Homework 6

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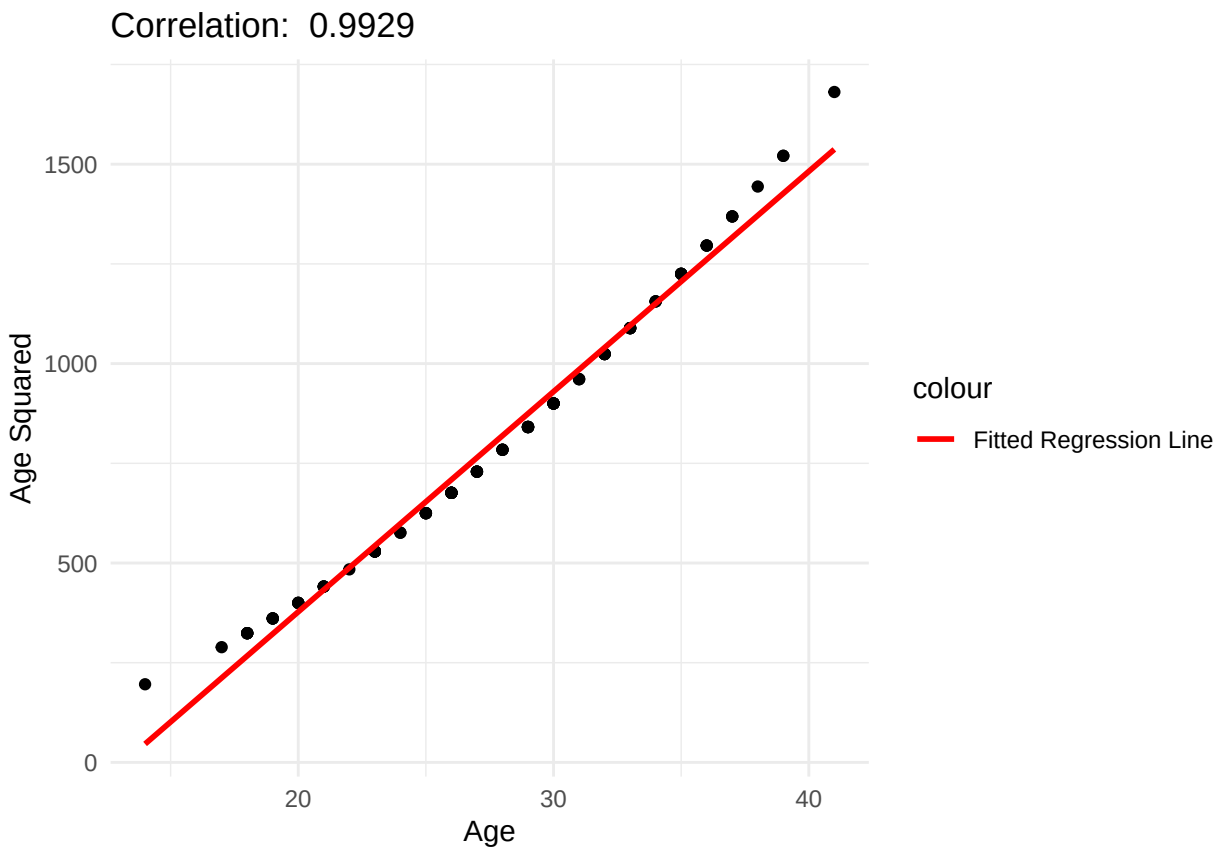
```
library(tidyverse)
library(kableExtra)
library(readxl)
library(gridExtra)
library(ggeffects)
library(mltools) # one hot encoding outside of caret package
library(data.table) # need this for mltools to work
```

