

Quantitative Input Feature Usage

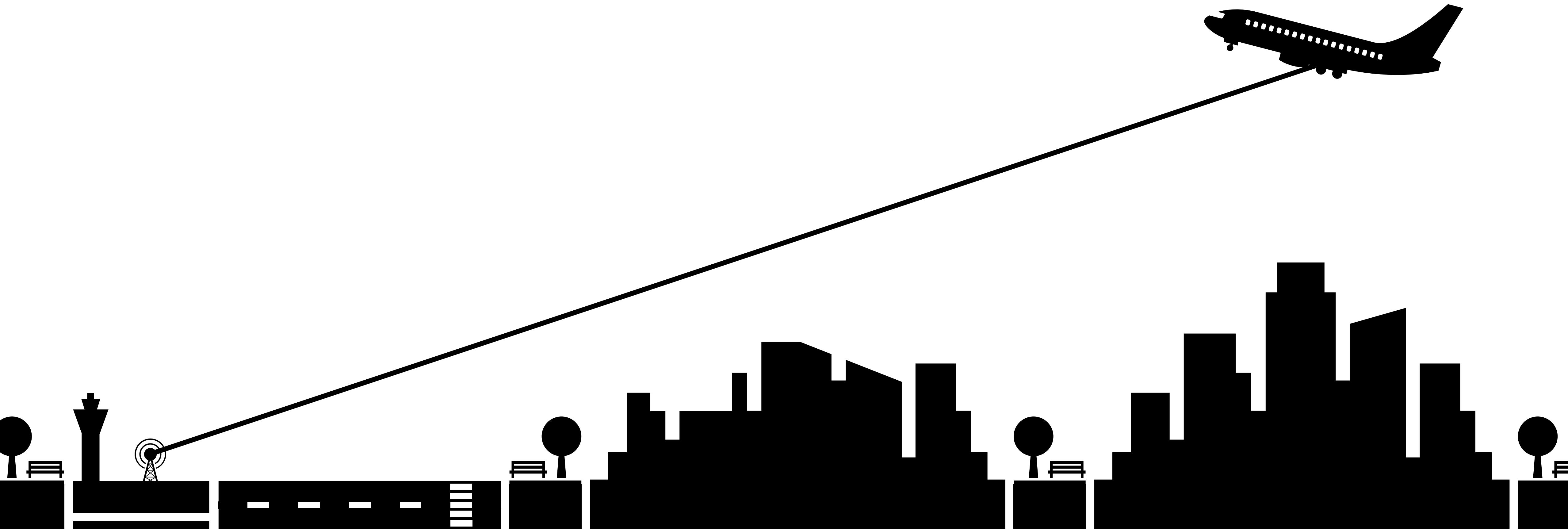
Denis Mazzucato, Marco Campion, Caterina Urban
17th March 2023 – weekly seminar ANTIQUE lab



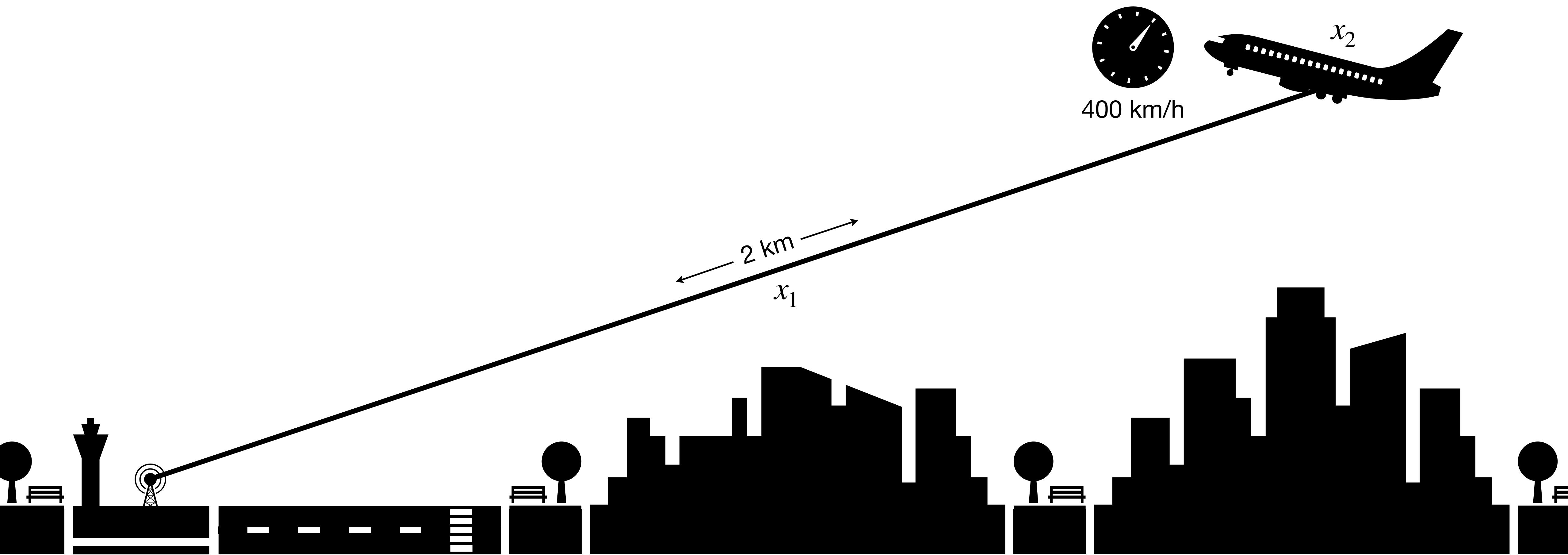
| PSL 

inria

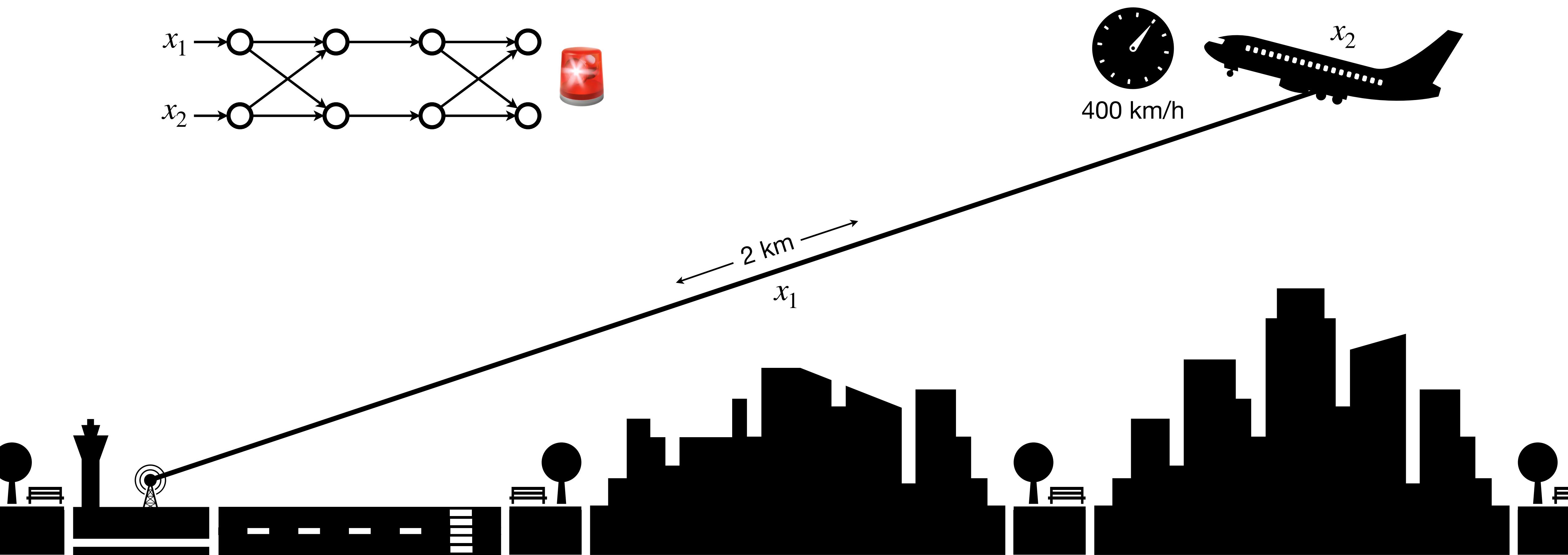
Landing Alarm System



Landing Alarm System



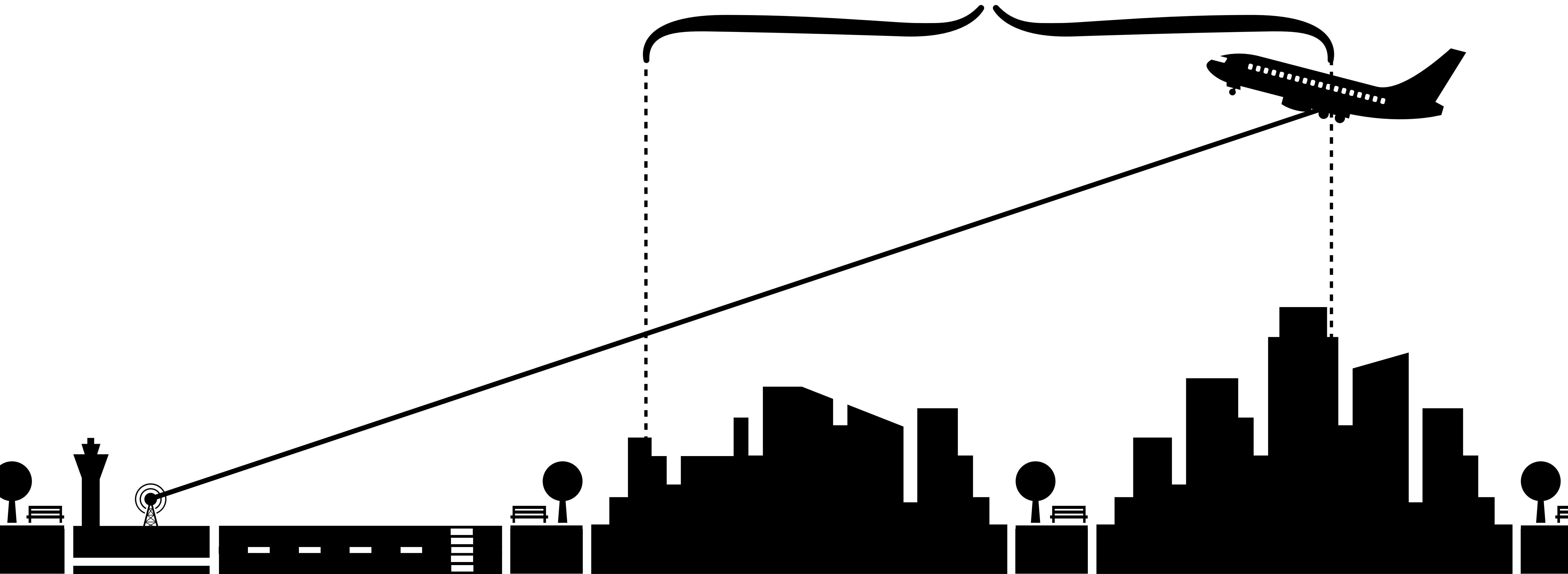
Landing Alarm System



Landing Alarm System



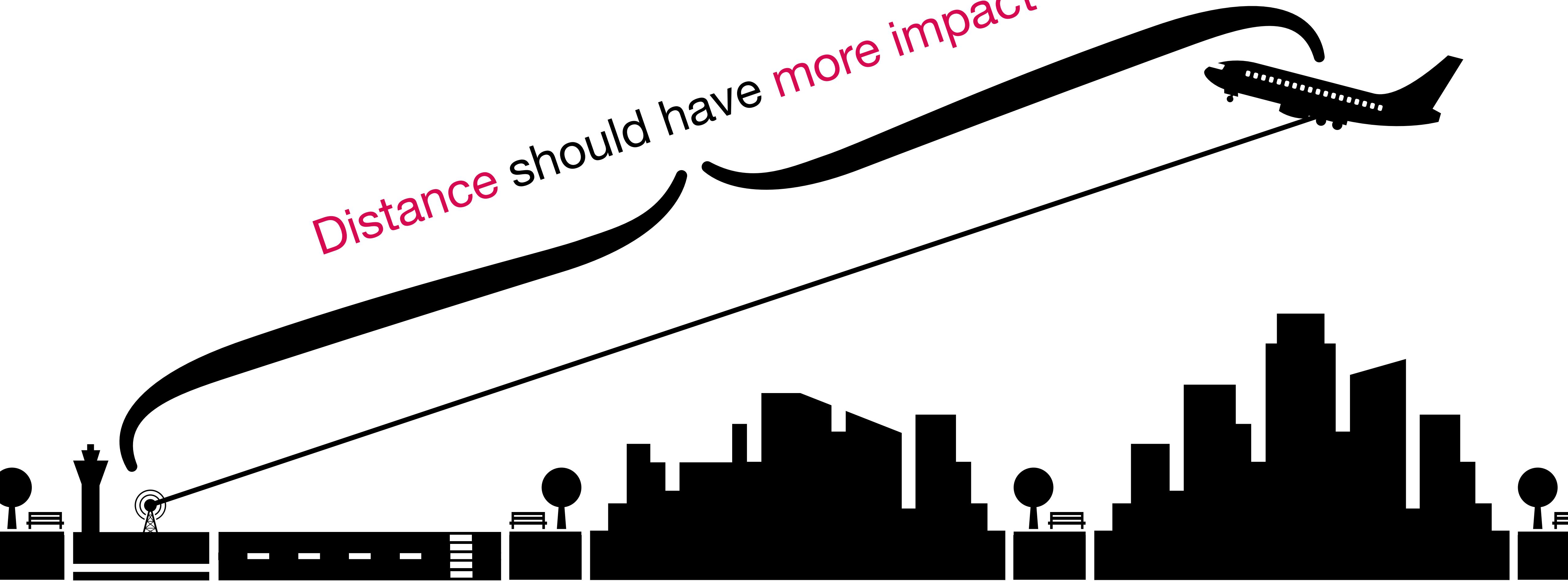
Speed should not be considered



Landing Alarm System



Distance should have more impact



Impact

The **impact** of an **input feature** is
its **computational influence** on the program outcome

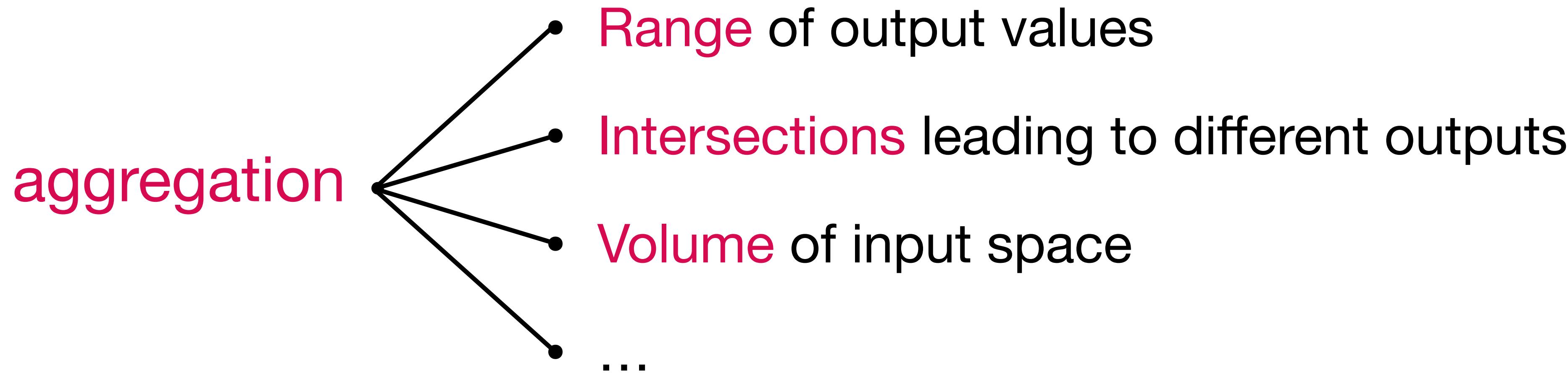
Impact

The **impact** of an **input feature** is
its **computational influence** on the program outcome

The **impact** of the i -th input feature is
the **aggregation** of all the possible **input perturbations** on the i -th variable
for all possible **inputs**

Impact

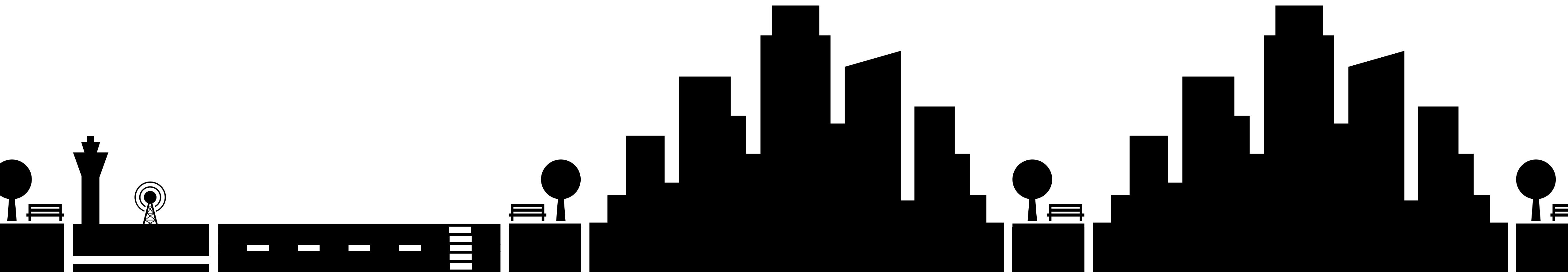
The **impact** of an **input feature** is
its **computational influence** on the program outcome



