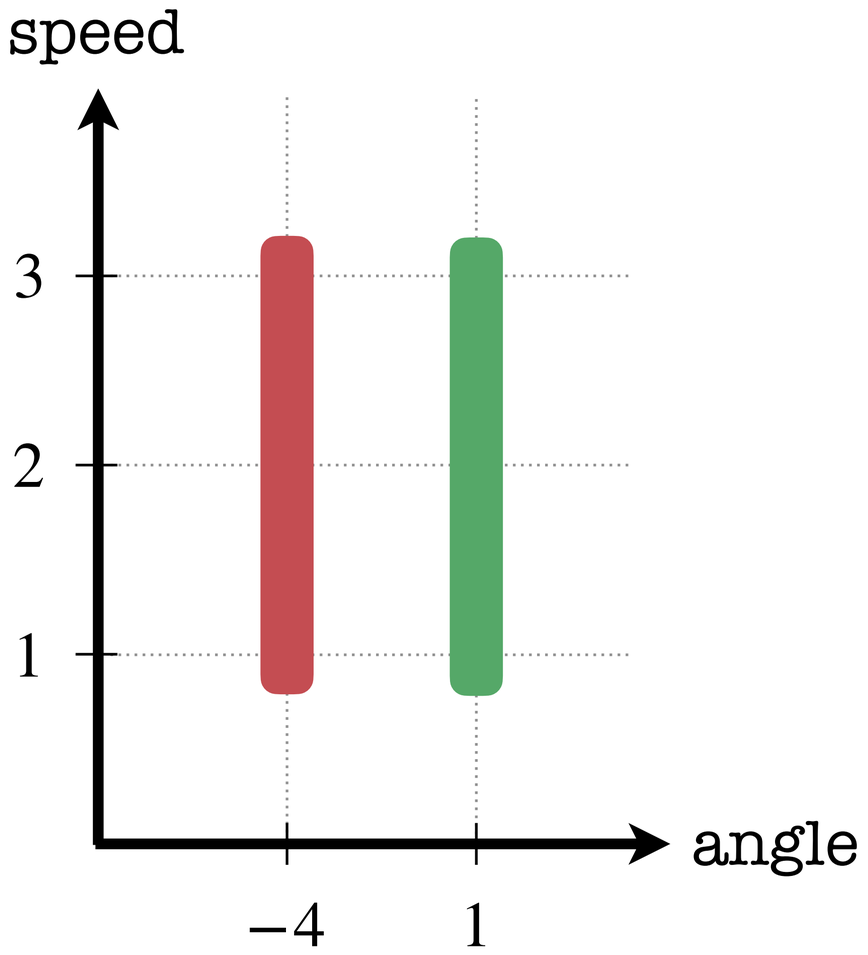
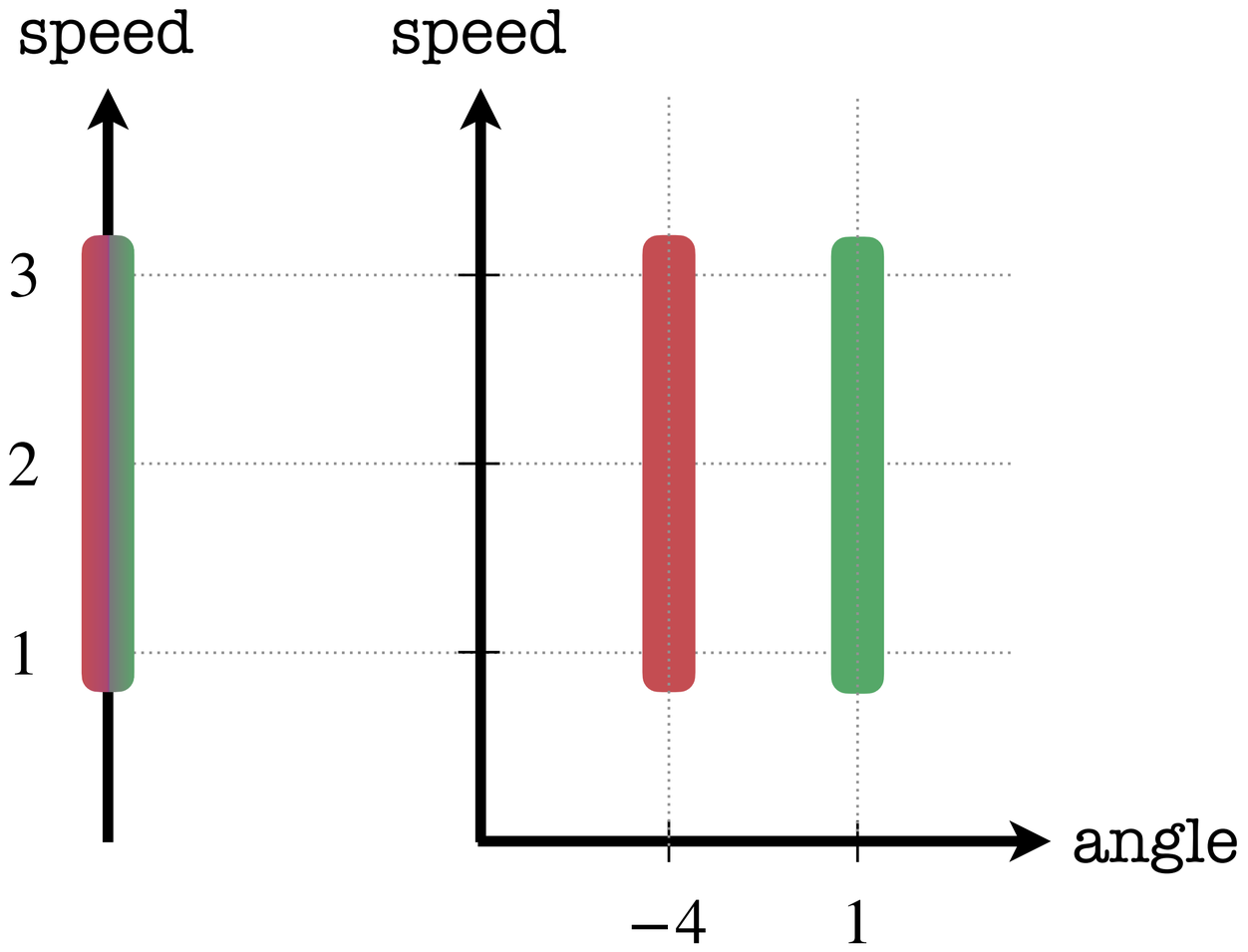




$$\text{angle} = -4 \;\wedge\; 1 \leq \text{speed} \leq 3$$

$$\text{angle} = 1 \;\wedge\; 1 \leq \text{speed} \leq 3$$



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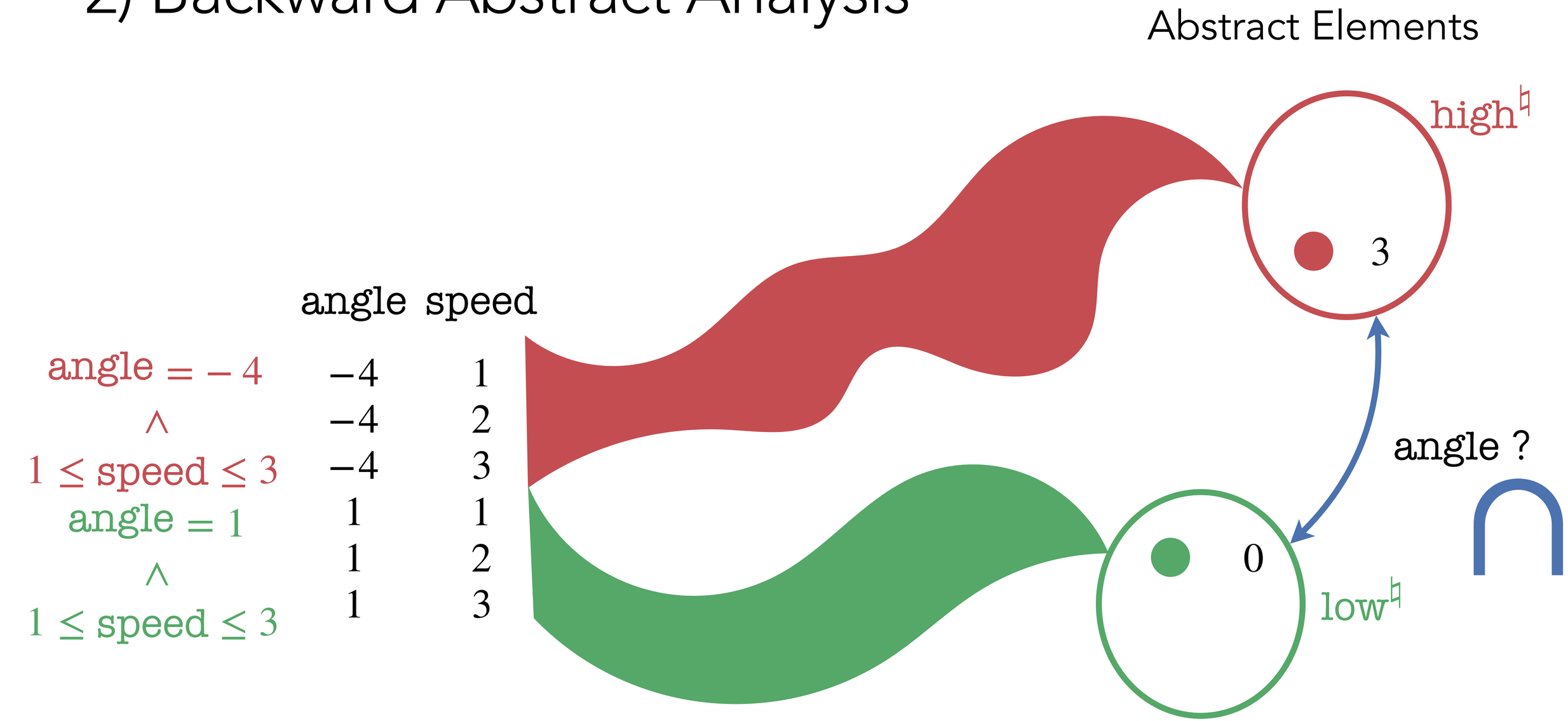

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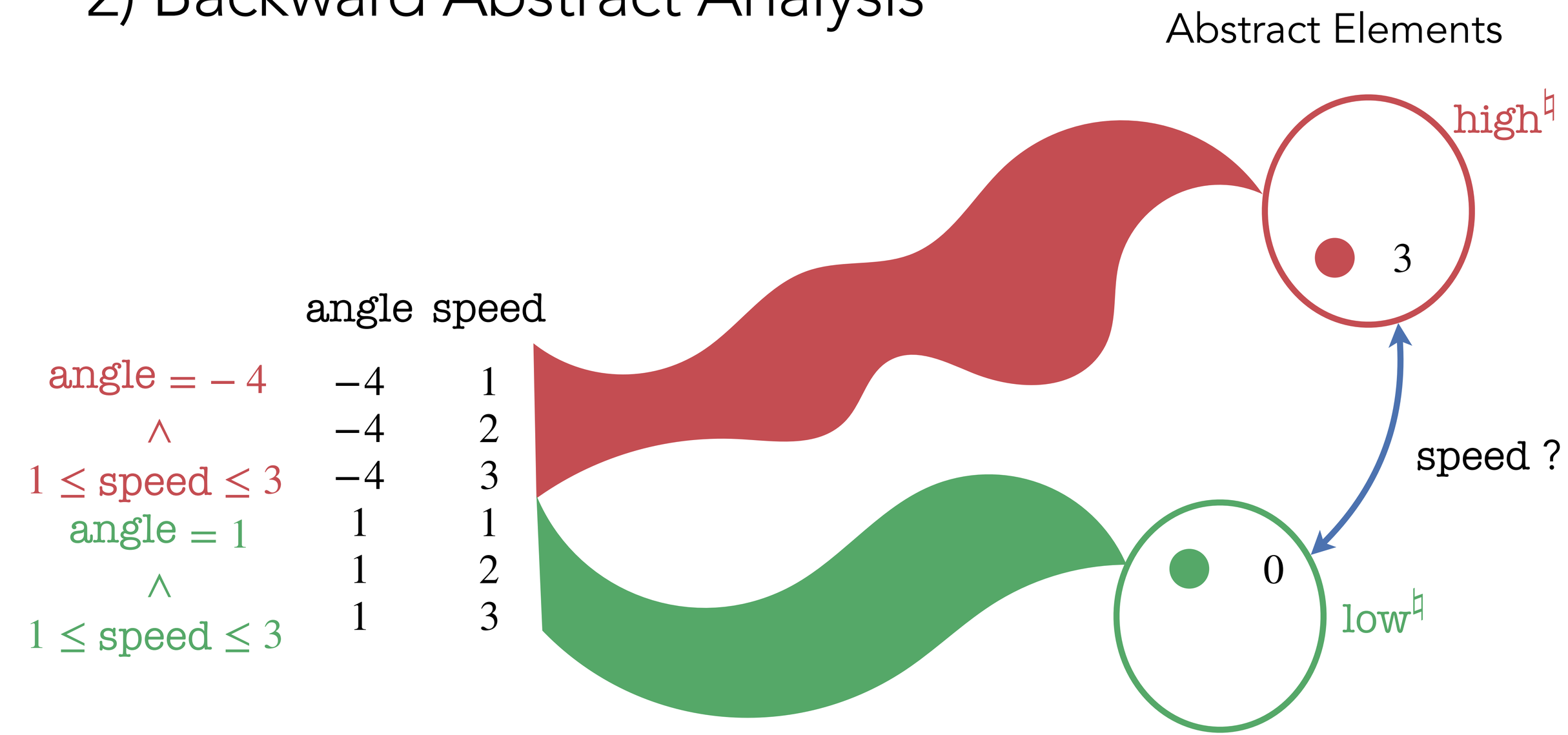