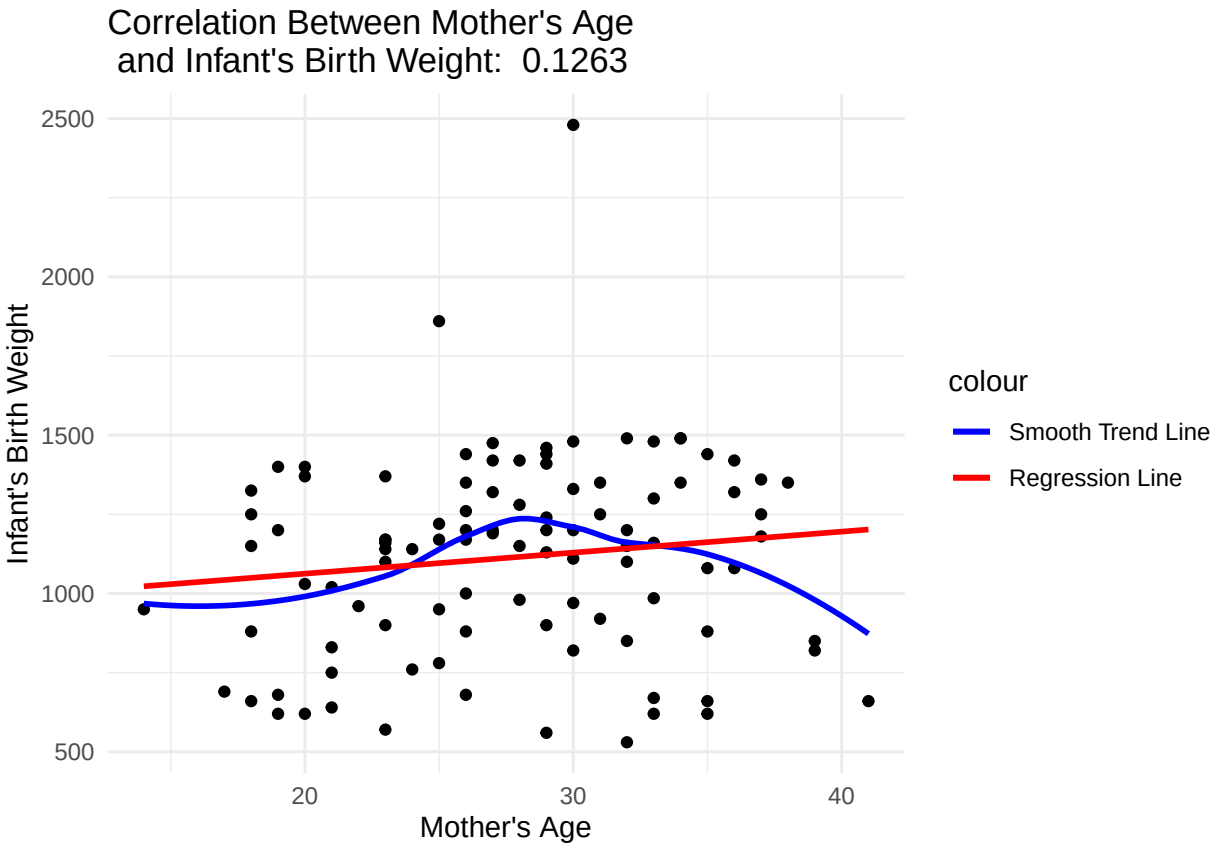
12.2

```
infants <- readxl::read_xls('/Users/denisostroushko/Desktop/UofM MS/MS Fall 2022/Puhb 7405/Data Sets/In

colnames(infants) <- c("head_c", "length", "gest_weeks", "birth_w", "m_age", "toxemia")

# process the data and keep variables for analysis

infants_f <- infants %>%
  select(birth_w, gest_weeks, m_age)
```


Correlation: 0.9929
Age Squared
1500
1000
500
0
20
30
40
Age
colour
Fitted Regression Line

12.2 - A

Model Specifications and T-tests



Model specification:

$$E[Y] = \hat{\beta}_0 + \hat{\beta}_1 * Gestional\ \ Weeks + \hat{\beta}_2 * Mother's\ \ Age + \hat{\beta}_3 * Mother's\ \ Age^2$$

Model Summary

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-1442.928	496.023	-2.909	0.005
Gestational Weeks	75.667	10.652	7.103	0.000
Mother's Age	30.252	36.813	0.822	0.413
Mother's Age Squared	-0.582	0.656	-0.887	0.377

Comemnts on Model summary:

