The University of Auckland

Department of Engineering Science

Part IV Project

Developing a Julia/JuMP Optimization Model for Plant Scheduling for an NZ Company

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Abstract

An unnamed NZ company has partnered with the University of Auckland to develop a spreadsheet model to evaluate possible schedules for a production plant. The purpose of the model was to generate appropriate schedules for the plant while minimizing some objective. The main objectives considered included minimizing purchased water for the cleaning of machines, and the retention of target volume levels in the silos to maintain a steady working operation at the plant. If implemented successfully, the unnamed NZ company could use the spreadsheet model to assist in scheduling machines at the product plant.

The Excel add-in, SolverStudio, was used to process the data from the production plant spreadsheet and visualize the results of the model. And Julia/JuMP was used to formulate and optimize the model. The optimization model solutions were presented as a brief schedule and the results of the solver in a table format.

The model generates feasible solutions for a week using the rolling horizon algorithm. These spreadsheet solution results regulate flow at the plant, establishing a working prototype for the plant that can be developed further to better represent the goals of the plant.

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1 Introduction

Production scheduling in the dairy industry has been a prominent theme over the past few decades [1]. Scheduling is the process of allocating resources, such as equipment, utilities, and labour to complete processing tasks to ensure production demands are met. Generally, scheduling decisions at the plants are often made by managers and operators in an ad hoc manner, meaning performance and efficiency heavily rely on their experience and how well they understand the processes and equipment [2].

In this project, an optimization model has been developed for a dairy plant to schedule its production processes efficiently. A spreadsheet of the current model was provided, which was used to evaluate the possible schedules at the plant manually. The implementation of an automated model would help the plant produce optimal, high-quality plant schedules. This will help reduce the involvement of a worker in decision-making. Consequently, this could help a company save on resources such as power and cost.

The optimization model will be developed using the Julia/JuMP optimization framework that interacts with the spreadsheet model to generate and display the solution schedules. The model will be solved mainly using the Gurobi solver. This is because Gurobi is a proven solver, having the fastest solve times for all model types, including Linear Programs and Mixed Integer Programs [3].

Furthermore, the practicalities of real-world plant operations were considered to ensure the solution produced can be applied to the dairy plant immediately if required for a various scenarios. This problem considered complexities such as minimizing the use of purchased water for cleaning purposes and the retention of target volume levels in the silos daily.

2 Literature Review

2.1 Machine Scheduling Models for Dairy Plants

Many different approaches have been taken to model machine schedules for dairy plants. Most models follow a Mixed-Integer-Linear-Programming (MILP) formulation, as they are flexible, capable, and effective [2]. These MILP formulations are often tailored to the details of the dairy plant's requirements to meet their resource, equipment, and demand requirements. This section gives insight into machine scheduling problems in the dairy industry that have been applied to a plant.

One of the main features of a standard dairy plant is that many products are similar, meaning the processing characteristics may be alike. Georgiadis et al. [2] provide good insight into grouping these products with similar properties into product families to simplify the problem by decreasing the number of active variables. The model addresses constraints such as storage limitations, machine capacity, and material balances, which are typically met in production scheduling formulations. The model's objective was to minimize the operating cost, including production, inventory, and changeover costs. The MILP model was solved using the CPLEX solver on the General Algebraic Modelling System (GAMS) interface and applied to a large dairy facility in Greece.

General scheduling methods have been deemed impractical for optimal production scheduling for dairy plants due to the nature and complexity of the systems [1]. To counteract this, Doganis and Sarimveis [1] have implemented a customized MILP model that considers various due dates, sequence-dependent set-up times, and storage management. The key characteristics of this model include that time is partitioned into periods representative of days, meaning that production tasks must start and end on the same day. Within these time periods, time is treated as a continuous variable, meaning it was not required to determine a suitable number of timeslots sequentially as other formulations may implement. This MILP model was applied to a large dairy plant in Greece and solved using the CPLEX solver on the GAMS interface.

Touil et al. [4] implemented a MILP model to address problems in production scheduling to minimize the completion time (makespan) of a multistage, multiproduct milk process. The significant constraints are sequence-dependent changeover times, machine speed, and capacity constraints. The model was applied to a leading dairy company in Morocco and solved using the IBM ILOG CPLEX solver.

2.2 Rolling Horizon Algorithm

A rolling horizon approach is often implemented in production scheduling as a flexibility tool for a time-dependent model where the planning interval is moved forward in time at every iteration of the algorithm [5]. This method, when applied, would allow a plant to plan for the future sequentially.

To account for changes in the production system, such as order modifications, new orders, and cancellations to orders, Georgiadis et al. [2] implemented a rolling horizon algorithm. This reactive scheduling method provides a solution to the volatility of product orders which is quite apparent in the dairy industry, ensuring optimal schedules after new information arrives. In figure 1, it can be seen how a full scheduling horizon was completed through each iteration of the rolling horizon algorithm. A portion of the prediction horizon would be taken as the control horizon, which was then fixed for the next iteration of the algorithm. This process would continue until the full scheduling horizon was completed.