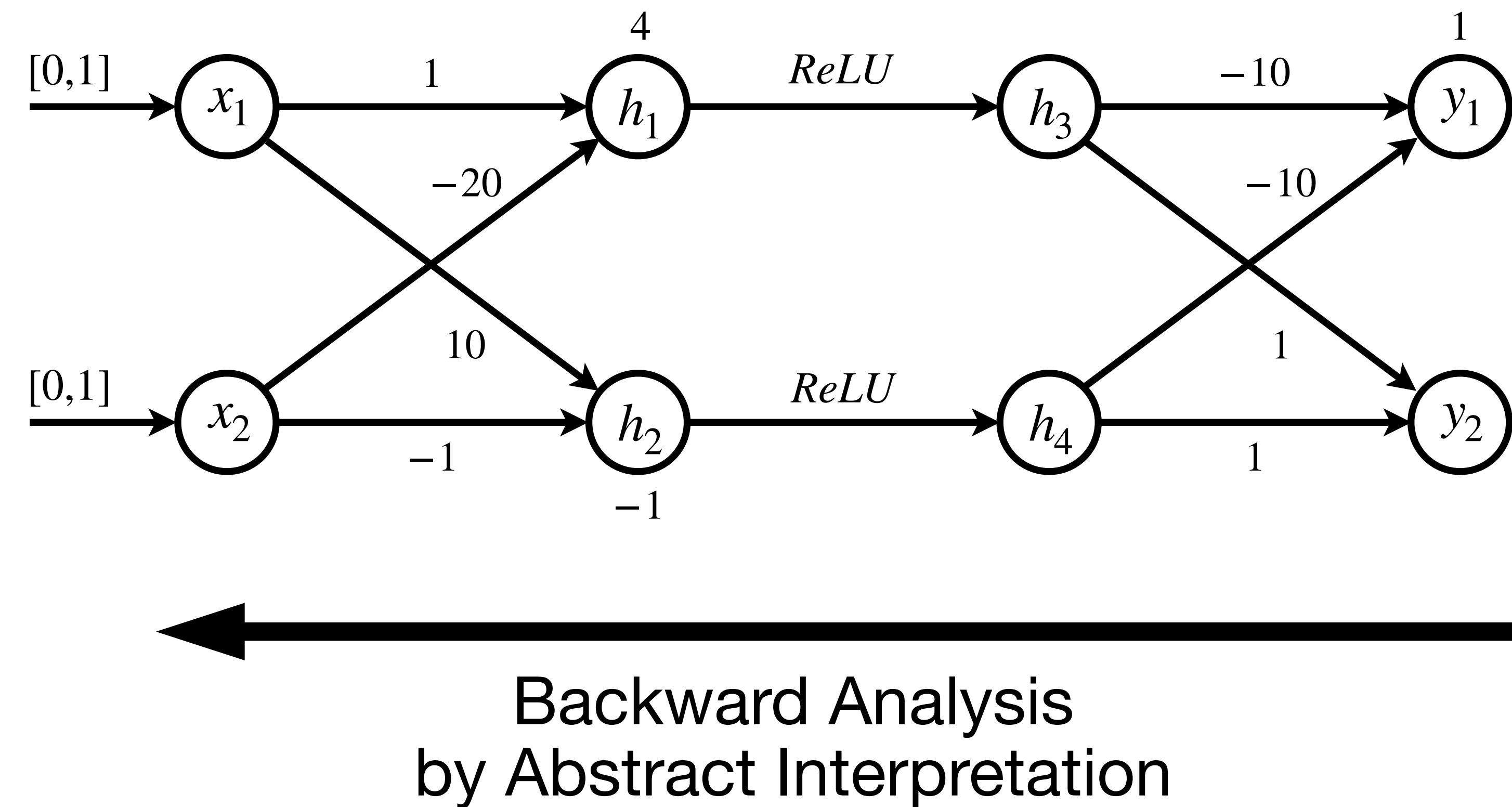
Quantitative Input Feature Usage

Discriminate between input features with **different impact**

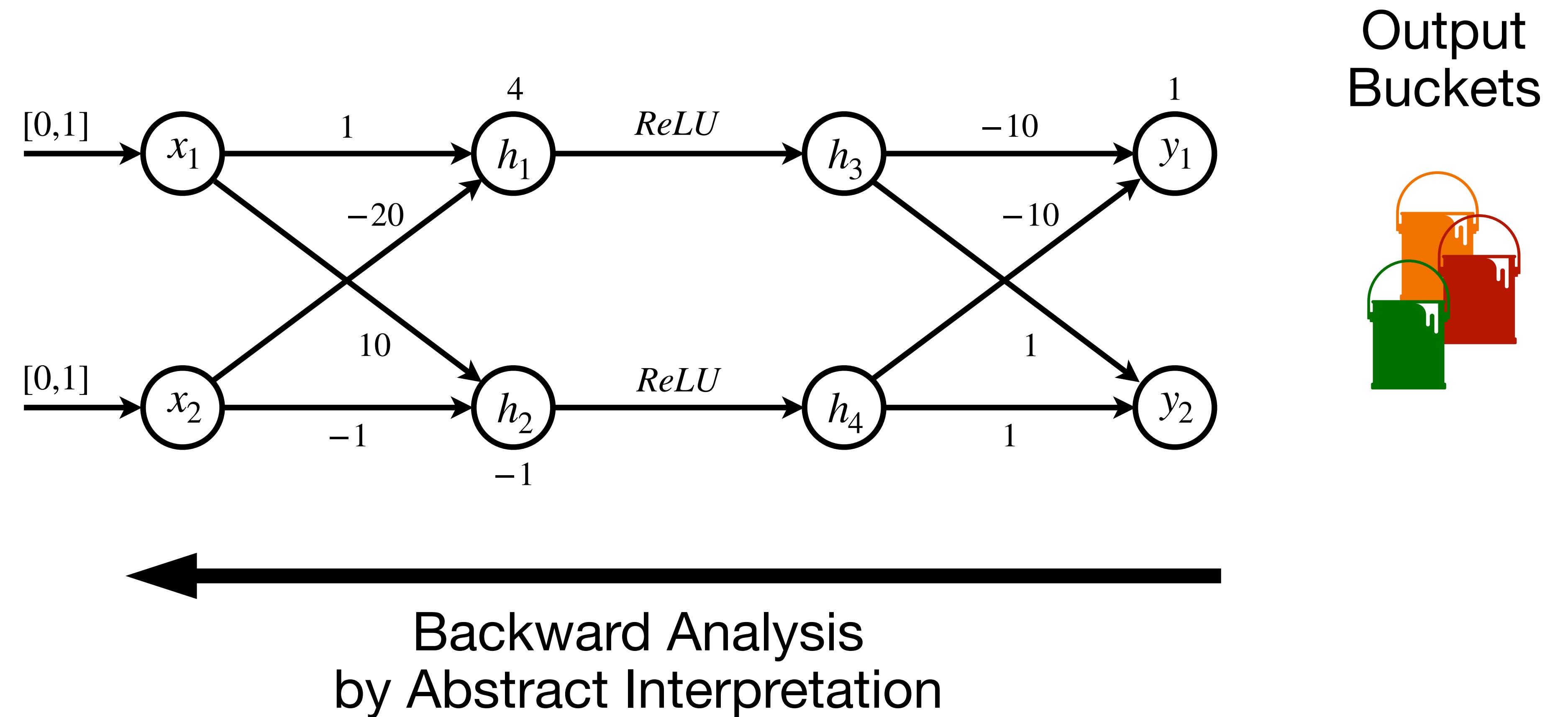
- Certify intended behaviours
- Reveal potential flaws