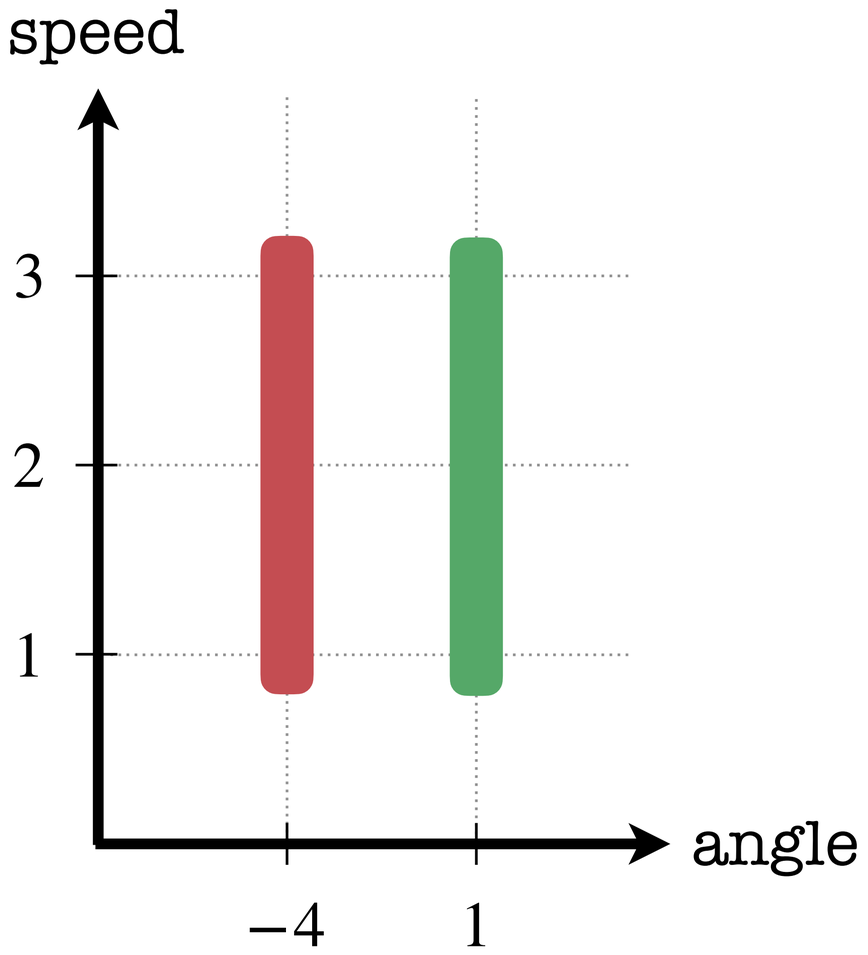

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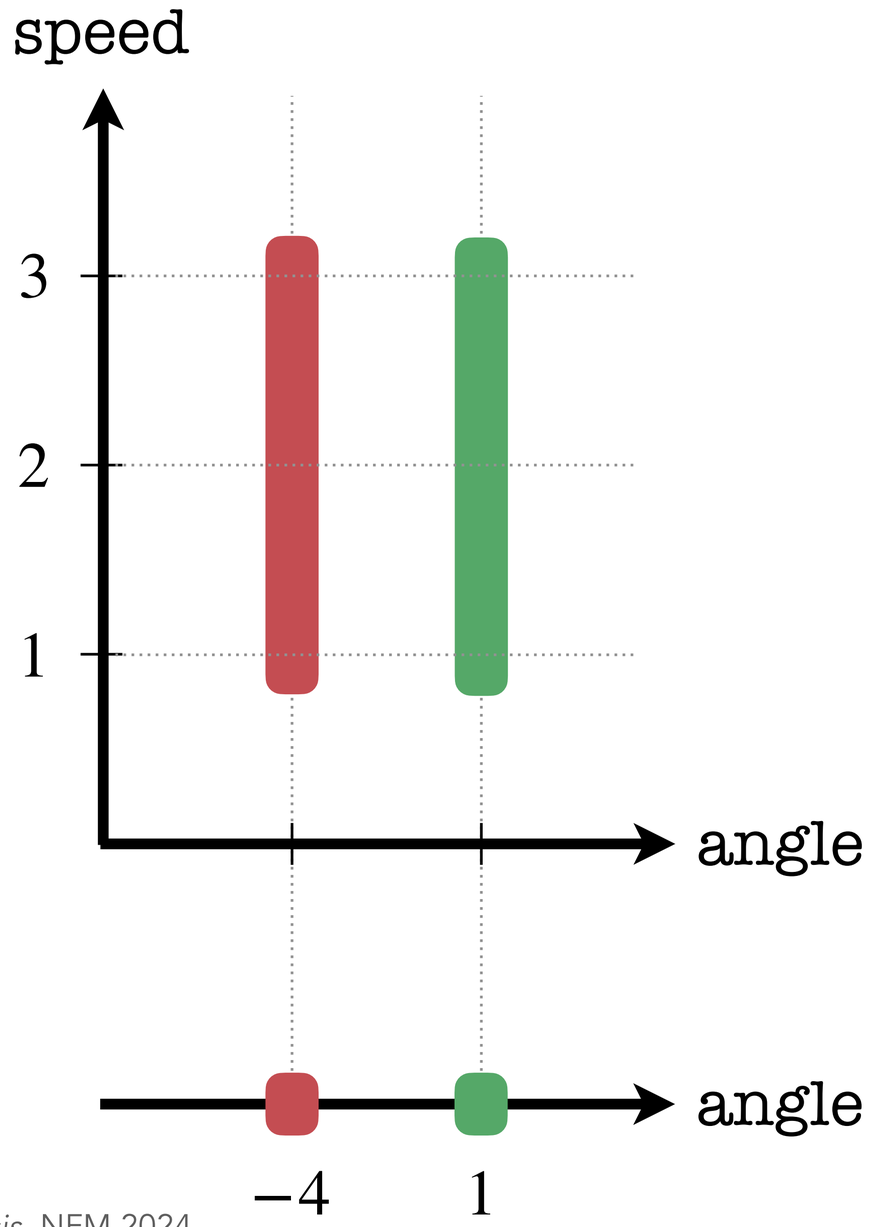

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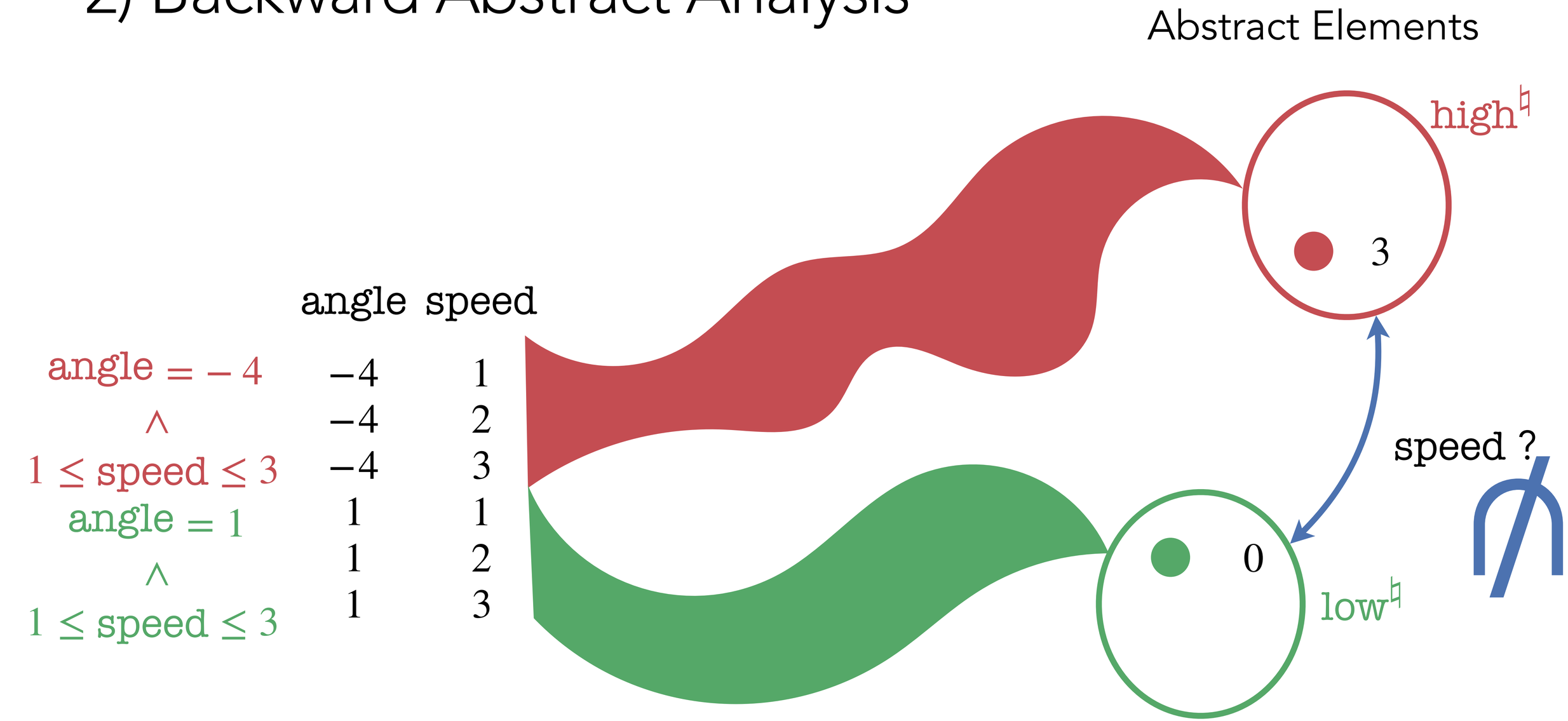

3) Abstract Implementation of Impact Definitions

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	∩	∩	∩	∩
speed			∩	

Abstract Impact **Outcomes**♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle		0, 3		
speed				