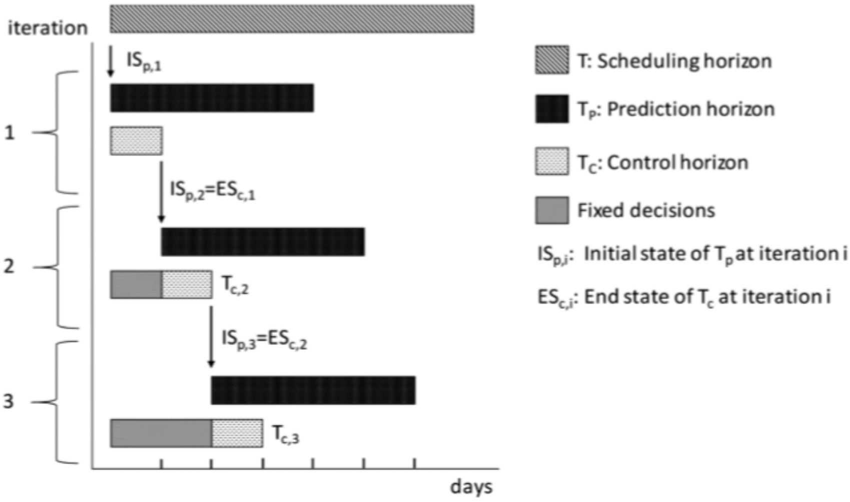

Figure 1 - Rescheduling via a rolling horizon approach [2]

2.3 Constraint Programming in Scheduling Models

As stated, most models for machine scheduling tend to follow a MILP model format to ensure optimal solutions and schedules. However, these methods prioritize optimality and do not always ensure feasible solutions; hence different methods were investigated for this case. A proposed method known to prioritize feasibility over optimality is constraint programming (CP).

Meng et al. [6] implemented a constraint programming model (among many other MILP models) for the distributed flexible job shop scheduling problem (DFJSP) due to its NP-hard classification. The performance and computation times of these models were compared to justify which models are most suitable to be used to solve the DFJSP problem. The CP model considers interval decision variables and domain filtering algorithms. The domain filtering algorithm ensures constraints are feasible by removing the values of the variables for the current domain that aren’t in a solution for that constraint. The proposed CP model was concluded to outperform the other algorithms in respect of efficiency and the quality of the solution. The CP model of the DFJSP was coded using IBM CPLEX Studio.

Constraint programming models can be solved using MiniZinc, a high-level program that allows the user to model constraint satisfaction [7]. MiniZinc allows users to model CP and optimization problems in a solver-independent manner while taking advantage of the large library base with pre-defined constraints. The compiled model can then be fed into several solvers, including Gurobi, which it is compatible with.

2.4 JuMP Program

JuMP is a modelling language used for mathematical optimization in Julia [8]. This framework allows users to formulate and solve many problems, such as linear and integer programs, making it relevant and useful for this project. JuMP achieves performance comparable to many commercial modelling tools. This is done using Julia's advanced features like syntactic macros and code generation [8]. The syntactic macros feature enables JuMP to provide a simple syntax for algebraic modelling without the implications of operator overloading. The code generation feature allows JuMP to generate low-level, yet efficient code as required using the LLVM compiler. Hence, the JuMP framework, in conjunction with Julia, will be used to formulate the scheduling for this project.

Another advantage of using JuMP is the many sources online with relevant column generation code online, which in turn takes on column generation theory. Applying this algorithm to a model could have a significant upside, in terms of saving on computational costs and being more efficient.

Column generation is essentially an algorithm that selects appropriate new columns to add to the base MILP, starting from an initial set of columns (decision variables) that make the problem feasible [9]. Appropriate new columns are chosen through a pricing problem that selects columns based on the criterion of having a significant impact on the objective function value. This method is particularly useful when solving large models, as optimality could be achieved without requiring enumeration of every column possible. Instead, using only the columns added to the MILP through column generation [9].

2.5 SolverStudio

SolverStudio [10] is an Excel add-in that allows users to write and solve optimization models within an Excel workbook using a range of different programming languages including Julia. A unique feature of the program is that it allows data on the spreadsheet to be used in the model as constants, parameters, and variables. The add-in also prints the solution of the model back into the spreadsheet after it has been solved within Excel. SolverStudio [10] calls Julia/JuMP [8] which is compatible with a number of different solvers, including the Gurobi [3], HiGHS [11] and CBC [12] solvers allowing solutions to be found in a timely manner.