Quantitative Input Feature Usage Analysis

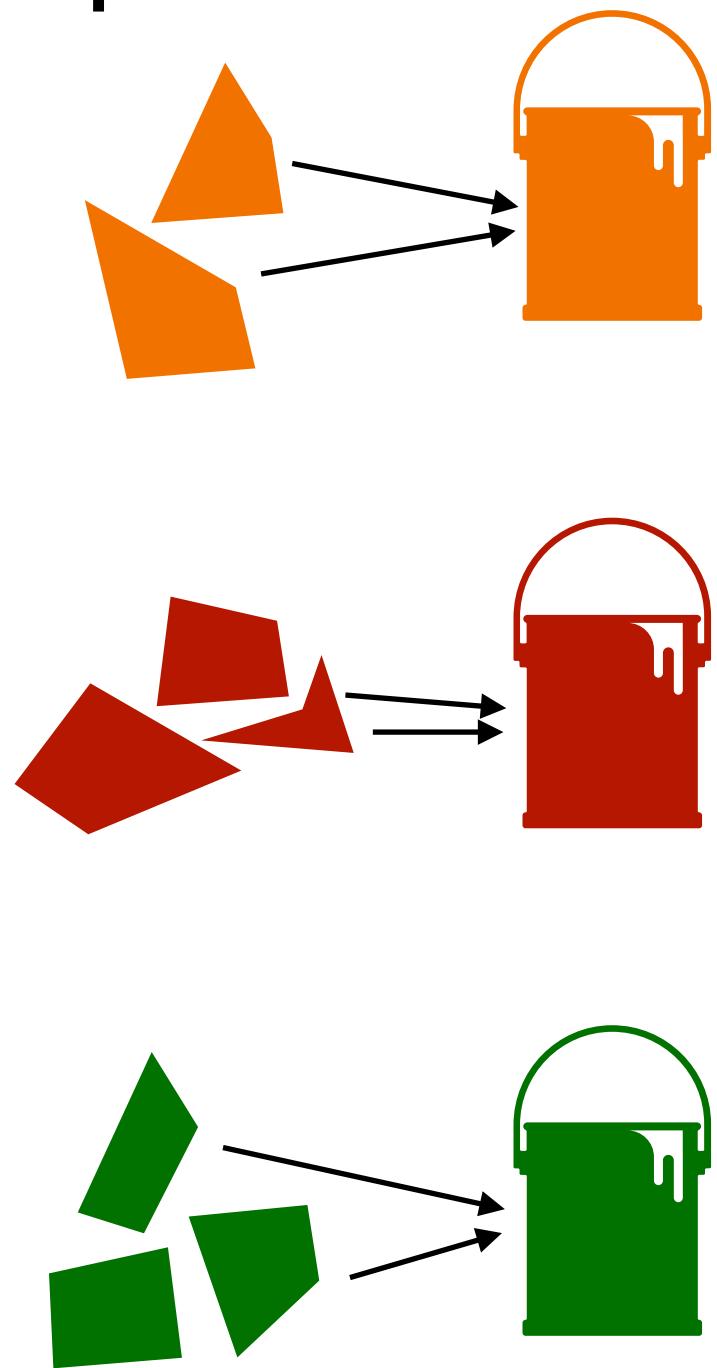


Quantitative Input Feature Usage Analysis



Quantitative Input Feature Usage Analysis

Input Space

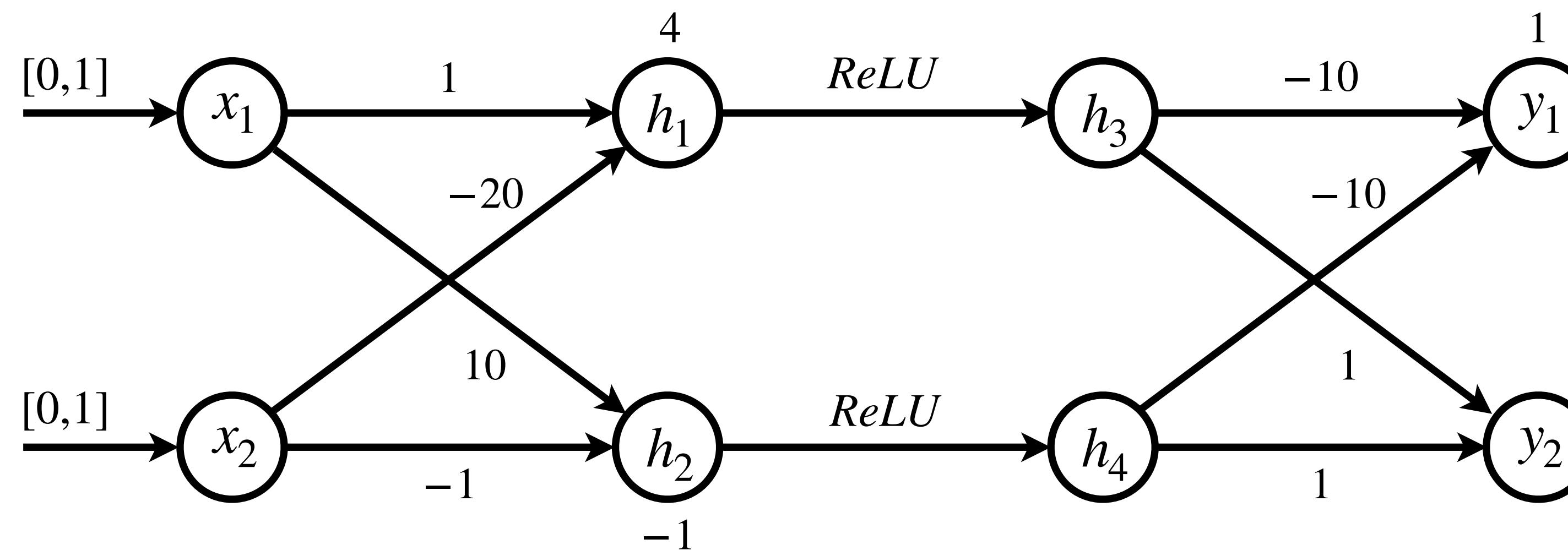


$[0,1]$

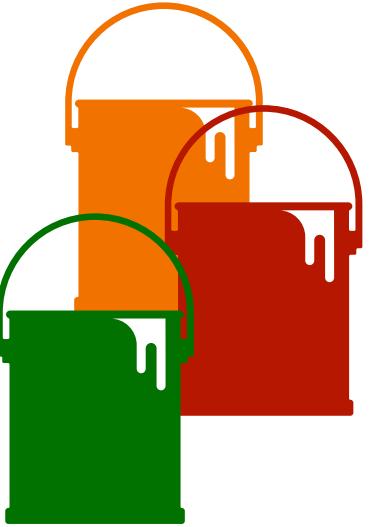
$[0,1]$



Backward Analysis
by Abstract Interpretation

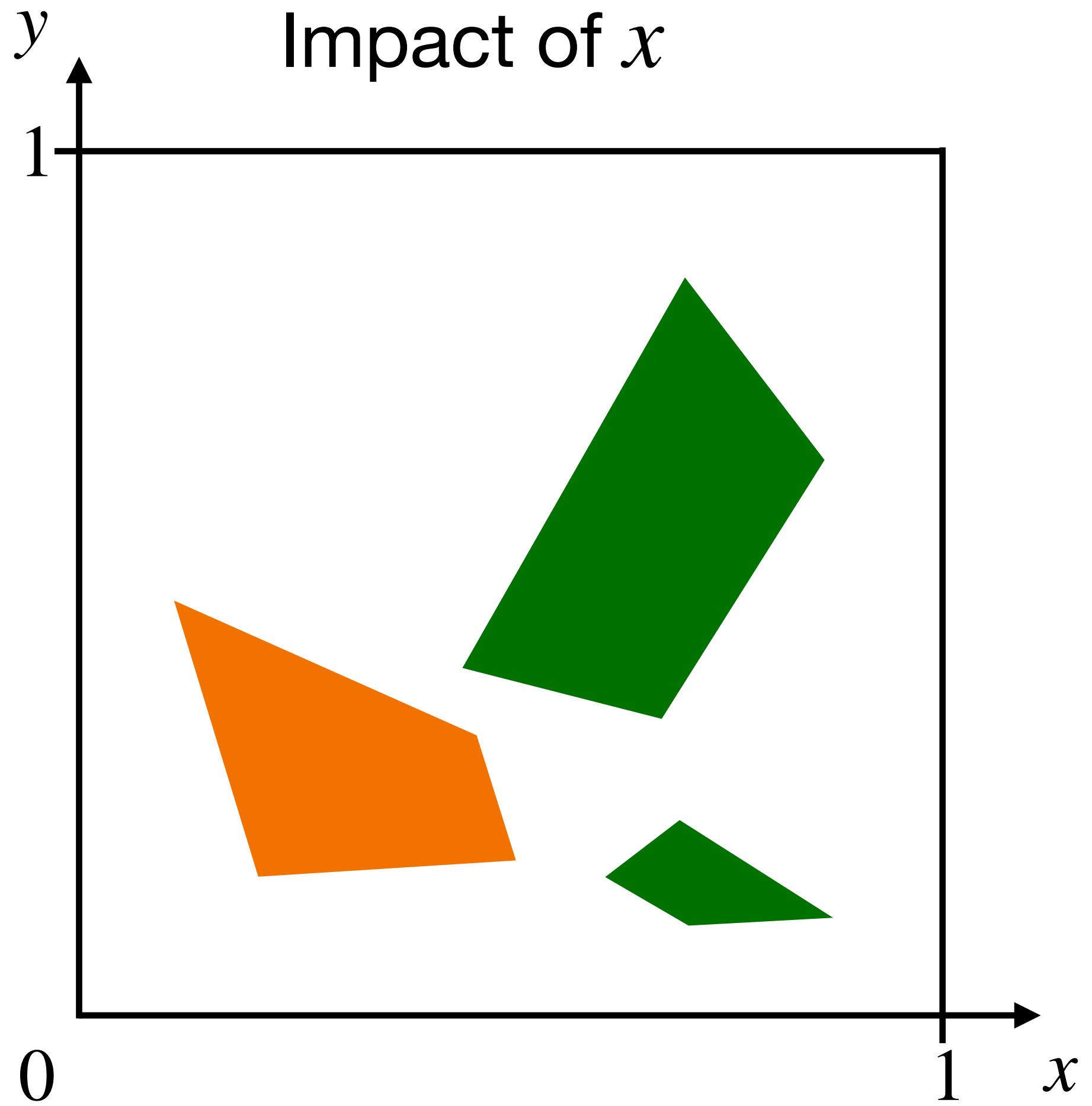


Output Buckets

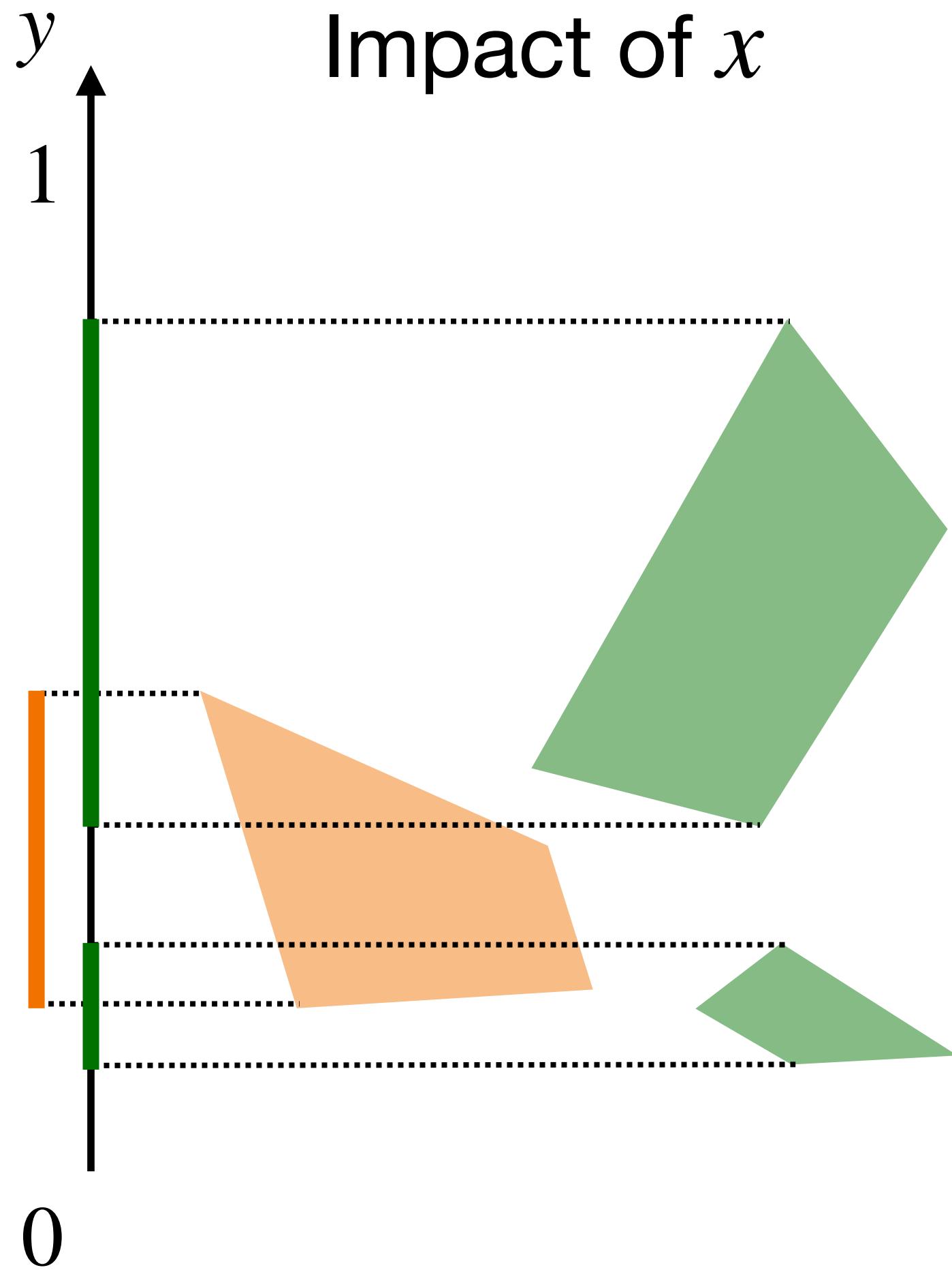


Over-approximation

Quantitative Input Feature Usage Analysis

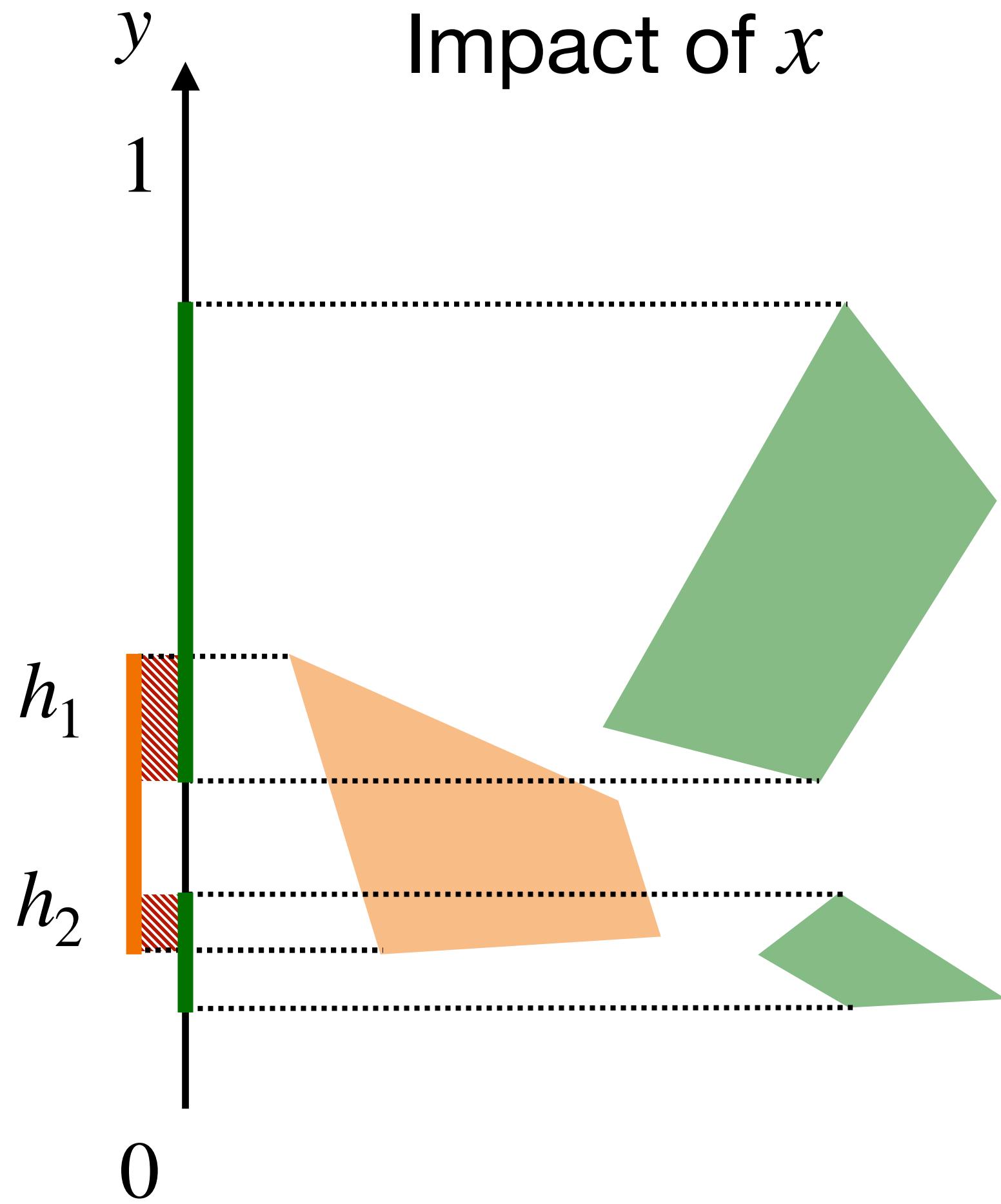


Quantitative Input Feature Usage Analysis

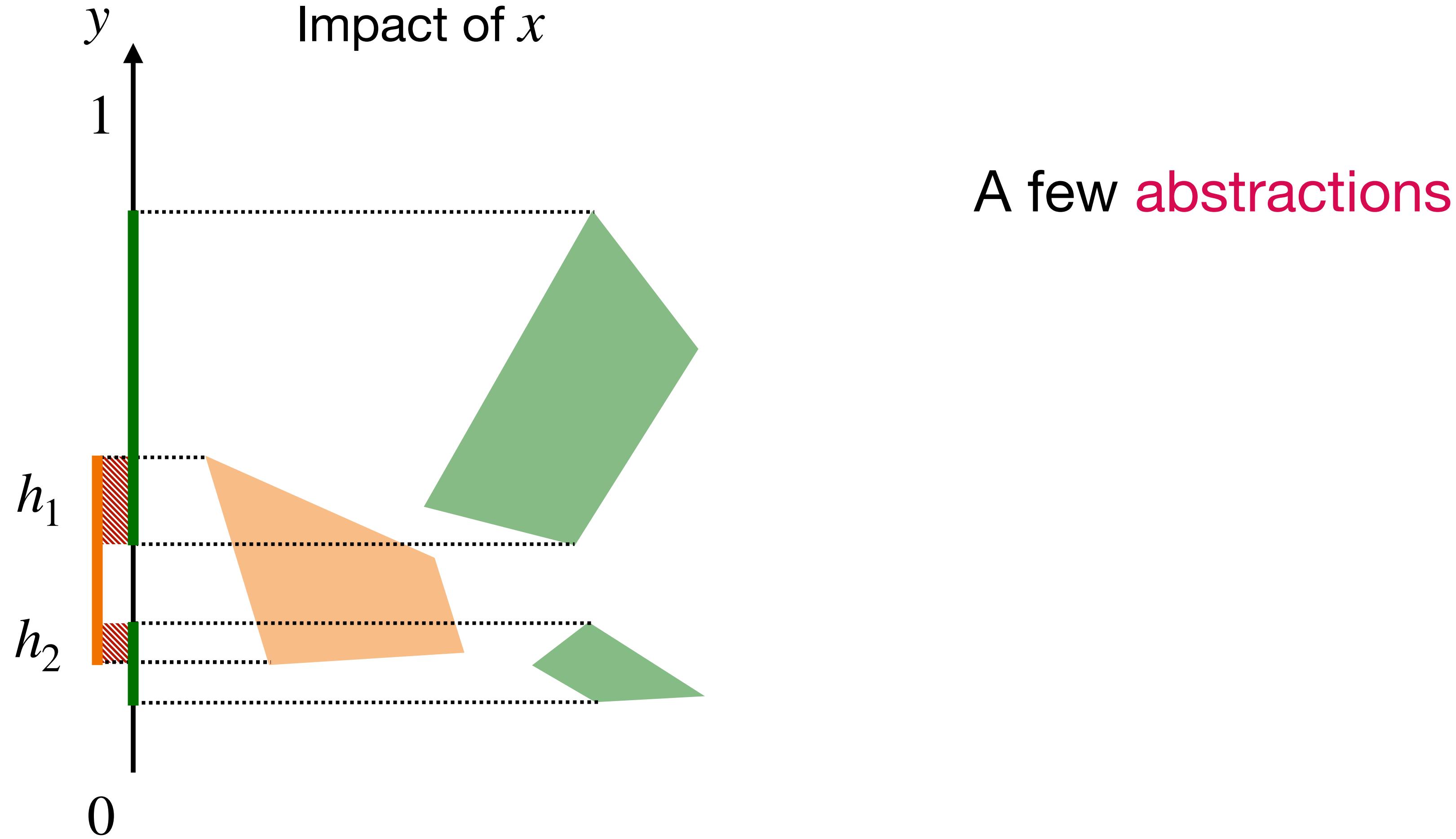


Quantitative Input Feature Usage Analysis

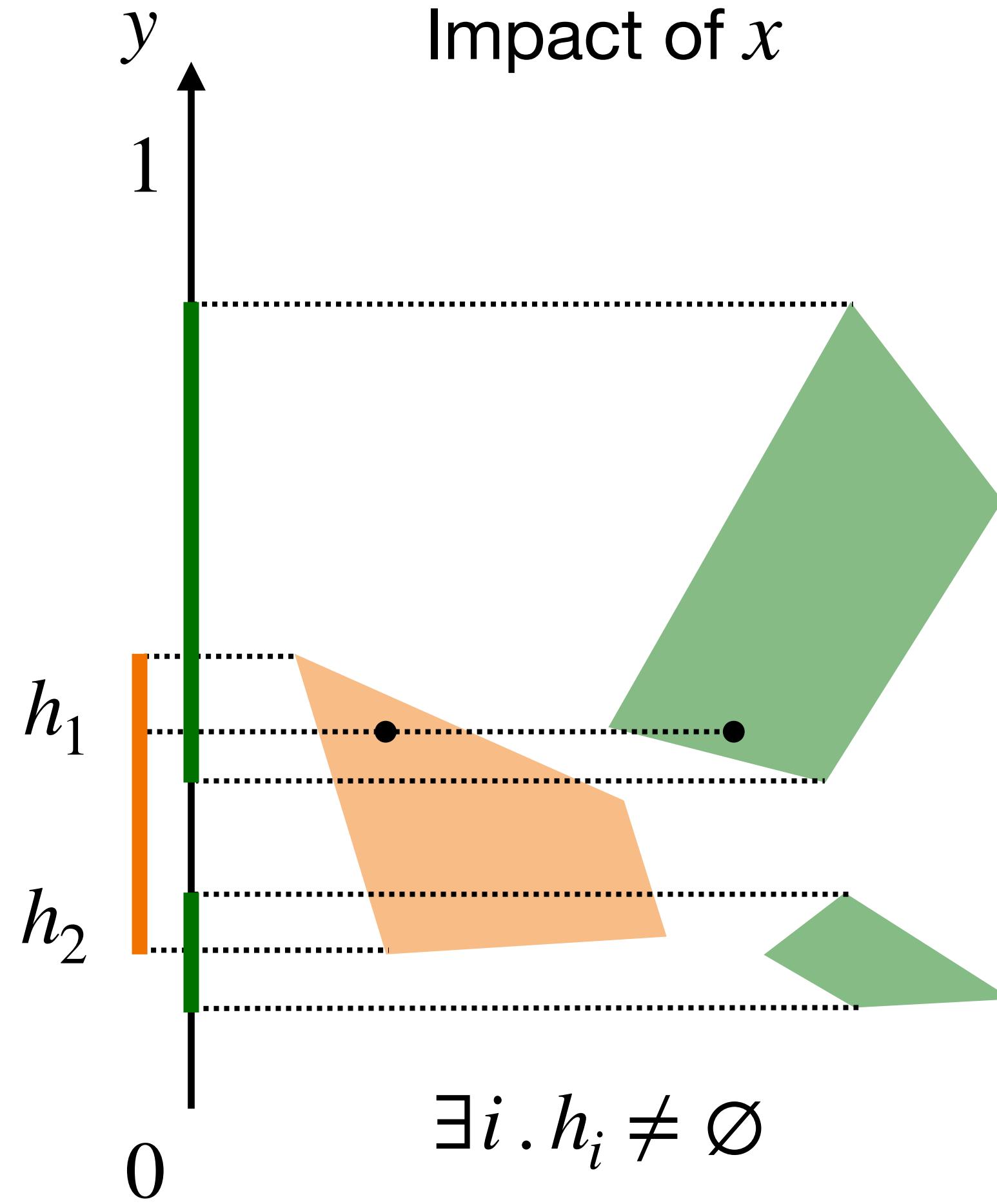
Intersections!



Quantitative Input Feature Usage Analysis



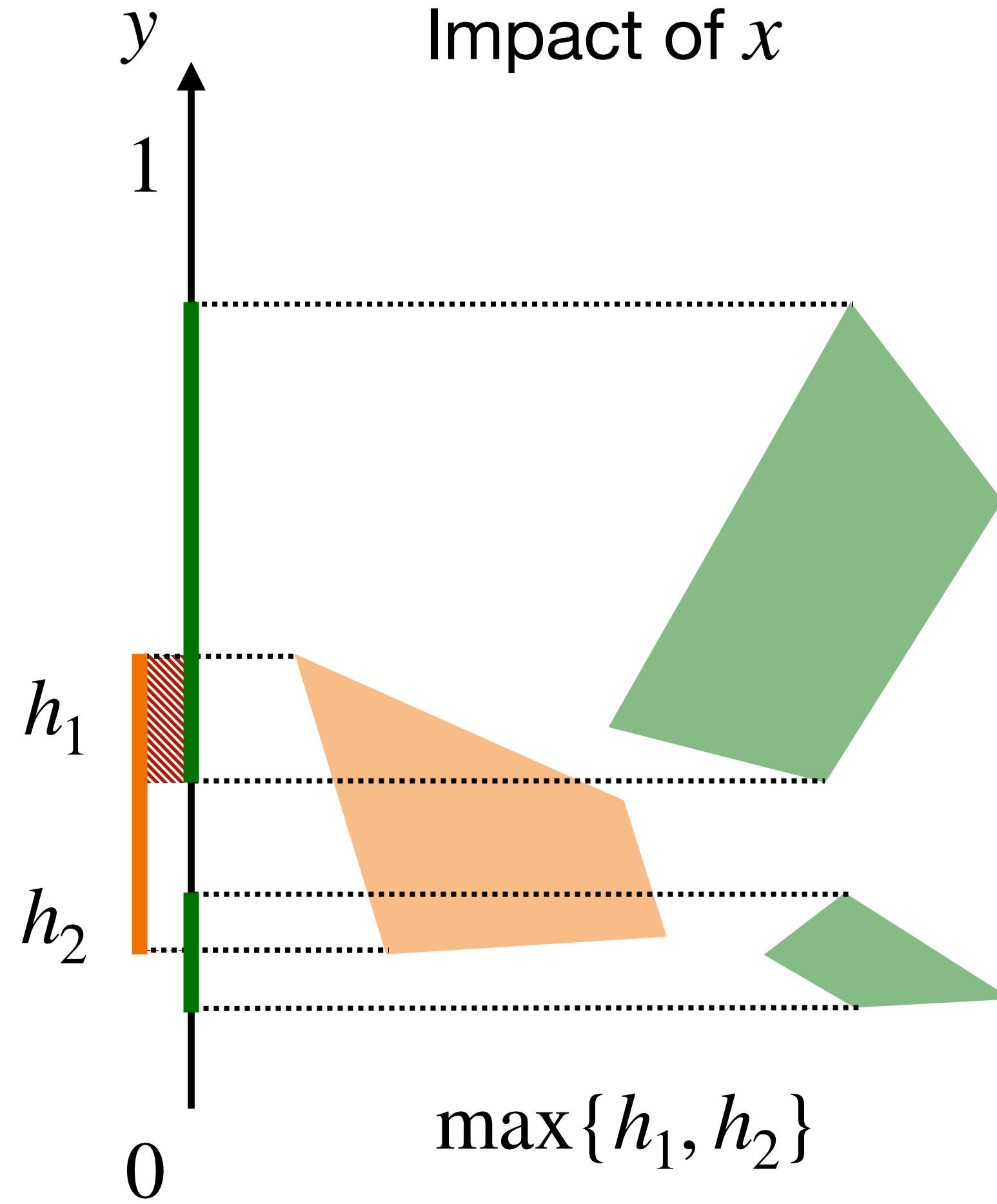
Quantitative Input Feature Usage Analysis



A few **abstractions**:

- **Intersection abstraction**

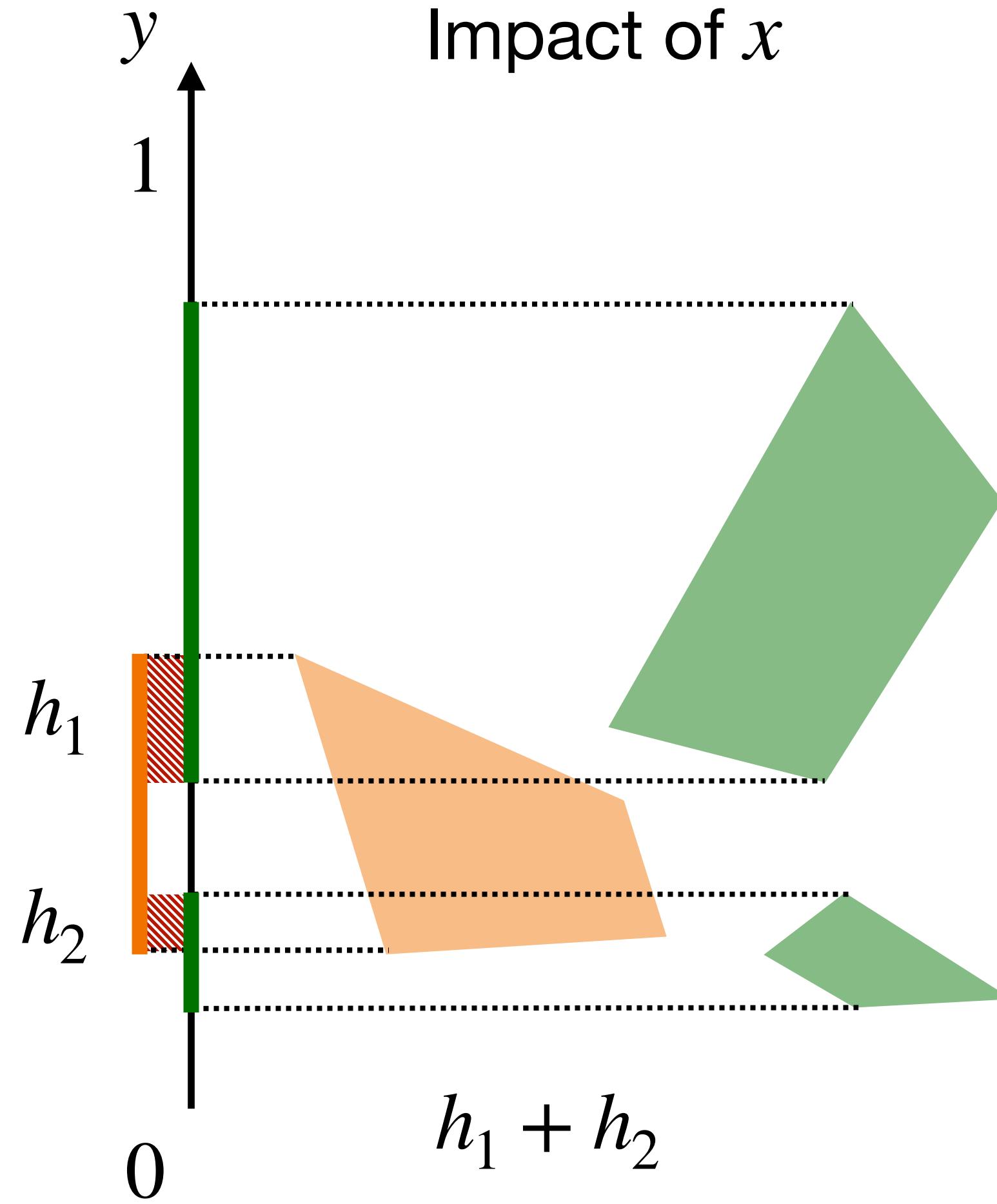
Quantitative Input Feature Usage Analysis



