

# HW1

## Problem B

### Step 1: uniform random numbers simulation

We draw 10,000 random numbers form a  $U(0, 1)$  distribution

Let  $X = 1$  if the coin comes up Heads, and  $X = 0$  if the coin comes up Tails. Let  $U$  be a uniform  $U(0, 1)$  variable.

We know that  $P(X = 1) = P(X = 0) = 0.5$ .

Therefore,

$$\mathbf{X} = \begin{cases} x_0 = 0, & U \leq P(X = 0) \\ x_1 = 1, & P(X = 0) < U \leq P(X = 0) + P(X = 1) \end{cases} \quad (1)$$

The first 30 uniform random numbers are:

```
[1] 0.28757752 0.78830514 0.40897692 0.88301740 0.94046728 0.04555650
[7] 0.52810549 0.89241904 0.55143501 0.45661474 0.95683335 0.45333416
[13] 0.67757064 0.57263340 0.10292468 0.89982497 0.24608773 0.04205953
[19] 0.32792072 0.95450365 0.88953932 0.69280341 0.64050681 0.99426978
[25] 0.65570580 0.70853047 0.54406602 0.59414202 0.28915974 0.14711365
```

Applying a rule from equation (1) produces coin tosses, the first 30 tosses are:

```
[1] 0 1 0 1 1 0 1 1 1 0 1 0 1 1 0 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0
```

Final proportion of Tails is 0.49

Running proportion of Tails is shown on Figure 1

Figure 1 shows that after about 1,250 coin tosses the proportion of heads and tails converges to around 0.5

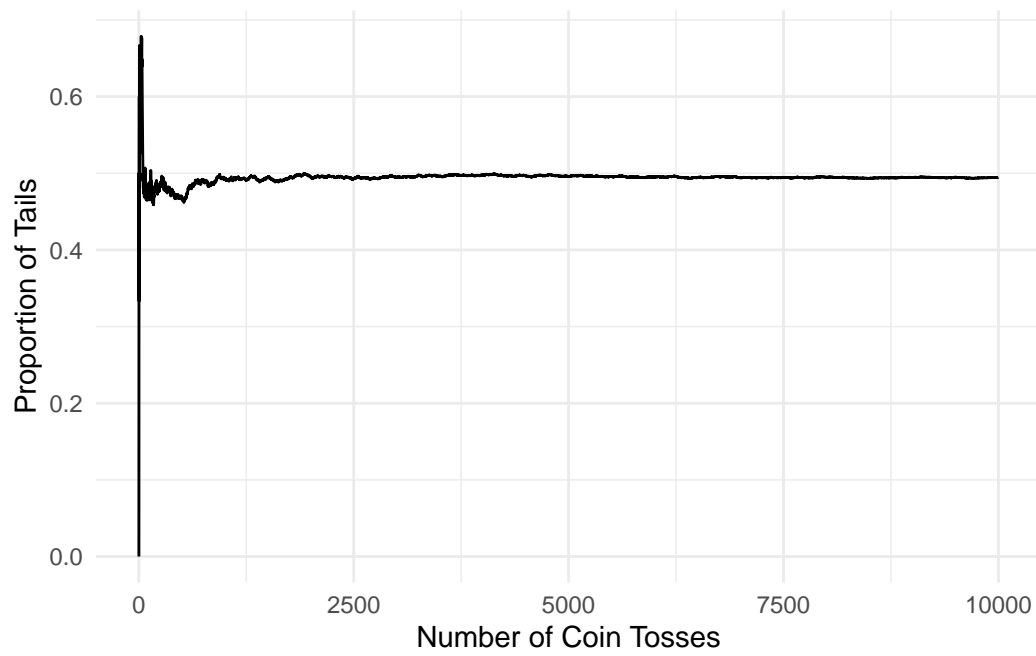


Figure 1: Long running proportions of Tails