Homework 6

Denis Ostroushko

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```
library(MASS)
require(tidyverse) # require instead of library to make sure that other packages do not overwrite tidyv
library(kableExtra)
library(readxl)
library(gridExtra)
library(ggeffects)
library(mltools) # one hot encoding outside of caret package
library(data.table) # need this for mltools to work
library(olsrr) # a better package for stepwise regression
```

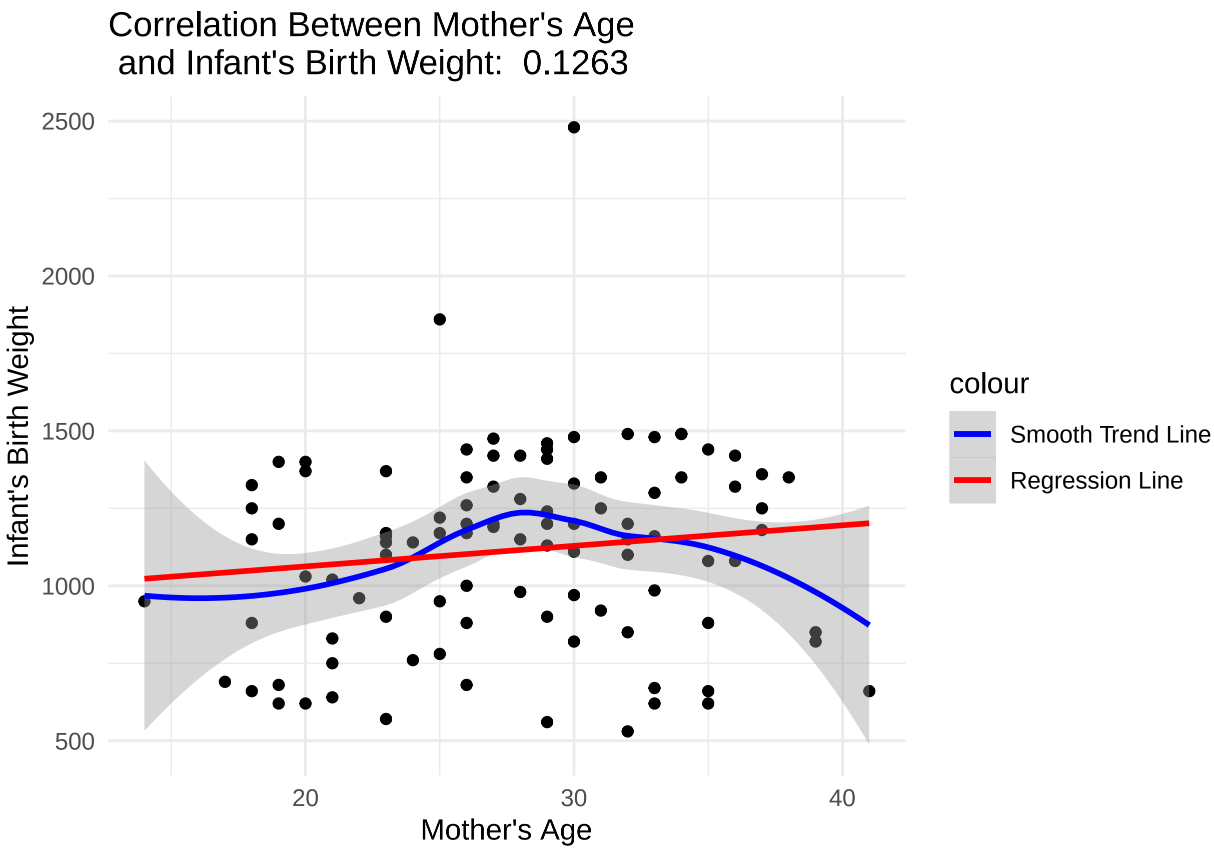
12.2

```
colnames(infants) <- c("head_c", "length", "gest_weeks", "birth_w", "m_age", "toxemia")