A few abstractions:

- Intersection abstraction
- Maximum Volume abstraction

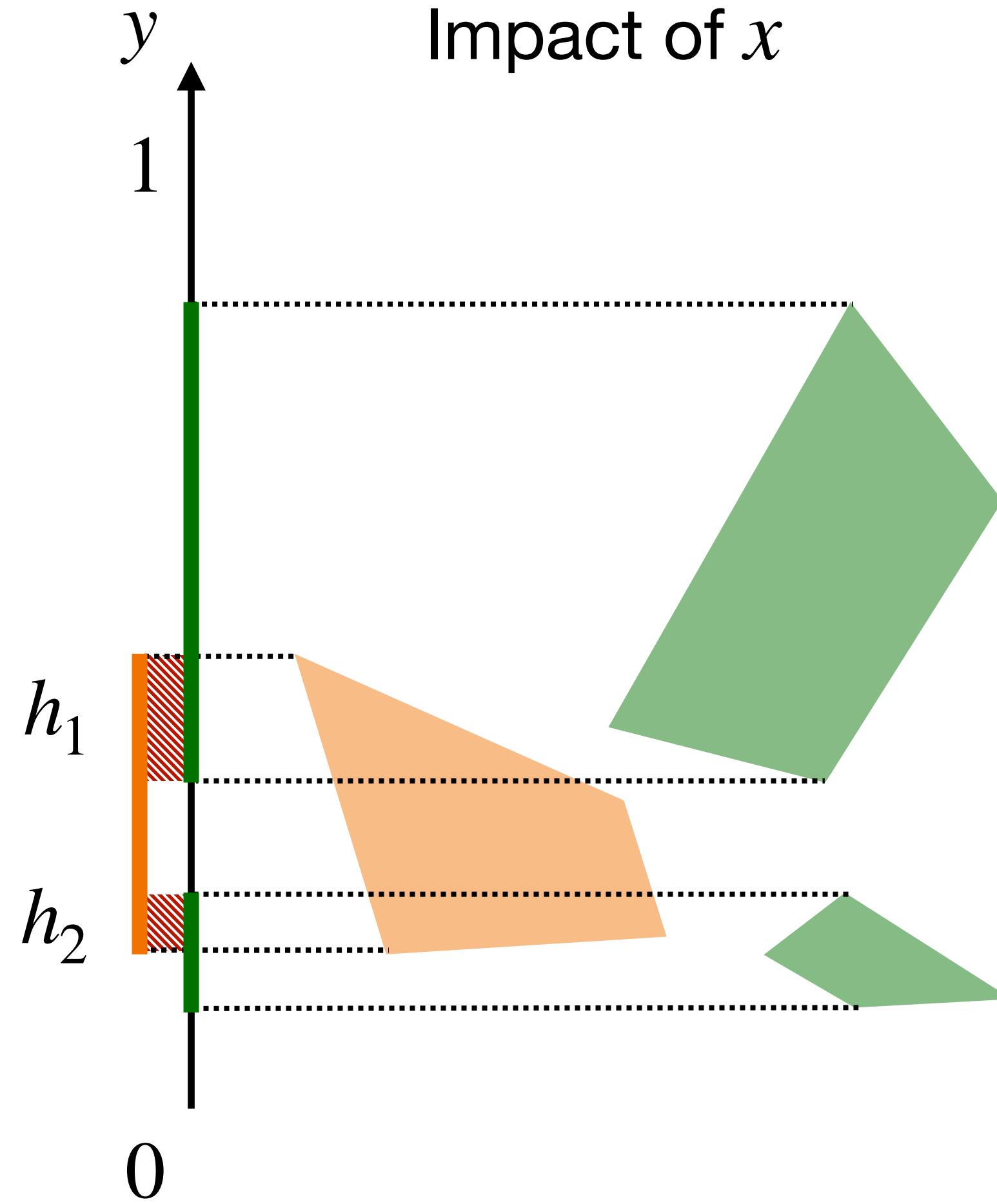
Quantitative Input Feature Usage Analysis



A few **abstractions**:

- Intersection abstraction
- Maximum Volume abstraction
- Total Volume abstraction

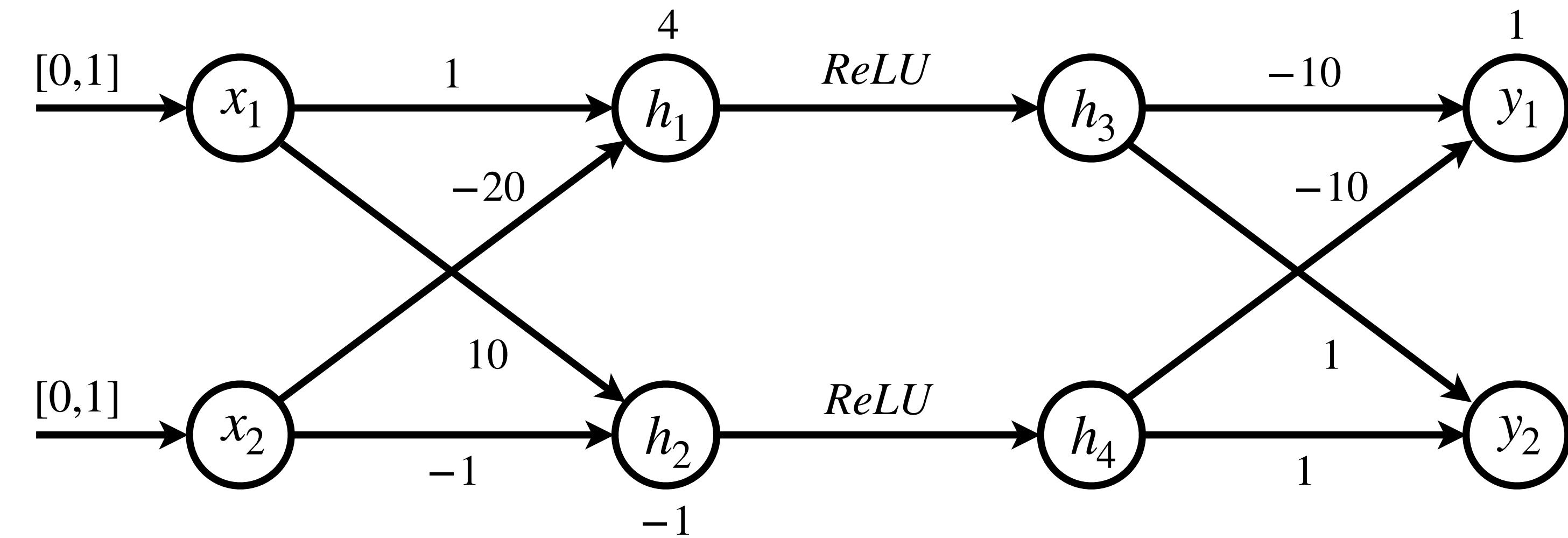
Quantitative Input Feature Usage Analysis



A few **abstractions**:

- **Intersection abstraction**
- **Maximum Volume abstraction**
- **Total Volume abstraction**
- ...

Backward Analysis

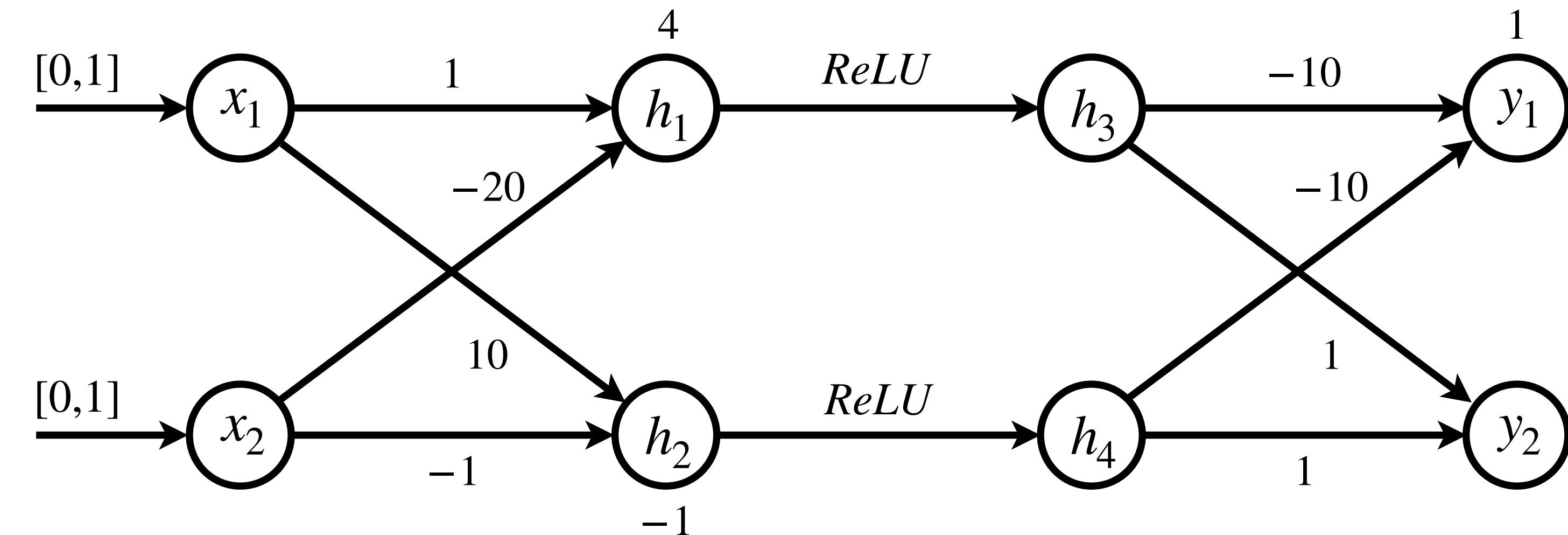


Output
Buckets

$$y_2 \leq y_1$$

$$y_1 \leq y_2$$

Backward Analysis

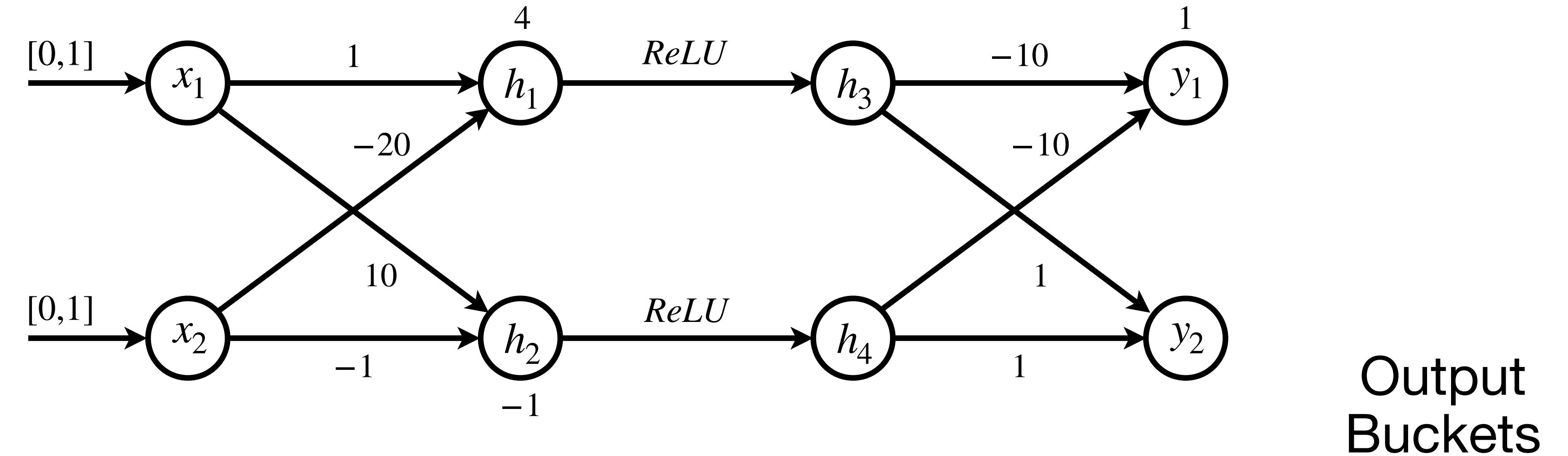


Output
Buckets

$$y_2 \leq y_1 \quad \leftarrow$$

$$y_1 \leq y_2$$

Backward Analysis

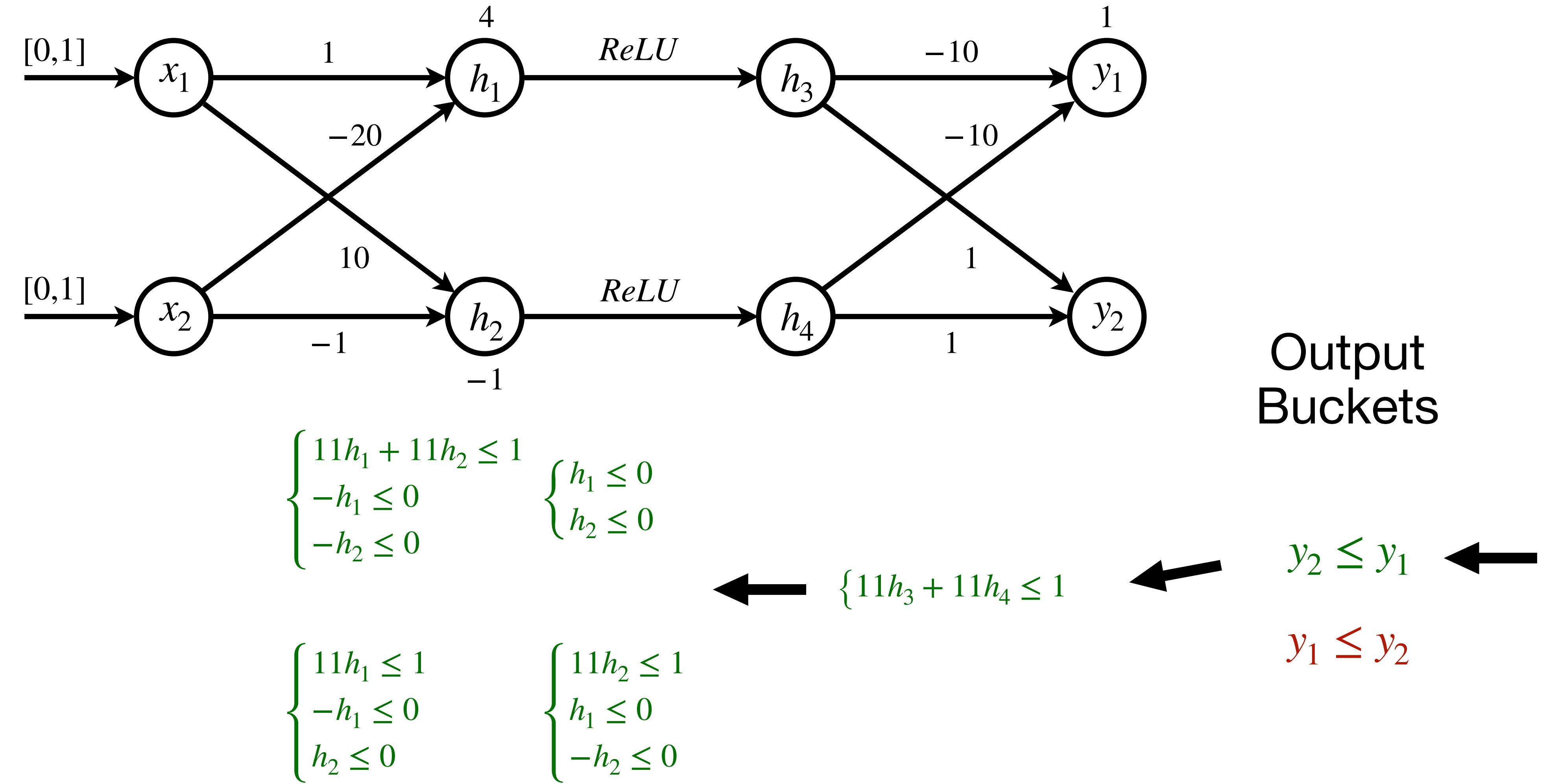


$$\{11h_3 + 11h_4 \leq 1\}$$

$$y_2 \leq y_1$$

$$y_1 \leq y_2$$

Backward Analysis



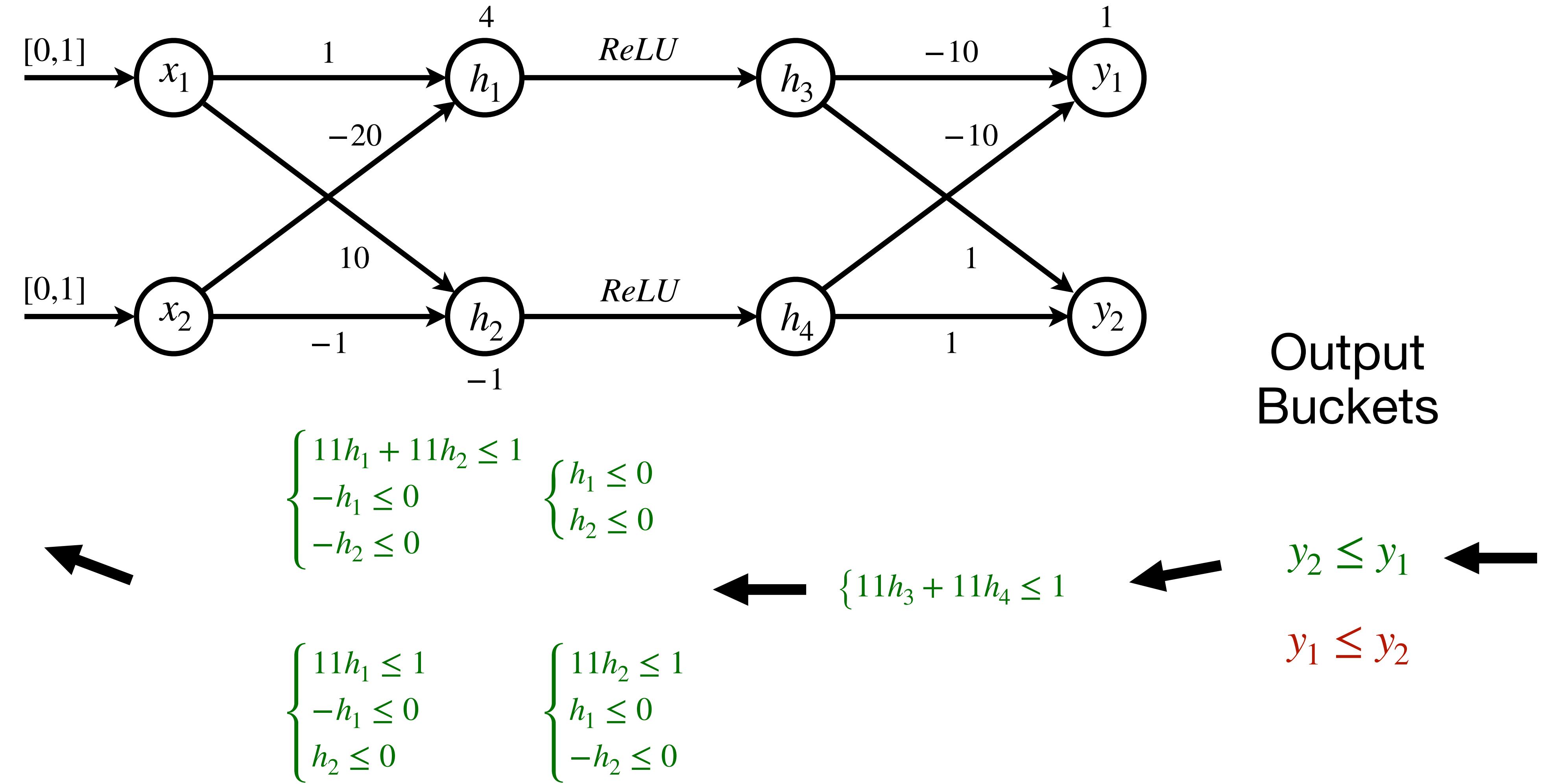
Backward Analysis

$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

$$\begin{cases} 121x_1 - 231x_2 \leq -32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

$$\begin{cases} x_1 - 20x_2 \leq -4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

$$\begin{cases} 110x_1 - 11x_2 \leq 12 \\ x_1 - 20x_2 \leq -4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