- R and Adjusted R: 0.3904 0.3714
- Coefficients for Age and Age ^ 2

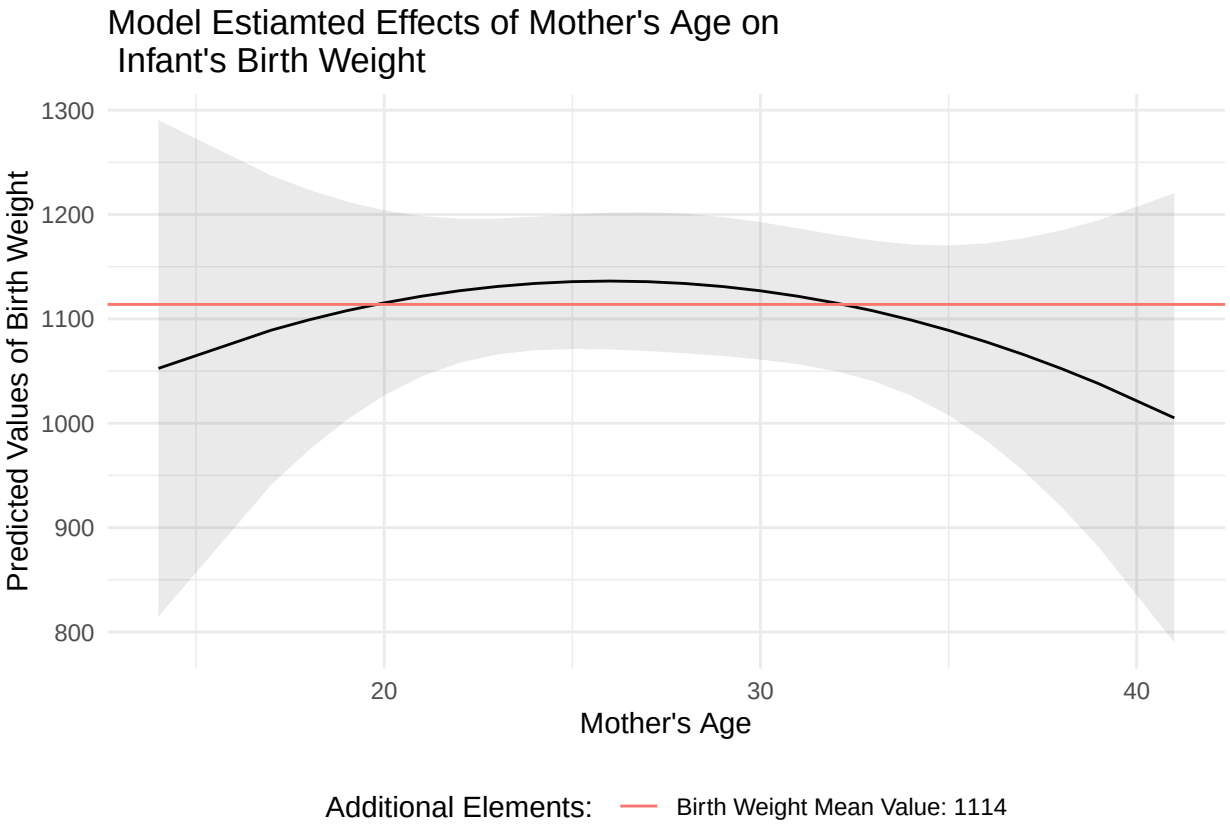
Evaluate Extra Sum of Squares

Focus: Evaluate SSR(Age^2 | Gest, Age)

Model Term	DF	SS	MS	F-statistic	P(F* > F)
Gestational Weeks	1	3755985.30	3755985.30	60.4451134	0.0000
Mother's Age	1	15505.20	15505.20	0.2495254	0.6186
Mother's Age Squared	1	48879.84	48879.84	0.7866239	0.3773
Residuals	96	5965322.40	62138.78	NA	NA

- Extra SS
- Extra R^2
- Connection with the t-test

Visualize Model Effects



- Comment on Standard Error and fit, we can fit a line with slope = +

Interpretation of Mother's Age Coefficients

From google, interpretation of the quadratic coefficient:

" A positive quadratic coefficient causes the ends of the parabola to point upward. A negative quadratic coefficient causes the ends of the parabola to point downward. The greater the quadratic coefficient, the narrower the parabola. The lesser the quadratic coefficient, the wider the parabola."

https://stats.stackexchange.com/questions/108657/how-to-interpret-coefficients-of-x-and-x2-in-same-regression

It may be useful to describe the effect of a unit change at some low value, some high value and somewhere in between.

12.2 - B

Correlation Transformation for variables $Y, X_1, ..., X_{p-1}$, denoted by V:

$$V^* = \frac{1}{\sqrt{n-1}} \times \left(\frac{V - \bar{V}}{sd(V)} \right)$$

```
correlation_transformation <-
  function(X, n = nrow(infants_f_cor_tr)){

    1/(sqrt(n - 1)) * (X - mean(X))/sd(X)

  }

infants_f$m_age_sq <- infants_f$m_age^2
infants_f_cor_tr <- infants_f

infants_f_cor_tr <- data.frame(lapply(infants_f_cor_tr, correlation_transformation))
```

Table 1: Original Scale Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	-1442.928	496.023	-2.909	0.005
Gestational Weeks	75.667	10.652	7.103	0.000
Mother's Age	30.252	36.813	0.822	0.413
Mother's Age Squared	-0.582	0.656	-0.887	0.377

Table 2: Correlation Transformation Regression Estimates

Model Term	Estimate	Std. Error	T-value	P-value
Intercept	0.000	0.008	0.000	1.000
Gestational Weeks	0.610	0.086	7.103	0.000
Mother's Age	0.576	0.701	0.822	0.413
Mother's Age Squared	-0.616	0.694	-0.887	0.377

- intercept is zero as expected in corr transformed
- P-values are different for m age
- Same conclusions apply

12.2 - C

Transformation back to the original scale:

For variables $X_1, ..., X_{p-1}$:

$$\hat{\beta}_i = \hat{\beta}_i^* \times \frac{sd(Y)}{sd(X_i)}$$

Table 3: Original Model Estiamtes and C.I.

