PubH 8432, Prob Models Homework 3, Due Nov 5, 2023

Question 1:

Consider the Markov chain in Figure 1. Assume that $1/2 < p < 1$. Does this chain have a limiting distribution? For all $i, j \in \{0, 1, \cdots\}$, find

$$\lim_{n \to \infty} P(X_n = j | X_0 = i)$$

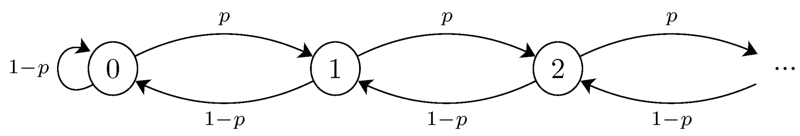

Figure 1:

Questions 2 - 3

Solve questions 70, 79 from chapter 4

Questions 4 - 10

Solve questions 2, 6, 9, 12, 14, 16, and 21 from chapter 5

Simulations

Consider a Hidden Markov Model with transition probability matrix and emission matrix given in Example 4.42 in the Textbook. Let (0,1) represent (acceptable, unacceptable) signals in the HMM.

- Code the Forward and Backward algorithms. Assume $\mathbf{s}_3 = (0, 1, 0)$. Print out $F_n(j)$ and compare your probabilities with that from Example 4.43. Also print out $B_k(i), i = 1, 2; k = 1, 2, 3$
- Using the Forward and Backward algorithms, code the conditional probability mass function

$$P(X_k = j|\mathbf{S^n} = \mathbf{s_n}) = \frac{P(\mathbf{S^n} = \mathbf{s_n}, X_k = j)}{P(\mathbf{S^n} = \mathbf{s_n})} = \frac{F_k(j)B_k(j)}{\sum_j F_k(j)B_k(j)}$$

  for an arbitrary length of signals. Assume $\mathbf{s}_3 = (0, 1, 0)$. Print out $P(X_3 = 1|\mathbf{s}_3)$ and compare your answer with question (a) in example 4.43.
- Predict the likely sequence of states given the sequence of signals $s_3 = (0, 1, 0, 1, 0)$ using the Viterbi algorithm. Print out your results.