Homework 3

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Probem 1

Logistic regression model for fitted data is given by:

$$logit(P(Cancer = Yes)) = -7 + 0.1 * A + 1.2 * S + 0.3 * R + 0.2 * R * S$$

YS conditional odds ratio equation

Conditional YS odds ratio is presented when we compare $R = 1$ to $R = 0$ and let S be a variable in the resulting odds ratio. Then, varying levels of smoking will further change odds ratio for $R = 1$ vs $R = 0$.

$$OR(R|S = s) = \frac{\frac{P(R=1|S=s)}{1-P(R=1|S=s)}}{\frac{P(R=0|S=s)}{1-P(R=0|S=s)}} =$$

Odds ratio for both numerator and denominator simplify to a single exponential term. We hold A constant while adjusting for it in our comparison. We let S = s be an arbitrary value of S that takes on value 0 or 1.

$$\frac{exp(-7 + 0.1 * A + 1.2 * s + 0.3 * 1 + 0.2 * s * 1)}{exp(-7 + 0.1 * A + 1.2 * s + 0.3 * 0 + 0.2 * s * 0)} =$$

$$exp(0.3 + 0.2 * s)$$

This odds ratio is the compares the effects of race on the likelihood of having cancer, while adjusting for smoking. For black smokers, we have the highest chance of getting cancer, and white non-smokers have the lowest chance of getting cancer.

More precisely, black non-smokers are $exp(0.3) = 1.3499$ times more likely to have cancer, while black smokers are $exp(0.3 + 0.2) = 1.6487$ times more likely to have cancer, after adjusting for other variables.

YR conditional odds ratio equation

Conditional YR odds ratio is presented when we compare $S = 1$ to $S = 0$ and let R be a variable in the resulting odds ratio. Then, varying levels of smoking will further change odds ratio for $S = 1$ vs $S = 0$.

$$OR(S|R = r) = \frac{\frac{P(S=1|R=r)}{1-P(S=1|R=r)}}{\frac{P(S=0|R=r)}{1-P(S=0|R=r)}} =$$

$$= \frac{exp(-7 + 0.1 * A + 1.2 + 0.3 * r + 0.2 * 1 * r)}{exp(-7 + 0.1 * A + 1.2 * 0 + 0.3 * r + 0.2 * 0 * r)}$$

$$exp(1.2 + 0.2 * r)$$

So, smokers are $exp(1.2) = 3.3201$ times more likely to have cancer when compared with non-smokers, after adjusting for other variables. Additionally, black smokers are $exp(1.2 + 0.2) = 4.0552$ times more likely to have cancer, after adjusting for other variables.

MORE TO FINISH THE PROBLEM

Problem 2

Stage 3 model summary table is:

	Estimate	Std..Error	z.value	Pr...z..
(Intercept)	0.3908113	0.0845813	4.620538	0.0000038
Eyes	-2.3960414	0.3878916	-6.177090	0.0000000
Pyes	-1.0994964	0.1786745	-6.153627	0.0000000
GMale	0.3088840	0.1458203	2.118252	0.0341538
Eyes:Pyes	1.7998744	0.5129536	3.508844	0.0004501

change all E, P, G,etc… to their real names for easier reading

Effect of G

interpret effects of 0.308884 and use 0.1458203 to get confidence intervals if independent then their interaction must be non-significant

According to comments from TA we also need to state every

test it:

1. Null hypothesis: $H_0 : \hat{\beta}_G = 0$

1.1 Mull hyp in english

2. Alternative hypothesis: $H_a : \hat{\beta}_G \neq 0$

2.1 Alt hyp in english

3. Z statistic: $(\frac{\hat{\beta}-0}{se(\hat{\beta})}) = 2.1183$
4. P-value: 0.0342
5. Conclusion: There is enough statistical evidence to conclude that the effect

Independence of E and P

if independent then their interaction must be non-significant

test it:

1. Null hypothesis: $H_0 : \hat{\beta}_{E\ and\ P} = 0$
2. Alternative hypothesis: $H_a : \hat{\beta}_{E\ and\ P} \neq 0$
3. Z statistic: $(\frac{\hat{\beta}-0}{se(\hat{\beta})}) = 3.5088$
4. P-value: 5×10^{-4}
5. Conclusion: Effects of E and P are not independent of each, as evidenced by the low p-value and big z-statistic. Therefore, we can conclude that effects of variable E have varying effects on the outcome M, depending on the levels of variable P, after adjusting for other variables.