# process the data and keep variables for analysis

infants_f <- infants %>%
  select(birth_w, gest_weeks, m_age)
```

12.2 - A

Model Specifications and T-tests

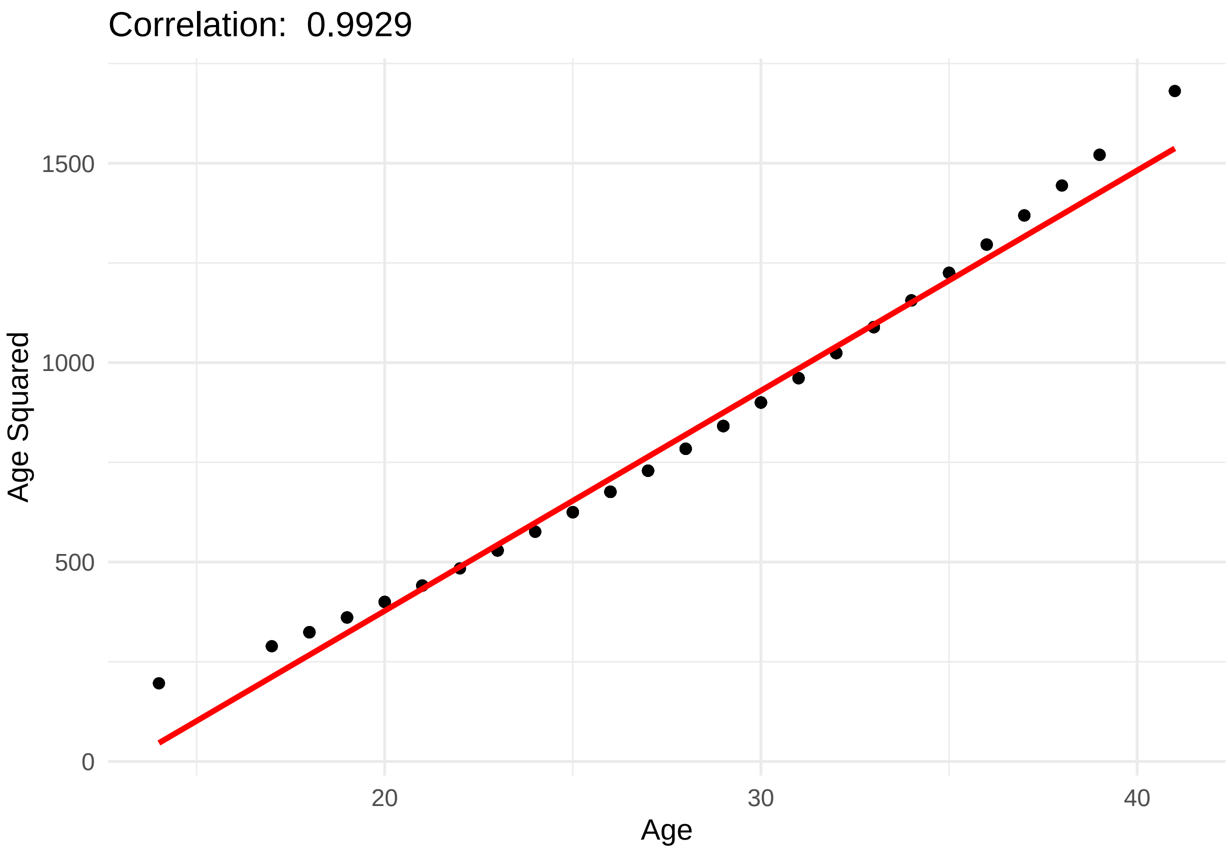
Before fitting the model, we wish to investigate the relationship between mother's age and infant's birth weight. Since the problem asks us to fit the model with age squared, we will have a second order polynomial relationship. We will look at the scatter plot to find any visual evidence that such model is justifiable.



We can see that we should fit the plynomial regression model because the smooth line shows a curved relationship between the two variables. However, the confidence bound around the smooth line suggest that potentially we may be able to fit a straight, first order, line in order to predict infant's birth weight. Overall, it is not very clear to what the verdict is, so we will fit the model with a higher order term and will use statistical tests to verify contribution of the squared term. Pearson's linear correlation estimate is low, so we should not expect to see string statistical evidence that mother's age is a strong predictor for infant's birth weight.

It is known that inclusion of higher order terms introduces multicollinearity issue to the model, which is hard to handle, and affects confidence intervals for predictors. Normally, we wish to perform another transformation of variables called *centering* in order to reduce the degree of linear correlation between the linear and higher order terms, however, I decided to include that into the appendix.

The plot below shows correlation between age and age squared.



Since the two variables are almost perfectly correlated, we expect that estimate for the standard error of $\hat{\beta}_i$ are higher in the model with no centering transformation applied. We verify it in the appendix section.

We are now ready to fit the model and explore the contribution of age-squared term. Model specification:

$$E[Y] = \hat{\beta}_0 + \hat{\beta}_1 * Gestional\ \ Weeks + \hat{\beta}_2 * Mother's\ \ Age + \hat{\beta}_3 * Mother's\ \ Age^2$$

We obtain model the estimates from the model and present them in the table below:

Table 1: Polymonial Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-1442.928	496.023	-2.909	0.005
Gestational Weeks	75.667	10.652	7.103	0.000
Mother's Age	30.252	36.813	0.822	0.413
Mother's Age Squared	-0.582	0.656	-0.887	0.377

Comments:

- R-squared is 0.3714 and Adjusted R-squared is 0.3904
- The number of gestational weeks is an extremely strong predictor of the infant's birth weight. Each additional week add an average of 75.667 units of measurement (not sure what they are in this problem) to infant's birth weight

- Both linear are quadratic terms for mother's age are not statistically significant, and therefore we do not have enough evidence to reject the null hypothesis and conclude that the coefficients for these predictors are statistically different from zero.
- An addition of a quadratic term turns the effect of age on birth weight from a straight line to the parabola. We can use estimates of the linear and quadratic terms to describe the shape of this parabola.
  - A positive quadratic coefficient causes the ends of the parabola to point upward. A negative quadratic coefficient causes the ends of the parabola to point downward. The greater the quadratic coefficient, the narrower the parabola. The lesser the quadratic coefficient, the wider the parabola.
  - In our case the coefficient is -0.582, so the effect can be visualized as a wide downward-pointing parabola.
  - A very wide parabola usualyy does not indicate a strong effect, and visually it should apprear closer to a straight line with a zero linear coefficient.

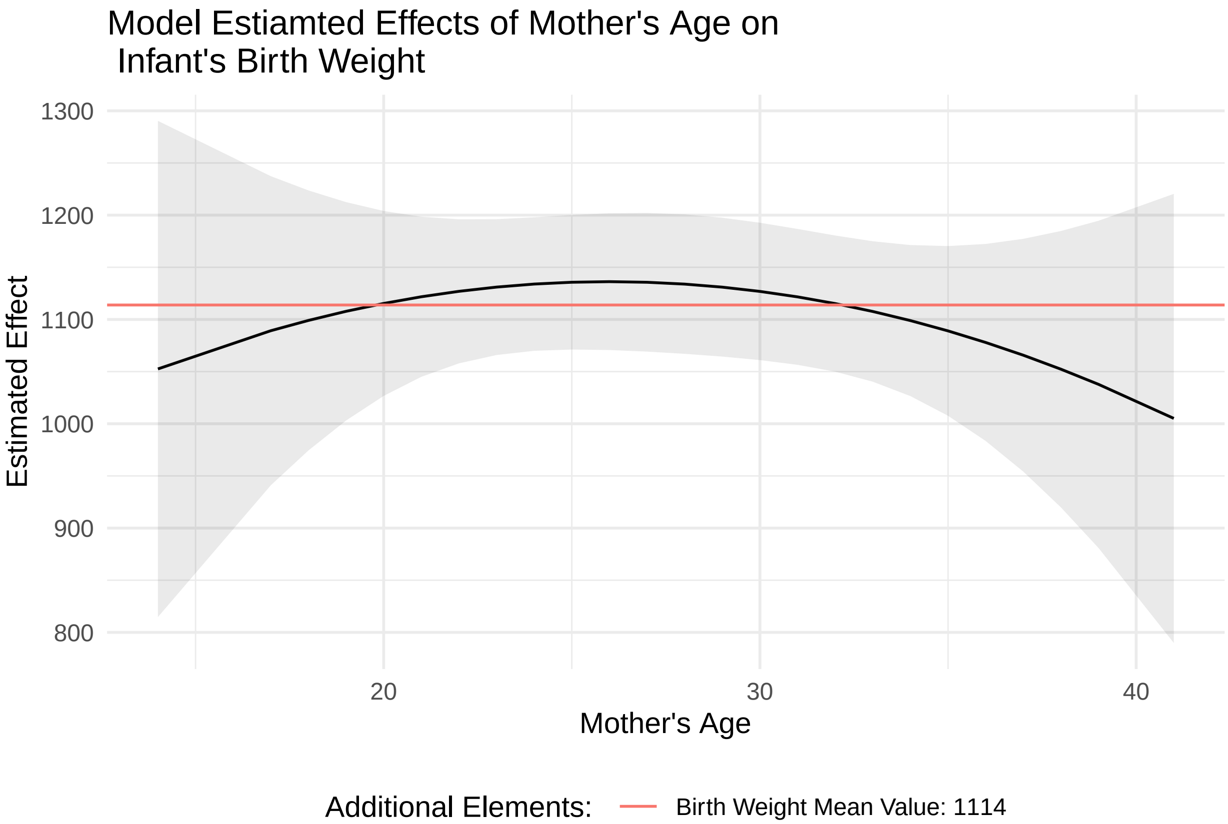
Evaluate Extra Sum of Squares

Focus: Evaluate SSR(Age^2 | Gest, Age)

Model Term	DF	SS	MS	F-statistic	P(F* > F)
Gestational Weeks	1	3755985.30	3755985.30	60.4451134	0.0000
Mother's Age	1	15505.20	15505.20	0.2495254	0.6186
Mother's Age Squared	1	48879.84	48879.84	0.7866239	0.3773
Residuals	96	5965322.40	62138.78	NA	NA

- Extra SS
- Extra R^2
- Connection with the t-test

Visualize Model Effects



- Comment on Standard Error and fit, we can fit a line with slope =

Interpretation of Mother's Age Coefficients

From google, interpretation of the quadratic coefficient:

" A positive quadratic coefficient causes the ends of the parabola to point upward. A negative quadratic coefficient causes the ends of the parabola to point downward. The greater the quadratic coefficient, the narrower the parabola. The lesser the quadratic coefficient, the wider the parabola."

https://stats.stackexchange.com/questions/108657/how-to-interpret-coefficients-of-x-and-x2-in-same-regression

It may be useful to describe the effect of a unit change at some low value, some high value and somewhere in between.

12.2 - B

Correlation Transformation for variables $Y, X_1, ..., X_{p-1}$, denoted by V:

$$V^* = \frac{1}{\sqrt{n-1}} \times \left(\frac{V - \bar{V}}{sd(V)}\right)$$