3 Methods

3.1 SolverStudio

Before beginning to model the problem, the SolverStudio files were updated to match the current version of Julia/JuMP, which was outdated. This included tasks such as adding catch to the end of try statements in the SSJuliaTools.jl file and fixing encoding errors by adding commands to the SolverStudio.jl file to ensure the results stored in the JuliaResults.py Python file had this appropriate encoding. Another issue that was fixed was applied directly to the model, where printing the Julia results from the model back onto the spreadsheet used outdated code. This was updated to use the correct JuMP commands for accessing the decision variable values.

3.2 Process Flow Diagram

To help visualize the processing plant, the process flow diagram in figure 2 was adopted to show how flow would move through the model and to state the assumptions made. By sequentially looking at the flow diagram, it can be seen where the inputs and outputs of every silo come from. Specifically, the process flow diagram helped to develop and guide the silo volume constraints, which kept silo volume levels within bounds.

Assumptions were vital to keeping the model manageable in complexity. These were decided during consultations with the company to ensure the agreement of the details in the formulation. Meetings were arranged to do this, and regular communication was maintained to ensure they were aware and approved of any assumptions made.

An assumption made was that there was no direct flow between machines 4a and 4b into machine 5, instead, the flow from machines 4a and 4b went through the buffer tank (silo 5) to then get to machine 5. This is because the procedure at the plant was to prioritize the flow from the buffer into machine 5 when machines 4a and 4b were turned on. Hence by ensuring the flow goes into the buffer first from machines 4a and 4b, the flow from the buffer always gets depleted and is the priority. Another assumption of the model was that flow could not go directly from silo 6 into silo 7, instead, the flow from silo 6 would be directed to machine 6 before it could reach silo 7. The current arrangement at the plant allows for this silo-to-silo flow to occur. However, on speaking to members of the team familiar with the plant this direct silo-to-silo flow was looking to get removed from the plant and was hence not included in the model developed.

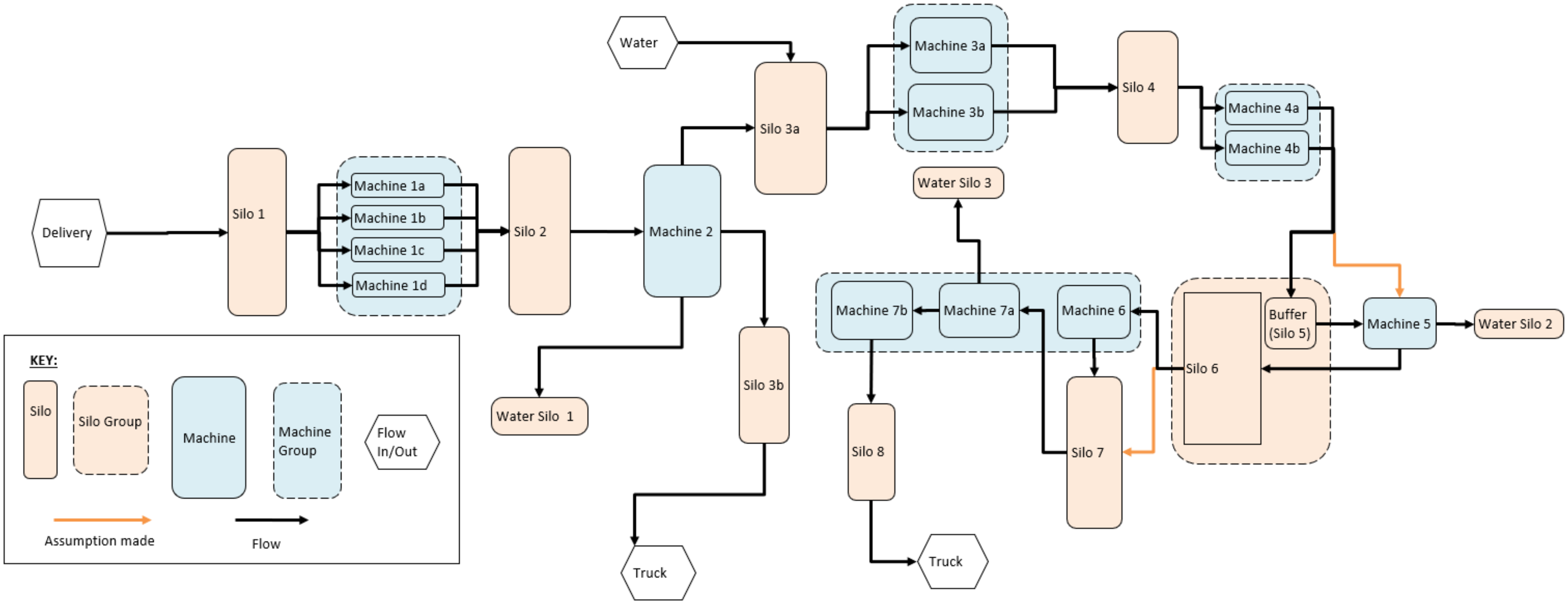

Figure 2 - Process Flow Diagram of the plant.