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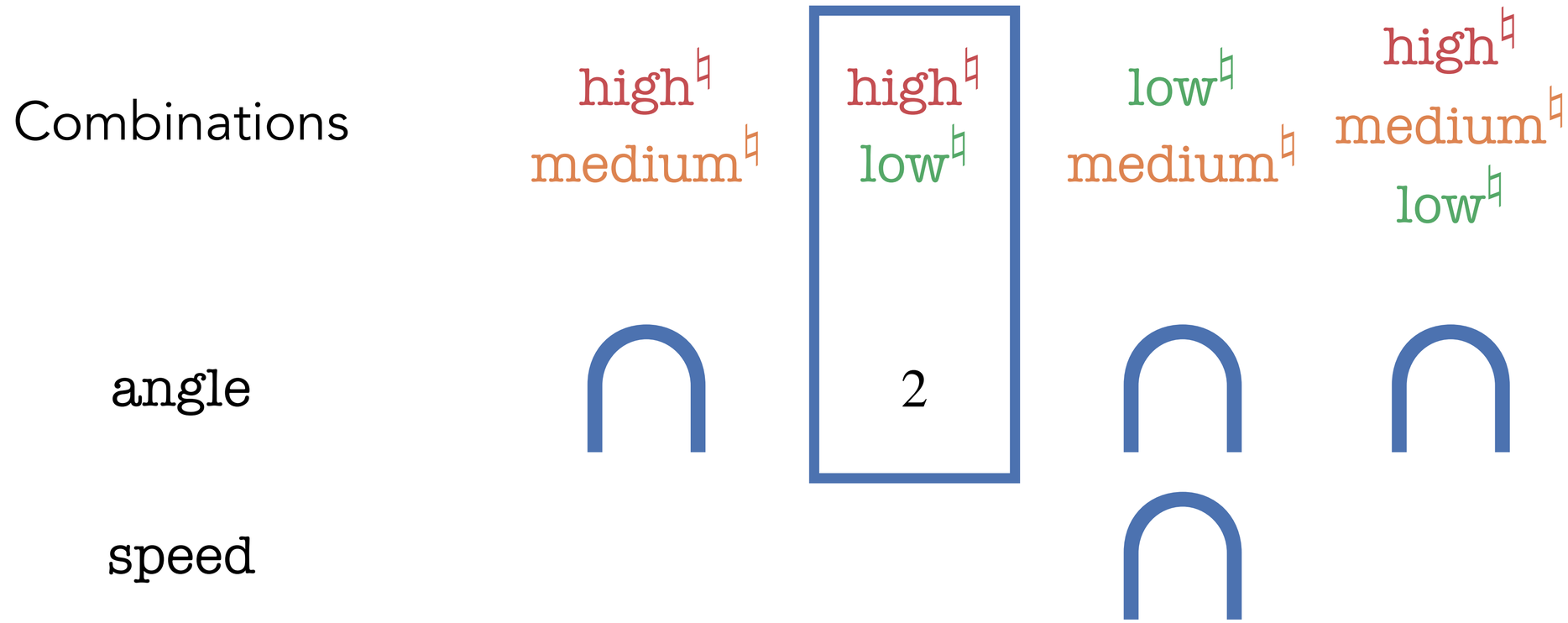
Abstract Impact **Outcomes**♮



Abstract Impact **Outcomes**♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	1,2, 3	2	∩	∩
speed			∩	

Abstract Impact **Outcomes**♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	3	2		
speed				

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Abstract Impact **Outcomes**♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	3	2	3	4
speed			3	

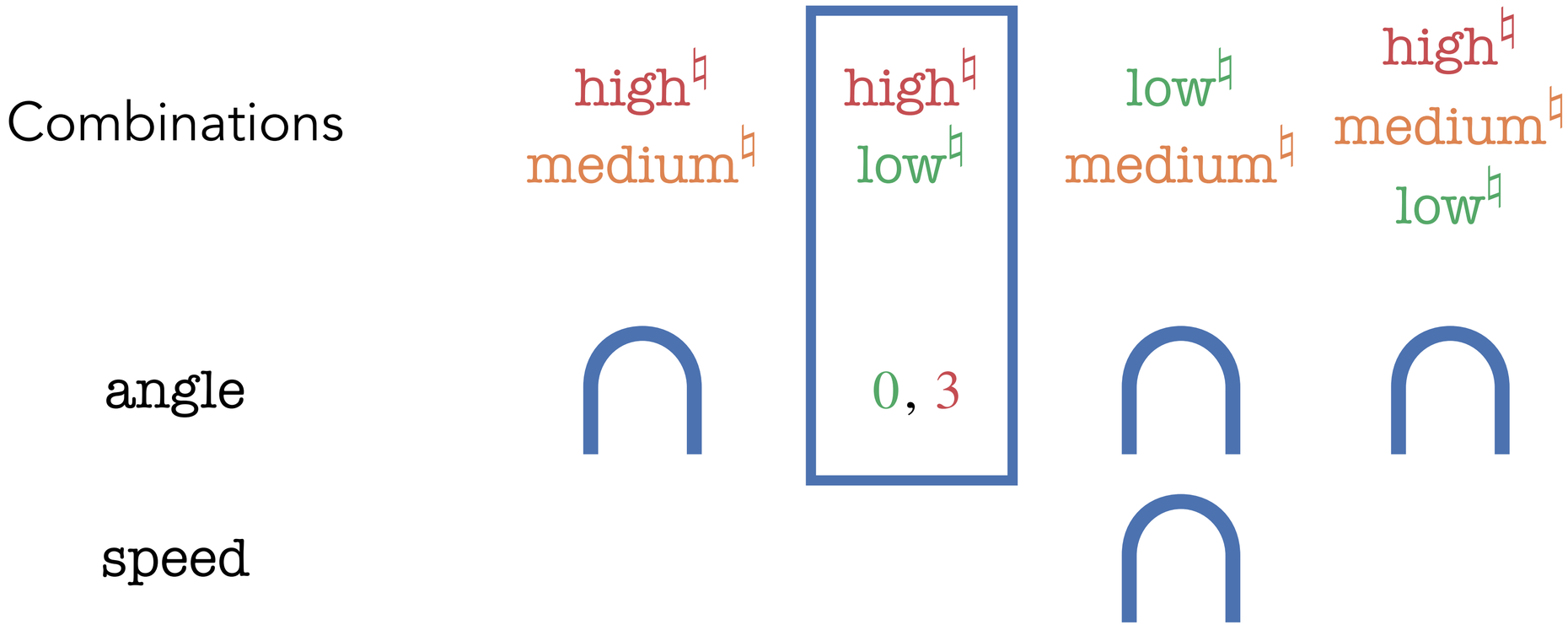
Abstract Impact **Outcomes**♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮	Outcomes♮
angle	3	2	3	4	⟹ 4
speed			3		⟹ 3

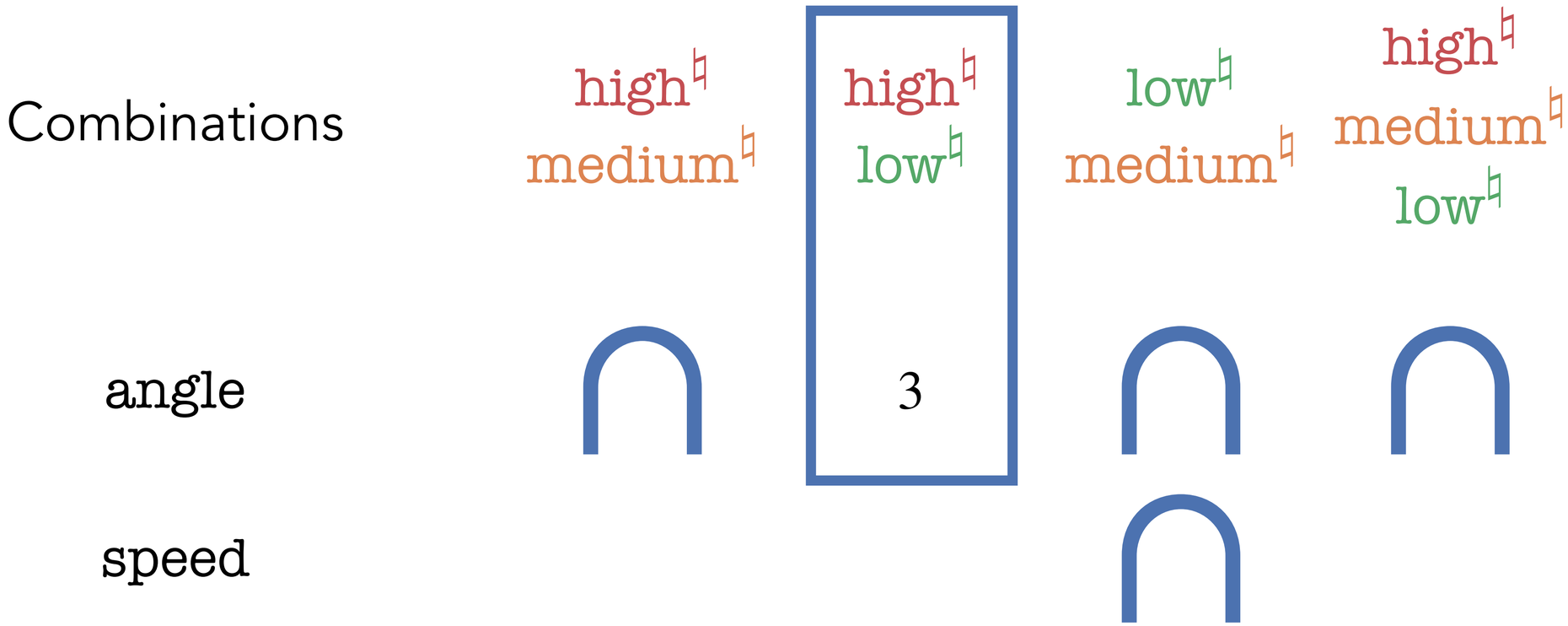
Abstract Impact Range♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle				
speed				

Abstract Impact Range♮



Abstract Impact Range♮



Abstract Impact Range♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	1,2, 3	3	∩	∩
speed			∩	

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Abstract Impact Range♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	2	3		
speed				

Abstract Impact Range♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮
angle	2	3	2	3
speed			2	

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Abstract Impact Range♮

Combinations	high♮ medium♮	high♮ low♮	low♮ medium♮	high♮ medium♮ low♮	Range♮
angle	2	3	2	3	⟹ 3
speed			2		⟹ 2

Goal: Quantify the impact of `speed` and `angle` on `risk`

	Range	Outcomes
angle	3	2
speed	2	3

Goal: Quantify the impact of speed and angle on risk

	RANGE	OUTCOMES	Range♮	Outcomes♮
angle	3	2	3	4
speed	2	3	2	3

Goal: Quantify the impact of speed and angle on risk

	Impact		Impact♮	
	Range	Outcomes	Range♮	Outcomes♮
angle	3	2	3	4
speed	2	3	2	3

Theorem (Soundness)

(i) Backward abstract analysis $\Lambda^{\natural}$ **over-approximates** the input-output relations

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$$\mathtt{Impact}^\natural_{\mathtt{i}}(\Lambda^\natural[\![P]\!]B^\natural) \leq k$$

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(iii) $\mathtt{Impact}^\natural$ is a **sound implementation** of IMPACT

$$\mathtt{Impact}^\natural_{\mathtt{i}}(\Lambda^\natural[\![P]\!]B^\natural) \leq k \implies$$

The variable i on the program P has an impact of **at most** k

Source of Imprecision

Abstraction of the Backward Analysis

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Abstraction of the Backward Analysis

Choice of the Output Buckets

Use Case: Reinhart and Rogoff

C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
American Economic Review 2010.

```
 1: def mean_growth_rate_60_90(
 2:     portugal1, portugal2, portugal3,
 3:     norway1,
 4:     uk1, uk2, uk3, uk4,
 5:     usa1, usa2, usa3):
 6:   portugal_avg = avg(portugal1, portugal2, portugal3)
 7:   norway_avg = avg(norway1)
 8:   uk_avg = avg(uk1, uk2, uk3, uk4)
 9:   usa_avg = avg(usa1, usa2, usa3)
10:   return avg(portugal_avg, norway_avg, uk_avg, usa_avg)
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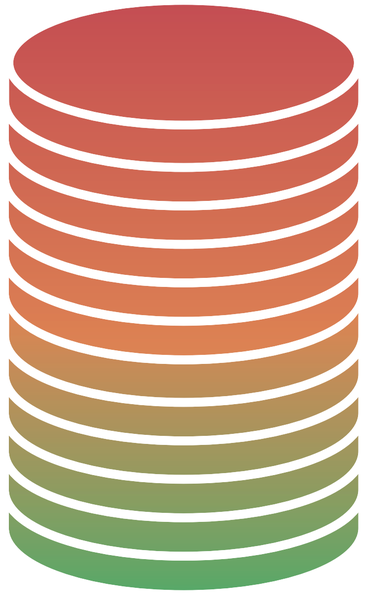
 9:   usa_avg = avg(usa1, usa2, usa3)
10:   return avg(portugal_avg, norway_avg, uk_avg, usa_avg)
```