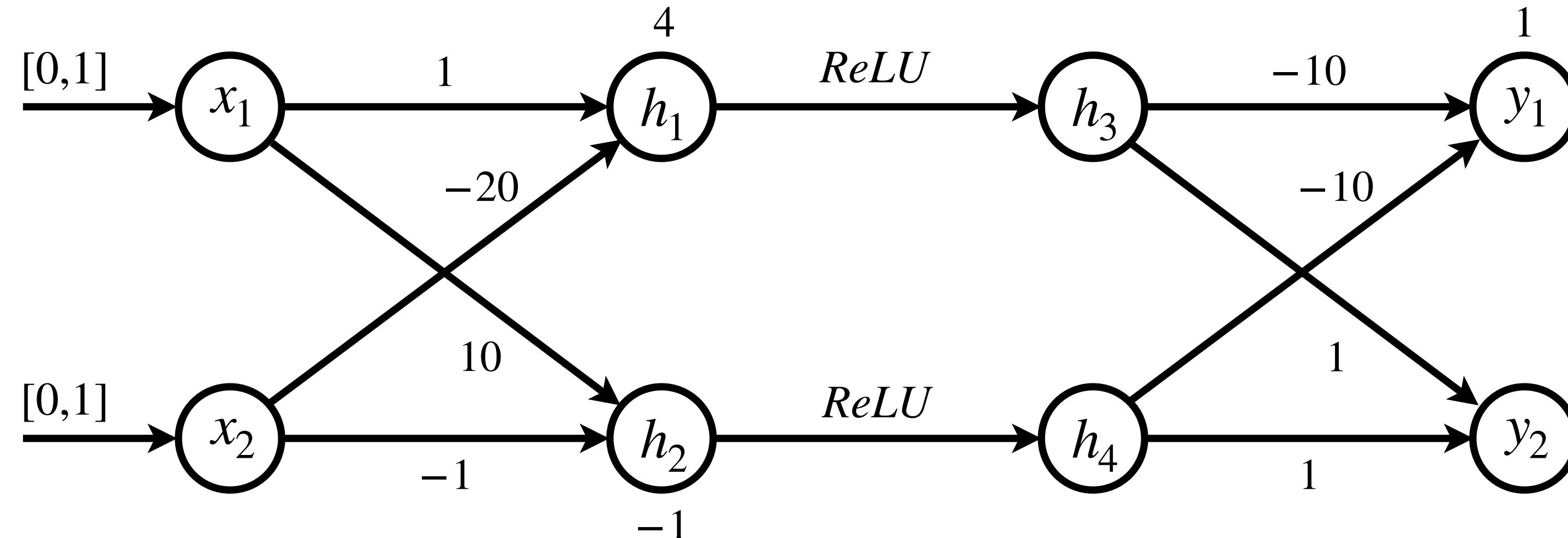


Backward Analysis

$$\begin{cases} -11x_1 + 220x_2 \leq 43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

$$\begin{cases} -121x_1 + 231x_2 \leq 32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

$$\begin{cases} -110x_1 + 11x_2 \leq -12 \\ x_1 - 20x_2 \leq -4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$



$$\begin{cases} -11h_1 - 11h_2 \leq -1 \\ -h_1 \leq 0 \\ -h_2 \leq 0 \end{cases}$$

$$\begin{cases} -11h_1 \leq -1 \\ -h_1 \leq 0 \\ h_2 \leq 0 \end{cases}$$

$$\begin{cases} -11h_2 \leq -1 \\ h_1 \leq 0 \\ -h_2 \leq 0 \end{cases}$$

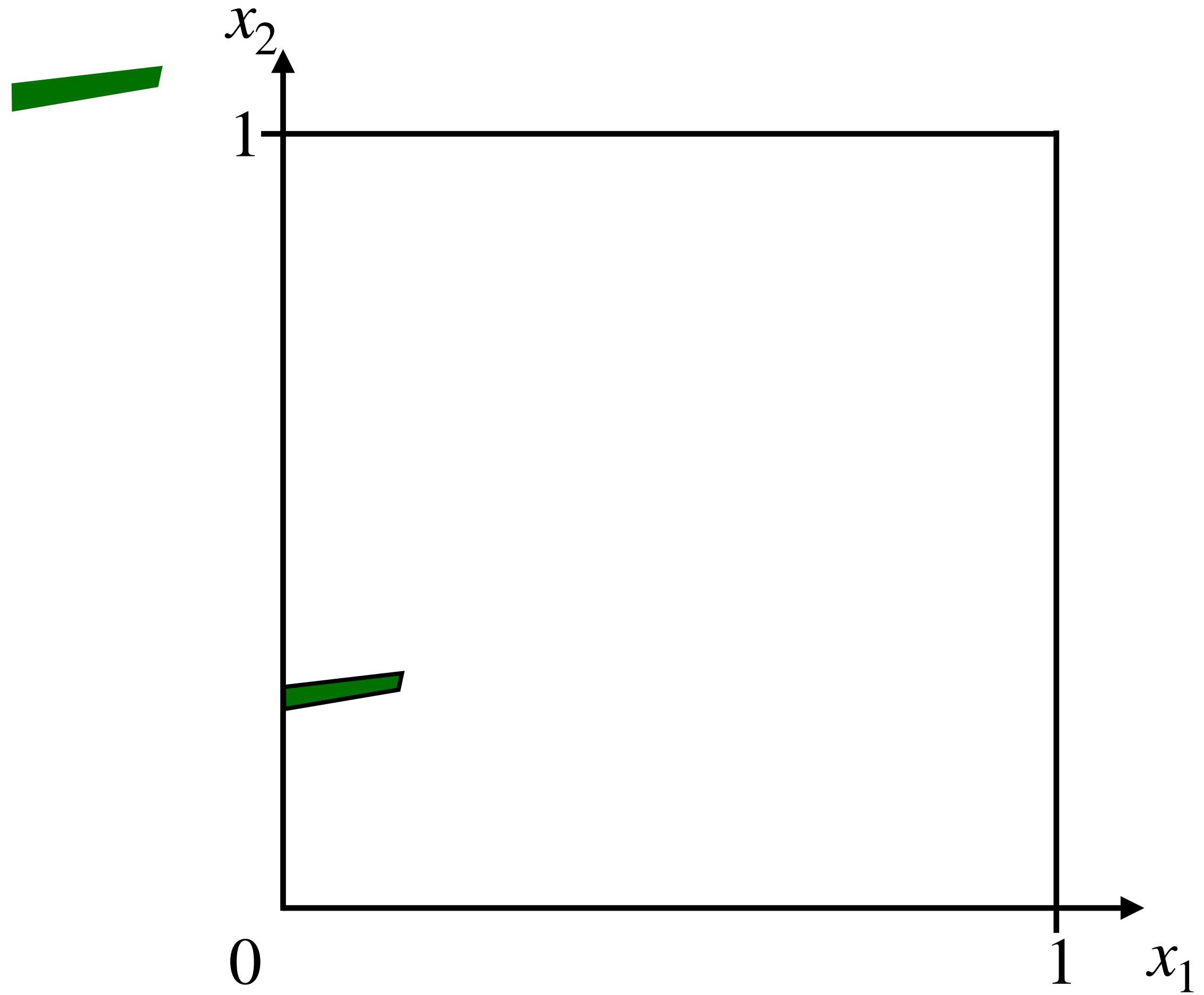
Output
Buckets

$$y_2 \leq y_1$$

$$y_1 \leq y_2$$

Input Space Composition

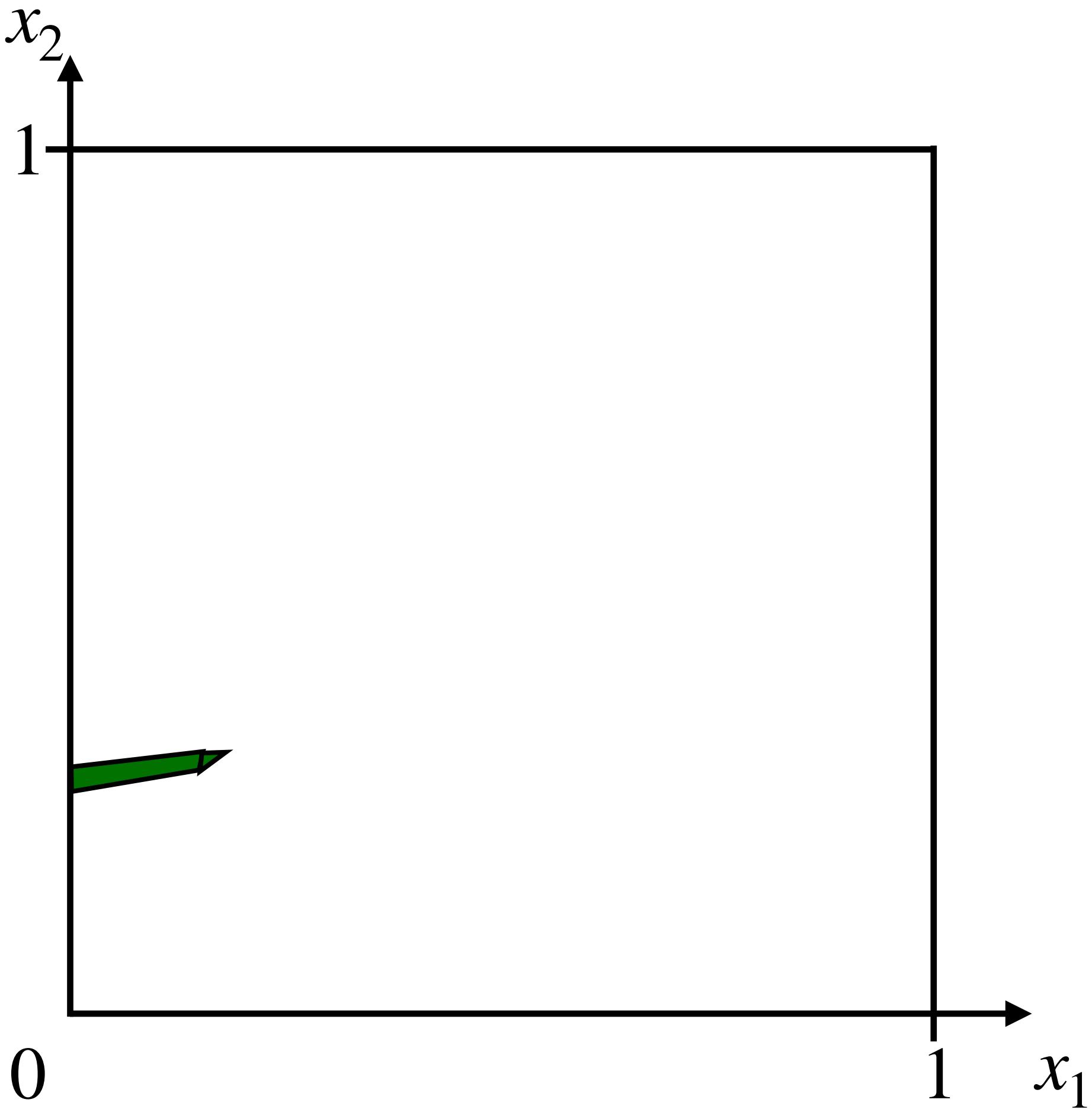
$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$



Input Space Composition

$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

► $\begin{cases} 121x_1 - 231x_2 \leq -32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$



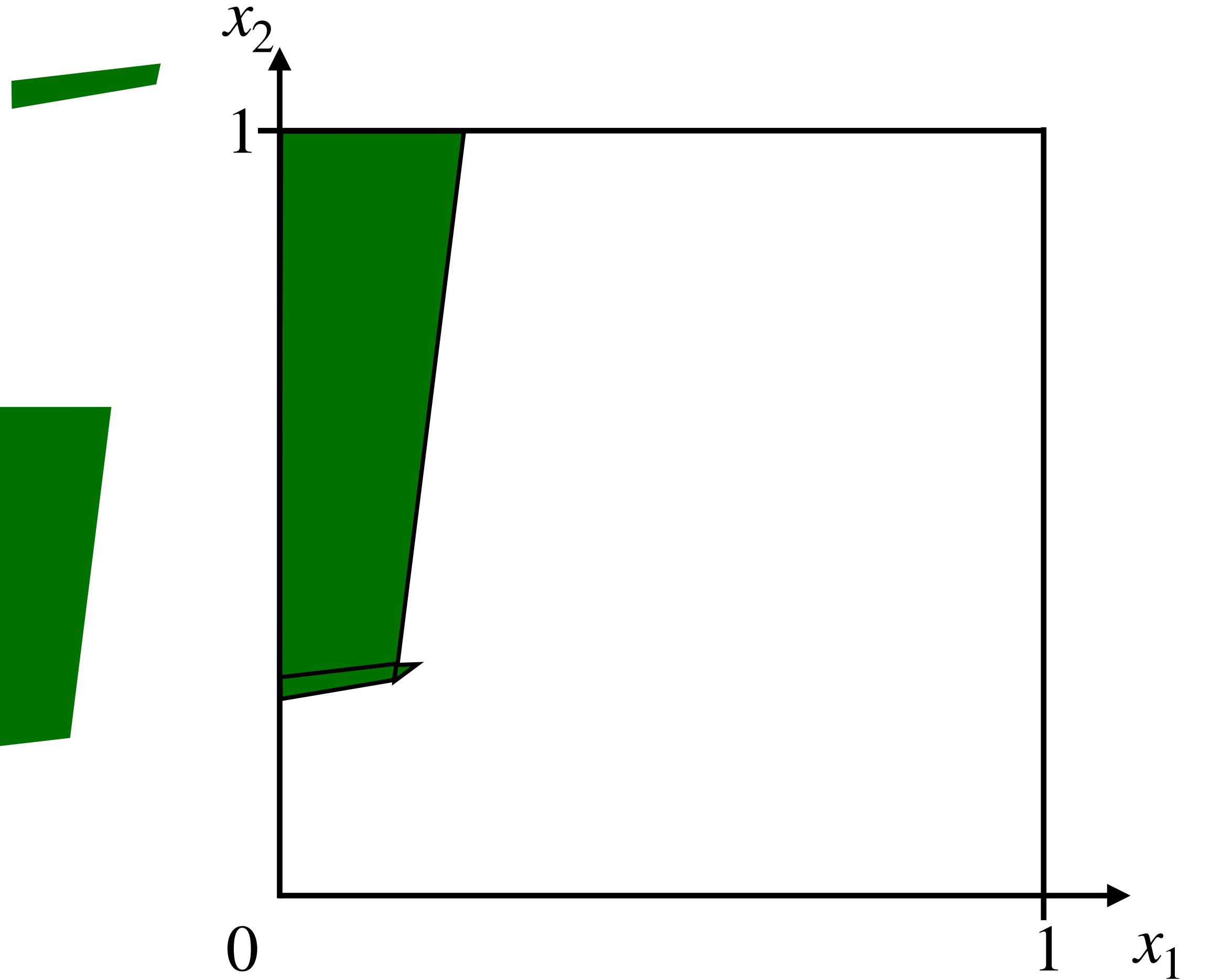
Input Space Composition

$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

►

$$\begin{cases} 121x_1 - 231x_2 \leq -32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

$$\begin{cases} x_1 - 20x_2 \leq -4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$



Input Space Composition

$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

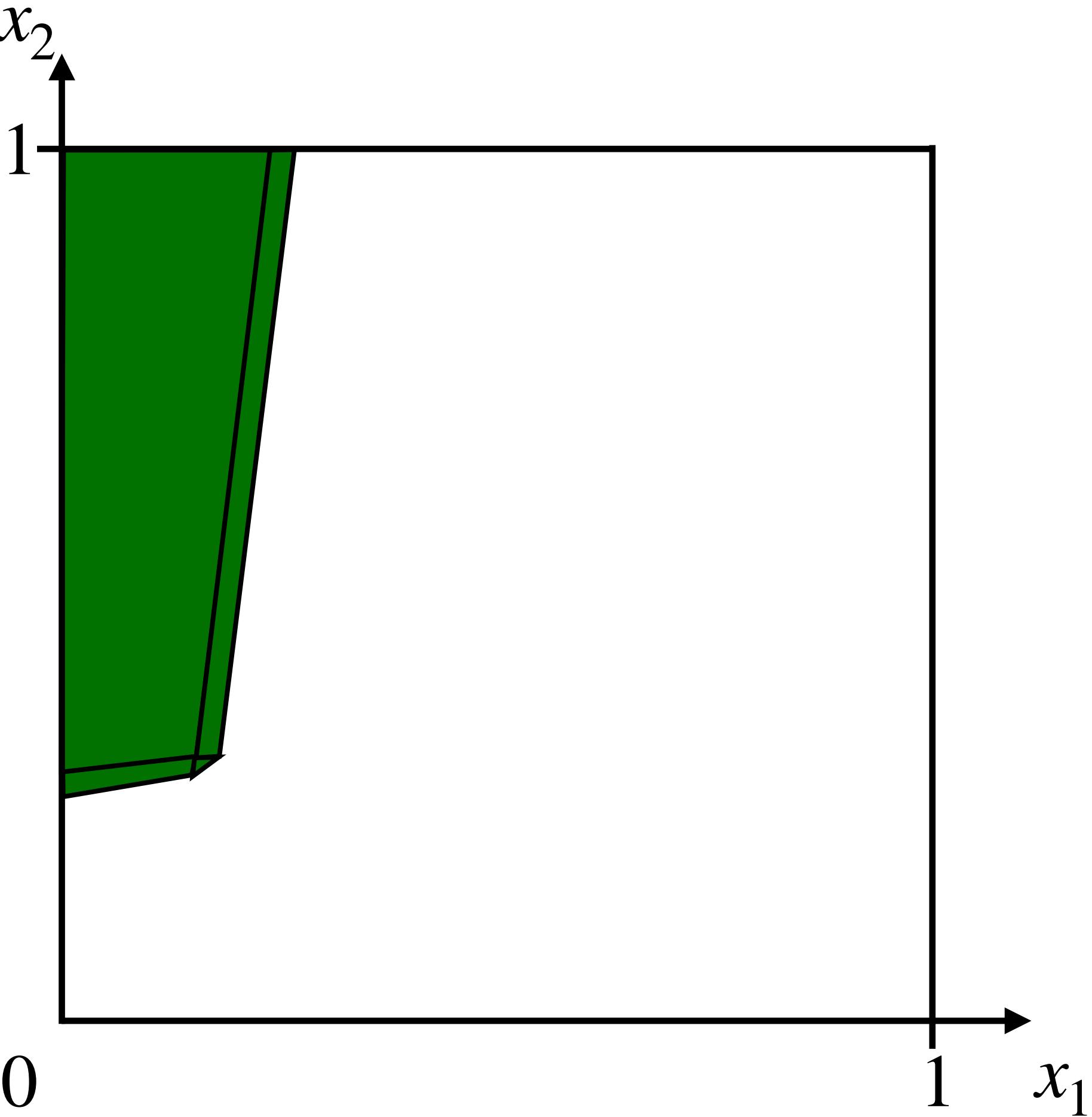
►

$$\begin{cases} 121x_1 - 231x_2 \leq -32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

$$\begin{cases} x_1 - 20x_2 \leq -4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

