Problem 1

- $Y_{i\alpha} \sim Pois(n_{i\alpha} \lambda_{i\alpha})$, $\lambda_{i\alpha} \sim Gamma(\gamma_{0\alpha}, \nu_{0\alpha})$

- $i \Rightarrow$ county number
  $\alpha \Rightarrow$ age group.

- $Y_{i\alpha}$ = death due to death stroke

  $n_{i\alpha}$ = population

  $\lambda_{i\alpha}$ = death rate

- $$p(Y_{i\alpha} \mid n_{i\alpha} \lambda_{i\alpha}) = \frac{e^{-(n_{i\alpha}\lambda_{i\alpha})} (n_{i\alpha}\lambda_{i\alpha})^{Y_{i\alpha}}}{(Y_{i\alpha})!}$$

- $$p(\lambda_{i\alpha} \mid \gamma_{0\alpha}, \nu_{0\alpha}) = \frac{\nu_{0\alpha}^{\gamma_{0\alpha}}}{\Gamma(\gamma_{0\alpha})} \cdot \lambda_{i\alpha}^{\gamma_{0\alpha}-1} e^{(-\nu_{0\alpha}\lambda_{i\alpha})}$$

- Posterior:

  $$p(\underline{\lambda_{i\alpha}} \mid \underline{Y_{i\alpha}}) \propto \frac{e^{-(n_{i\alpha}\underline{\lambda_{i\alpha}})} (n_{i\alpha}\underline{\lambda_{i\alpha}})^{\underline{Y_{i\alpha}}}}{(\underline{Y_{i\alpha}})!} \times$$

$$\times \frac{\nu_{0\alpha}^{\gamma_{0\alpha}}}{\Gamma(\gamma_{0\alpha})} \cdot \lambda_{i\alpha}^{(\gamma_{0\alpha}-1)} e^{(-\nu_{0\alpha}\lambda_{i\alpha})} \propto$$

$$\frac{e^{-(\lambda_{i\alpha})} \lambda_{i\alpha}^{y_{i\alpha}} \lambda_{i\alpha}^{\gamma_{0\alpha}-1} e^{(-\nu_{0\alpha}\lambda_{0\alpha})}}{(y_{i\alpha})!} \propto$$

$$\lambda_{i\alpha}^{(y_{i\alpha}+\gamma_{0\alpha})-1} e^{-(\nu_{0\alpha}+\nu_{i\alpha})\lambda_{i\alpha}}$$

This resembles a kernel of a gamma distribution, so, we conclude that a posterior distribution of $\lambda_{i\alpha}$ is given by

$$\lambda_{i\alpha} | y_{i\alpha} \sim \text{Gamma}(\gamma_{0\alpha}+y_{i\alpha}, \nu_{0\alpha}+\nu_{i\alpha})$$

So, a full conditional distribution can be written as

$$p(\lambda_{i\alpha} | y_{i\alpha}, \gamma_{0\alpha}, \nu_{i\alpha}, \nu_{0\alpha}) =$$

$$= \frac{(n_{0\alpha} + n_{i\alpha})^{y_{i\alpha} + y_{0\alpha}}}{\Gamma(y_{i\alpha} + y_{0\alpha})} \times \lambda_{i\alpha}^{(y_{0\alpha} + y_{i\alpha} - 1)} e^{-(n_{0\alpha} + n_{i\alpha})\lambda_{i\alpha}}$$