Model Term	Coefficient	95% C.I. Lower Bound	95% C.I. Upper Bound
Gestation Weeks	75.667	54.522	96.811
Mother's Age	30.252	-42.821	103.324
Mother's Age Squared	-0.582	-1.884	0.721

Table 4: Estimaes obtained via Back-Trnasformation and C.I.

Model Term	Coefficient	95% C.I. Lower Bound	95% C.I. Upper Bound
Gestation Weeks	75.678	54.522	96.811
Mother's Age	30.268	-42.821	103.324
Mother's Age Squared	-0.582	-1.884	0.721

```
transform_back <-
  function(Beta_star, s_x, s_y){
    Beta_star * (s_y / s_x)
  }

S_Y <- sd(infants_f$birth_w)
```

Hid code to prepare the table.

recall the the original model with the transformed variables was called `inf_lm`. Used it for Extra SS, t-tests and model effects. We can obtain standard errors and confidence intervals for the estimates to compare with the transformation back from the correlation transformation procedure.

```
conf <- data.frame(confint(inf_lm)) # just the confidence intervals
conf <- cbind(coefficients(inf_lm), conf )
```

so we can use linear transformations good to know

13.4

```
cig <- read_xlsx('/Users/denisostroushko/Desktop/UofM MS/MS Fall 2022/Puhb 7405/Data Sets/E-CID-3.xlsx')

cig$Y1 <- with(cig, log(NNAL_vt4_creat / NNAL_vt0_creat))
cig$Y2 <- with(cig, log(TNE_vt4_creat / TNE_vt0_creat))

cig <- cig %>%
  select(Y1, Y2, arm, age, gender, white, educ2, income30, FTND)

colnames(cig)[length(cig)] <- "ftnd"
```

13.4 - A

- Arm will result in 4 -1 variables
- Age is untouched
- FTND is treated as continuous

- Others need to be converted to factor variables

```
cig <- cig %>% select(
  Y1, Y2, age, arm, gender, educ2, income30, ftnd
)

cig$arm <- as.factor(cig$arm)

cig <- data.frame(one_hot(as.data.table(cig))) %>% select(-arm_5)

cig[,4:(length(cig)-1)] <- lapply(cig[,4:(length(cig)-1)], as.factor)
```

```
n_unique <- function(x){length(unique(x))}

meta_data <-

  data.frame(
    class = sapply(cig, class),
    n_unique = sapply(cig, n_unique)
  )
```

Table 5: Sumamry of Covariates

Predictors	Assigned Class	N of Unique Values
age	numeric	51
arm_6	factor	2
arm_7	factor	2
arm_8	factor	2
gender	factor	2
educ2	factor	2
income30	factor	2
ftnd	numeric	8

```
## [1] 8
```

13.4 - B

Regression on Y1

```
y1_lm1 <- lm(Y1 ~ ., data = cig %>% select(-Y2))
summary(y1_lm1)
```

```
##
## Call:
## lm(formula = Y1 ~ ., data = cig %>% select(-Y2))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.39111 -0.34404  0.01998  0.42870  2.59734
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.026537   0.281062   0.094 0.924880
```

```
## age         -0.002896   0.004130  -0.701 0.484025
## arm_61      -0.689983   0.175115  -3.940 0.000115 ***
## arm_71      -0.068278   0.174321  -0.392 0.695743
## arm_81      -0.425510   0.178753  -2.380 0.018303 *
## gender2     -0.112320   0.108917  -1.031 0.303768
## educ22      -0.066044   0.112269  -0.588 0.557069
## income302   -0.228841   0.119094  -1.922 0.056196 .
## ftnd         0.045966   0.042042   1.093 0.275657
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.751 on 186 degrees of freedom
## Multiple R-squared:  0.1628, Adjusted R-squared:  0.1268
## F-statistic: 4.521 on 8 and 186 DF,  p-value: 0.00004854
```

```
sum2 <- data.frame(summary(y1_lm1)$coefficients)

sum2$names <- c("Intercept", "Arm 6", "Arm 7", "Arm 8",
                "Age", "Gender", "White")

rownames(sum2) <- NULL

sum2 <- sum2 %>%  dplyr::select(names, everything())

round_3 <- function(x){round(x,3)}
sum2[,2:5] <- lapply(sum2[,2:5], round_3)

colnames(sum2) <-c("Model Term", "Estimate", "Std. Error", "T-value", "P-value")

kbl(sum_data, booktabs = T, caption = "Original Scale Regression Estimates") %>%
  kable_styling(latex_options = c("striped", "HOLD_position"))
```