$\text{avg}^{\natural} = 20\,\%$



$\text{avg}^{\natural} = -20\,\%$

41 Output Buckets

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C. M. Reinhart and K. S. Rogoff. Growth in a time of debt.
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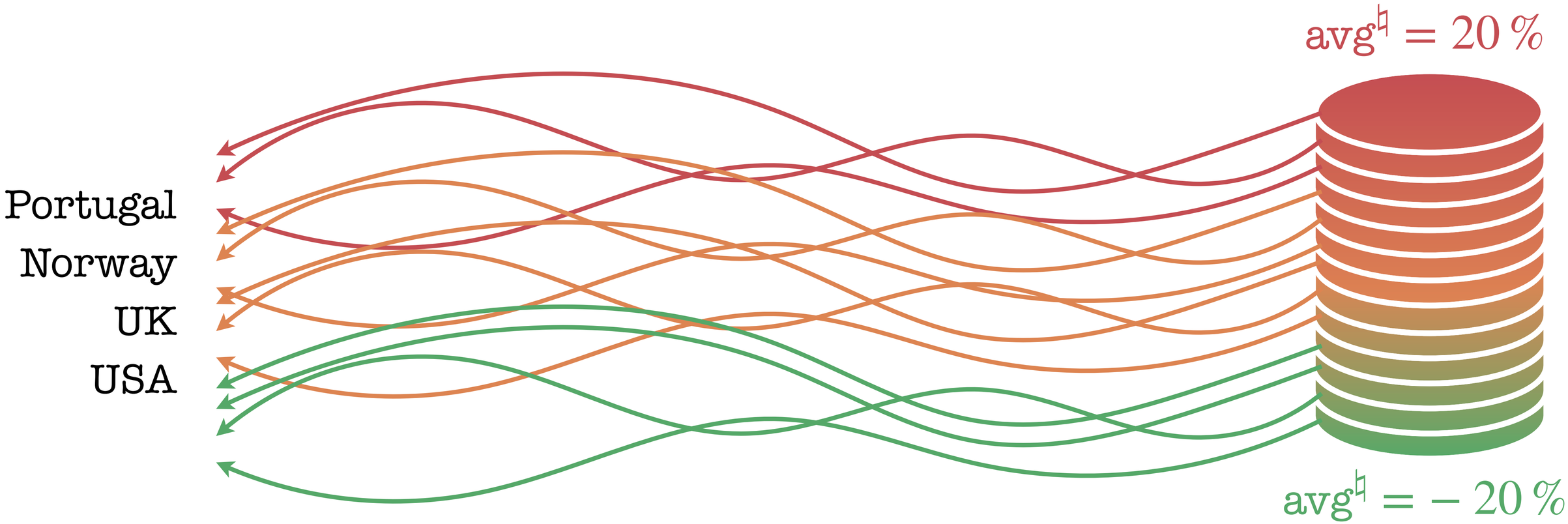

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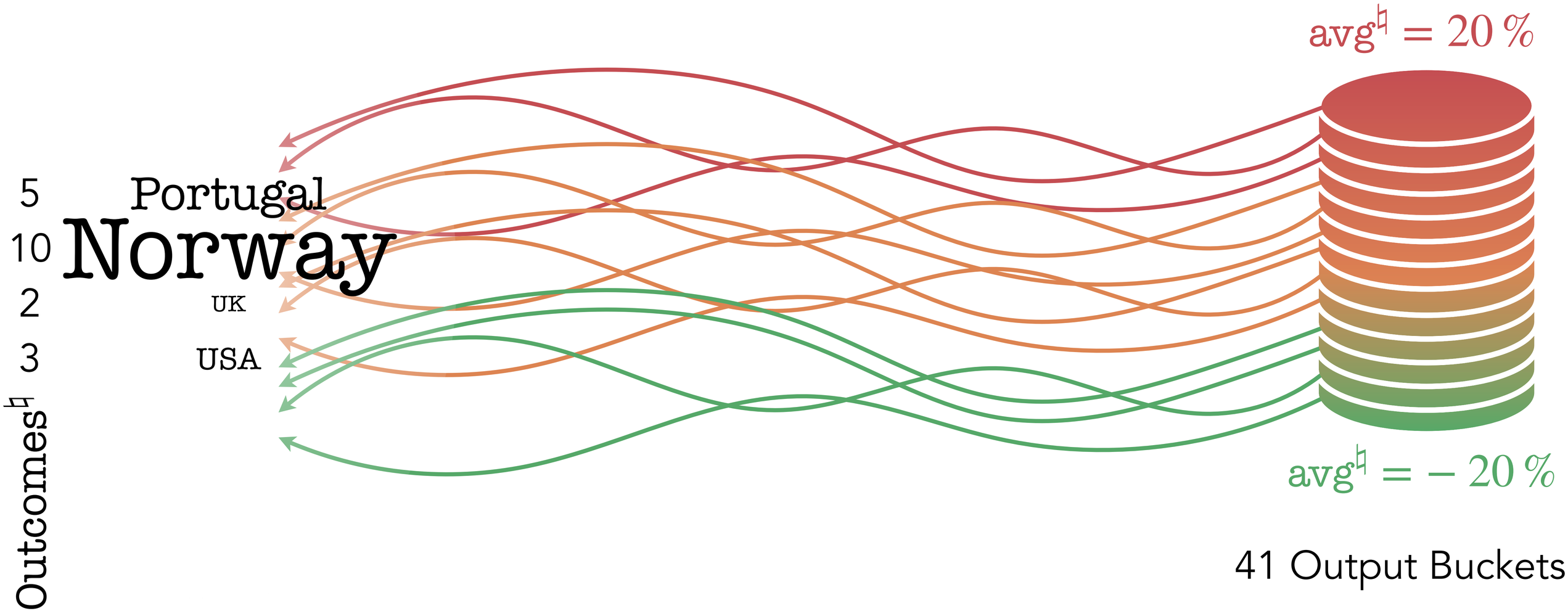

41 Output Buckets

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```

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American Economic Review 2010.

+1.7%

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 1: def mean_growth_rate_60_90(
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 4:     uk1, uk2, uk3, uk4,
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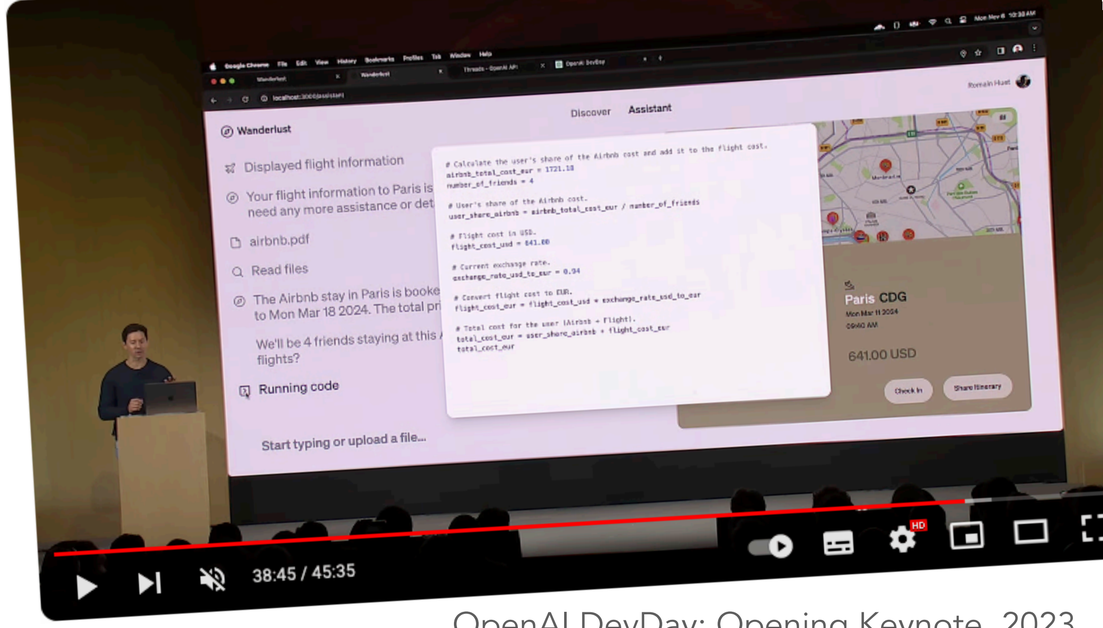
```

Use Case: GPT-4 Turbo



OpenAI DevDay: Opening Keynote. 2023
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9I

```
1: def share_division(
2:     airbnb_total_cost_eur,
3:     flight_cost_usd,
4:     number_of_friends):
5:   share_airbnb = airbnb_total_cost_eur / number_of_friends
6:   usd_to_eur = 0.92
7:   flight_cost_eur = flight_cost_usd * usd_to_eur
8:   total_cost_eur = share_airbnb + flight_cost_eur
9:   return total_cost_eur
```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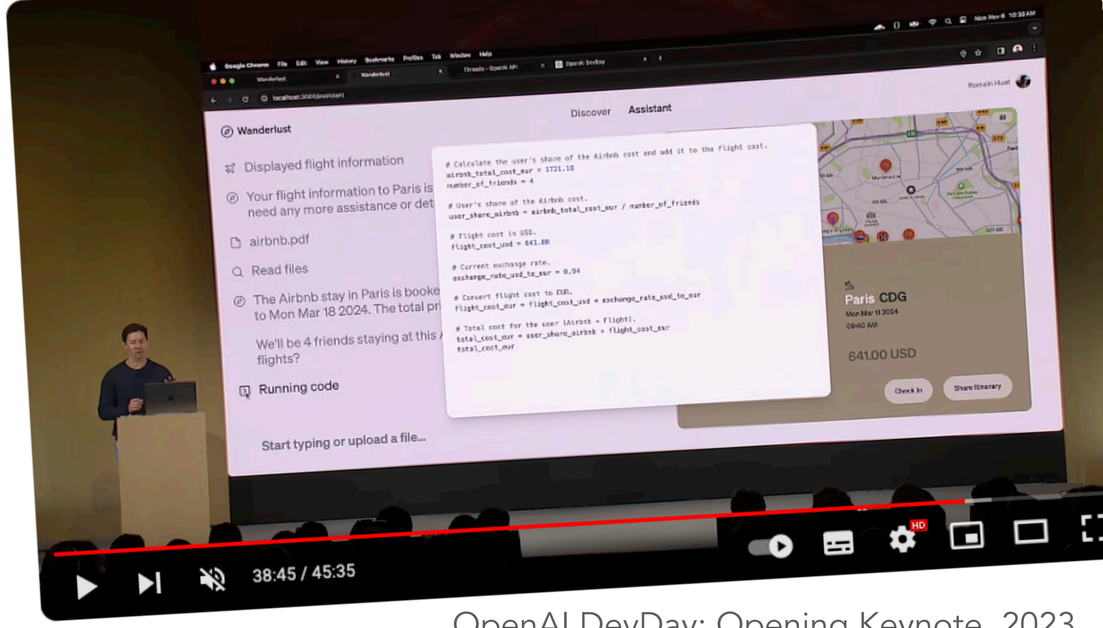
Use Case: GPT-4 Turbo