```
correlation_transformation <-
  function(X, n = nrow(infants_f_cor_tr)){

    1/(sqrt(n - 1)) * (X - mean(X))/sd(X)
```

```
  }

infants_f$m_age_sq <- infants_f$m_age^2
infants_f_cor_tr <- infants_f

infants_f_cor_tr <- data.frame(lapply(infants_f_cor_tr, correlation_transformation))
```

Table 2: Original Scale Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-1442.928	496.023	-2.909	0.005
Gestational Weeks	75.667	10.652	7.103	0.000
Mother's Age	30.252	36.813	0.822	0.413
Mother's Age Squared	-0.582	0.656	-0.887	0.377

Table 3: Correlation Transformation Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	0.000	0.008	0.000	1.000
Gestational Weeks	0.610	0.086	7.103	0.000
Mother's Age	0.576	0.701	0.822	0.413
Mother's Age Squared	-0.616	0.694	-0.887	0.377

- intercept is zero as expected in corr transformed
- P-values are different for m age
- Same conclusions apply

12.2 - C

Transformation back to the original scale:

For variables $X_1, ..., X_{p-1}$:

$$\hat{\beta}_i = \hat{\beta}_i^* \times \frac{sd(Y)}{sd(X_i)}$$

```
transform_back <-
  function(Beta_star, s_x, s_y){
    Beta_star * (s_y / s_x)
  }

S_Y <- sd(infants_f$birth_w)
```

Hid code to prepare the table.

recall the the original model with the transformed variables was called `inf_lm`. Used it for Extra SS, t-tests and model effects. We can obtain standard errors and confidence intervals for the estimates to compare with the transformation back from the correlation transformation procedure.

Table 4: Original Model Estiamtes and C.I.

Model Term	Coefficient	95% C.I. Lower Bound	95% C.I. Upper Bound
Gestation Weeks	75.667	54.522	96.811
Mother's Age	30.252	-42.821	103.324
Mother's Age Squared	-0.582	-1.884	0.721

Table 5: Estimaes obtained via Back-Trnasformation and C.I.

Model Term	Coefficient	95% C.I. Lower Bound	95% C.I. Upper Bound
Gestation Weeks	75.678	54.522	96.811
Mother's Age	30.268	-42.821	103.324
Mother's Age Squared	-0.582	-1.884	0.721

```
conf <- data.frame(confint(inf_lm)) # just the confidence intervals
conf <- cbind(coefficients(inf_lm), conf )
```

so we can use linear transformations good to know

13.4

```
cig$Y1 <- with(cig, log(NNAL_vt4_creat / NNAL_vt0_creat))
cig$Y2 <- with(cig, log(TNE_vt4_creat / TNE_vt0_creat))

cig <- cig %>%
  select(Y1, Y2, arm, age, gender, white, educ2, income30, FTND)

colnames(cig)[length(cig)] <- "ftnd"
```

13.4 - A

- Arm will result in 4 -1 variables
- Age is untouched
- FTND is treated as continuous
- Others need to be converted to factor variables

```
cig <- cig %>% select(
  Y1, Y2, age, arm, gender, educ2, income30, ftnd
)

cig$arm <- as.factor(cig$arm)

cig <- data.frame(one_hot(as.data.table(cig))) %>% select(-arm_5)

cig[,4:(length(cig)-1)] <- lapply(cig[,4:(length(cig)-1)], as.factor)
```

```
n_unique <- function(x){length(unique(x))}
```

```r
meta_data <-

  data.frame(
    class = sapply(cig, class),
    n_unique = sapply(cig, n_unique)
  )
```

Table 6: Sumamry of Covariates

Predictors	Assigned Class	N of Unique Values
age	numeric	51
arm_6	factor	2
arm_7	factor	2
arm_8	factor	2
gender	factor	2
educ2	factor	2
income30	factor	2
ftnd	numeric	8

13.4 - B

Regression on Y1

add model specification

```r
y1_lm1 <- lm(Y1 ~ . - Y2, data = cig )
```

Table 7: Original Scale Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	0.027	0.281	0.094	0.925
Age	-0.003	0.004	-0.701	0.484
Arm 6	-0.690	0.175	-3.940	0.000
Arm 7	-0.068	0.174	-0.392	0.696
Arm 8	-0.426	0.179	-2.380	0.018
Gender	-0.112	0.109	-1.031	0.304
Education	-0.066	0.112	-0.588	0.557
Income >= $30K	-0.229	0.119	-1.922	0.056
FTND	0.046	0.042	1.093	0.276

- Bonferroni Adjustments
  - $P - value = 0.05$
  - $Bonferroni\ adjusted\ P - value = 0.0063$

corrected p-value = p-value / number of predictors