- Bonferroni Adjustments
- HOLM Adjustments
- Hochberg Adjustments

Regression on Y2

```
y2_lm1 <- lm(Y2 ~ ., data = cig %>% select(-Y1))
summary(y2_lm1)
```

```
##
## Call:
## lm(formula = Y2 ~ ., data = cig %>% select(-Y1))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.0046 -0.1844  0.2342  0.5819  1.6352
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.183014   0.438043  -0.418    0.677
## age         -0.001567   0.006437  -0.243    0.808
## arm_61      -0.277558   0.272922  -1.017    0.310
```

```
## arm_71       0.195040   0.271684   0.718    0.474
## arm_81      -0.094934   0.278592  -0.341    0.734
## gender2     -0.096213   0.169751  -0.567    0.572
## educ22      -0.197596   0.174974  -1.129    0.260
## income302   -0.218290   0.185612  -1.176    0.241
## ftnd         0.056018   0.065523   0.855    0.394
##
## Residual standard error: 1.171 on 186 degrees of freedom
## Multiple R-squared:  0.05224,    Adjusted R-squared:  0.01148
## F-statistic: 1.282 on 8 and 186 DF,  p-value: 0.2554
```

- Bonferroni Adjustments
- HOLM Adjustments
- Hochberg Adjustments

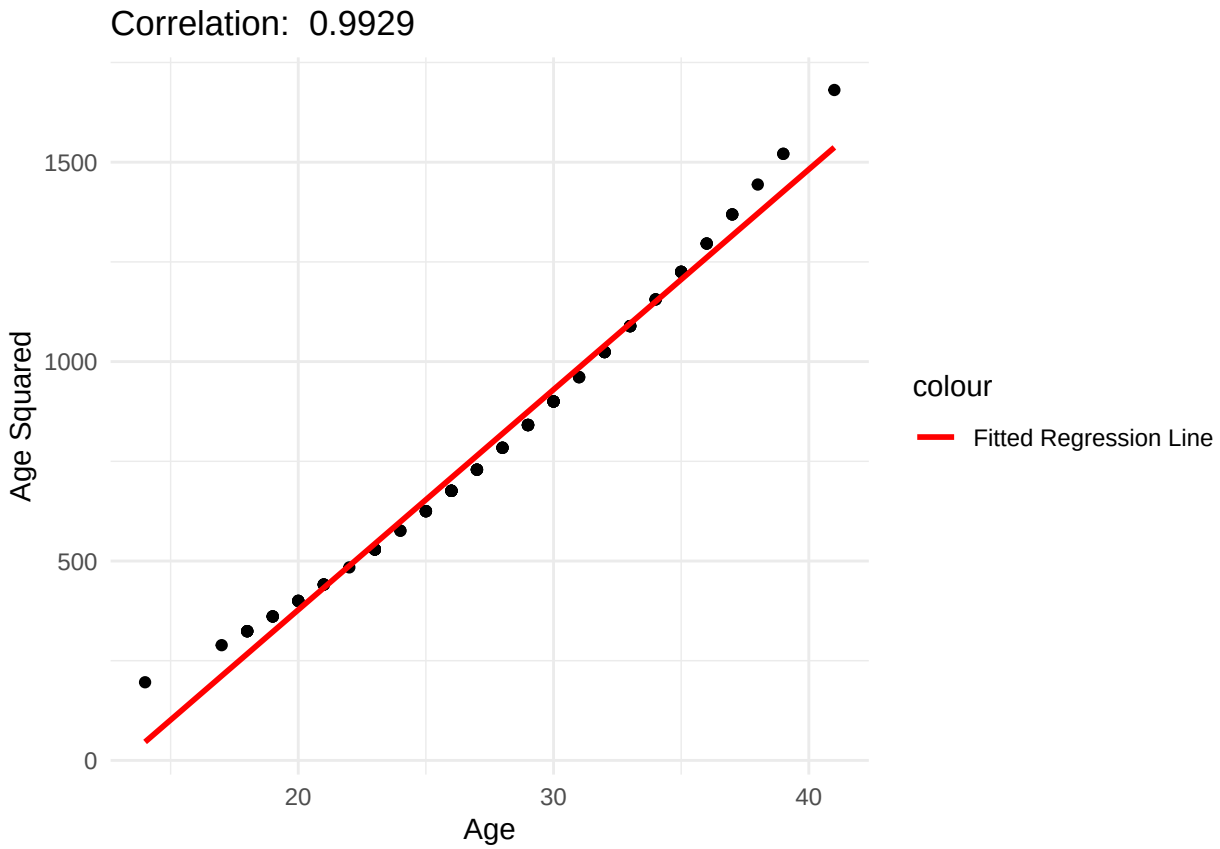
13.4 - C

Step Wise Regression on Y1