```
1: def share_division(
2: 500 ≤ airbnb_total_cost_eur ≤ 2000
3:  50 ≤ flight_cost_usd ≤ 1000
4:   2 ≤ number_of_friends ≤ 10
```



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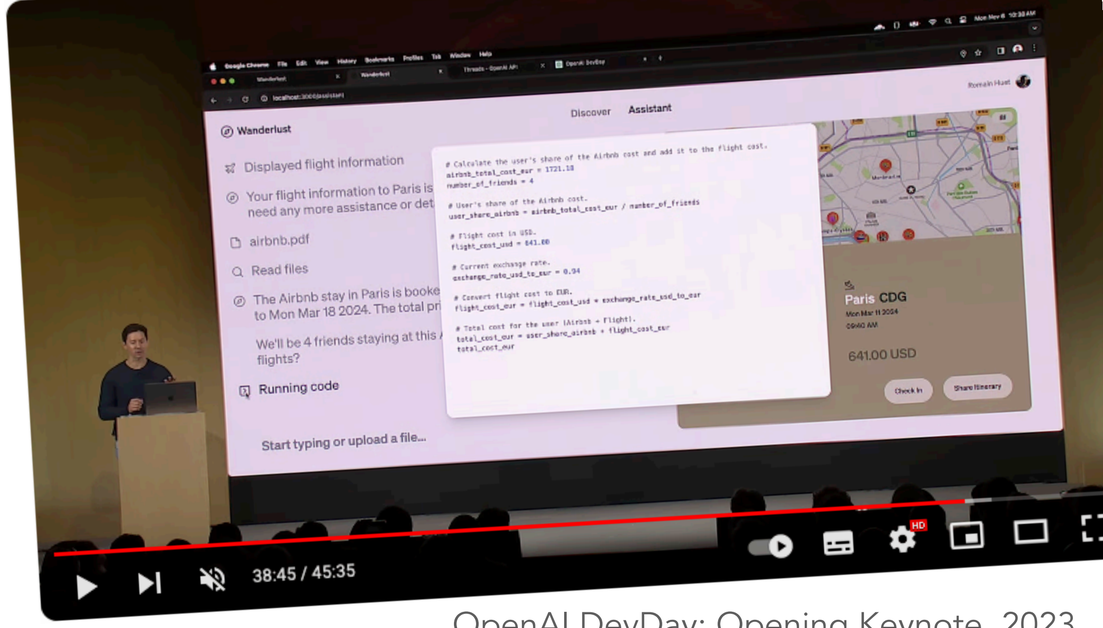

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1: def share_division(
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3:  50 ≤ flight_cost_usd ≤ 1000
4:   2 ≤ number_of_friends ≤ 10
5:   share_airbnb = airbnb_total_cost_eur / number_of_friends
6:   usd_to_eur = 0.92
7:   flight_cost_eur = flight_cost_usd * usd_to_eur
8:   total_cost_eur = share_airbnb + flight_cost_eur
9:   return total_cost_eur
```
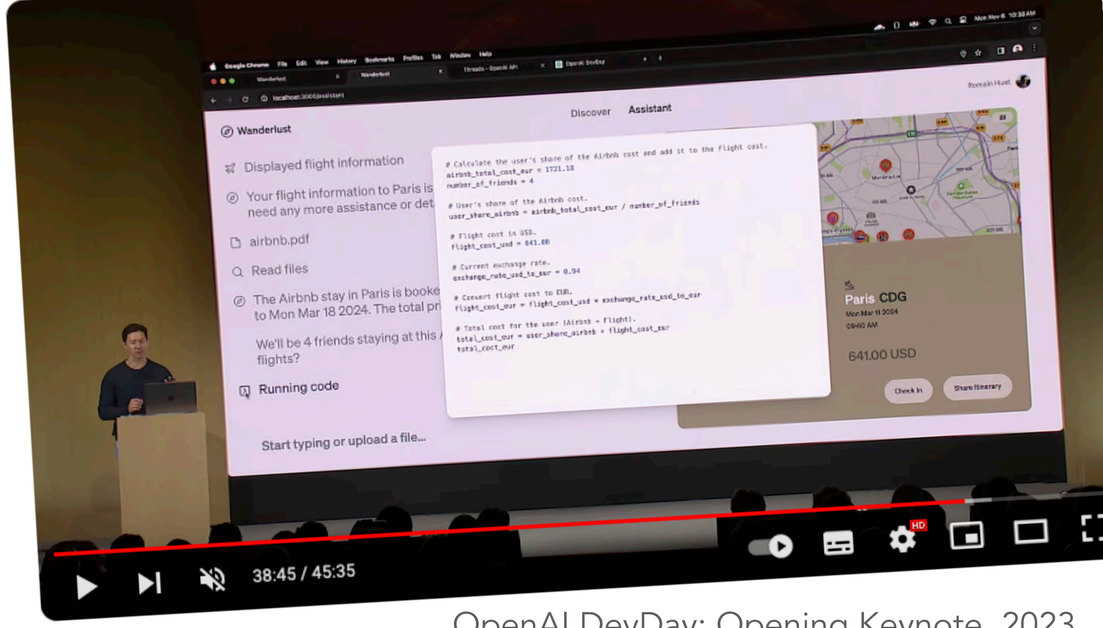

Use Case: GPT-4 Turbo



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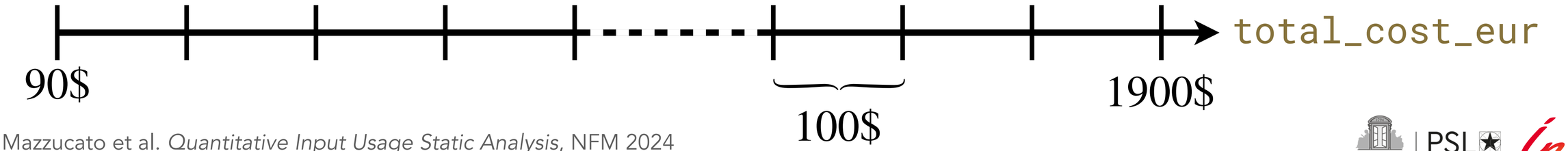
https://www.youtube.com/live/U9mJuUkhUzk?si=vH5gZI3YUR53ep9I

```
1: def share_division(
2: 500 ≤airbnb_total_cost_eur ≤ 2000
3:  50 ≤flight_cost_usd ≤ 1000
4:   2 ≤number_of_friends ≤ 10
5:    share_airbnb = airbnb_total_cost_eur / number_of_friends
6:    usd_to_eur = 0.92
7:    flight_cost_eur = flight_cost_usd * usd_to_eur
8:    total_cost_eur = share_airbnb + flight_cost_eur
9:    return total_cost_eur
```

20 Output Buckets



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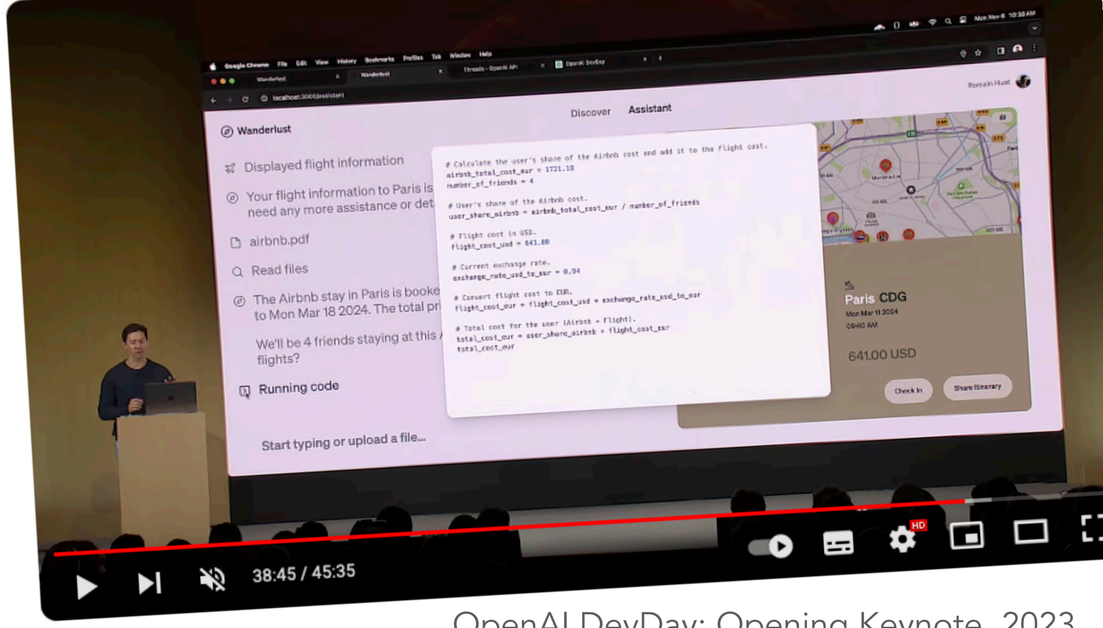
Use Case: GPT-4 Turbo

airbnb_total_cost_eur
flight_cost_usd
number_of_friends



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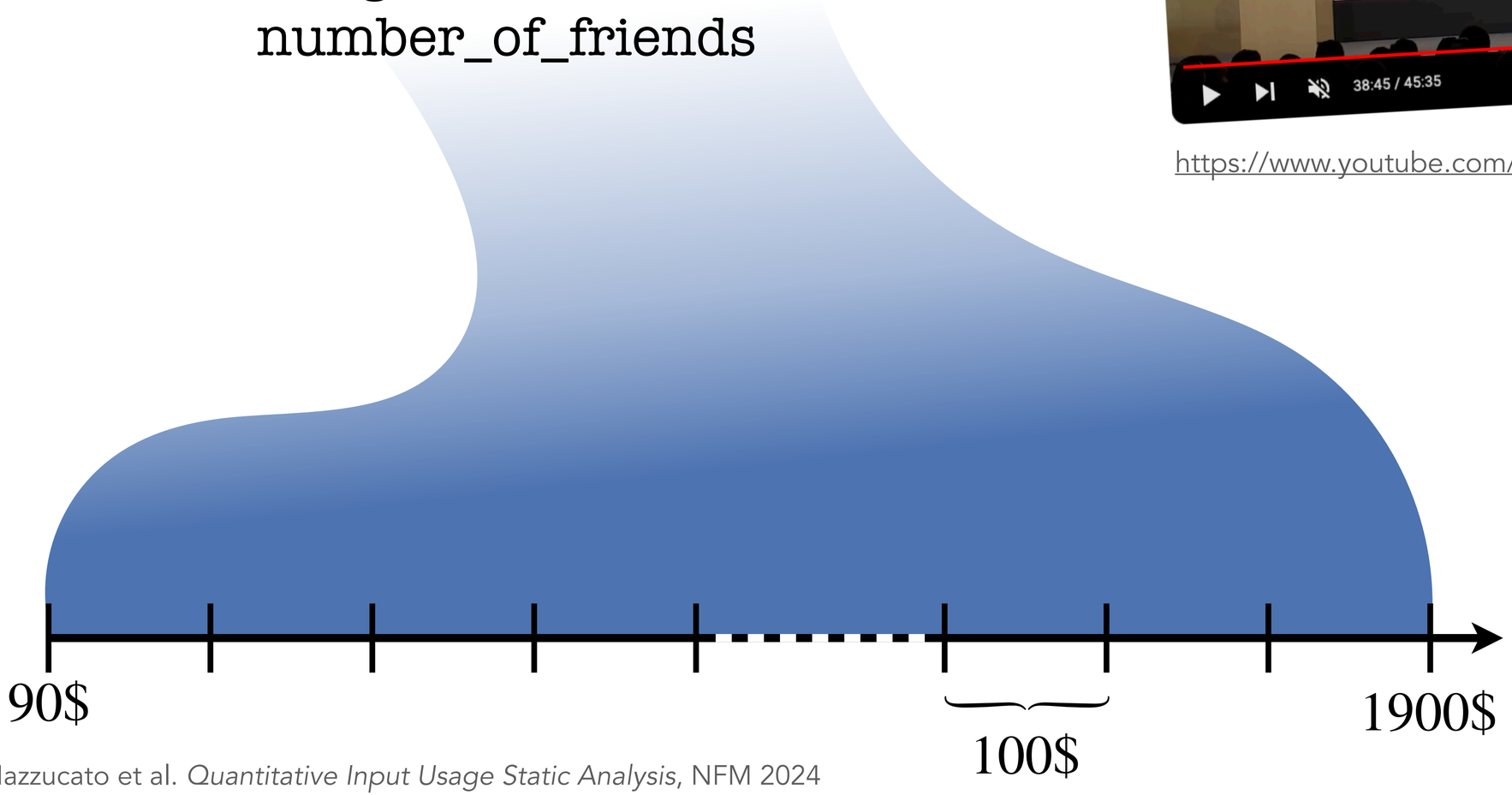

20 Output Buckets

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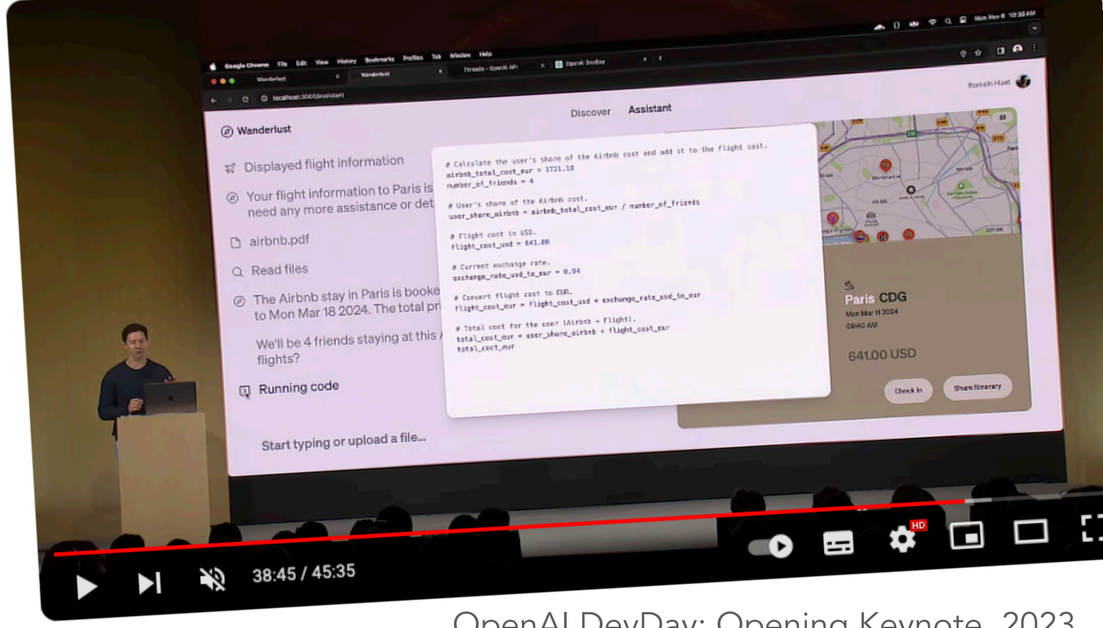