```r
sum_bonf_adj <- sum2 %>% select(`Model Term`, `P-value`)
sum_bonf_adj$`Significant at Adj. Level` =
  with(sum_bonf_adj,
       ifelse(`P-value` < 0.05 / n_predictors , "*", ""))
```

```r
      )

sum_bonf_adj %>%
  kbl( booktabs = T, caption = "Regression of Y1 Bonferroni Adjusted Comparison") %>%
    kable_styling(latex_options = c("striped", "HOLD_position")) %>%
    column_spec(3, width = "2cm")
```

Table 8: Regression of Y1 Bonferroni Adjusted Comparison

Model Term	P-value	Significant at Adj. Level
Intercept	0.925	
Age	0.484	
Arm 6	0.000	*
Arm 7	0.696	
Arm 8	0.018	
Gender	0.304	
Education	0.557	
Income >= $30K	0.056	
FTND	0.276	

- HOLM Adjustments
  - order p-values smallest to largest
  - if first p-value if smaller than $0.05/8 = 0.0063$ then conclude significance, and move to next predictor, otherwise stop, none are significant
  - next predictor will be tested at $0.05/7 = 0.0071$

```r
holm_data <-
  sum2 %>% select(`Model Term`, `P-value`) %>% arrange(`P-value`) %>%
  filter(`Model Term` != "Intercept")

holm_data$`Comparison P-value` <- 1
holm_data$`Significant at Adj. Level` <- ""

cur_adj_n <- n_predictors

for(i in 1:nrow(holm_data)){

  cur_level <- 0.05 / cur_adj_n
  holm_data[i,3] <- cur_level

  if(holm_data[i,2] <= cur_level ){
    cur_adj_n <- cur_adj_n - 1
    holm_data[i,3] <- cur_level
    holm_data[i,4] <- "*"
  }
}

holm_data[,2:3] <- lapply(holm_data[,2:3], round_3)

holm_data %>%
```

```r
  kbl( booktabs = T, caption = "Regression of Y1 HOLM Adjusted Comparison") %>%
    kable_styling(latex_options = c("striped", "HOLD_position")) %>%
    column_spec(c(3,4), width = "2cm")
```

Table 9: Regression of Y1 HOLM Adjusted Comparison

Model Term	P-value	Comparison P-value	Significant at Adj. Level
Arm 6	0.000	0.006	*
Arm 8	0.018	0.007	
Income >= $30K	0.056	0.007	
FTND	0.276	0.007	
Gender	0.304	0.007	
Age	0.484	0.007	
Education	0.557	0.007	
Arm 7	0.696	0.007	

- Hochberg Adjustments
  - Sort P-values largest to smallest
  - Compare the largest to 0.05, if significant, declare all significant
  - Otherwise, compare the next one to $0.05/2 = 0.025$
  - Keep comparing to 0.05/3, 0.05/4, etc.. until we find a comparison where

```r
hoch_data <-
  sum2 %>% select(`Model Term`, `P-value`) %>% arrange(-`P-value`) %>%
  filter(`Model Term` != "Intercept")

hoch_data$`Comparison P-value` <- 0.05
hoch_data$`Significant at Adj. Level` <- ""

cur_adj_n <- 1

for(i in 1:nrow(hoch_data)){

  cur_level <- 0.05 / cur_adj_n
  hoch_data[i,3] <- cur_level

  if(hoch_data[i,2] > cur_level){
    cur_adj_n <- cur_adj_n + 1

    holm_data[i,3] <- cur_level
  }
}

hoch_data[,4] <- ifelse(hoch_data[,2] < hoch_data[,3], "*", "")

hoch_data[,2:3] <- lapply(hoch_data[,2:3], round_3)

hoch_data %>%
  kbl( booktabs = T, caption = "Regression of Y1 HOCHBERG Adjusted Comparison") %>%
    kable_styling(latex_options = c("striped", "HOLD_position")) %>%
    column_spec(c(3,4),  width = "2cm")
```

Table 10: Regression of Y1 HOCHBERG Adjusted Comparison

Model Term	P-value	Comparison P-value	Significant at Adj. Level
Arm 7	0.696	0.050	
Education	0.557	0.025	
Age	0.484	0.017	
Gender	0.304	0.013	
FTND	0.276	0.010	
Income >= $30K	0.056	0.008	
Arm 8	0.018	0.007	
Arm 6	0.000	0.006	*

- SUMMARY OF COEFFICIENT SELECTION FOR Y1 REGRESSION

Regression on Y2