Note: the orange flows represent the assumptions that have not been included in the model.

Figure 3 shows a process flow diagram for the wastewater constraints. The purple arrows indicate water that flows out of machines 2, 5, and 7a into their respective water silos after being purified. The maroon arrows show the water from the respective water silos used to clean specific machines. The information given at the plant was that the water silos could only be used to clean machines close to this silo. Hence it was assumed that the following machines in the diagram were the only ones that could be cleaned from the recycled water in the water silos.

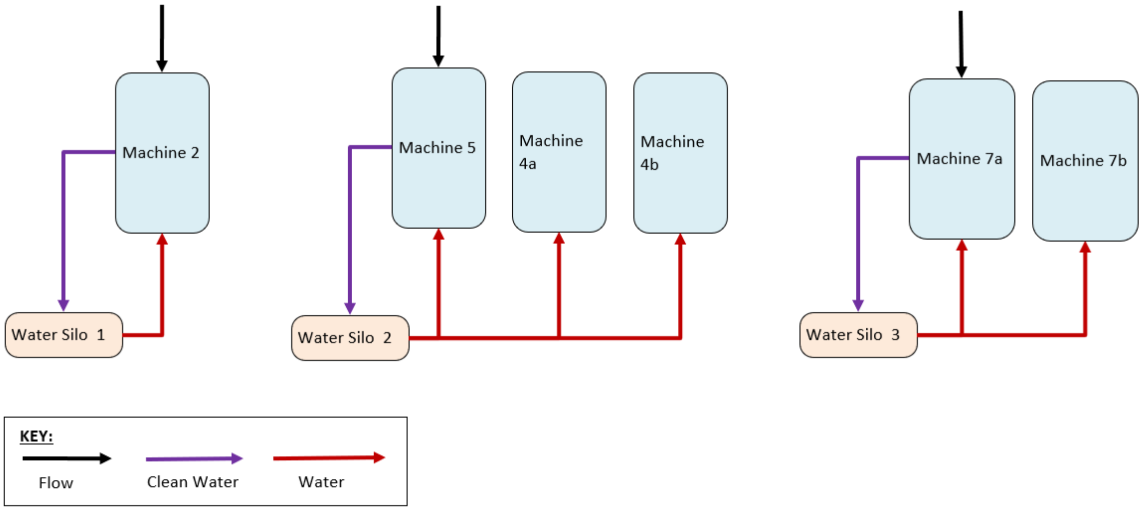

Figure 3 - Process Flow Diagram of the cleaning at the plant.

3.3 Model Definitions

3.3.1 Set Definitions

$$\text{Time Periods}: \; T = \{1,2, \ldots ,36\}$$

$$\text{Stages/Silos}: \quad S \;=\; \{\{1,2,3a,3b,4,5,6,7,8\}\}$$

P_s is the group of predecessor machines at stage s that feed into the silo s,

$$\mathrm{P}_2 = \{1a,1b,1c,1d\},\; P_{3a} = \{2\},\; P_{3b} = \{2\},\; \mathrm{P}_4 = \{3a,3b\},$$
$$\mathrm{P}_5 = \{4a,4b\},\; \mathrm{P}_6 = \{5\},\; \mathrm{P}_7 = \{6\}, \mathrm{P}_8 \;= \{7b\}$$

S_s is the group of successor machines at stage s that the silo feeds into the silo s,

$$S_1 = \{1a,1b,1c,1d\},\; S_2 = \{2\},\; S_{3a} = \{3a,3b\},$$
$$S_4 = \{4a,4b\},\; S_5 = \{5\},\; S_6 = \{6\},\; S_7 \;= \{7a\}$$

$$\text{Truck Type}: \quad J \;=\; \{\{\text{Truck Type 1, Truck Type 2}\}\}$$

$$W \;=\; \{\{\text{Water Silo 1, Water Silo 2, Water Silo 3}\}\}$$

3.3.2 Decision Variable Definitions

$$m_{i,t} = \text{ 1 if machine i is on for the duration of time period t, and 0 otherwise}$$

$$m_t^5 = \text{ Continuous flow rate for machine 5 if it is on for the duration of time period t, and 0 otherwise}$$

$$m_{i,t}^{\text{start}} = \text{1 if machine i starts operating at the start of period t after being off during period t-1, and 0 otherwise}$$

$$m_{i,t}^{\text{finish}} = \text{1 if machine i finishes operating at the start of period t after being on during period t-1, and 0 otherwise}$$

$$p_t = \text{ 1 if there is exactly one machine running between machines 4a and 4b for the duration of time period t, and 0 otherwise}$$