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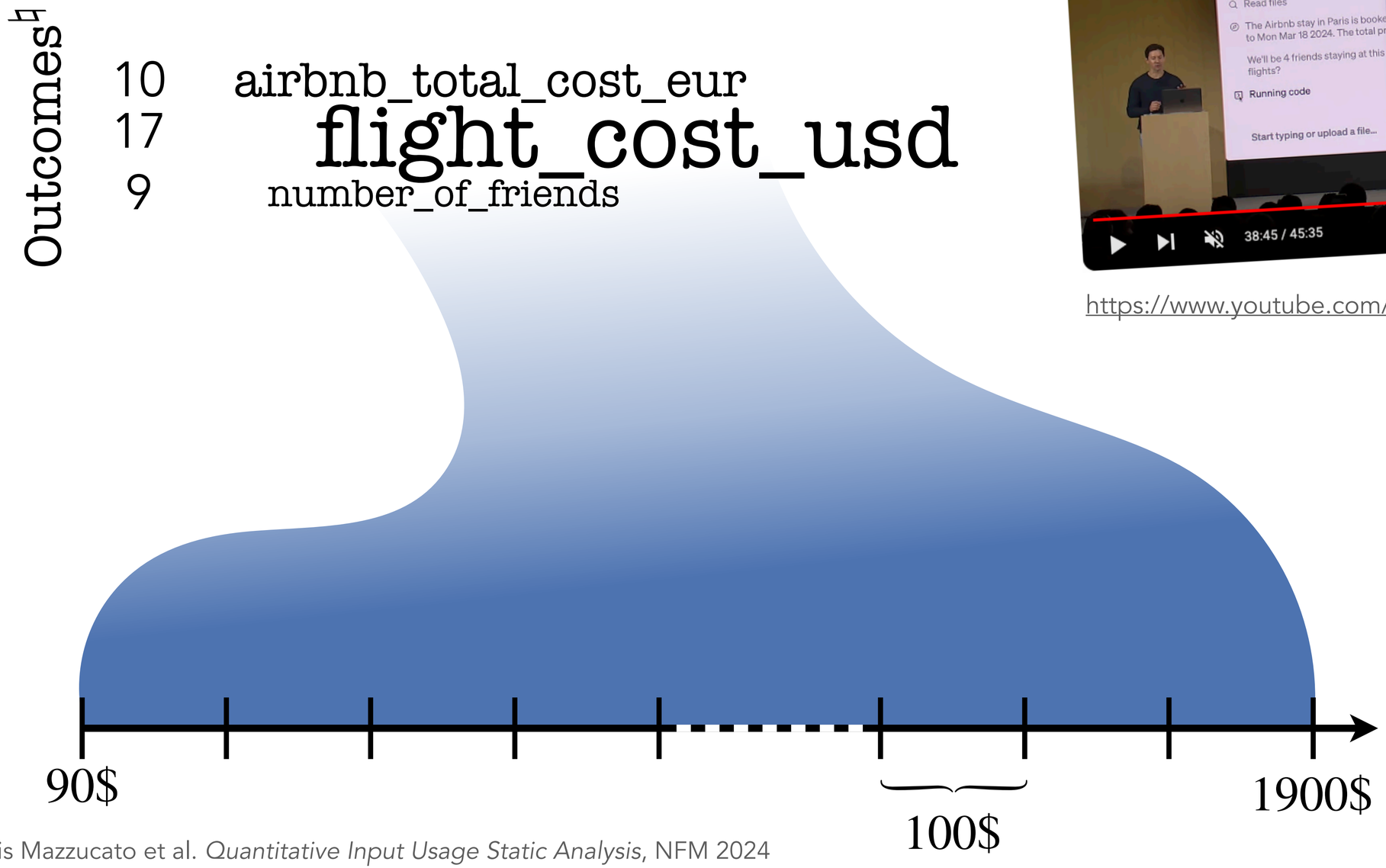

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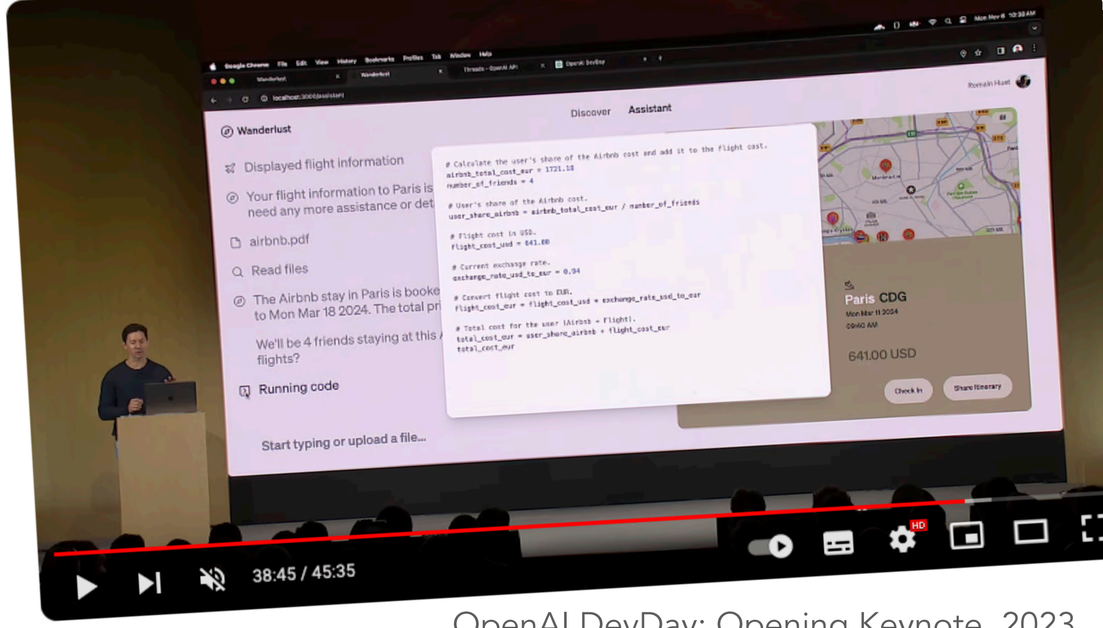

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Conclusion

Goal: Quantify the impact of speed and angle on risk

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1: landing_coeff = abs(angle) + speed
2: if landing_coeff < 2 then
3:   risk = 0
4: else if landing_coeff > 5 then
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6: else
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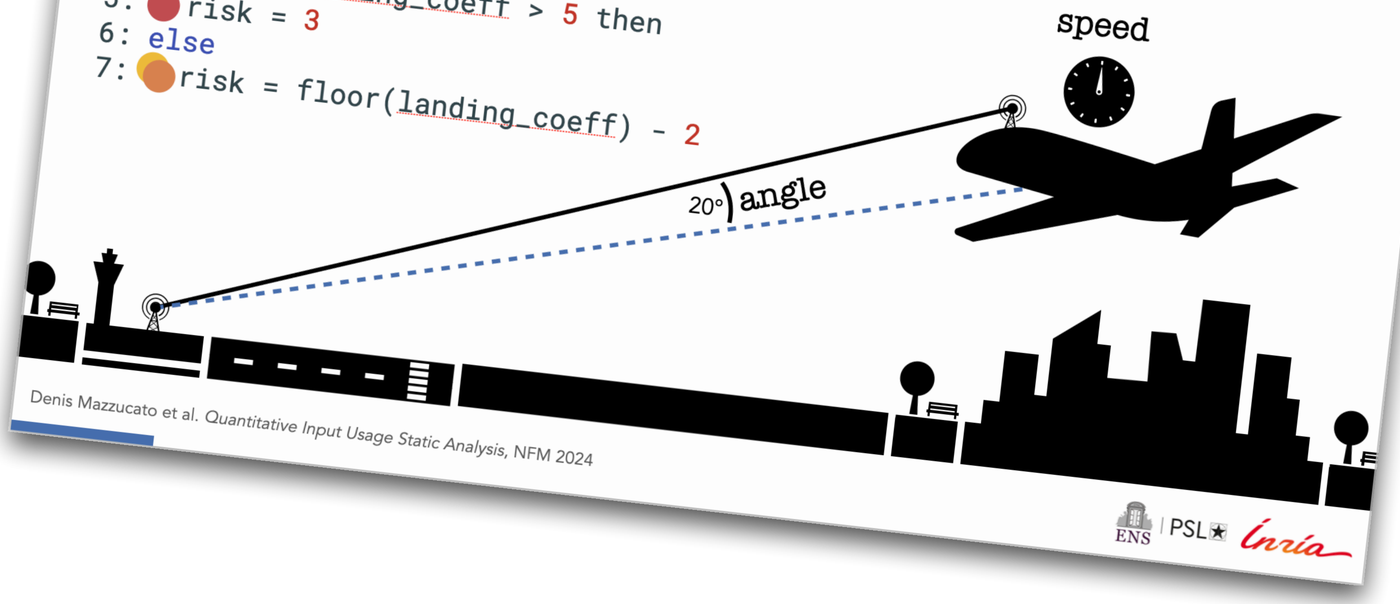

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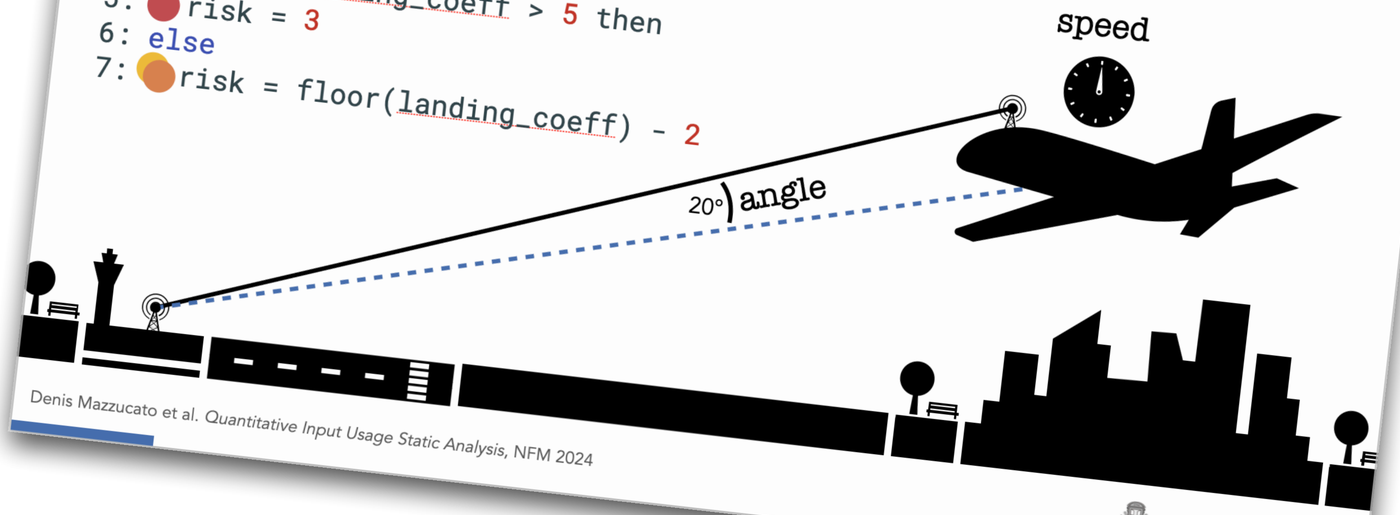