```
y2_lm1 <- lm(Y2 ~ . - Y1, data = cig )
```

Table 11: Original Scale Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-0.183	0.438	-0.418	0.677
Age	-0.002	0.006	-0.243	0.808
Arm 6	-0.278	0.273	-1.017	0.310
Arm 7	0.195	0.272	0.718	0.474
Arm 8	-0.095	0.279	-0.341	0.734
Gender	-0.096	0.170	-0.567	0.572
Education	-0.198	0.175	-1.129	0.260
Income >= $30K	-0.218	0.186	-1.176	0.241
FTND	0.056	0.066	0.855	0.394

- Bonferroni Adjustments

Table 12: Regression of Y2 Bonferroni Adjusted Comparison

Model Term	P-value	Significant at Adj. Level
Intercept	0.677	
Age	0.808	
Arm 6	0.310	
Arm 7	0.474	
Arm 8	0.734	
Gender	0.572	
Education	0.260	
Income >= $30K	0.241	
FTND	0.394	

- HOLM Adjustments

Table 13: Regression of Y2 HOLM Adjusted Comparison

Model Term	P-value	Comparison P-value	Significant at Adj. Level
Income >= $30K	0.241	0.006	
Education	0.260	0.006	
Arm 6	0.310	0.006	
FTND	0.394	0.006	
Arm 7	0.474	0.006	
Gender	0.572	0.006	
Arm 8	0.734	0.006	
Age	0.808	0.006	

- Hochberg Adjustments

Table 14: Regression of Y2 HOCHBERG Adjusted Comparison

Model Term	P-value	Comparison P-value	Significant at Adj. Level
Age	0.808	0.050	
Arm 8	0.734	0.025	
Gender	0.572	0.017	
Arm 7	0.474	0.013	
FTND	0.394	0.010	
Arm 6	0.310	0.008	
Education	0.260	0.007	
Income >= $30K	0.241	0.006	

13.4 - C

Step Wise Regression on Y1

comment that we have 2^8 possible models, and these are candidates for the best possible model

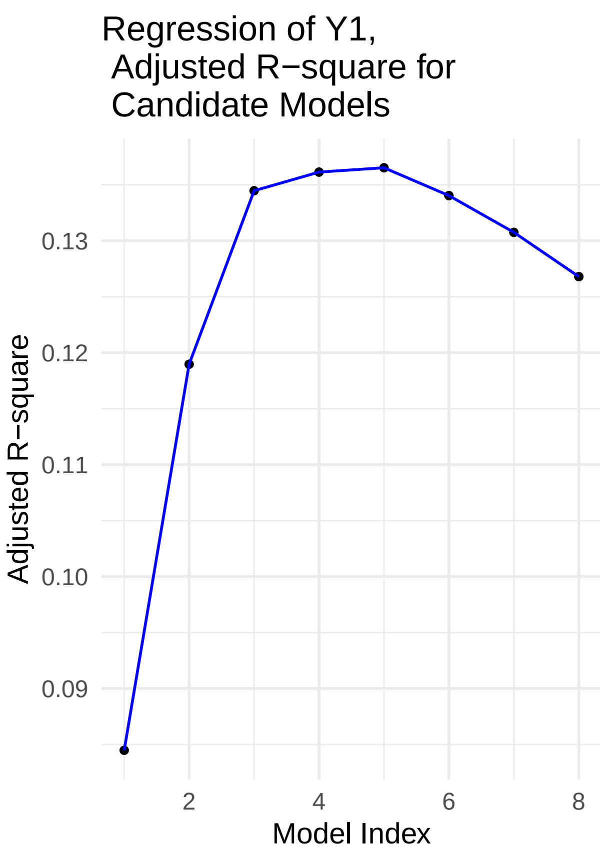
```
k <- ols_step_best_subset(y1_lm1)

k %>% dplyr::select(n, predictors) %>%
  kbl(booktabs = T,
      caption = "Regression of Y1, Best Candidate Models") %>%
  kable_styling(latex_options = c("striped", "HOLD_position"))
```

Table 16: Regression of Y1, Parameters of Selected Model

Predictors	R-squared	Adj. R-squared	AIC
arm_6 arm_8 income30	0.148	0.134	445.969

Table 15: Regression of Y1, Best Candidate Models

	n	predictors
2	1	arm_6
17	2	arm_6 arm_8
65	3	arm_6 arm_8 income30
139	4	arm_6 arm_8 gender income30
210	5	arm_6 arm_8 gender income30 ftnd
231	6	age arm_6 arm_8 gender income30 ftnd
252	7	age arm_6 arm_8 gender educ2 income30 ftnd
255	8	age arm_6 arm_7 arm_8 gender educ2 income30 ftnd



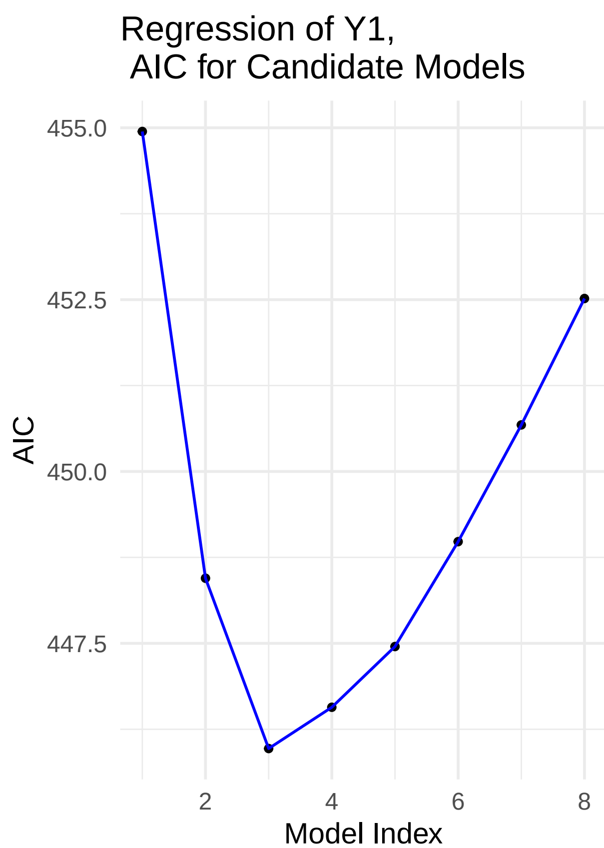


Table 17: Regression of Y1,

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-0.077	0.086	-0.897	0.371
Arm 6	-0.645	0.128	-5.033	0.000
Arm 8	-0.403	0.132	-3.045	0.003
Income >= $30K	-0.246	0.117	-2.106	0.036

Step Wise Regression on Y2

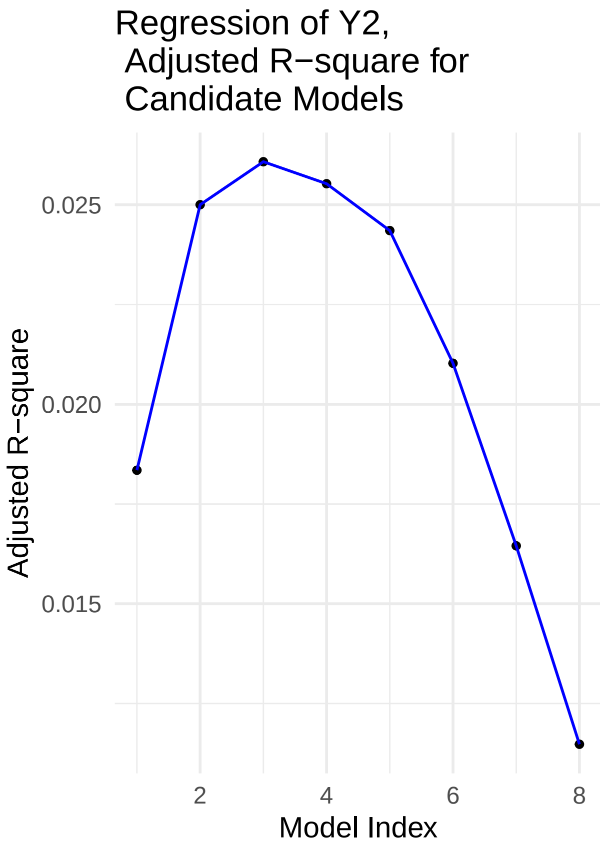
```
k <- ols_step_best_subset(y2_lm1)