Problem 3

(i)

INTERPRET ON ODDS SCALE, SO USE EXPOENENTIATED COEFFICIENTS

Radiation Level	Estimate	Std. Error	Z-value	P-value
Intercept	-3.370	0.282	-11.947	0.000
radiation1to9	-0.319	0.533	-0.598	0.550
radiation10to49	-0.038	0.535	-0.071	0.944
radiation50to99	0.618	0.659	0.939	0.348
radiation100to199	1.322	0.602	2.198	0.028
radiation200plus	2.764	0.407	6.795	0.000

Radiation Level	Estimate	Std. Error	Z-value	P-value
Intercept	-3.566	0.212	-16.800	0
radiation_midpoint	0.012	0.001	7.819	0

(ii)

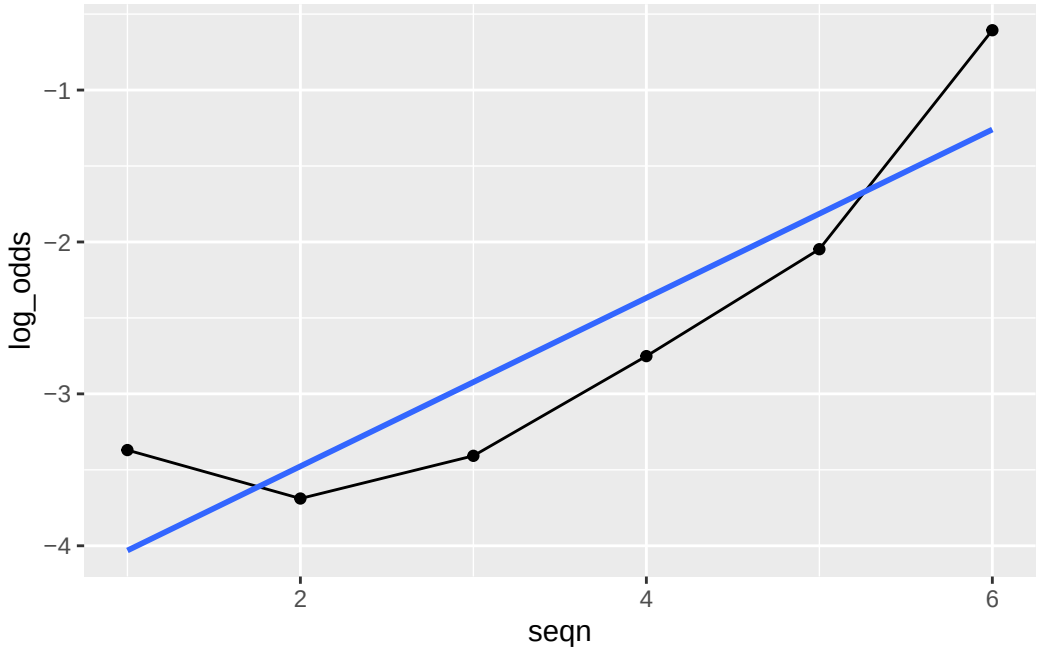
(iii)

COMPARE to model prob3_glm

Prefer this one:

becuase claerly lienar trend is better

Avoid multiple comparisons



Deviance Comparison

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Analysis of Deviance Table

Model: binomial, link: logit

Response: cbind(leukemia = prob3$leukemia, other = prob3$other)

Terms added sequentially (first to last)

                Df Deviance Resid. Df Resid. Dev
NULL                                5     54.351
prob3$radiation  5   54.351         0      0.000


Analysis of Deviance Table

Model: binomial, link: logit

Response: cbind(leukemia = prob3$leukemia, other = prob3$other)

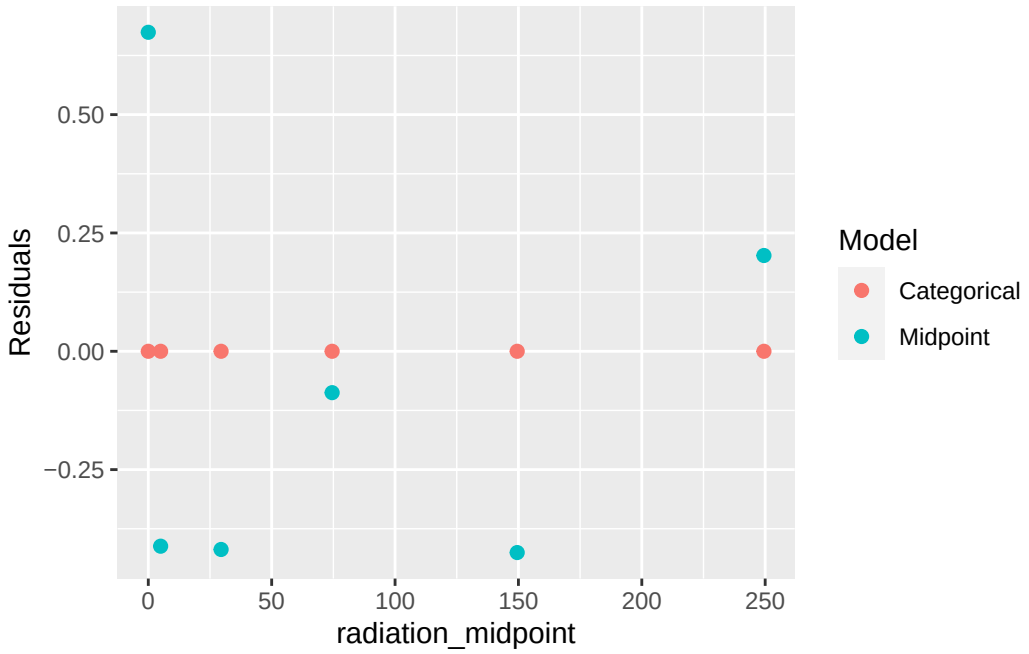
Terms added sequentially (first to last)

