Calculate the response of ideal J2 plasticity material model by given strain tensors. Material – structural steel with yield strength of 500MPa.

Input parameters: $E = 210\text{GPa}$; $\nu = 0{,}3$; $\sigma_y = 500\text{MPa}$; $\underline{\varepsilon}^{el} = \underline{0}$; $\underline{\varepsilon}^{pl} = \underline{0}$; $\Delta\varepsilon = \begin{pmatrix} 0{,}01 & 0 & 0 \\ 0 & -0{,}004 & 0 \\ 0 & 0 & -0{,}004 \end{pmatrix}$; $\underline{\sigma}$–? ; $\Delta\underline{\varepsilon}^{el}$– ? ; $\Delta\underline{\varepsilon}^{pl}$– ?; $\Delta\bar{\varepsilon}^{pl}$– ? .

Remark: tensor component numbering and Voigt notation for an increment strain tensor $\Delta\underline{\varepsilon} = \begin{pmatrix} \Delta\varepsilon_{11} & \Delta\varepsilon_{12} & \Delta\varepsilon_{13} \\ \Delta\varepsilon_{21} & \Delta\varepsilon_{22} & \Delta\varepsilon_{23} \\ \Delta\varepsilon_{31} & \Delta\varepsilon_{32} & \Delta\varepsilon_{33} \end{pmatrix} = \begin{pmatrix} \Delta\varepsilon_1 & \Delta\varepsilon_4 & \Delta\varepsilon_6 \\ \Delta\varepsilon_4 & \Delta\varepsilon_2 & \Delta\varepsilon_5 \\ \Delta\varepsilon_6 & \Delta\varepsilon_5 & \Delta\varepsilon_3 \end{pmatrix} \rightarrow \begin{pmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\varepsilon_3 \\ 2\Delta\varepsilon_4 \\ 2\Delta\varepsilon_5 \\ 2\Delta\varepsilon_6 \end{pmatrix}$

Ideal J2 Plasticity in 9 steps – calculation scheme

Step 1: Calculation of trial stress at the begin of increment $\underline{\boldsymbol{\sigma}}^{tr}$

$\underline{\sigma}^{tr} = \underline{\underline{C}}^{el}(\underline{\varepsilon}^{el} + \Delta\underline{\varepsilon})$; $\underline{\tilde{\varepsilon}} = \underline{\varepsilon}^{el} + \Delta\underline{\varepsilon}$

$$\underline{\sigma}^{tr} = \begin{pmatrix} \sigma_1^{tr} \\ \sigma_2^{tr} \\ \sigma_3^{tr} \\ \sigma_4^{tr} \\ \sigma_5^{tr} \\ \sigma_6^{tr} \end{pmatrix} = \begin{pmatrix} \lambda+2\mu & 0 & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix} \begin{pmatrix} \tilde{\varepsilon}_1 \\ \tilde{\varepsilon}_2 \\ \tilde{\varepsilon}_3 \\ \tilde{\varepsilon}_4 + \tilde{\varepsilon}_4 \\ \tilde{\varepsilon}_5 + \tilde{\varepsilon}_5 \\ \tilde{\varepsilon}_6 + \tilde{\varepsilon}_6 \end{pmatrix} = \begin{pmatrix} (\lambda+2\mu)\tilde{\varepsilon}_1 + \lambda\tilde{\varepsilon}_2 + \lambda\tilde{\varepsilon}_3 \\ \lambda\tilde{\varepsilon}_1 + (\lambda+2\mu)\tilde{\varepsilon}_2 + \lambda\tilde{\varepsilon}_3 \\ \lambda\tilde{\varepsilon}_1 + \lambda\tilde{\varepsilon}_2 + (\lambda+2\mu)\tilde{\varepsilon}_3 \\ 2\mu\tilde{\varepsilon}_4 \\ 2\mu\tilde{\varepsilon}_5 \\ 2\mu\tilde{\varepsilon}_6 \end{pmatrix}$$

Remark: $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$; $\mu = \frac{E}{2(1+\nu)}$; $\underline{\tilde{\varepsilon}}$ – trial strain tensor.

Step 2: Calculation of $\underline{\mathbf{S}}^{tr}$ deviatoric part of trial stress tensor

$\underline{S}^{tr} = \underline{\sigma}^{tr} - \sigma_m^{tr}\underline{I}$; $\underline{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | $\sigma_m^{tr} = \frac{1}{3}(\sigma_1^{tr} + \sigma_2^{tr} + \sigma_3^{tr})$ | $S_1^{tr} = \sigma_1^{tr} - \sigma_m^{tr}$; $S_2^{tr} = \sigma_2^{tr} - \sigma_m^{tr}$; $S_3^{tr} = \sigma_3^{tr} - \sigma_m^{tr}$; $S_4^{tr} = \sigma_4^{tr}$; $S_5^{tr} = \sigma_5^{tr}$; $S_6^{tr} = \sigma_6^{tr}$