$$q_t = \text{ 1 if there are exactly two machines running between machines 4a and 4b for the duration of time period t, and 0 otherwise}$$

$$c_{it} = \text{ 1 if machine i is being cleaned for the duration of time period t, and 0 otherwise}$$

$$c_{i,t}^{\text{start}} = \text{ 1 if machine i starts cleaning at the start of period t after being cleaned during period t-1, and 0 otherwise}$$

$$c_{i,t}^{\text{finish}} = \text{1 if machine i finishes cleaning at the start of period t after being cleaned during period t-1, and 0 otherwise}$$

$$d_{i,t} = \text{ 1 if machine i dirty at the start of time period t, and 0 otherwise}$$

$$n_t^j = \text{ Number of trucks of type j arriving at the start of time period t}$$

$$v_t^s = \text{ Volume for silo s at the start of time period t}$$

$$u_t^w = \text{ Volume for Waste Water Silo w at the start of time period t}$$

$$w_{i,t}^{\text{r}} = \text{Recycled water used at the start of time period t for machine i}$$

$$w_{i,t}^{\text{p}} = \text{Purchased water used at the start of time period t for machine i}$$

$$m_{i,t}^{\text{switch}} = \text{ 1 if machine i starts operating at the start of period t after being off during period t-1, -1 if machine i ...}$$

finishes operating at the start of period t after being on during period t-1 and 0 otherwise

3.3.3 Definition of Constants

$$F_i = \text{Flow rate into the silo from the predecessor machine i}$$

$$O_i = \text{Flow rate out of the silo into the successor machine i}$$

$$f_1 = \text{constant flow rate for exactly one machine running between machines 4a and 4b}$$

$$f_2 = \text{constant flow rate for exactly two machines running between machines 4a and 4b}$$

$$D_t = \text{ Milk delivery to the plant at the start of time period t}$$

$$V^s = \text{ Max silo volume for silo s at the start of time period t}$$

$$V^j = \text{ The truck volume for truck j}$$

$$r_i^{\max} = \text{Maximum runtime for machine i}$$

$$r_i^{\min} = \text{Minimum runtime for machine i}$$

$$c_i^{\text{length}} = \text{Cleaning runtime length for machine i}$$

$$M_i = \text{Water used in the cleaning for machine i}$$

$$g^s = \text{Target volume goal for silo } s$$

3.4 Model Formulation

3.4.1 Silo Volumes and Truck Constraints

The first set of constraints that were developed were the silo volume constraints. These constraints linked the set of machines according to the full process flow diagram to ensure a mass balance throughout. Generally, the silo volume constraints followed the generic mass balance equation below that was adapted for specific silos.

$$\text{Inflow - Outflow} + \text{Carryover Volume} = \text{Volume}$$

$$D_t - \sum_{i \in S_1} \left(O_i \cdot m_{i,t}\right) + v_{t-1}^1 = v_t^1 \quad \forall\, t \in T \tag{1}$$

$$\sum_{i \in S_1} m_{i,t}^{\text{start}} \leq 2 \quad \forall\, t \in T \tag{2}$$

$$0 \leq v_t^s \leq V^s \quad \forall\, t \in T,\, s \in S \tag{3}$$

The first step of the production plan was to model the delivery of flow into the first silo, also known as stage 1 of the plant. Constraint 1 has an inflow volume D_t delivered at the start of time period t and subtracts the sum of the outflow flow rate O_i for every machine $i \in S_1$ that follows the silo if machine i is turned on for the duration of time period t. The volume at the start of the previous time period v_{t-1} was then added to get the volume of the silo at the start of the current time period v_t.

Constraint 2 was created to ensure that the sum of machines $i \in S_1$ that follows silo 1 if machine i starts operating at the start of time period t after being off for time period t-1 is less than equal to two. Therefore, only two of the machines in this group can start at any one point in time. Constraint 3 ensures that the volume of each silo v_t^s for the duration of time period $t \in T$ for all silos $s \in S$ stays between 0 and the max silo volume of silo s, V^s. This constraint was applied for all the stages of the production constraints.

$$\sum_{i \in P_s} \left(F_i \cdot m_{i,t}\right) - \sum_{i \in S_s} \left(O_i \cdot m_{i,t}\right) + v_{t-1}^s = v_t^s \quad \forall\, t \in T,\, s \in S \tag{4}$$

Constraint 4 was the general silo volume constraint that was applied to silos 2, 3a, and 7 and adapted appropriately for the rest of the silos. The only difference to the above delivery formulation is that instead of a delivery inflow volume into the silo at the start of time period t, the inflow volume was the sum of the inflow flow rate F_i for every machine $i \in P_s$ that precedes the silo if machine i is turned on for the duration of time period t. Note: slack and surplus variables were added to each of the general silo volume constraints for feasibility purposes if a solution for the model couldn't be found initially.