k %>% dplyr::select(n, predictors) %>%
  kbl(booktabs = T,
      caption = "Regression of Y2, Best Candidate Models") %>%
  kable_styling(latex_options = c("striped", "HOLD_position"))
```

Table 18: Regression of Y2, Best Candidate Models

	n	predictors
3	1	arm_7
25	2	arm_7 income30
80	3	arm_7 educ2 income30
135	4	arm_6 arm_7 educ2 income30
207	5	arm_6 arm_7 educ2 income30 ftnd
244	6	arm_6 arm_7 gender educ2 income30 ftnd
254	7	arm_6 arm_7 arm_8 gender educ2 income30 ftnd
255	8	age arm_6 arm_7 arm_8 gender educ2 income30 ftnd

Table 19: Regression of Y1, Parameters of Selected Model

Predictors	R-squared	Adj. R-squared	AIC
arm_7 income30	0.035	0.025	617.082



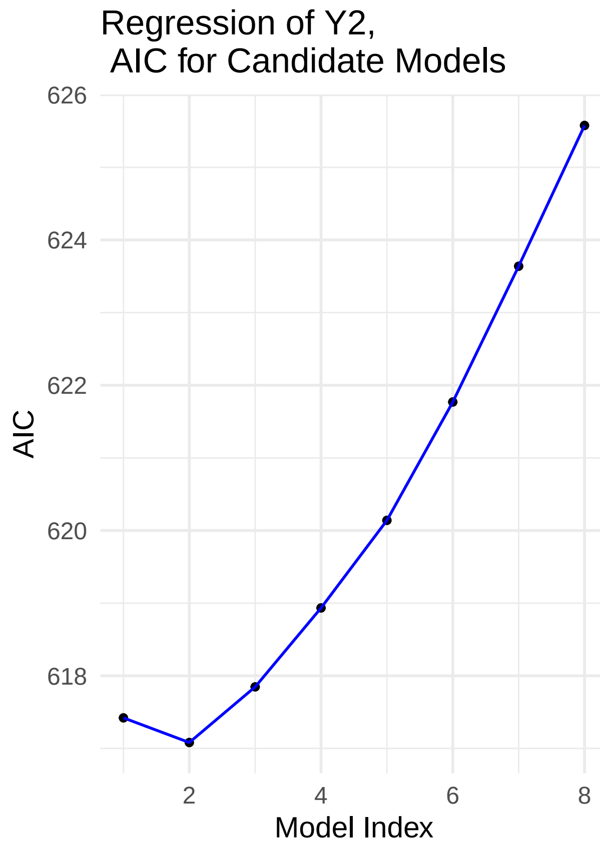
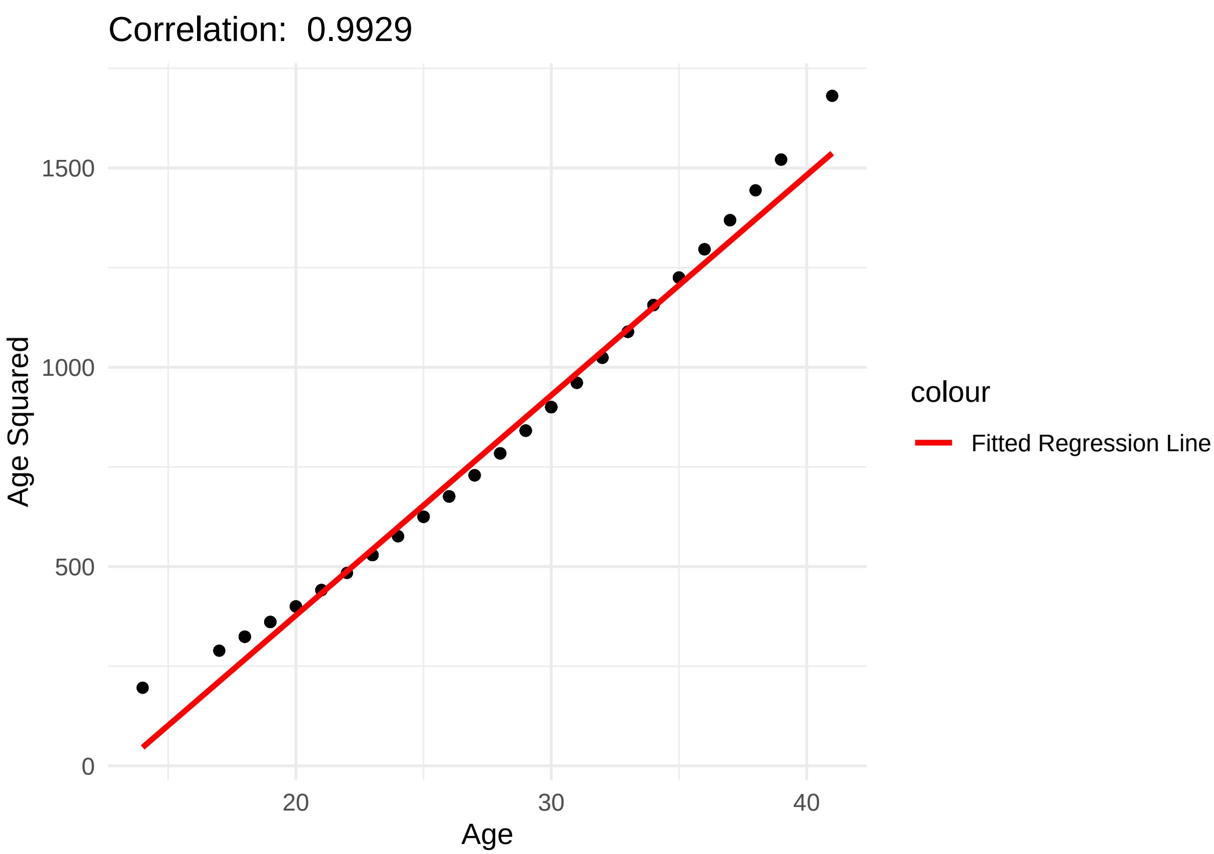


Table 20:

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-0.372	0.115	-3.224	0.001
Arm 7	0.382	0.180	2.130	0.034
Income >= $30K	-0.275	0.180	-1.522	0.130

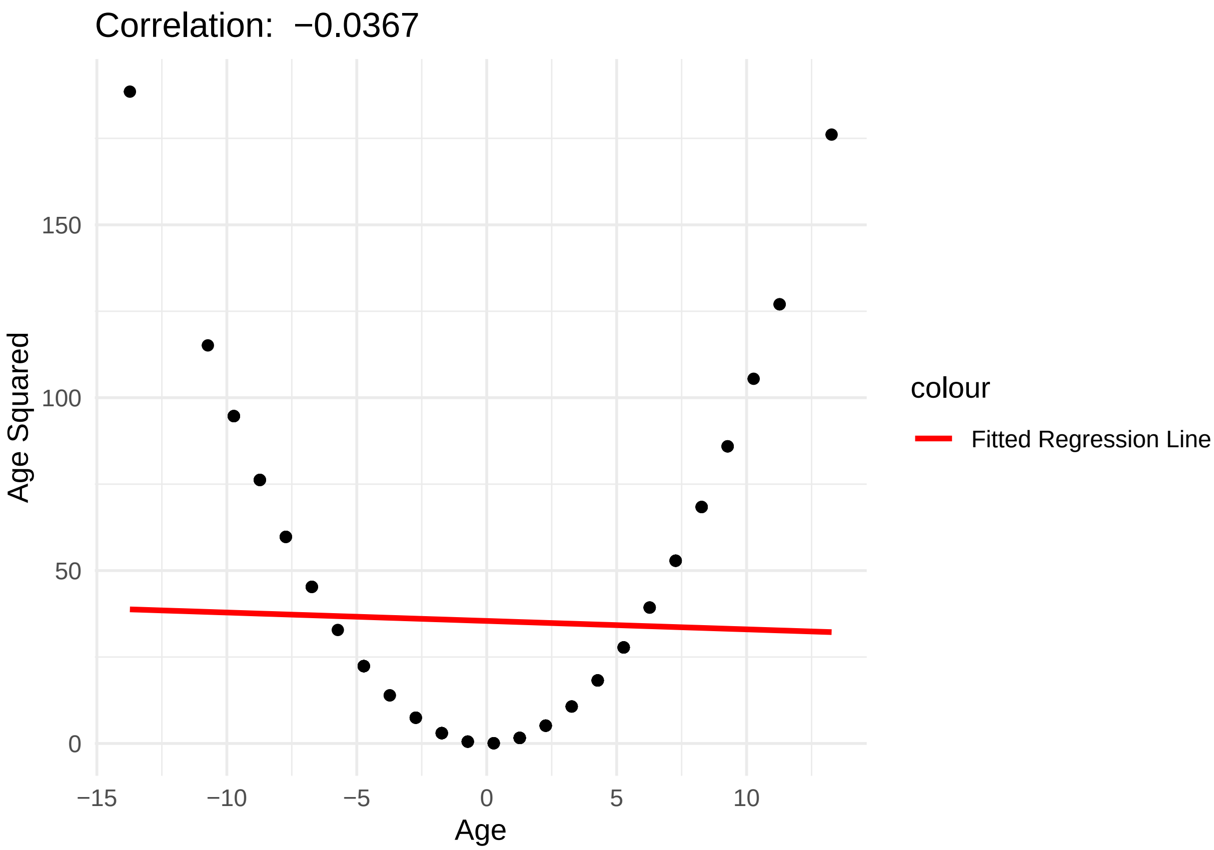
Appendix: 12.2