Step 3: Calculation of $\boldsymbol{\sigma_v}$

$\sigma_v = \sqrt{\frac{3}{2}}\|\underline{S}^{tr}\|$; $\sigma_v = \sqrt{\frac{3}{2}}\sqrt{(S_1^{tr})^2 + (S_2^{tr})^2 + (S_3^{tr})^2 + 2(S_4^{tr})^2 + 2(S_5^{tr})^2 + 2(S_6^{tr})^2}$ | Remark: $\|\underline{A}\| = \sqrt{A_{ij}.A_{ij}} = \sqrt{\sum_{ij} A_{ij}{}^2}$

Step 4: Comparison of $\boldsymbol{\sigma_v}$ and $\boldsymbol{\sigma_y}$

IF $\sigma_v < \sigma_y$ THEN elastic step, go to step 5;
IF $\sigma_v > \sigma_y$ THEN plastic step, go to step 6.

Step 5: Elastic step

$\underline{\sigma} = \underline{\sigma}^{tr}$; $\underline{\varepsilon}^{el} = \underline{\tilde{\varepsilon}}$; $\Delta\underline{\varepsilon}^{pl} = \underline{0}$; exit the calculation (skip steps 6-9).

Step 6: Calculation of flow direction $\underline{\mathbf{n}}$

$\underline{n} = \frac{\underline{S}^{tr}}{\|\underline{S}^{tr}\|}$ | $n_1 = \frac{S_1^{tr}}{\|\underline{S}^{tr}\|}$; $n_2 = \frac{S_2^{tr}}{\|\underline{S}^{tr}\|}$; $n_3 = \frac{S_3^{tr}}{\|\underline{S}^{tr}\|}$; $n_4 = \frac{S_4^{tr}}{\|\underline{S}^{tr}\|}$; $n_5 = \frac{S_5^{tr}}{\|\underline{S}^{tr}\|}$; $n_6 = \frac{S_6^{tr}}{\|\underline{S}^{tr}\|}$ | Remark: $\|\underline{S}^{tr}\|$ is known from **Step 3**

Step 7: Calculation of plastic strain multiplier $\boldsymbol{\gamma}$

$\gamma = \frac{\|\underline{S}^{tr}\| - \sqrt{\frac{2}{3}}\sigma_y}{2\mu}$; $\Delta\bar{\varepsilon}^{pl} = \sqrt{\frac{2}{3}}\gamma$;

Remark: $\begin{cases} \Delta\underline{\varepsilon}^{pl} = \underline{n}\gamma \\ \Delta\bar{\varepsilon}^{pl} = \sqrt{\frac{2}{3}}\gamma \end{cases}$

Step 8: Calculation of $\underline{\boldsymbol{\varepsilon}}^{el}$ (plastic corrector $\underline{\mathbf{n}}\boldsymbol{\gamma}$)

$\underline{\varepsilon}^{el} = \underline{\tilde{\varepsilon}} - \underline{n}\gamma$ | $\varepsilon_1^{el} = \tilde{\varepsilon}_1 - n_1\gamma$; $\varepsilon_2^{el} = \tilde{\varepsilon}_2 - n_2\gamma$; $\varepsilon_3^{el} = \tilde{\varepsilon}_3 - n_3\gamma$; $\varepsilon_4^{el} = \tilde{\varepsilon}_4 - n_4\gamma$; $\varepsilon_5^{el} = \tilde{\varepsilon}_5 - n_5\gamma$; $\varepsilon_6^{el} = \tilde{\varepsilon}_6 - n_6\gamma$

Step 9: Calculation of updated stress $\underline{\boldsymbol{\sigma}}$ after plastic correction

$$\underline{\sigma} = \underline{\underline{C}}^{el}(\underline{\tilde{\varepsilon}} - \underline{n}\gamma) = \underline{\underline{C}}^{el}\underline{\varepsilon}^{el} ; \quad \underline{\sigma} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix} = \begin{pmatrix} (\lambda+2\mu)\varepsilon_1^{el} + \lambda\varepsilon_2^{el} + \lambda\varepsilon_3^{el} \\ \lambda\varepsilon_1^{el} + (\lambda+2\mu)\varepsilon_2^{el} + \lambda\varepsilon_3^{el} \\ \lambda\varepsilon_1^{el} + \lambda\varepsilon_2^{el} + (\lambda+2\mu)\varepsilon_3^{el} \\ 2\mu\varepsilon_4^{el} \\ 2\mu\varepsilon_5^{el} \\ 2\mu\varepsilon_6^{el} \end{pmatrix} ; \quad \underline{\varepsilon}^{el} = \underline{\tilde{\varepsilon}} - \underline{n}\gamma ; \quad \Delta\underline{\varepsilon}^{pl} = \underline{n}\gamma ; \quad \Delta\bar{\varepsilon}^{pl} = \sqrt{\frac{2}{3}}\gamma$$ exit the calculation.

Hint: by given increment strain tensor the material will give plastic response.