$$\sum_{i \in P_4} \left(F_i \cdot m_{i,t}\right) - \left(f_1 \cdot p_t + (f_2 - f_1) \cdot q_t\right) + v_{t-1}^4 = v_t^4 \quad \forall\, t \in T$$

(5)

$$(f_1 \cdot p_t + (f_2 - f_1) \cdot q_t) - m_t^5 + v_{t-1}^5 = v_t^5 \qquad \forall\, t \;\in T$$

(6)

$$\sum_{i \in S_4} m_{i,t} = p_t + q_t \qquad \forall\, t \;\in T$$

(7)

$$p_t \geq q_t \qquad \forall\, t \;\in T$$

(8)

	Ex. 1	Ex. 2	Ex. 3
p_t	0	1	1
q_t	0	0	1
Machine(s) Running	0	1	2
Flow	0	f_1	f_2

Table 1 - Condition table for machines 4a and 4b

Constraints 5-8 were created for a portion of the plant where the machines carrying flow between silos 4 and 5, machines 4a and 4b, had different conditions to the other machines in the plant. The conditions in table 1 show the effect that the number of machines running had on the outflow of silo 4 and inflow of silo 5.

The outflow for constraint 6 works slightly different to the general silo constraint. Instead of summing over constant flow rates multiplied by the machine's running variable m_{it}, the outflow of constraint 6 is the continuous flow rate m_t^5 where 0 is the lower bound and also an indicator that the machine is off. This variable is slightly different to the other machine running variables m_{it}.

$$m_t^5 - \sum_{i \in S_6} \left(O_i \cdot m_{i,t}\right) + v_{t-1}^6 = v_t^6 \qquad \forall\, t \;\in T$$

(9)

Constraint 9 was another implementation of the general silo volume constraint, where the outflow follows the same formulation only for the specific set of machines following silo 6. Much like the outflow of constraint 6, the inflow of constraint 9 is the continuous flow rate of machine 5 m_t^5.

$$\sum_{i \in P_s} \left(F_i \cdot m_{i,t}\right) - \left(V^j \cdot n_t^j\right) + v_{t-1}^s = v_t^s \quad \forall\, t \;\in T,\, s \;\in S, j \;\in J$$

(10)

$$\sum_{j \in J} n_t^j \leq 2 \quad \forall\, t \;\in T$$

(11)

Constraint 10 was the final implementation of the general silo volume constraint, which was applied to silos 3b and 8. This case was slightly different as instead of an outflow going into another silo it would be filling up a truck. The outflow was specified as truck j's volume V^j multiplied by the number of trucks n_t^j of type j that arrived at the start of time period t.

This was done to ensure that trucks were filled up at a flow rate equivalent to its volume so that the truck, at the end of time period t, be ready to leave the plant thereafter. Constraint 11 ensured that the number of trucks n_t^j of type $j \in J$ that had arrived at the plant at the start of time period t was no larger than 2 meaning that no more than two trucks could arrive to the plant at any one point in time.

3.4.2 Initial Model Machine Constraints

Initially a set of machine constraints were developed to ensure correct runtimes were set in the model for machines which were later revised with simpler formulations for the model. The following were the initial runtime constraints used in the model to generate solutions.

$$m_{i,t}^{\text{switch}} = m_{i,t} - m_{i,t-1} \quad \forall\, t \in T,\, i \in S_s \tag{12}$$

Constraint 12 was created as a switch constraint, which was calculated by finding the difference between the machine running variable $m_{i,t}$ for machine $i \in S_s$ at the start of time period t and t-1 for all $t \in T$. The machine switch variable $m_{i,t}^{\text{switch}}$ would have a 1 where the machine started operating at the start of time period t and a -1 where the machine stops operating at the start of period t after being on during time period t-1 for $i \in S_s$.

$$\sum_{j=t}^{j=t+r_i^{\min}-1} m_{i,j} \geq r_i^{\min} \cdot m_{i,t}^{\text{switch}} \quad \forall\, t \in T,\, i \in S_s \tag{13}$$

$$\sum_{j=t}^{j=t+r_i^{\max}} m_{i,j} \leq r_i^{\max} \quad \forall\, t \in T,\, i \in S_s \tag{14}$$

Constraints 13 and 14 were the model's minimum and maximum runtime constraints. These constraints ensured that the runtime for machine $i \in S_s$ was greater than the minimum machine runtime $r_i^{\min}$, and no greater than the maximum runtime $r_i^{\max}$. Setting minimum and maximum runtime constraints were important to the model to ensure realistic solutions were generated. By having no minimum or maximum runtime constraints, the machines wouldn't be required to run nor stop running which is highly unrealistic at a production plant.

$$\sum_{j=t}^{j=t+r_i^{\min}-1} m_{i,j} \leq c_i^{\text{length}} \cdot \left(1 + m_{i,t}^{\text{switch}}\right) \quad \forall\, t \in T,\, i \in S_s \tag{15}$$

Constraint 15 ensured that that there was the suitable amount of time between a machine $i \in S_s$ being turned off and on again for cleaning to occur.