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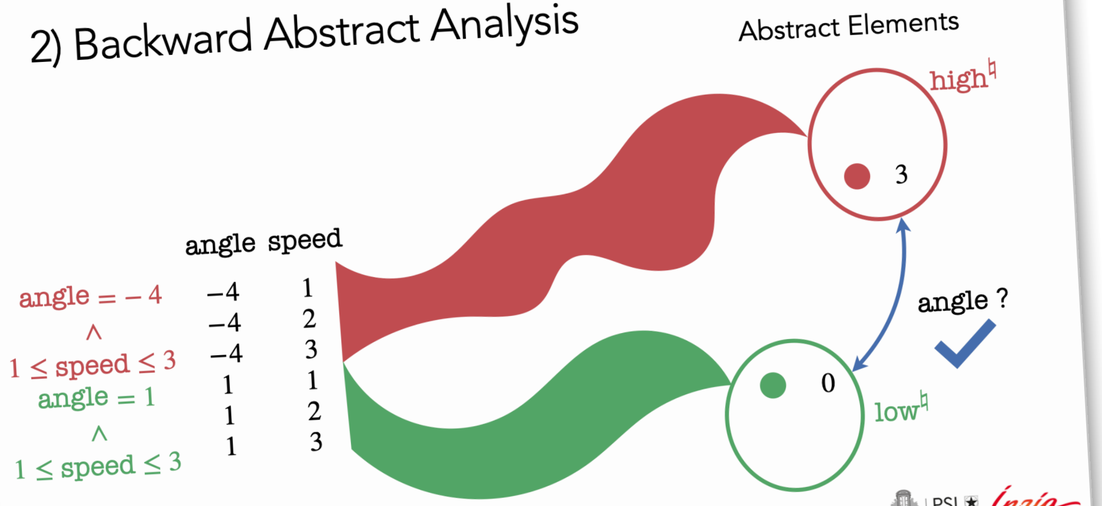
2) Backward Abstract Analysis



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Conclusion

Use Case: Reinhart and Rogoff

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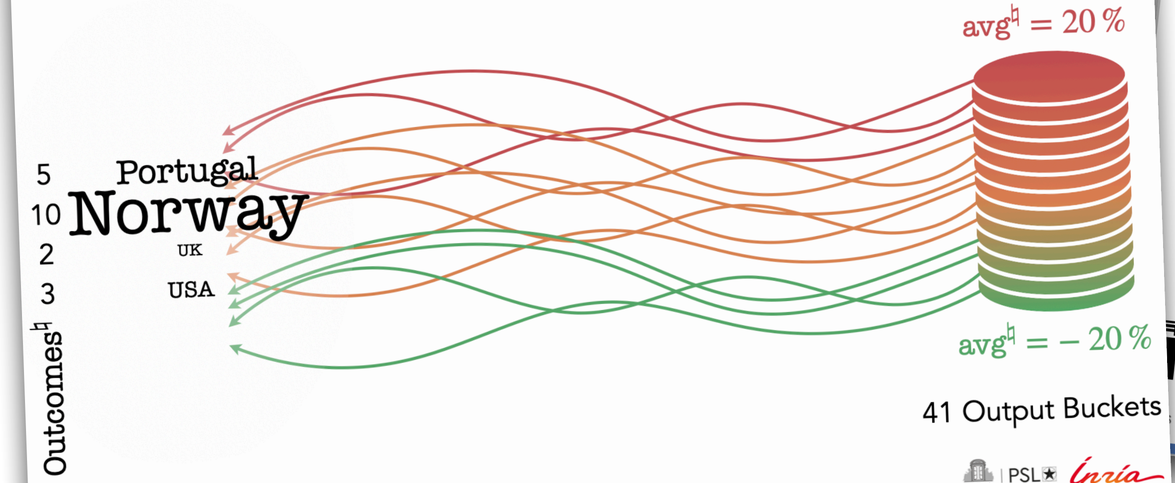

41 Output Buckets

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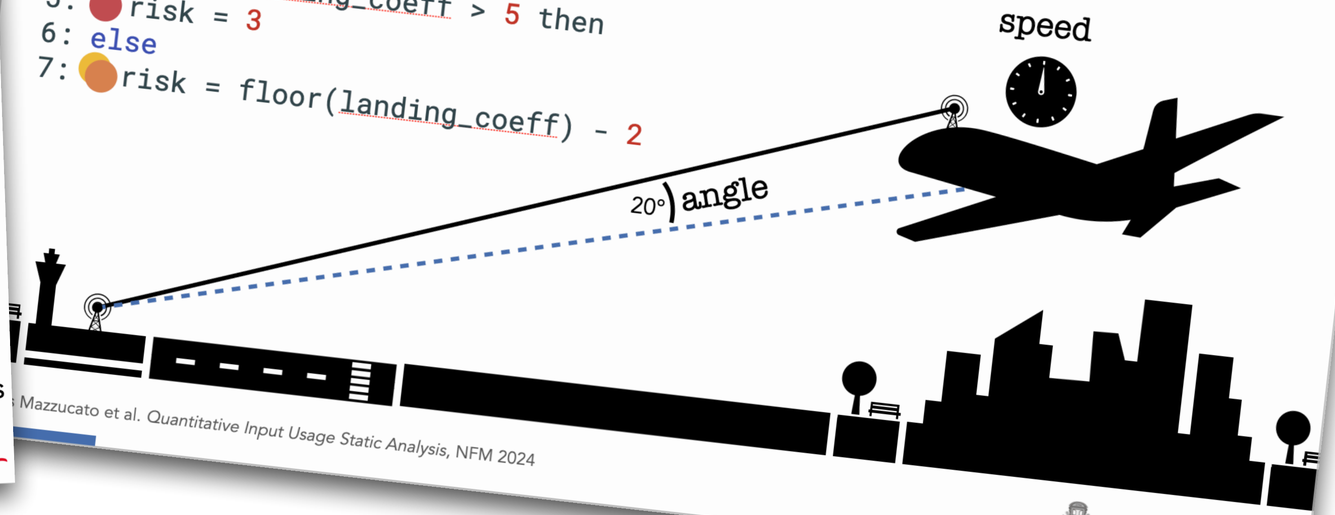

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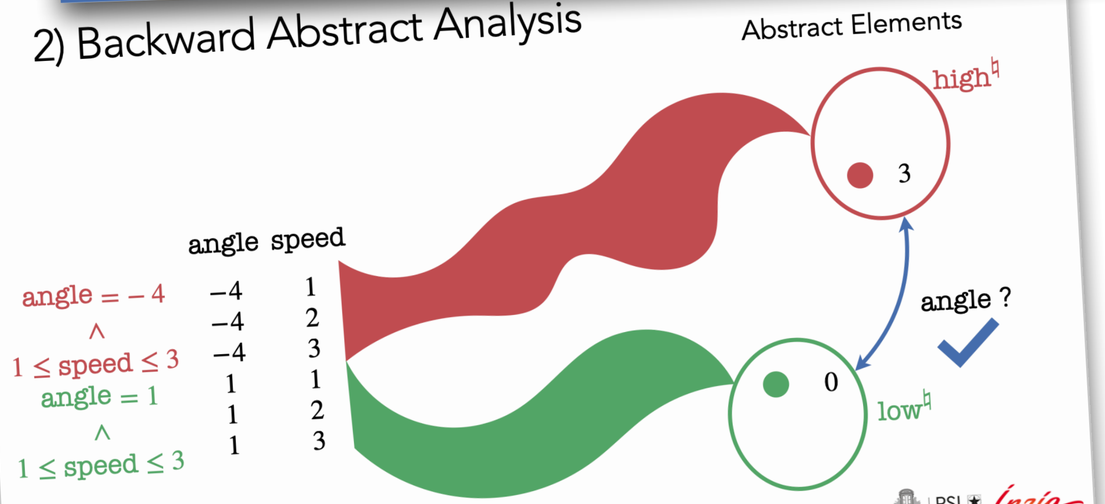

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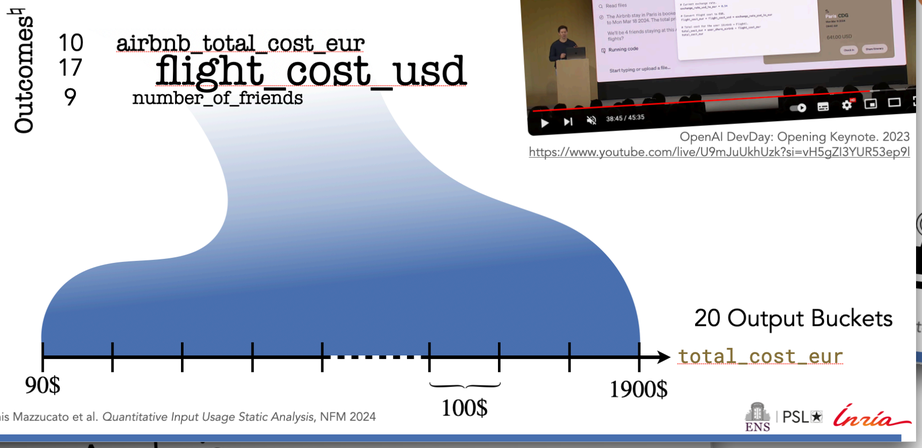
Use Case: GPT-4 Turbo



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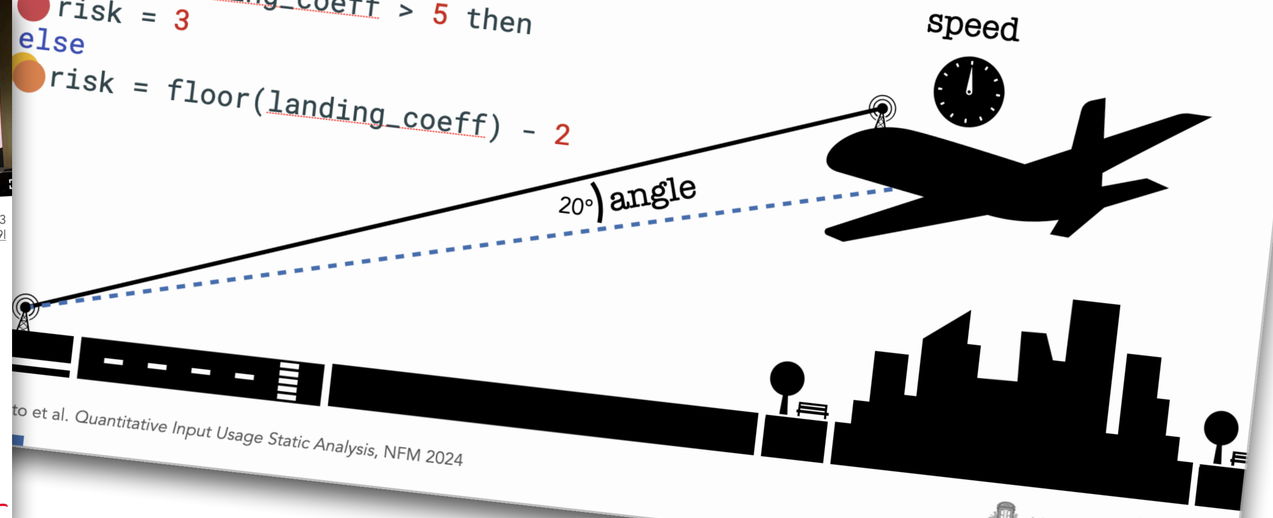