↙

$$\begin{cases} 110x_1 - 11x_2 \leq 12 \\ x_1 - 20x_2 \leq -4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$



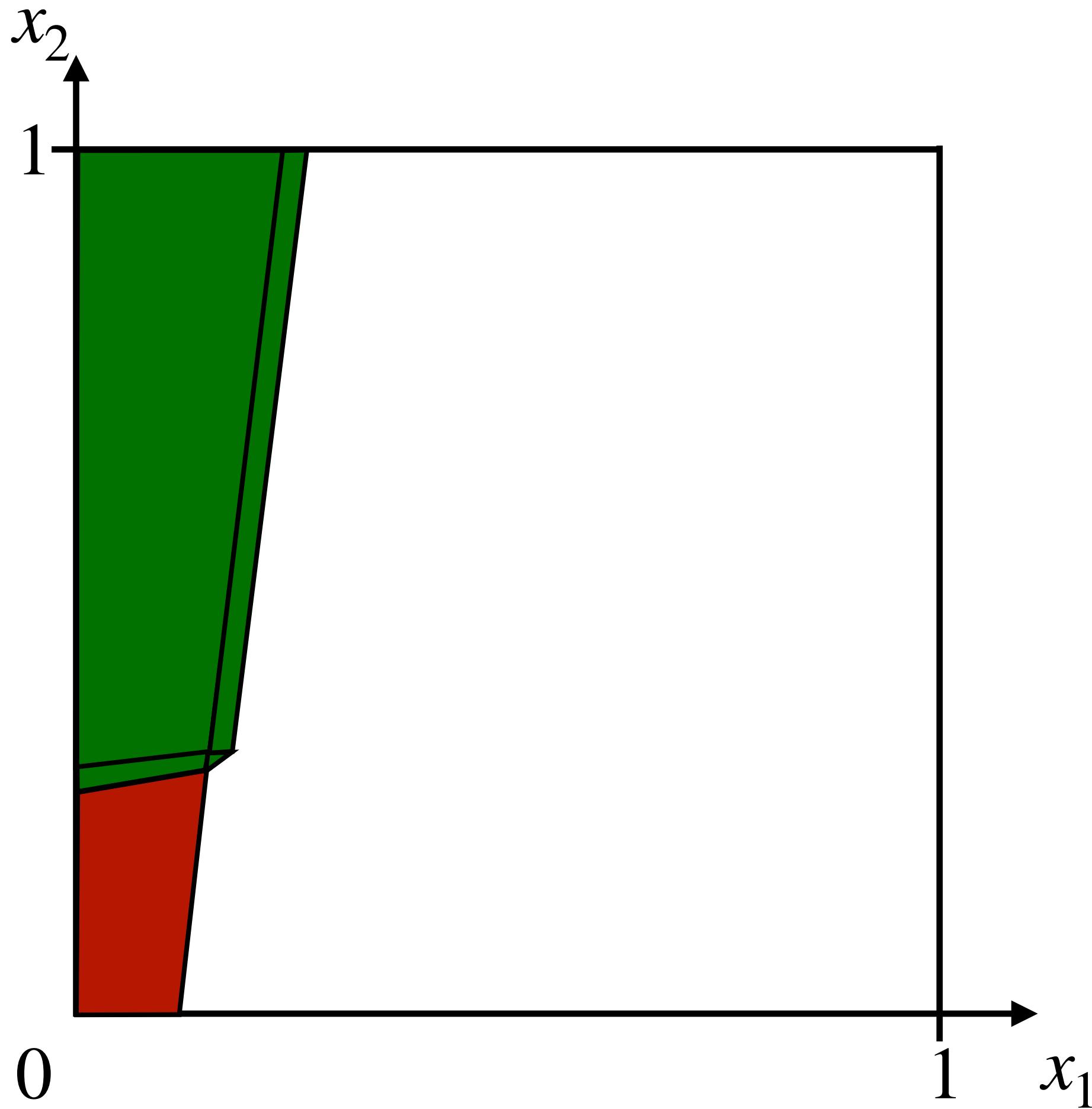
Input Space Composition

$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

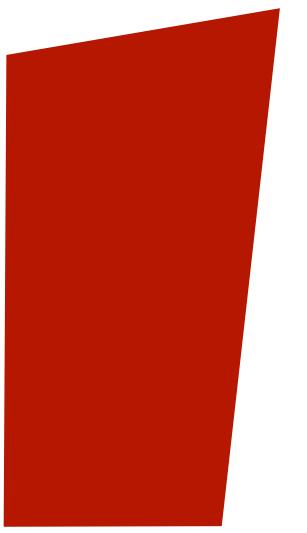
$$\begin{cases} 121x_1 - 231x_2 \leq -32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

$$\begin{cases} x_1 - 20x_2 \leq -4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

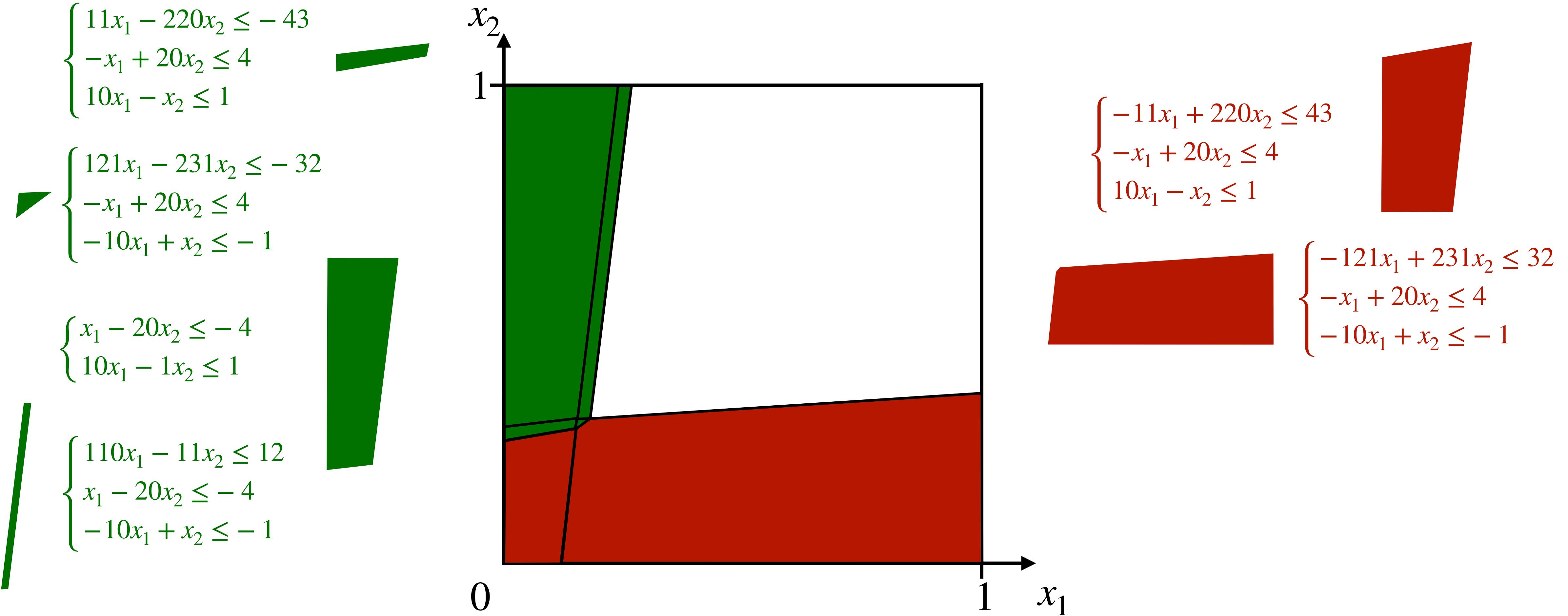
$$\begin{cases} 110x_1 - 11x_2 \leq 12 \\ x_1 - 20x_2 \leq -4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$



$$\begin{cases} -11x_1 + 220x_2 \leq 43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$



Input Space Composition



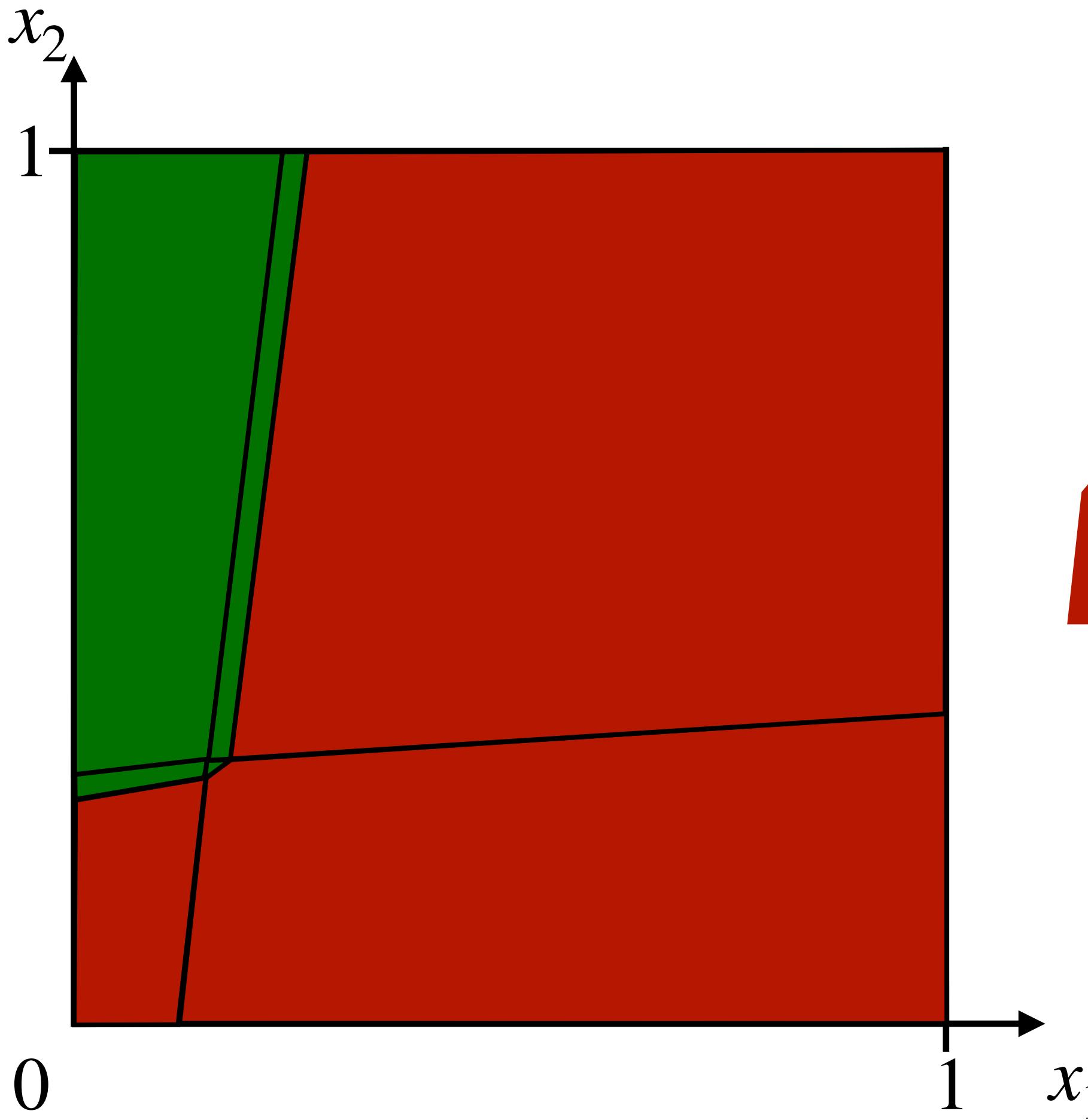
Input Space Composition

$$\begin{cases} 11x_1 - 220x_2 \leq -43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

$$\begin{cases} 121x_1 - 231x_2 \leq -32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

$$\begin{cases} x_1 - 20x_2 \leq -4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

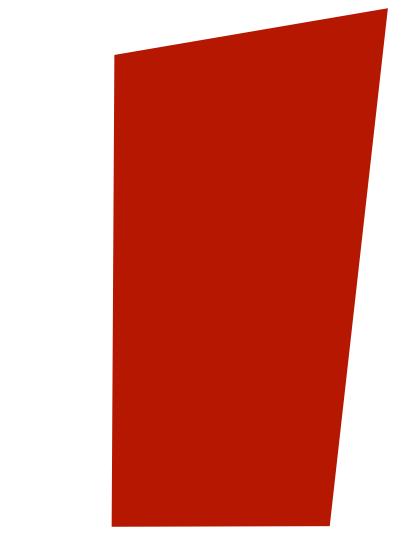
$$\begin{cases} 110x_1 - 11x_2 \leq 12 \\ x_1 - 20x_2 \leq -4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$



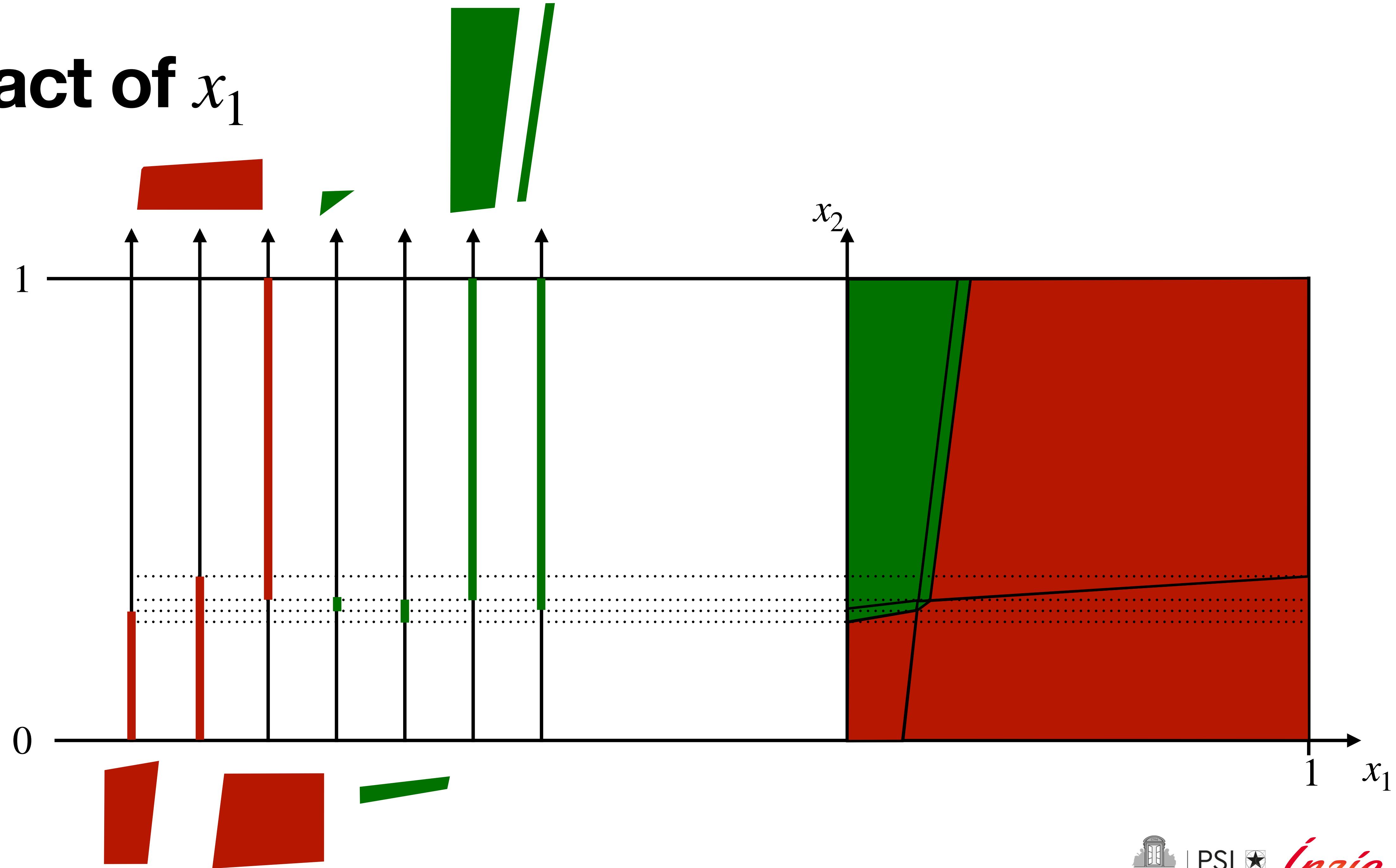
$$\begin{cases} -11x_1 + 220x_2 \leq 43 \\ -x_1 + 20x_2 \leq 4 \\ 10x_1 - x_2 \leq 1 \end{cases}$$

$$\begin{cases} -121x_1 + 231x_2 \leq 32 \\ -x_1 + 20x_2 \leq 4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$