3.4.3 Revised Model Machine Constraints

After reviewing the initial machine constraints, it was decided that they could be made tighter and improved using a different method with separate start and stop decision variables. The revised model used the following revised machine runtime constraints.

$$m_{i,t} \leq m_t^5 \quad \forall\, t \in T,\, i \in S_5 \tag{16}$$

$$f_2 \cdot m_{i,t} \geq m_t^5 \quad \forall\, t \in T,\, i \in S_5 \tag{17}$$

Constraints 16 and 17 were conversion constraints of the continuous flow rate runtime variable m_t^5 into the binary machine runtime variable $m_{i,t}$ for machine 5. This was done to ensure that the machine runtime constraints could formulate the runtime constraints.

$$m_{i,t-1} + m_{i,t}^{\text{start}} - m_{i,t}^{\text{finish}} = m_{i,t} \quad \forall\, t \in T,\, i \in S_s \tag{18}$$

$$m_{i,t}^{\text{start}} \leq 1 - m_{i,t-1} \quad \forall\, t \in T,\, i \in S_s \tag{19}$$

$$m_{i,t}^{\text{finish}} \leq m_{i,t-1} \quad \forall\, t \in T,\, i \in S_s \tag{20}$$

T	1	2	3	4	5	6
$m_{i,t}$	0	1	1	1	1	0
$m_{i,t}^{\text{start}}$	0	1	0	0	0	0
$m_{i,t}^{\text{finish}}$	0	0	0	0	0	1

Table 2 - Example spreadsheet for runtime constraints

The combination of constraints 18 to 20 ensure that the machine start time $m_{i,t}^{\text{start}}$ for $i \in S_s$ can only be 1 if machine *i* starts operating at the start of period *t* after being off during time period *t-1*. They also ensure that the machine finish time $m_{i,t}^{\text{finish}}$ for $i \in S_s$ can only be 1 if machine *i* finishes operating at the start of period *t* after being on during time period *t-1*. The following can be seen in table 2.

$$\sum_{j=t+r_i^{\min}+1}^{j=t} m_{i,j}^{start} \leq m_{i,t} \quad \forall\, t \in T,\, i \in S_s \tag{21}$$

$$\sum_{j=t}^{j=t+r_i^{\max}} m_{i,j} \leq r_i^{\max} \quad \forall\, t \in T,\, i \in S_s \tag{22}$$

Constraints 21 and 22 were the model's minimum and maximum runtime constraints. These constraints ensured that the runtime for machine $i \in S_s$ was greater than the minimum machine runtime $r_i^{\min}$, and no greater than the maximum runtime $r_i^{\max}$.

$$m_{i,t} = m_{j,t-k} \quad \forall\, t \in T,\, i,j \in S_s,\, i \neq j,\, k \in (1,2) \tag{23}$$

Constraint 23 was created for the specific case where machines had to be turned on 1-2 hours after each other. This constraint was applied to machines 6, 7a and 7b, to ensure that machine 7a started one hour after machine 6, and machine 7b started one hour after machine 7a.

$$m_{i,t} + c_{i,t} \leq 1 \quad \forall\, t \in T,\, i \in S_s \tag{24}$$

Constraint 24 ensures that cleaning could not be scheduled when the machine was running by having the sum of the machine running variable and cleaning running variable for machine $i \in S_s$ less than equal to 1.

3.4.4 Cleaning Constraints

Cleaning was introduced in the model and incorporated similar constraints to the revised machine start and stop variables to ensure consistency.

$$c_{i,t-1} + c_{i,t}^{\text{start}} - c_{i,t}^{\text{finish}} = c_{i,t} \quad \forall\, t \in T,\, i \in S_s \tag{25}$$

$$c_{i,t}^{\text{start}} \leq 1 - c_{i,t-1} \quad \forall\, t \in T,\, i \in S_s \tag{26}$$

$$c_{i,t}^{\text{finish}} \leq c_{i,t-1} \quad \forall\, t \in T,\, i \in S_s \tag{27}$$

$$\sum_{j=t+c_i^{length}+1}^{t} c_{i,j}^{start} \leq c_{i,t} \quad \forall\, t \in T,\, i \in S_s \tag{28}$$

$$\sum_{j=t}^{j=t+c_i^{length}} c_{i,j} \leq c_i^{\text{length}} \quad \forall\, t \in T,\, i \in S_s \tag{29}$$

