

Homework 1

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Question 1

a)

Outcomes for 20th subject are shown in Table 1

Table 1: Outcomes for 20th subject in the study

Y20
0.101
0.074
0.059
0.019
0.082

b)

In the long format of the data, where we have 107 unique participants, and 5 observations per Id, the full length of responses \mathbf{Y} is 535

c)

We are given that for a random variable that generate observed values for the 20th participant, $Var(Y_{20\ j}) = 0.05$, so variance is constant for each observation $j = 1, 2, 3, 4, 5$.

Covariance of i^{th} and j^{th} observations is also constant at 0.2.

Covariance of i^{th} observation with itself is variance, an is 0.05.

Table 2 displays resulting variance-covariance matrix.

Table 2: Variance-covariance matrix for 20th participant

0.05	0.02	0.02	0.02	0.02
0.02	0.05	0.02	0.02	0.02
0.02	0.02	0.05	0.02	0.02
0.02	0.02	0.02	0.05	0.02
0.02	0.02	0.02	0.02	0.05

We can now convert this matrix into the correlation matrix.

$$\rho_{ij} = \frac{Cov(Y_{ij}, Y_{jk})}{\sqrt{Var(Y_{ij}) * Var(Y_{jk})}}$$

Since all variances are equal to 0.05, the calculation is pretty straightforward.

Resulting correlation matrix is shown in Table 3

Table 3: Correlation matrix for 20th individual

1.0	0.4	0.4	0.4	0.4
0.4	1.0	0.4	0.4	0.4
0.4	0.4	1.0	0.4	0.4
0.4	0.4	0.4	1.0	0.4
0.4	0.4	0.4	0.4	1.0

Each observation on the diagonal is equal to 1, which makes the results more credible

d)

A variance-covariance matrix for each individual is 5×5 in size, and we have 107 individuals, therefore the size of full matrix is $(5 \times 107) \times (5 \times 107)$, which means that the dimension of variance covariance matrix Σ is 535×535 .

e)

We consider trial number, age, and sex of a participant as predictors. Values provided in the dataset are given in the Table 4.

Table 4: Covariates for 20th participant

Trial Number	Age	Sex
1	31	0
2	31	0
3	31	0
4	31	0
5	31	0

However, we also need a column of 1s in order to estimate $\hat{\beta}_0$. A full matrix \mathbf{X}_{20} for the 20th participant is given in Table 5

Table 5: Full matrix for model estimation

1	1	31	0
1	2	31	0
1	3	31	0
1	4	31	0
1	5	31	0

f)

Including intercept column, the full dimension of \mathbf{X} is 535×4 . Without intercept, the size is \mathbf{X} is 535×3 .