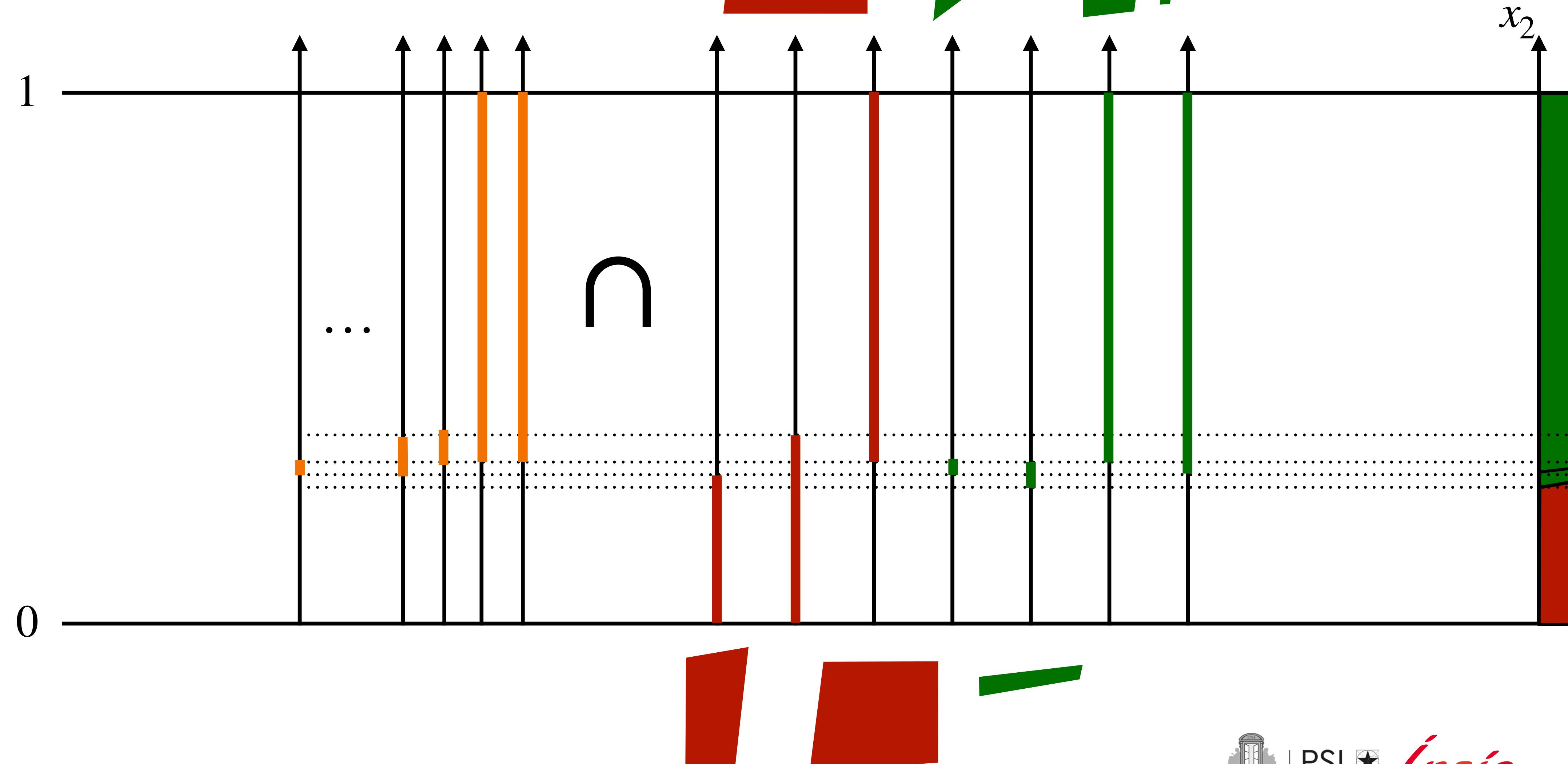
$$\begin{cases} -110x_1 + 11x_2 \leq -12 \\ x_1 - 20x_2 \leq -4 \\ -10x_1 + x_2 \leq -1 \end{cases}$$