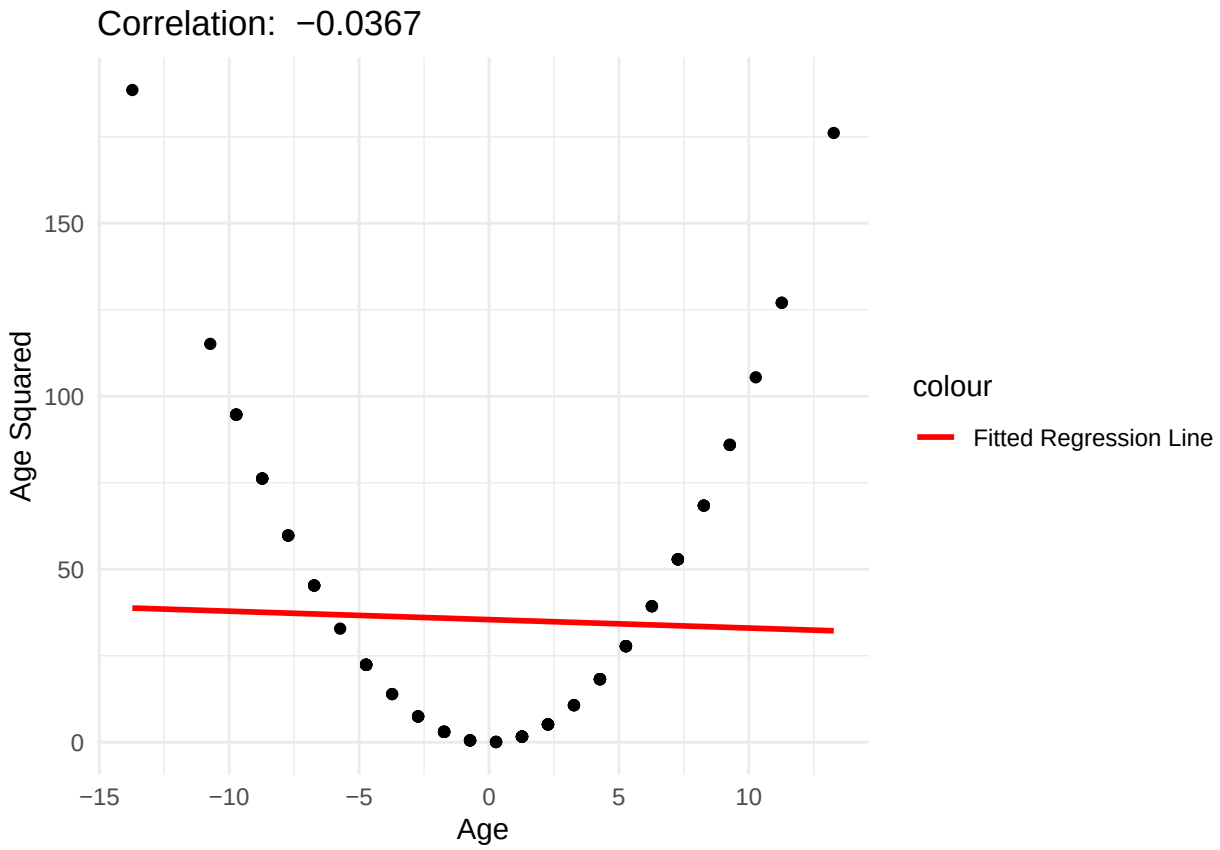
Step Wise Regression on Y1

Appendix: 12.2

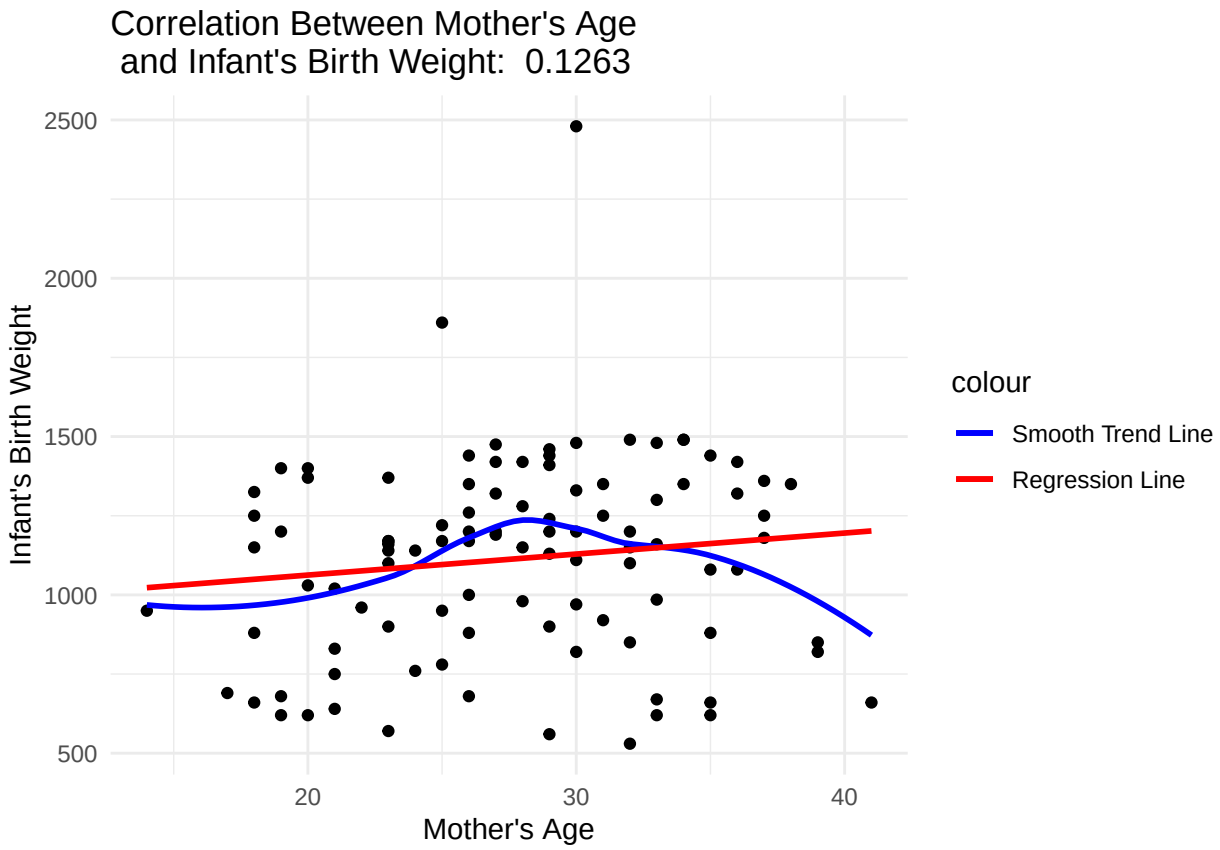
```
# look at the correlation between age and age^2
ggplot(data = infants_f,
       aes(x = m_age,
           y = m_age^2 )) + geom_point() +

  stat_smooth(method = "lm", se = F, aes(color = "Fitted Regression Line")) +
  scale_color_manual(values = c("Fitted Regression Line" = "red")) +

  xlab("Age") +
  ylab("Age Squared") +
  ggtitle(paste("Correlation: ", round(cor(infants_f$m_age, infants_f$m_age^2),4))) +
  theme_minimal()
```