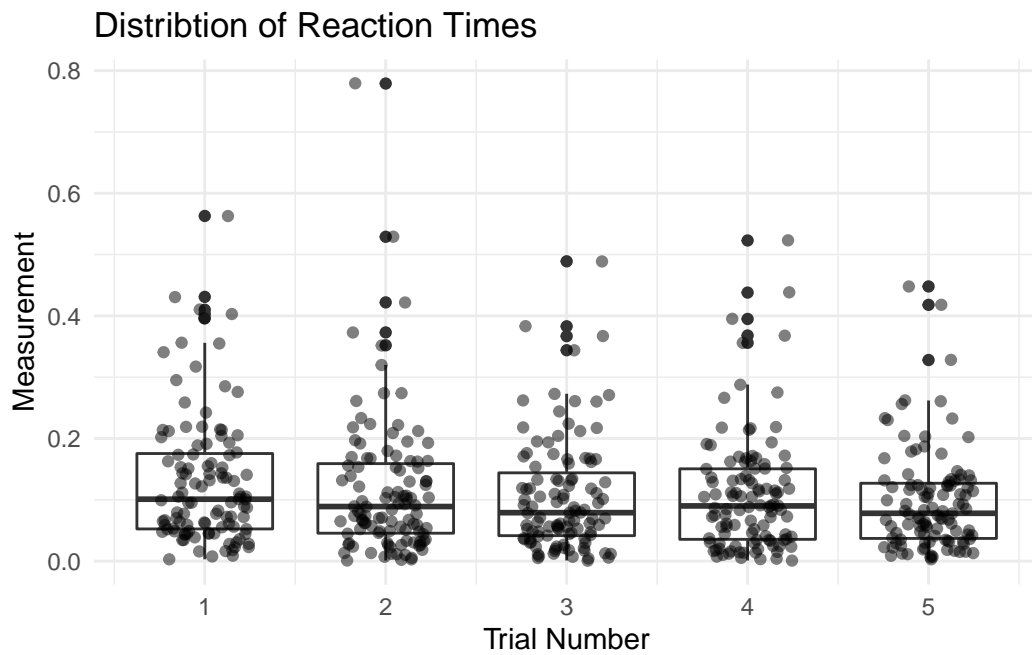


Figure 1: Distribution of Reaction Measurements in Each Trial

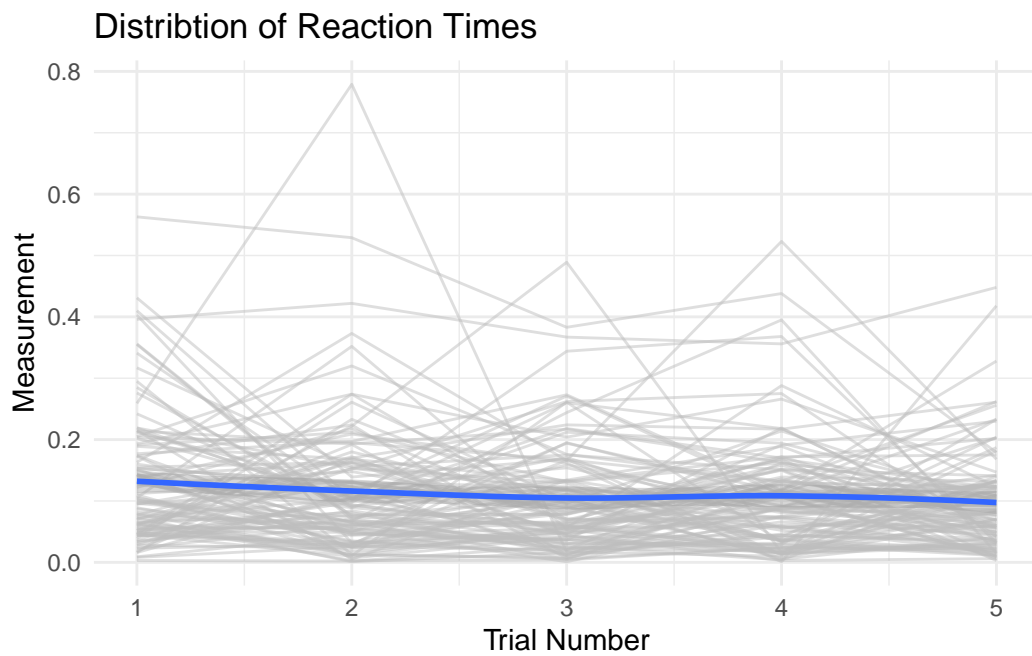


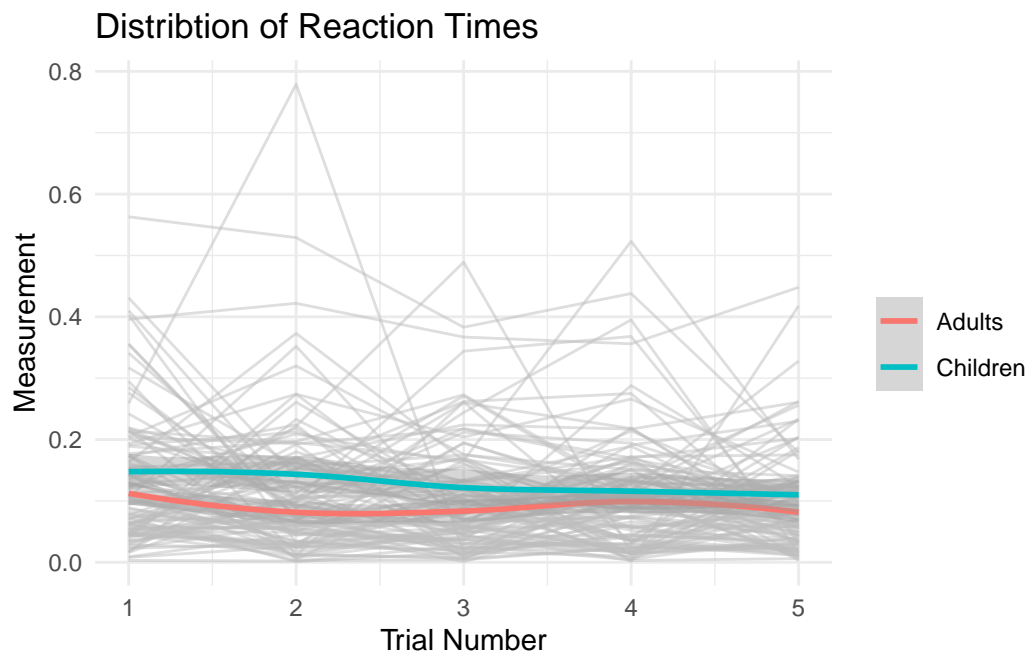
Figure 2: Individual Profiles of Measurements for each participant

Question 2

a)

b)

c)



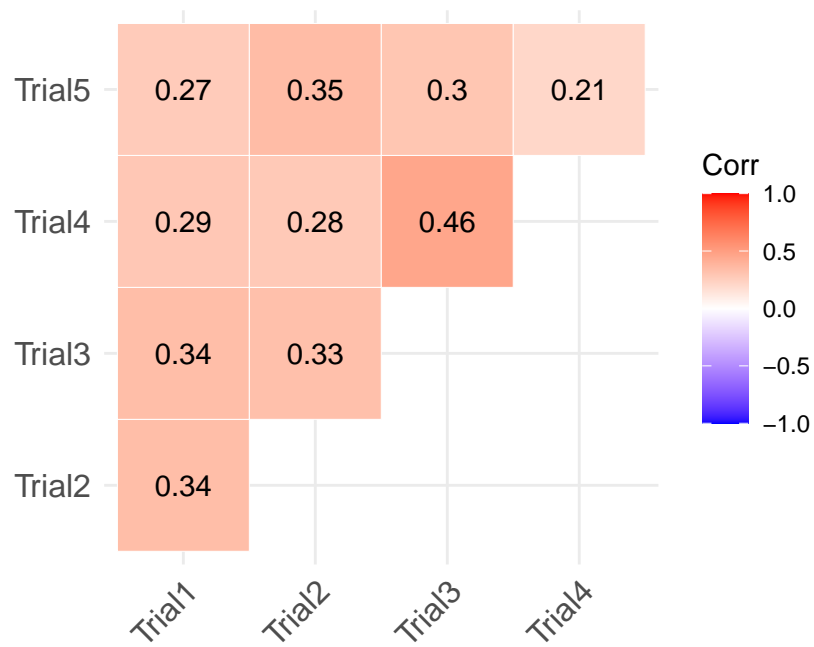


Figure 3: ?(caption)

Question 3

a)

b)

Question 4

a)

b)

i)

ii)

iii)