```r
# look at the correlation between age and age^2
ggplot(data = infants_f,
       aes(x = m_age,
           y = m_age^2 )) + geom_point() +

  stat_smooth(method = "lm", se = F, aes(color = "Fitted Regression Line")) +
  scale_color_manual(values = c("Fitted Regression Line" = "red")) +

  xlab("Age") +
  ylab("Age Squared") +
  ggtitle(paste("Correlation: ", round(cor(infants_f$m_age, infants_f$m_age^2),4))) +
  theme_minimal()
```

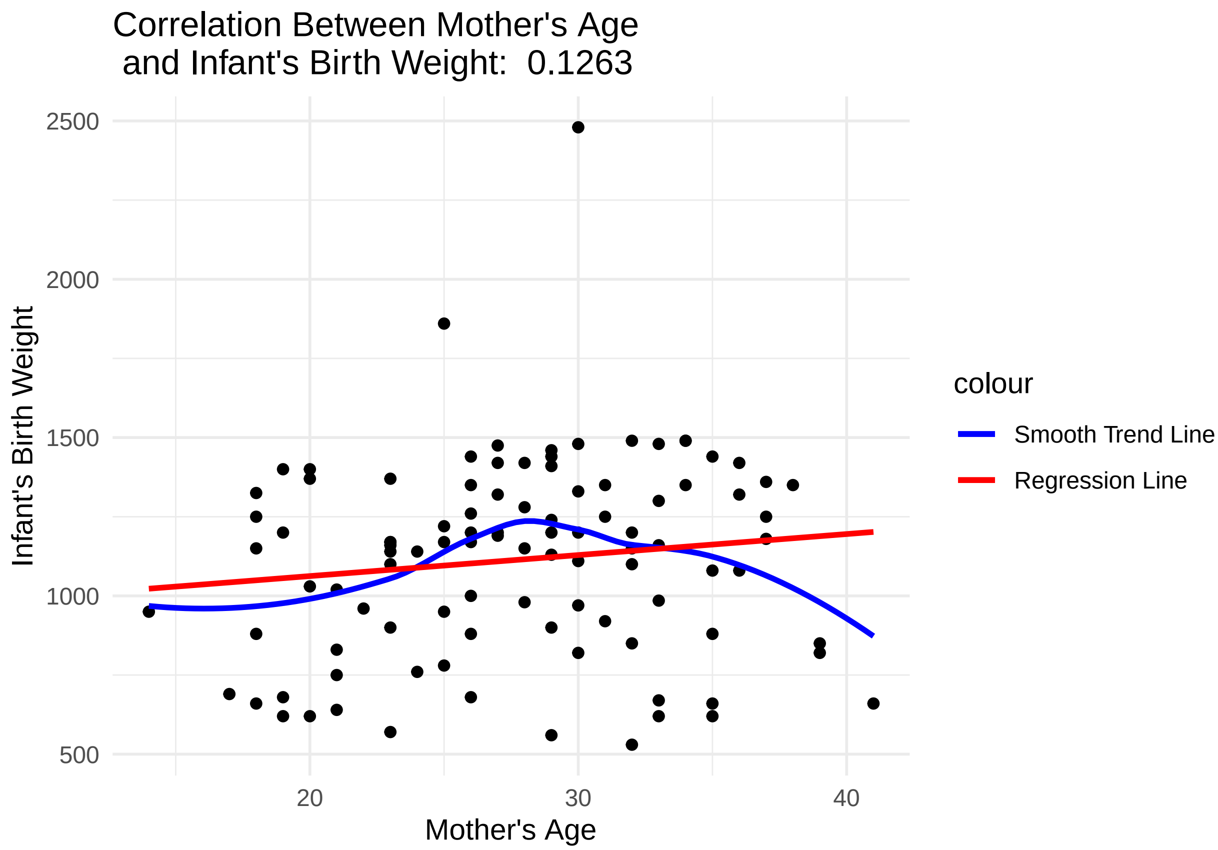



