Homework 4

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Problem 1

(a)

Proportional Hazards Model: $h(t|Z) = h_0(t)exp(Z'\hat{\beta})$,

where $Z'\hat{\beta}$ is a linear combination of predictors. In our case, $\hat{\beta}$ is a vector of length one, a single model estimate that governs estimated differences between groups.

So, the full expression for the model is: $h(t|Z) = h_0(t)exp(\hat{\beta} * I(Group\ 2))$, thus $Group$ 1 is a reference level.

(b)

We create a dummy variable to conduct regression analysis, set indicator to 0 when an individual is in an 'Untreated' group, and 1 when in 'Radiated' group

To write down the partial likelihood for the model, we need to specify the hazard function for the survival times. For Cox proportional hazards model we assume that the hazard function for each group is proportional to the hazard function for the other group, with a constant hazard ratio over the entire observed timeline.

Generally, partial likelihood for the Cox model is given by:

$$L = \prod_{i=1}^{N} h_0(T_i)^{\delta_i} * [exp(\beta' X_i)]^{\delta_i} * exp(-H_0(T_i)\ exp(\beta' X_i))$$

where:

1. N is the sample size
2. β is the log hazard ratio for the radiated group compared to the untreated group

3. X_i is the value of the dummy variable for the ith individual (i.e., 0 for untreated, 1 for radiated)
4. T_i is the observed survival time for the ith individual,
5. δ_i is an indicator variable that takes the value 1 if the survival time is uncensored

For this case with one dummy predictor the likelihood function has a simpler form

$$L = \prod_{i=1}^{12} h_0(T_i)^{\delta_i} * [exp(\beta * X_i)]^{\delta_i} * exp(-H_0(T_i)\ exp(\beta * X_i))$$

(c)

(d)

The baseline hazard rate for the two models will be different due to different time values that go into the hazard equation. However, variance and betas should be the same due to similar pattern and ordering of events and corresponding groups.

Problem 2

(a)

Summary of fitted Cox PH model is given below. We will use the tets statistics and corresponding p-values to conduct the tests we are required.

```
fit <- coxph(Surv(time, delta) ~ type, method="breslow", data=tongue)
summary(fit)
```

```
Call:
coxph(formula = Surv(time, delta) ~ type, data = tongue, method = "breslow")

  n= 80, number of events= 53

       coef exp(coef) se(coef)     z Pr(>|z|)
type 0.4610    1.5856   0.2805 1.643      0.1

     exp(coef) exp(-coef) lower .95 upper .95
type     1.586     0.6307     0.915     2.748
```

```
Concordance= 0.564  (se = 0.036 )
Likelihood ratio test= 2.61  on 1 df,   p=0.1
Wald test            = 2.7  on 1 df,   p=0.1
Score (logrank) test = 2.75  on 1 df,   p=0.1
```

Breslow

1. Null hypothesis: no difference in survival rates between groups
2. Alternative hypothesis: some difference in survival rates between groups
3. P-value: 0.1
4. Conclusion: we do not reject null hypothesis due to lack of overwhelming statistical evidence. Although, these results might be suggestive of the fact that there is some difference between the groups

Score test

1. Null hypothesis: no difference in survival rates between groups
2. Alternative hypothesis: some difference in survival rates between groups
3. P-value: 0.1
4. Conclusion: we do not reject null hypothesis due to lack of overwhelming statistical evidence. Although, these results might be suggestive of the fact that there is some difference between the groups

(b)

95% confidence interval is given below

```r
exp(confint(fit))
```

```
         2.5 %  97.5 %
type 0.9149511 2.74778
```

(c)

```r
fit2 <- coxph(Surv(time, delta) ~ type, method="exact", data=tongue)
summary(fit2)
```

```
Call:
coxph(formula = Surv(time, delta) ~ type, data = tongue, method = "exact")

  n= 80, number of events= 53 

       coef exp(coef) se(coef)     z Pr(>|z|)  
type 0.4684    1.5974   0.2828 1.656   0.0977 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     exp(coef) exp(-coef) lower .95 upper .95
type     1.597      0.626    0.9177     2.781

Concordance= 0.564  (se = 0.036 )
Likelihood ratio test= 2.65  on 1 df,   p=0.1
Wald test            = 2.74  on 1 df,   p=0.1
Score (logrank) test = 2.79  on 1 df,   p=0.09
```

P-value: 0.0977

(d)

Wald is just regular model without special method specification

```
Call:
coxph(formula = Surv(time, delta) ~ type, data = tongue)

  n= 80, number of events= 53 

       coef exp(coef) se(coef)     z Pr(>|z|)  
type 0.4664    1.5942   0.2804 1.663   0.0963 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     exp(coef) exp(-coef) lower .95 upper .95
```

```
type     1.594     0.6273    0.9201     2.762

Concordance= 0.564  (se = 0.036 )
Likelihood ratio test= 2.67  on 1 df,   p=0.1
Wald test            = 2.77  on 1 df,   p=0.1
Score (logrank) test = 2.81  on 1 df,   p=0.09
```

P-value: 0.0963

(e)

Breslow

```
     coef exp(coef)  se(coef)
0.4609544 1.5855865 0.2805353

     coef exp(coef)  se(coef)
0.4663742 1.5942034 0.2804476
```

Problem 3

(a)

```
        coef exp(coef)  se(coef)         z     Pr(>|z|)
Z1 -1.811971 0.1633320 0.5597120 -3.237327 1.206552e-03
Z2 -3.557371 0.0285137 0.7582538 -4.691530 2.711695e-06
```

(b)

there is no intercept in a cox ph model, so to compare radiated to untreated we need to use estiamte and variance for radiated group

```
       Z1
-1.811971

[1] 0.3132775

        Z1         Z1
-2.9090060 -0.7149351
```

(c)

```
Analysis of Deviance Table
 Cox model: response is  Surv(time, death)
 Model 1: ~ 1
 Model 2: ~ Z1 + Z2
   loglik  Chisq Df Pr(>|Chi|)
1 -73.016
2 -59.331 27.371  2  1.139e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Wald test:
----------

Chi-squared test:
X2 = 22.0, df = 1, P(> X2) = 2.7e-06
```

(d)

```
Wald test:
----------

Chi-squared test:
X2 = 10.5, df = 1, P(> X2) = 0.0012
```

(d)

```
      Z1
1.487844

      Z1
0.136792
```

(e)

```
        Z1          Z1
 0.5747103 57.0934426
```

(e) again

same as part (c)

```
Analysis of Deviance Table
 Cox model: response is  Surv(time, death)
 Model 1: ~ 1
 Model 2: ~ Z1 + Z2
   loglik  Chisq Df Pr(>|Chi|)
1 -73.016
2 -59.331 27.371  2  1.139e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(g)

same as part (c)

```
Wald test:
----------

Chi-squared test:
X2 = 22.0, df = 1, P(> X2) = 2.7e-06
```