PUBH 7440: Intro to Bayesian Analysis
Homework from Week 2 — Due Feb 6

For the unknown parameters in each of the following scenarios, *find/show* that something is a conjugate prior for the unknown parameter in each model (i.e., write $p(\theta \mid Y) \propto p(Y \mid \theta) \times p(\theta)$, show a couple of steps, then show the posterior). If you can't find a conjugate prior, explain what you might look for in a prior (e.g., τ^2 must be greater than 0, so a gamma distribution might be appropriate; $\theta \in (0,1)$, so a beta distribution might be appropriate; etc.). For each prior, provide an interpretation of the prior parameters. Vaguely describe what might make for a relatively noninformative prior.

1. $x \sim \text{Bin}(n, \theta)$, n known
2. $x \sim \text{NegBin}(r, \theta)$, r known
3. $\mathbf{x} \sim \text{Mult}(n, \boldsymbol{\theta})$, n known
4. $x \sim \text{Gam}(\alpha, \beta)$, α known
5. $x \sim \text{Gam}(\alpha, \beta)$, β known
6. **OPTIONAL**: $\mathbf{x} \sim \text{Norm}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma}$ known
7. **OPTIONAL**: $\mathbf{x} \sim \text{Norm}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\boldsymbol{\mu}$ known (this one is fun)

Hint: You can use the table on the next page to refresh your memory what different mass/density functions look like.

GLORIOUS (YET INCOMPLETE) TABLE OF DENSITY/MASS FUNCTIONS

Distribution	Density/Mass Function
$x \sim \text{Bern}(\pi)$	$p(x \mid \pi) = \pi^x (1-\pi)^{1-x}$
$x \sim \text{Geometric}(\pi)$	$p(x \mid \pi) = (1-\pi)^{x-1}\pi$
$x \sim \text{Binomial}(n,\pi)$	$p(x \mid \pi) = \binom{n}{x}\pi^x (1-\pi)^{n-x}$
$x \sim \text{NegBin}(r,\pi)$	$p(x \mid \pi) = \binom{x+r-1}{x}(1-\pi)^{x}\pi^{r}$
$x \sim \text{Pois}(\lambda)$	$p(x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$
$x \sim \text{Laplace}(\mu,b)$	$p(x \mid \mu,b) = \frac{1}{2b}e^{-\lvert x-\mu\rvert/b}$
$x \sim \text{Norm}(\mu,\sigma^2)$	$p(x \mid \mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
$x \sim \text{Gam}(\alpha,\beta)$	$p(x \mid \alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$
$x \sim \text{Exp}(\lambda)$	$p(x \mid \lambda) = \lambda e^{-\lambda x}$
$x \sim \text{IG}(a,b)$	$p(x \mid a,b) = \frac{b^a}{\Gamma(a)}x^{-a-1}e^{-b/x}$
$\mathbf{x} \sim \text{Mult}(n,\boldsymbol{\theta})$	$p(\mathbf{x} \mid n,\boldsymbol{\theta}) = \frac{n!}{\prod_i x_i!}\prod_i \theta_i^{x_i}$
$\mathbf{x} \sim \text{Dir}(\boldsymbol{\alpha})$	$p(\mathbf{x} \mid \boldsymbol{\alpha}) \propto \prod_i x_i^{\alpha_i-1}$
$\mathbf{x} \sim \text{Norm}(\boldsymbol{\mu},\boldsymbol{\Sigma})$	$p(\mathbf{x} \mid \boldsymbol{\mu},\boldsymbol{\Sigma}) \propto \lvert\boldsymbol{\Sigma}\rvert^{-1/2}\exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]$
$\mathbf{X} \sim \text{IW}(\nu,\mathbf{G})$	$p(\mathbf{X} \mid \nu,\mathbf{G}) \propto \lvert\mathbf{X}\rvert^{-(\nu+p+1)/2}\exp\left[-\frac{1}{2}\text{tr}(\mathbf{G}\mathbf{X}^{-1})\right]$

Table 1: Table of density/mass functions for selected distributions. Note that these parameterizations are consistent with those on Wikipedia but may differ from those used in Casella & Berger and other textbooks. I made a table like this for the Biostat PhD exam, but I figured since you might not have a textbook for referencing, it might be good to provide you some guidance here...