                          Df Deviance Resid. Df Resid. Dev
NULL                                         5     54.351
prob3$radiation_midpoint  1   53.322         4      1.029


Analysis of Deviance Table

Model 1: cbind(leukemia = prob3$leukemia, other = prob3$other) ~ prob3$radiation
Model 2: cbind(leukemia = prob3$leukemia, other = prob3$other) ~ prob3$radiation_midpoint
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
1         0     0.0000
2         4     1.0287 -4  -1.0287   0.9054
```

Residual Plots



(iv)

Given a logistic regression model with an intercept and one predictor, odds ratio for a one unit change in predictor X are given by $e^{\hat{\beta}_1*((x+1)-x)} = e^{\hat{\beta}_1}$

Therefore, for two values of X that are more than one unit apart, denoted as W_1 and W_2 are given by $e^{\hat{\beta}_1*(W_1-W_2)}$

Using a full notation and including intercepts we have $e^{\hat{\beta}_0-\hat{\beta}_0+\hat{\beta}_1*(W_1-W_2)}$

Now we can take a ratio of odds ratios, keeping intercepts in the notation. We denote levels of X from the first odds ratio as W_1 and W_2, and levels of X from the second odds ratio as Z_1 and Z_2. In each case we compare odds for level with subscript 1 to level with subscript 2.

We then take a ratio of odds ratio for $W's$ to odds ratio for $Z's$. Estimator is given below:

$$e^{\hat{\beta}_0(1-1-1+1)+\hat{\beta}_1*(W_1-W_2-Z_1+Z_2)}$$

We can see that algebraic signs follow a patter, and we have one real number as a multiplier for each model parameters. Note that a real number for $\hat{\beta}_0$ is zero, however, it is convenient to keep it there as for the derivation of a matrix form calculation.

Therefore, as per Jared's tip, a ratio of odds ratios is a function of four values of X, and can be represented as

$$e^{\mathbf{a}^T\hat{\beta}}$$

, where $\mathbf{a}^T = [1-1-1+1, W_1 - W_2 - Z_1 + Z_2]$

So, the ratio of the odds of having leukemia comparing a radiation level of '100 to 199' and a radiation level of '50 to 99' is given by $e^{\hat{\beta}_0(1-1-1+1)+\hat{\beta}_1*(100-199-50+99)} = 0.559218$

(v)

In order to obtain a confidence interval for the ratio of odds ratio we can take two approaches:

1. Calculate confidence interval for $\mathbf{a}^T\hat{\beta}$, and then exponentiate the interval
2. Use the delta method to calculate $Var(e^{\mathbf{a}^T\hat{\beta}})$ and then calculate the 95% confidence interval using a standard error of the odds scale directly.

For my own reference, I will use both methods, and validate that the results indeed match.

Log odds confidence interval

Using Jared's tip, we can calculate $Var(\mathbf{a}^T\hat{\beta})$ directly by taking $a^T\mathbf{V}a$ where V is a variance-covariance matrix of the fitted logistic regression model. Variance-covariance estimates are given for model estimates including the intercept.

Using `R` output we estimate $Var(\mathbf{a}^T\hat{\beta}) = 0.005525$

Then, the 95% confidence interval on the original scale is $\mathbf{a}^T\hat{\beta} \pm 1.96 * \sqrt{Var(\mathbf{a}^T\hat{\beta})}$ = -0.581215 $\pm$1.96$*$ 0.07433

Taking exponential of interval end point given us a confidence interval for the ratio of odds ratios. The 95% confidence interval is (0.483404, 0.646923)