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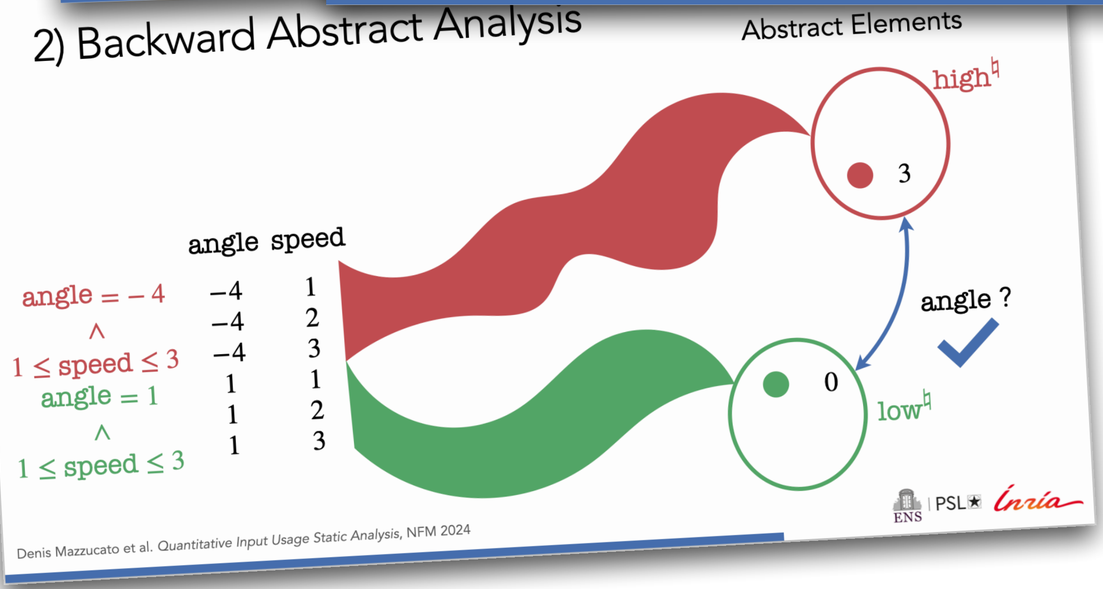

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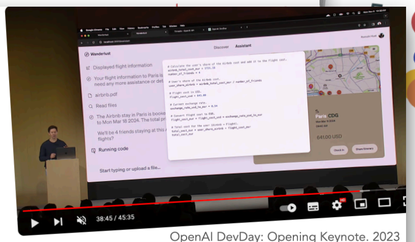

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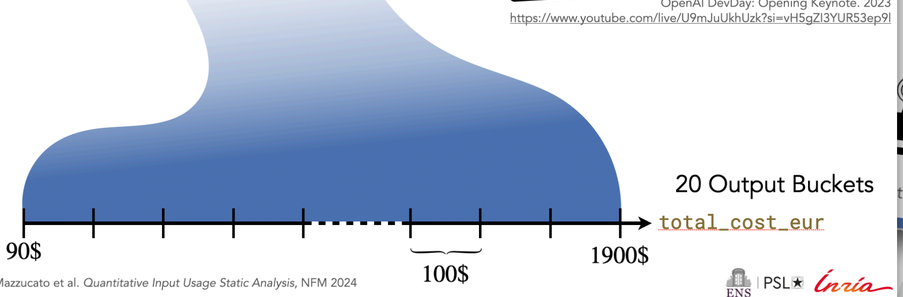

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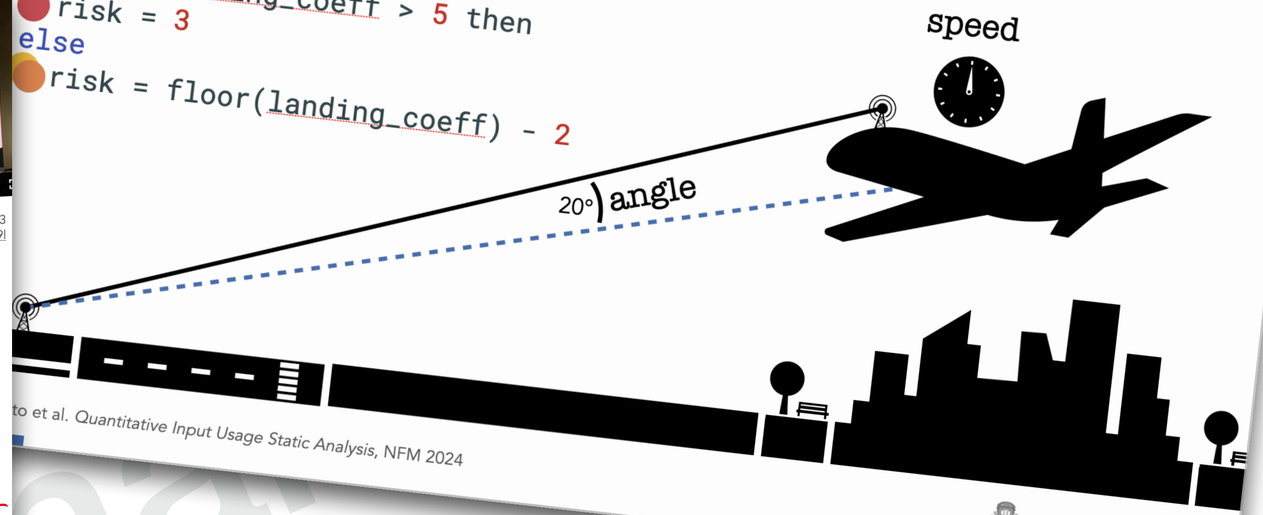

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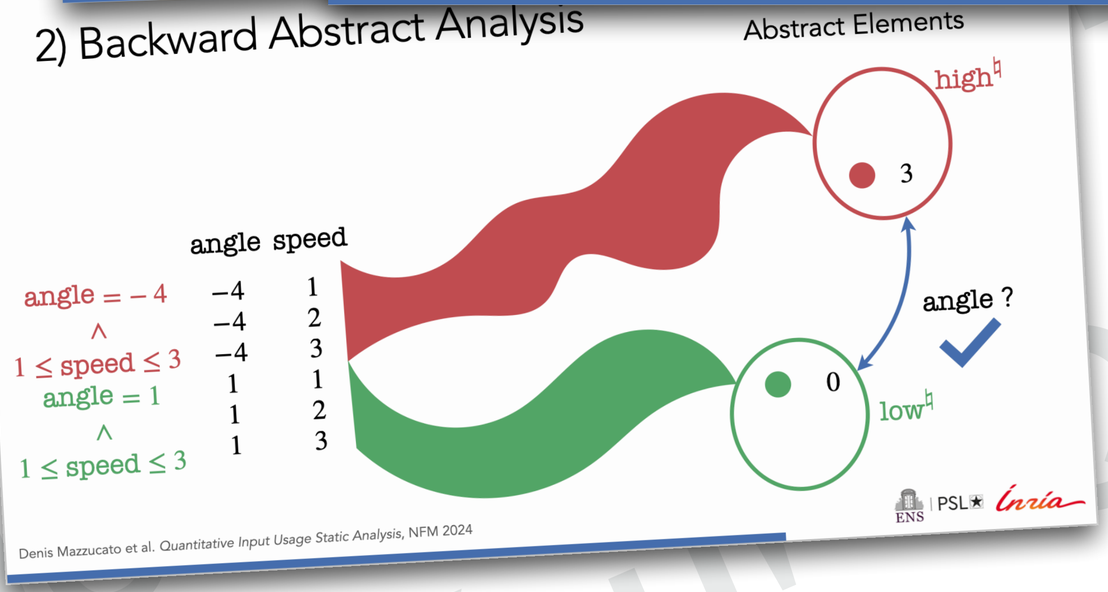

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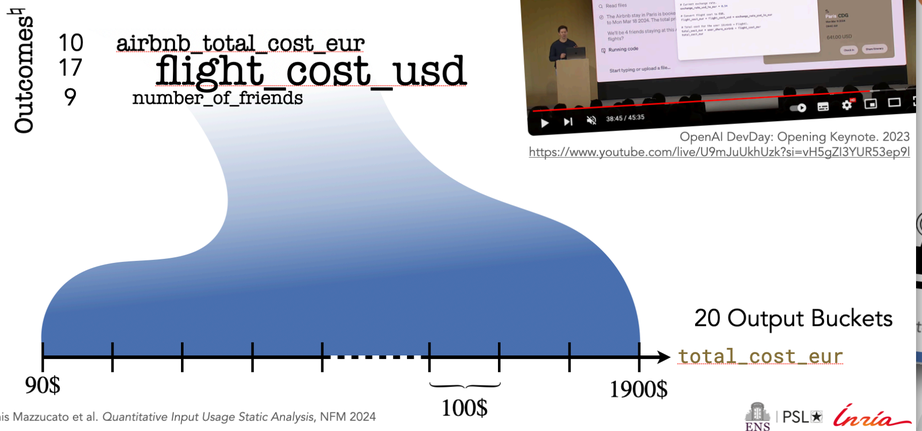

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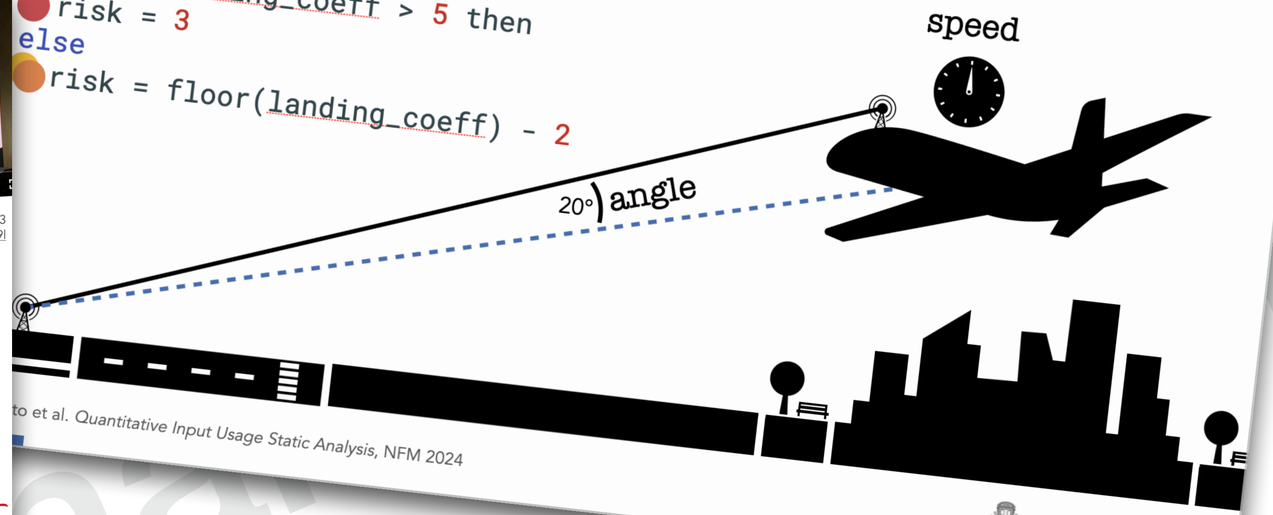

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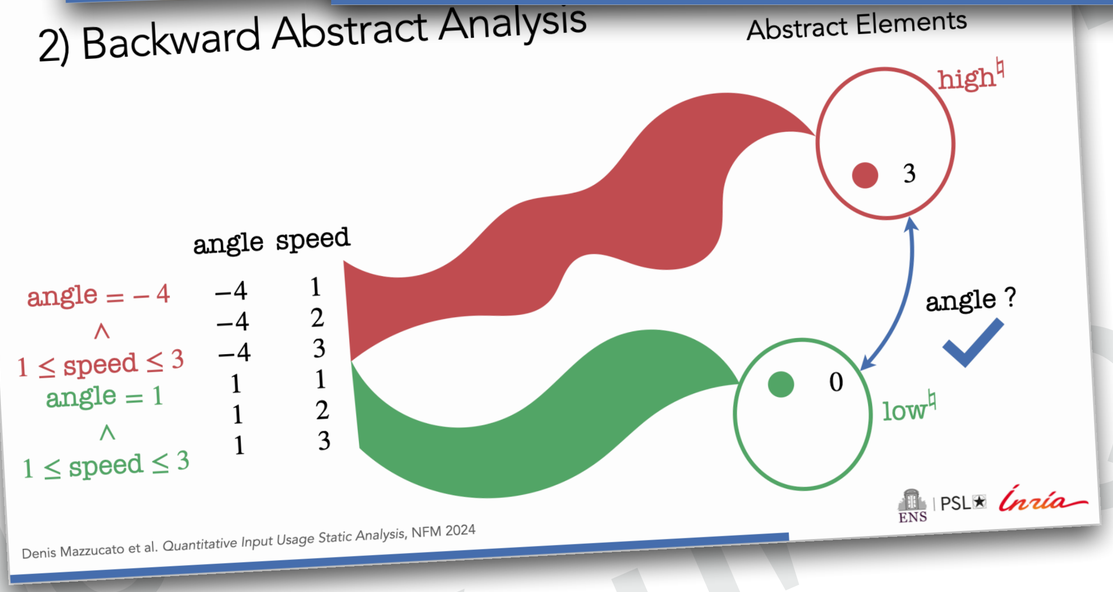

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Future Work

Quantify the Impact on Timing Behavior

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