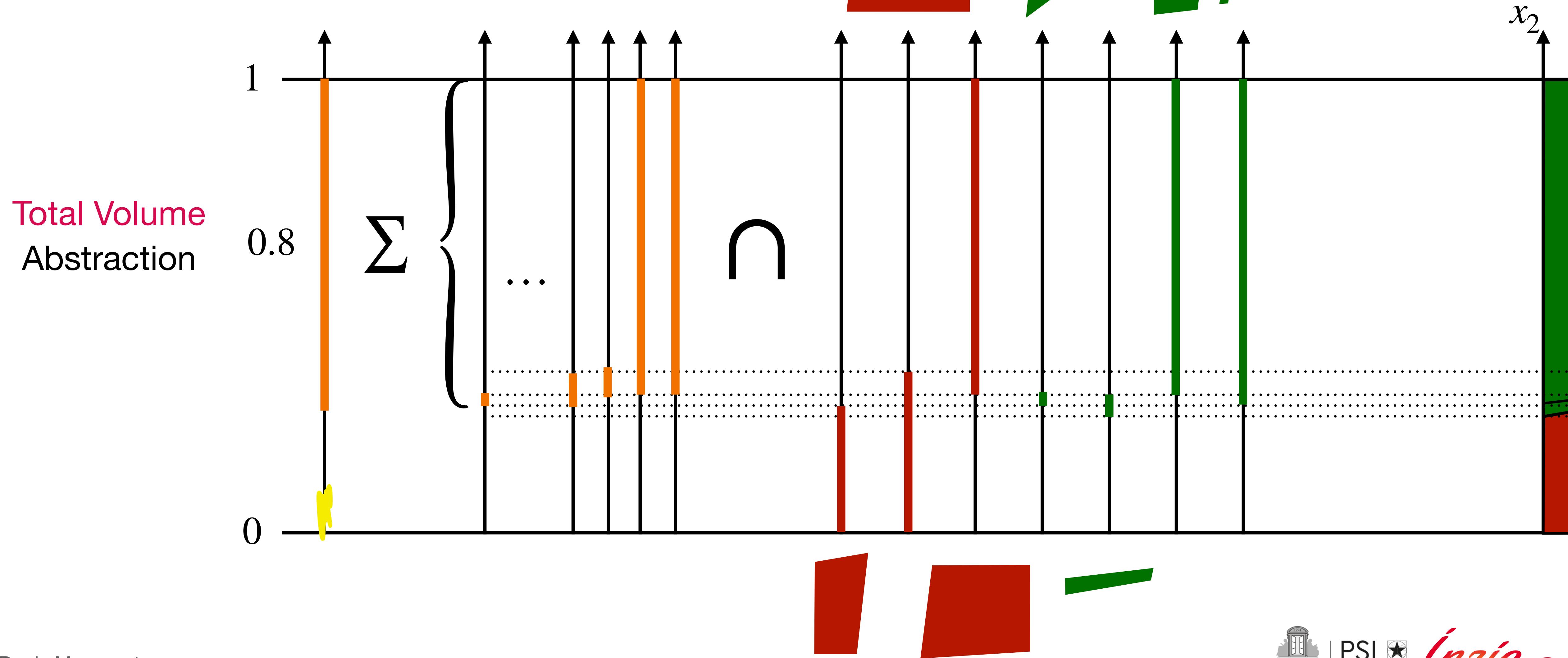
Impact of x_1



Impact of x_1

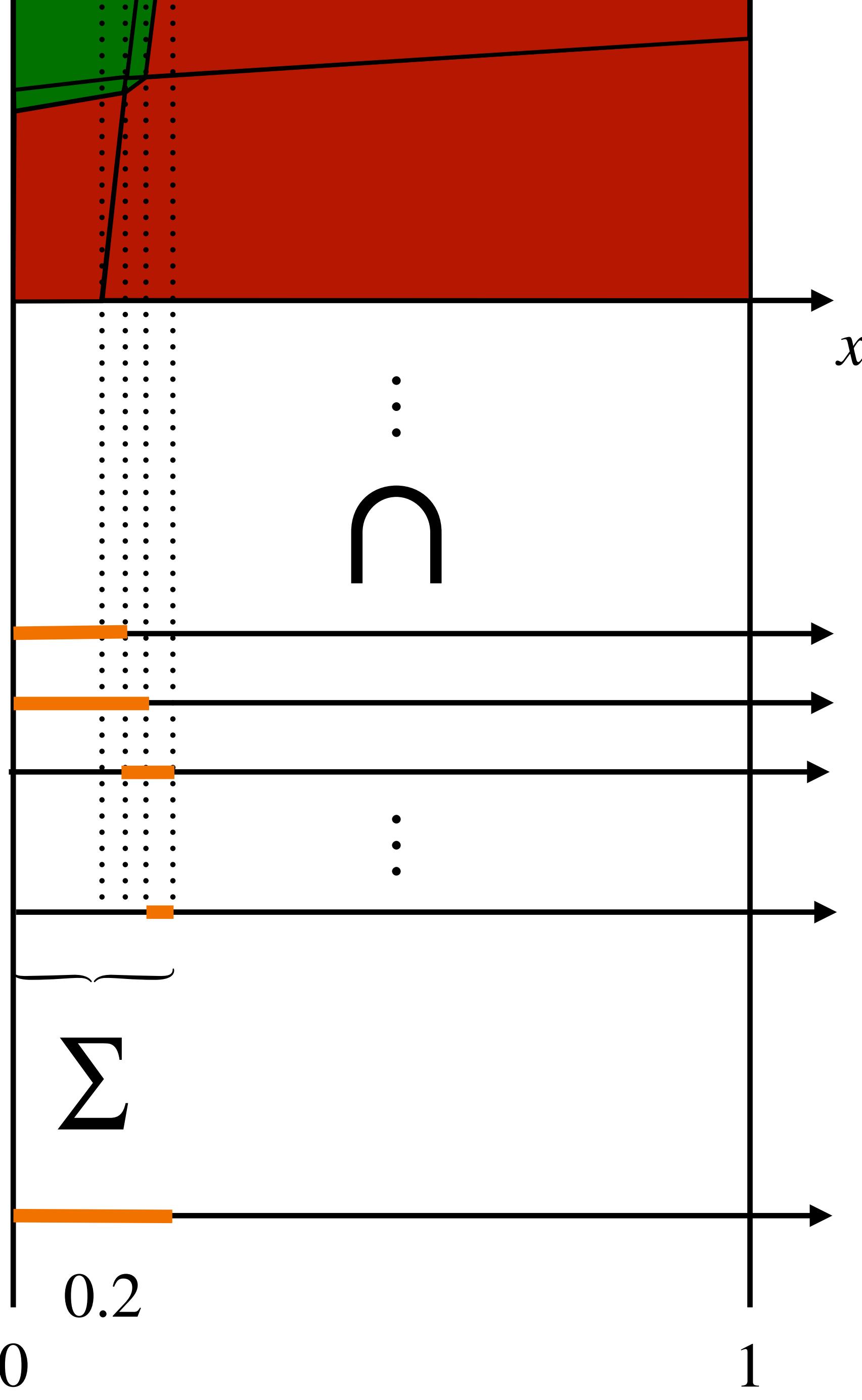


Impact of x_1

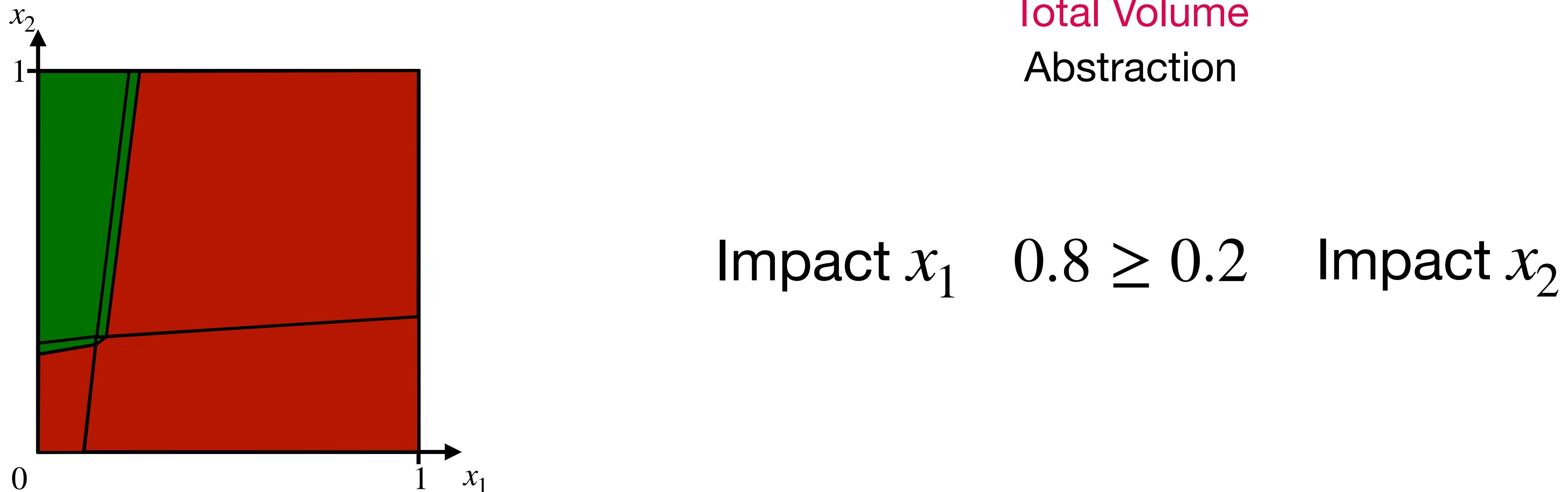


Impact of x_2

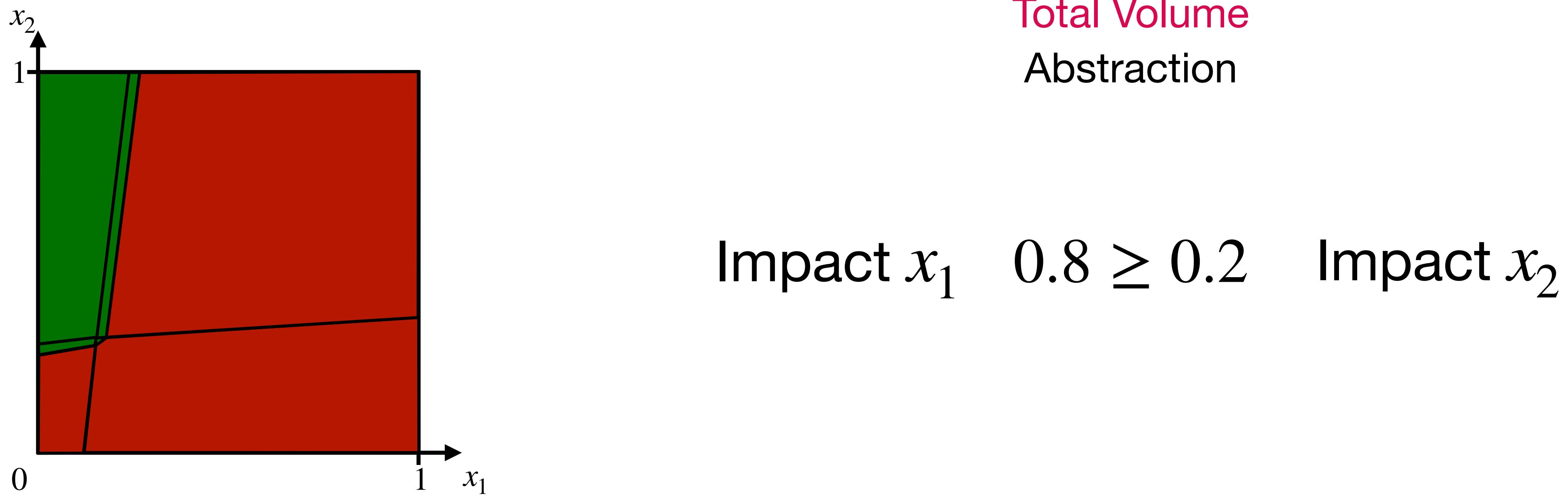
Total Volume
Abstraction



Quantitative Input Feature Usage Analysis

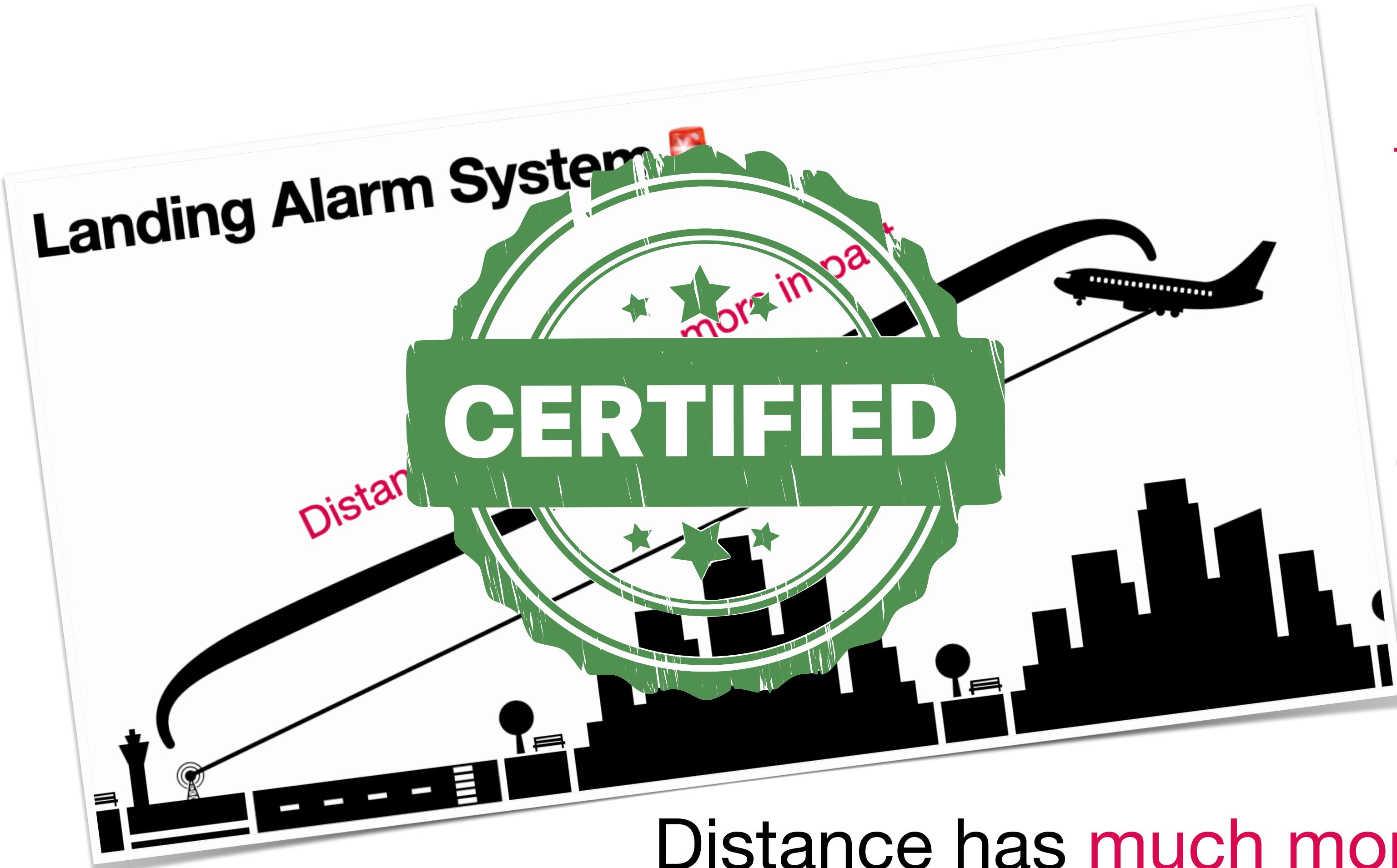


Quantitative Input Feature Usage Analysis



Distance has much more impact than the speed

Quantitative Input Feature Usage Analysis

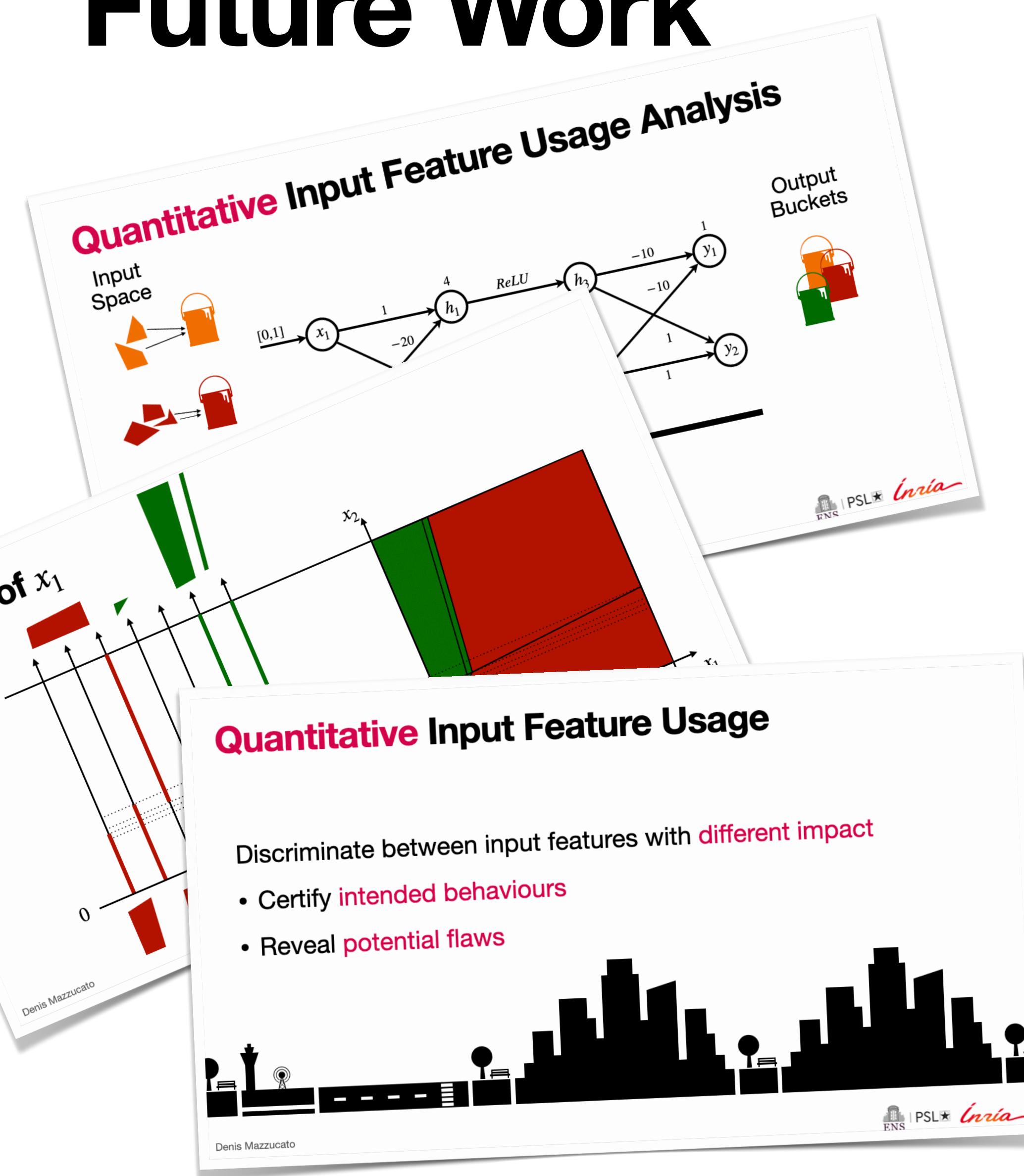


Total Volume
Abstraction

$$0.8 \geq 0.2 \quad \text{Impact } x_2$$

Distance has much more impact than the speed

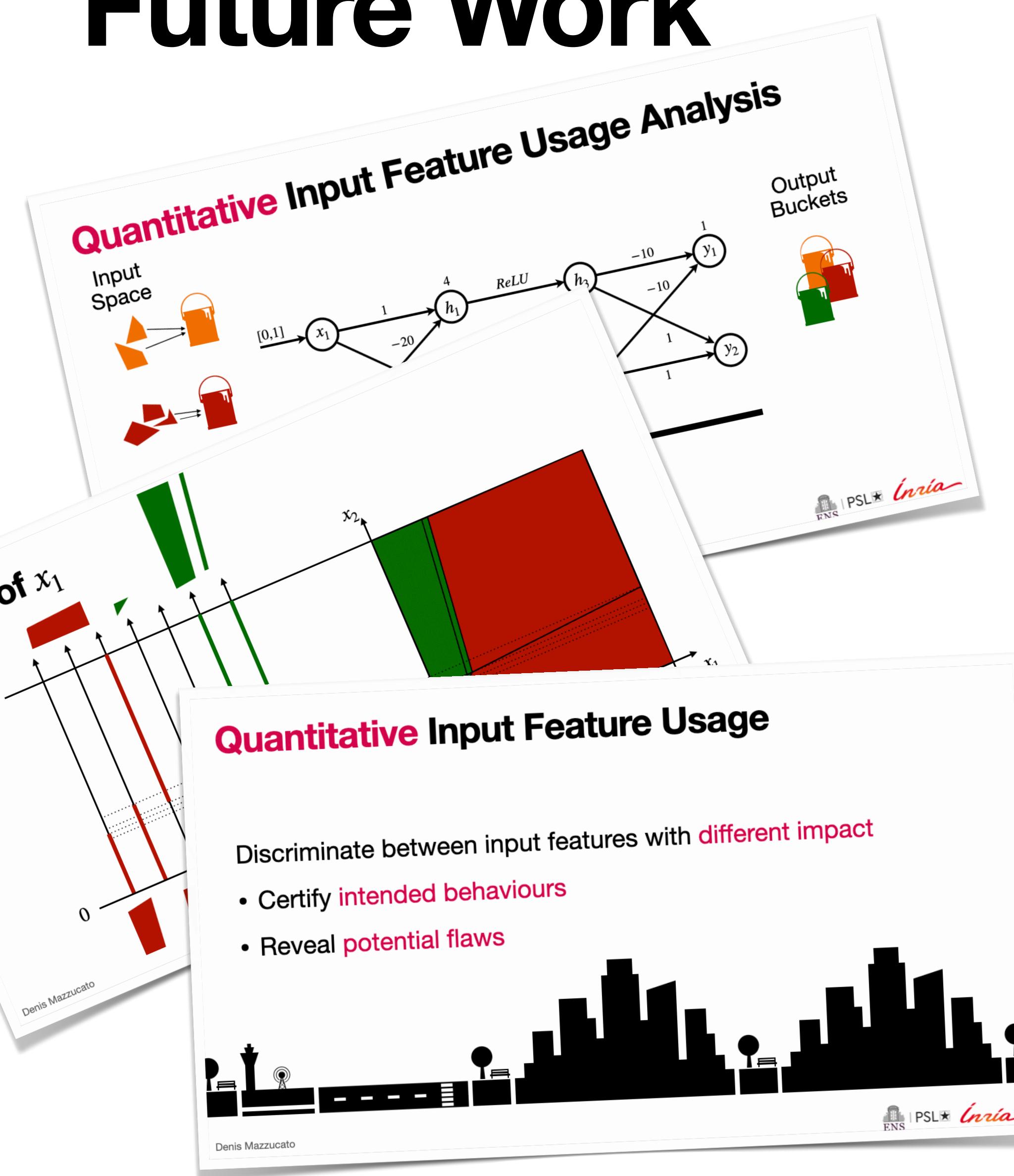
Future Work



Analysis improvements:

- Approximate abstract domains
- Forward pre-analysis

Future Work

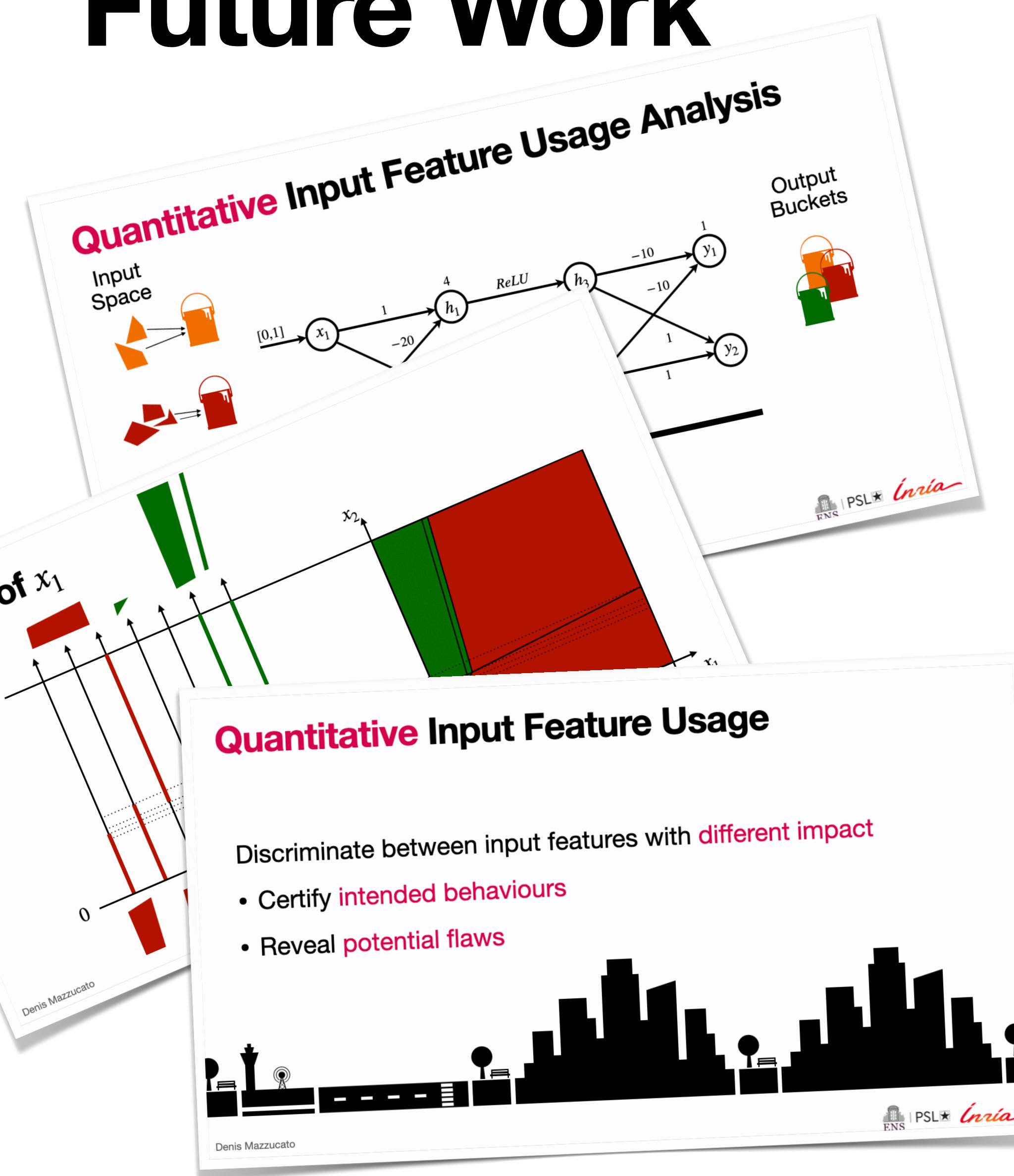


Analysis improvements:

- **Approximate abstract domains**
- **Forward pre-analysis**

What remains next:

Future Work



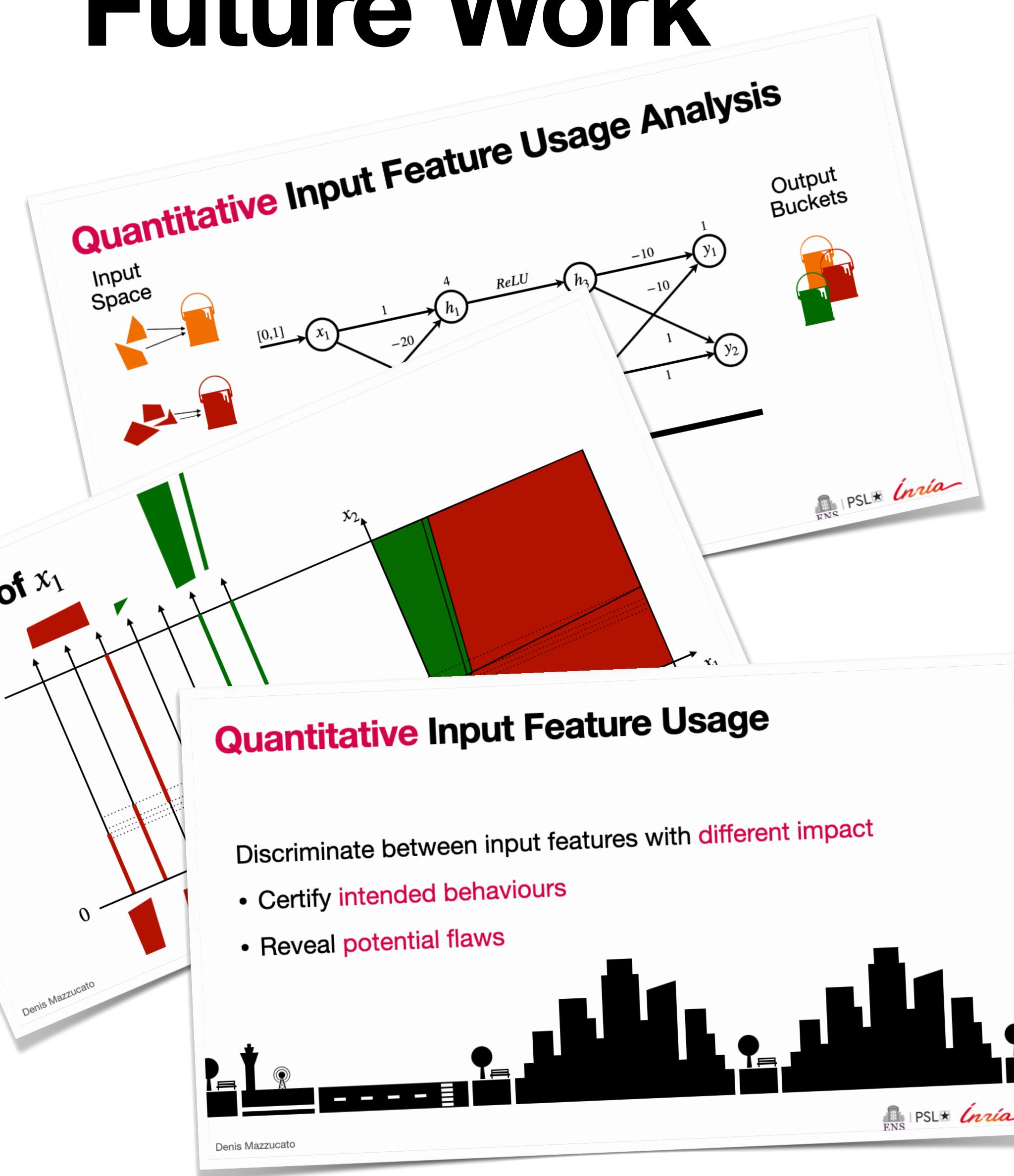
Analysis improvements:

- Approximate abstract domains
- Forward pre-analysis

What remains next:

- To write the paper

Future Work



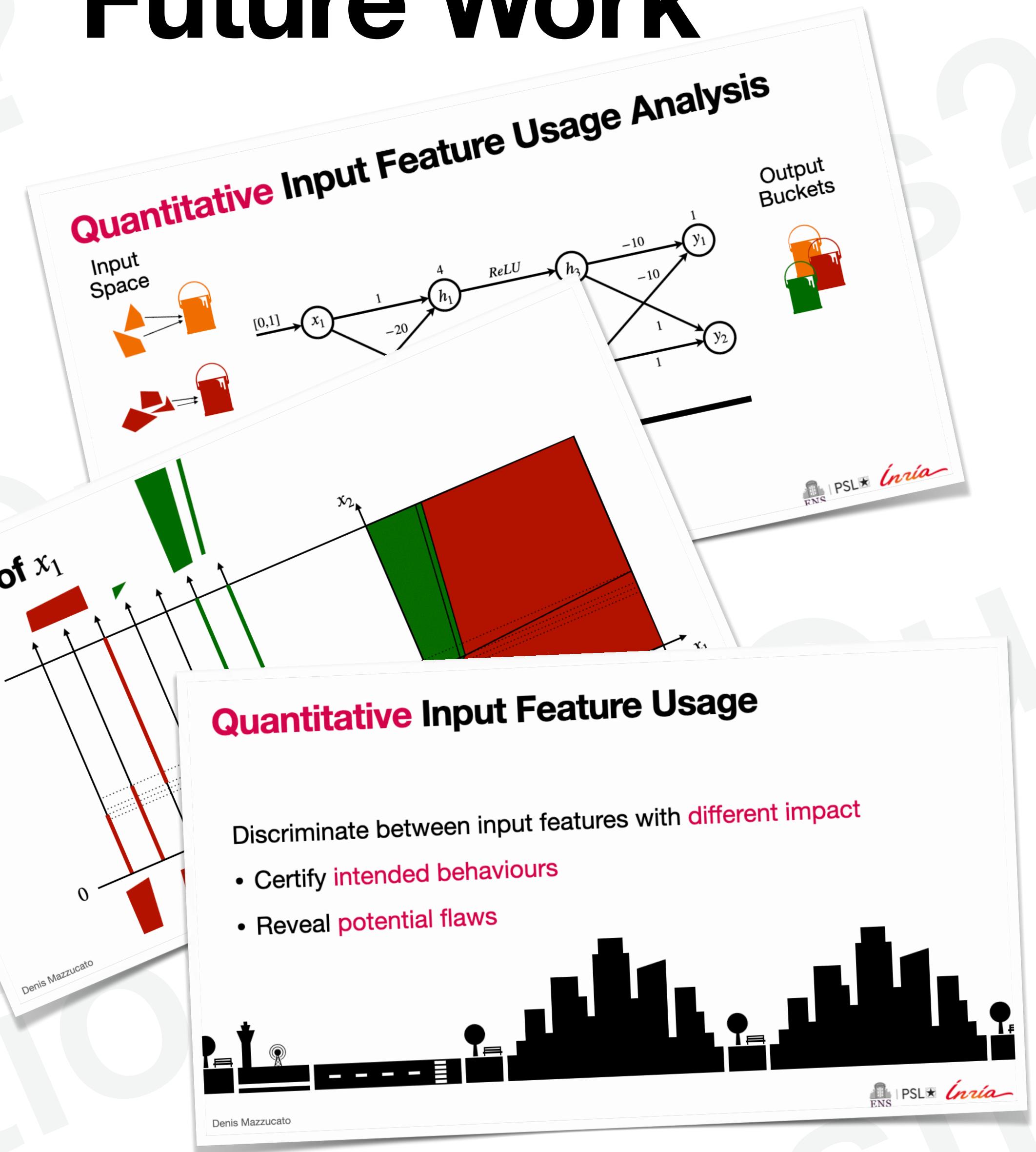
Analysis improvements:

- **Approximate abstract domains**
- **Forward pre-analysis**

What remains next:

- Comparison with **Feature Importance Metrics**
- **Identify unbiased features** in fairness studies
- **Non-trivial control flow**

Future Work



Analysis improvements:

- Approximate abstract domains
- Forward pre-analysis

What remains next:

- Comparison with Feature Importance Metrics
- Identify unbiased features in fairness studies
- Non-trivial control flow