HW1

```r
library(tidyverse)
```

Problem B

Step 1: uniform random numbers simulation

We draw 10,000 random numbers form a $U(0,1)$ distribution

```r
set.seed(123)
n_simulations <- 10000
uniform_numbers <- runif(n = n_simulations, min = 0, max = 1)
```

Let $X = 1$ if the coin comes up Heads, and $X = 0$ if the coin comes up Tails. Let U be a uniform $U(0,1)$ variable.

We know that $P(X = 1) = P(X = 0) = 0.5$.

Therefore,

$$\mathbf{X} = \begin{cases} x_0 = 0, \ U \leq P(X = 0) \\ x_1 = 1, \ P(X = 0) < U \leq P(X = 0) + P(X = 1) \end{cases} \tag{1}$$

The first 30 uniform random numbers are:

```r
head(uniform_numbers, 30)
```

```
 [1] 0.28757752 0.78830514 0.40897692 0.88301740 0.94046728 0.04555650
 [7] 0.52810549 0.89241904 0.55143501 0.45661474 0.95683335 0.45333416
[13] 0.67757064 0.57263340 0.10292468 0.89982497 0.24608773 0.04205953
[19] 0.32792072 0.95450365 0.88953932 0.69280341 0.64050681 0.99426978
[25] 0.65570580 0.70853047 0.54406602 0.59414202 0.28915974 0.14711365
```

```r
p <- 0.5

# Simulate the coin toss using the Bernoulli distribution
coin_toss <- ifelse(uniform_numbers <= p, 0, 1)
```

Applying a rule from equation (1) produces coin tosses, the first 30 tosses are:

```r
head(coin_toss, 30)
```

```
 [1] 0 1 0 1 1 0 1 1 1 0 1 0 1 1 0 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0
```

Final proportion of Tails is 0.49

Running proportion of Tails is shown on Figure 1

```r
toss = seq(from = 1, to = length(coin_toss), by = 1)

ggplot(
  data = data.frame(
    toss = toss,
    runprop = cumsum(coin_toss)/toss
  ),
   aes(
    x = toss,
    y = runprop
      )
) + geom_line() +
  theme_minimal() +
  ylab("Proportion of Tails") +
  xlab("Number of Coin Tosses")
```

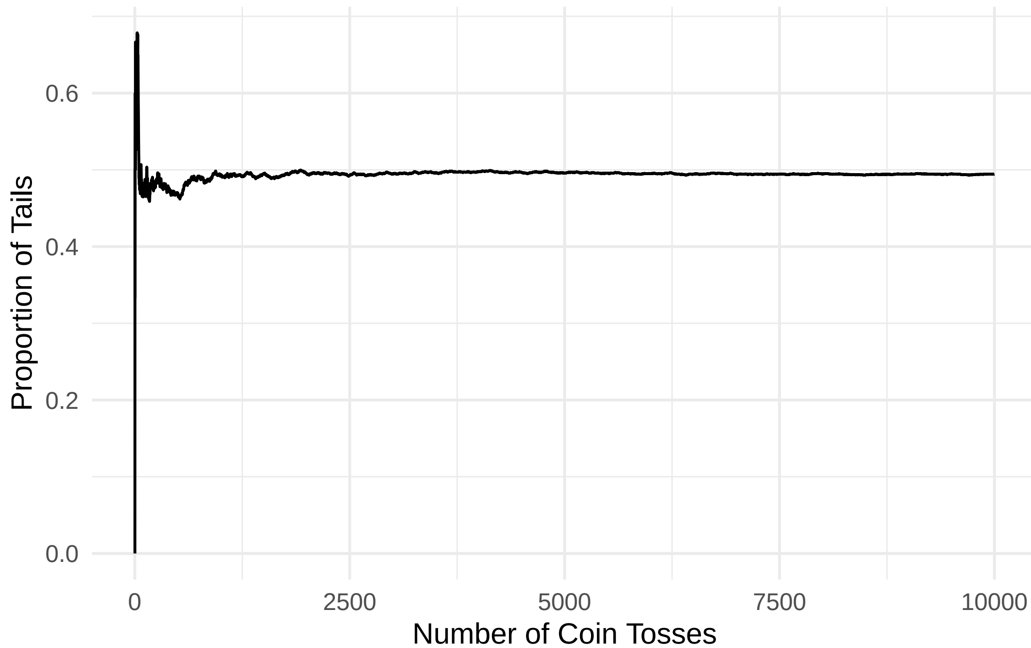

Figure 1: Long running proportions of Tails

Figure 1 shows that after about 1,250 coin tosses the proportion of heads and tails converges to around 0.5