```r
# now apply centering:

infants_f$m_age_centered <- with(infants_f, m_age - mean(m_age))
infants_f$gest_weeks_centered <- with(infants_f, gest_weeks - mean(gest_weeks))

ggplot(data = infants_f,
       aes(x = m_age_centered,
           y = m_age_centered^2 )) + geom_point() +

  stat_smooth(method = "lm", se = F, aes(color = "Fitted Regression Line")) +
  scale_color_manual(values = c("Fitted Regression Line" = "red")) +
```

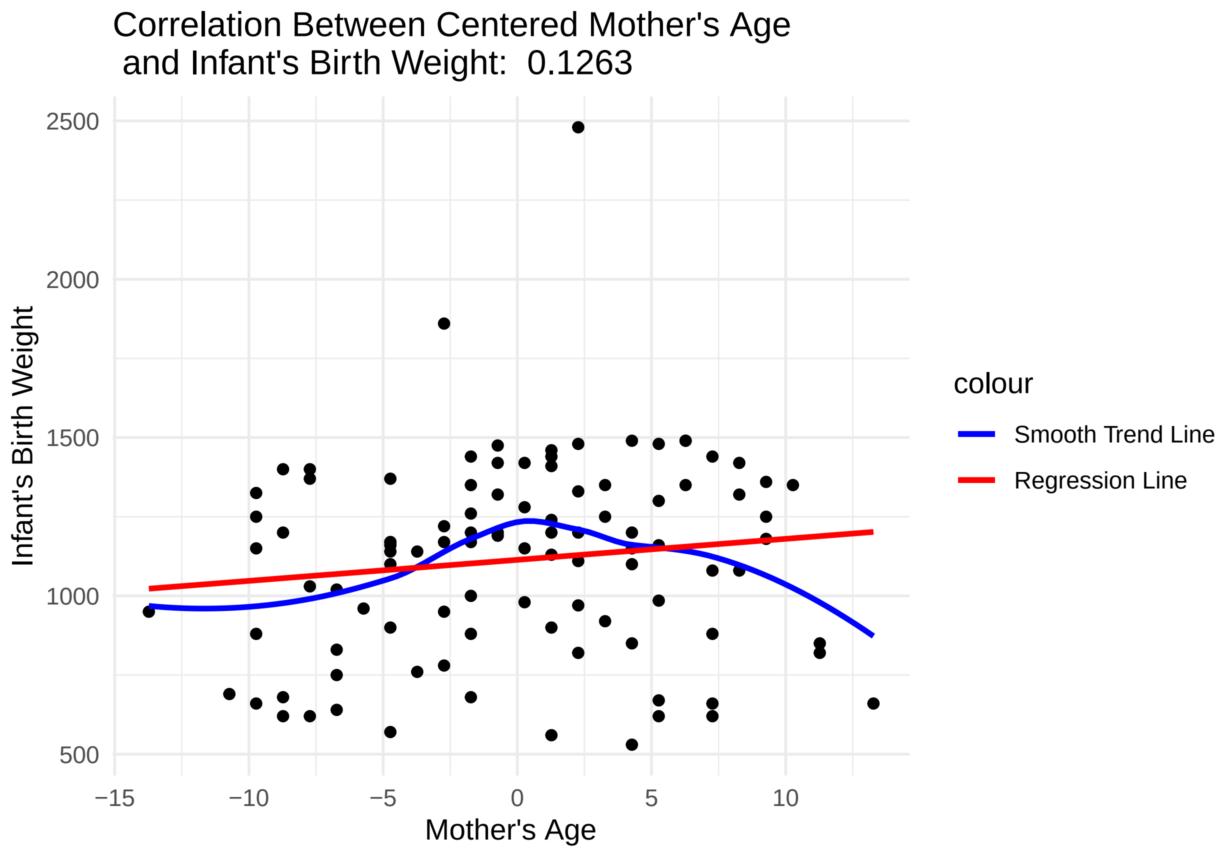
```r
  xlab("Age") +
  ylab("Age Squared") +
  ggtitle(paste("Correlation: ", round(cor(infants_f$m_age_centered, infants_f$m_age_centered^2),4))) +
  theme_minimal()
```




```r
ggplot(data = infants_f,
       aes(x = m_age,
           y = birth_w)) + geom_point() +

  stat_smooth(se = F, aes(color = "Smooth Trend Line")) +
  stat_smooth(se = F, method = "lm", aes(color = "Regression Line")) +

  scale_color_manual(values = c("Smooth Trend Line" = "blue",
                                "Regression Line" = "red")) +
  xlab("Mother's Age") +
  ylab("Infant's Birth Weight") +
  ggtitle(paste("Correlation Between Mother's Age  \n and Infant's Birth Weight: ",
                round(cor(infants_f$m_age,
                          infants_f$birth_w), 4))) +
  theme_minimal()
```




```
ggplot(data = infants_f,
       aes(x = m_age_centered,
           y = birth_w)) + geom_point() +

  stat_smooth(se = F, aes(color = "Smooth Trend Line")) +
  stat_smooth(se = F, method = "lm", aes(color = "Regression Line")) +

  scale_color_manual(values = c("Smooth Trend Line" = "blue",
                                "Regression Line" = "red")) +
  xlab("Mother's Age") +
  ylab("Infant's Birth Weight") +
  ggtitle(paste("Correlation Between Centered Mother's Age  \n and Infant's Birth Weight: ",
              round(cor(infants_f$m_age_centered,
                        infants_f$birth_w), 4))) +
  theme_minimal()
```


Correlation Between Centered Mother's Age
and Infant's Birth Weight: 0.1263
Infant's Birth Weight
2500
2000
1500
1000
500
−15
−10
−5
0
5
10
Mother's Age
colour
Smooth Trend Line
Regression Line