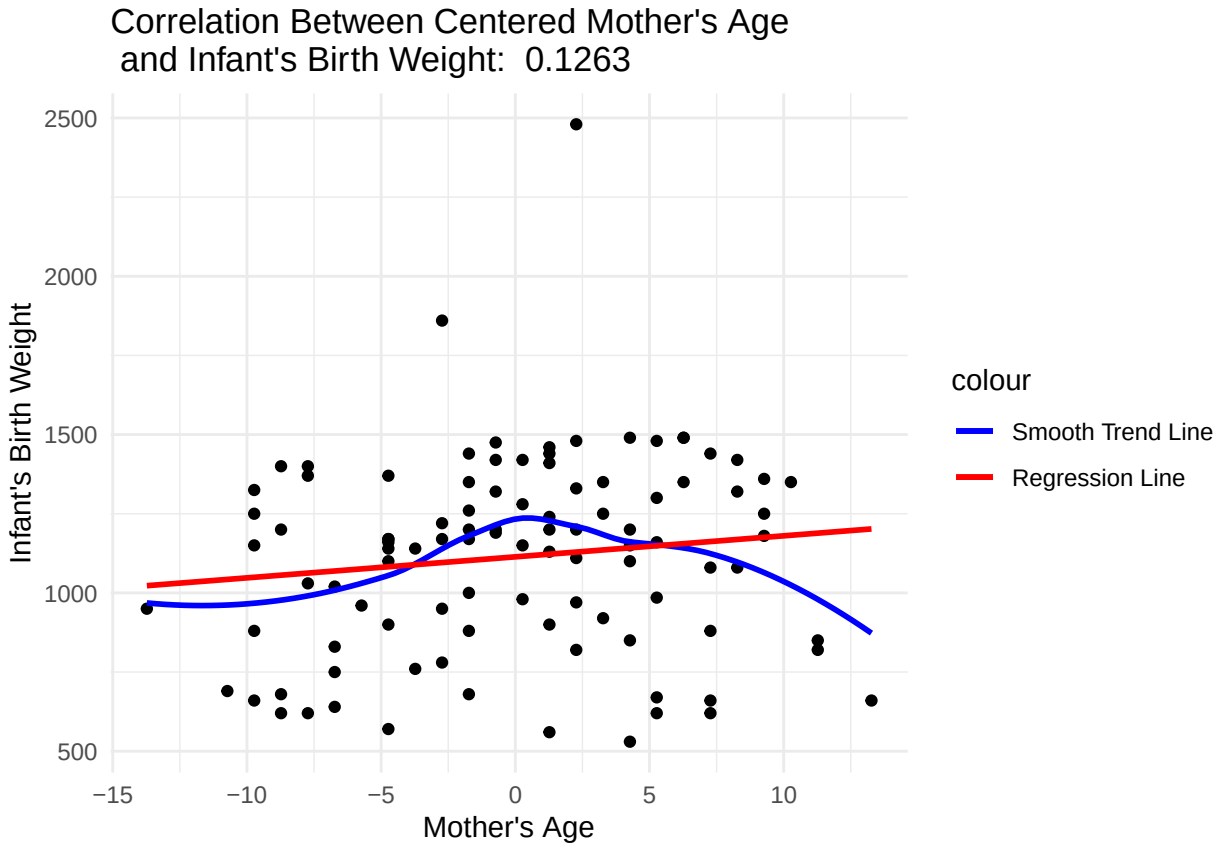

```
# now apply centering:

infants_f$m_age_centered <- with(infants_f, m_age - mean(m_age))
infants_f$gest_weeks_centered <- with(infants_f, gest_weeks - mean(gest_weeks))

ggplot(data = infants_f,
       aes(x = m_age_centered,
           y = m_age_centered^2 )) + geom_point() +

  stat_smooth(method = "lm", se = F, aes(color = "Fitted Regression Line")) +
  scale_color_manual(values = c("Fitted Regression Line" = "red")) +

  xlab("Age") +
  ylab("Age Squared") +
  ggtitle(paste("Correlation: ", round(cor(infants_f$m_age_centered, infants_f$m_age_centered^2),4))) +
  theme_minimal()
```




```
ggplot(data = infants_f,
       aes(x = m_age,
           y = birth_w)) + geom_point() +

  stat_smooth(se = F, aes(color = "Smooth Trend Line")) +
  stat_smooth(se = F, method = "lm", aes(color = "Regression Line")) +

  scale_color_manual(values = c("Smooth Trend Line" = "blue",
                                "Regression Line" = "red")) +
  xlab("Mother's Age") +
  ylab("Infant's Birth Weight") +
  ggtitle(paste("Correlation Between Mother's Age  \n and Infant's Birth Weight: ",
                round(cor(infants_f$m_age,
                          infants_f$birth_w), 4))) +
  theme_minimal()
```




```
ggplot(data = infants_f,
       aes(x = m_age_centered,
           y = birth_w)) + geom_point() +

  stat_smooth(se = F, aes(color = "Smooth Trend Line")) +
  stat_smooth(se = F, method = "lm", aes(color = "Regression Line")) +

  scale_color_manual(values = c("Smooth Trend Line" = "blue",
                                "Regression Line" = "red")) +
  xlab("Mother's Age") +
  ylab("Infant's Birth Weight") +
  ggtitle(paste("Correlation Between Centered Mother's Age  \n and Infant's Birth Weight: ",
                round(cor(infants_f$m_age_centered,
                          infants_f$birth_w), 4))) +
  theme_minimal()
```


Correlation Between Centered Mother's Age
and Infant's Birth Weight: 0.1263
Infant's Birth Weight
2500
2000
1500
1000
500
−15
−10
−5
0
5
10
Mother's Age
colour
Smooth Trend Line
Regression Line