The cleaning runtime constraints, constraints 25-29, are identical to the revised machine runtime constraints, constraints 18-22. This was done to ensure that the cleaning start time $c_{i,t}^{\text{start}}$ and cleaning finish time $c_{i,t}^{\text{finish}}$ work like the machine parallels respectively. Constraints 28 and 29 were also added to ensure cleaning runtime does not exceed the required cleaning time c_i^{length}.

$$d_{i,t-1} - c_{i,t}^{\text{start}} + m_{i,t}^{\text{finish}} \leq d_{i,t} \quad \forall\, t \in T,\, i \in S_s \tag{30}$$

$$d_{i,t} + m_{i,t}^{\text{start}} \leq 1 \quad \forall\, t \in T,\, i \in S_s \tag{31}$$

T	1	2	3	4	5	6
$d_{i,t}$	0	1	1	1	0	0
$c_{i,t}^{\text{start}}$	0	0	0	0	1	0
$m_{i,t}^{\text{finish}}$	0	1	0	0	0	0

Table 3 - Condition table for dirty machines

Constraint 30 ensures that when machine $i \in S_s$ finishes running at the start of time period *t*, it is considered dirty at the start of time period *t* and only becomes clean (not dirty) when the cleaning procedure begins at the start of a different time period *t*. This can be seen in table 3.

3.4.5 Wastewater Constraints

To quantify the amount of water being used in the cleaning, and where it was coming from, wastewater constraints were defined.

$$w_{i,t}^{\mathrm{r}} + w_{i,t}^{\mathrm{p}} = M_i \cdot c_{i,t} \quad \forall\, t \in T,\, i \in S_s \tag{32}$$

Constraint 33 ensures the amount of recycled water $w_{i,t}^{r}$ and purchased water $w_{i,t}^{p}$ used at the start of time period *t* for machine *i* was equivalent to the water used in the cleaning process M_i for for $i \in S_s$ when the cleaning process was underway.

$$\sum_{i \in 2} (F_i - O_i) \cdot m_{i,t} - \sum_{i \in 2} w_{i,t}^{\mathrm{r}} + u_{t-1}^{w} \geq u_t^{w} \quad \forall\, t \in T,\, w \in W \tag{33}$$

$$\sum_{\mathrm{i} \in 5} (1 - 1/\mathrm{O_i}) \cdot m_{i,t}^{5} - \sum_{\mathrm{i} \in (4\mathrm{a},4\mathrm{b},5)} w_{i,t}^{\mathrm{r}} + \mathrm{u}_{\mathrm{t}-1}^{\mathrm{w}} \geq \mathrm{u}_{\mathrm{t}}^{\mathrm{w}} \quad \forall\, \mathrm{t} \in \mathrm{T},\, \mathrm{w} \in \mathrm{W} \tag{34}$$

$$\sum_{i \in 7a} (F_i - O_i) \cdot m_{i,t} - \sum_{i \in (7a,7b)} w_{i,t}^{\mathrm{r}} + u_{t-1}^{w} \geq u_t^{w} \quad \forall\, t \in T,\, w \in W \tag{35}$$

The water used for cleaning in the plant came from 3 different silos, silos 2, 5 and 7a, all in separate water storage tanks. This water was then used to clean silos close to this storage. In the formulation, the water that came from silo 2, was used to clean silo 2 (constraint 34), the water that came from silo 5, was used to clean silos 4a, 4b and 5 (constraint 35), and the water that came from silo 7a, was used to clean silos 7a and 7b (constraint 36). Cleaning was scheduled for the other machines, but the recycled water usage was not considered.

Constraints 34 to 36 followed the generic mass balance equation for the 3 different water silos. The inflow was formulated as the remainder of flow from the silo that didn't proceed to be processed as the key milk flow. This was written slightly differently in constraint 35 due to the machine decision variable for machine 5 m_t^5 already being a flow. The outflow for all 3 constraints was the recycled water $w_{i,t}^{r}$ returning back into the machine for cleaning for machine i stated in the respective constraints. The volume of the water silo at the start of the previous time period u_{t-1} was then added to get the volume of the water silo at the start of the current time period u_t.

3.4.6 Objective Functions

A wide variety of different objective function components were considered in the problem, all with different goals of the plant taken into consideration.

$$\text{Min} \sum_{t \in T} \sum_{s \in S} (\text{suplus}_t^s \text{ - slack}_t^s) \tag{36}$$

Objective Component 1 was written to minimize the difference of surplus and slack variables when added to the silo volume constraints. This component only came into action when a solution couldn't be found initially.

$$\text{Min} \sum_{t \in T} v_t^5 \tag{37}$$

Objective component 2 minimized the flow in silo 5, as it was found to provide a faster turnover in the silo, which had a slightly different formulation to the rest of the silos.

$$\text{Min} \sum_{s \in S} |\text{g}^s - v_t^s| \tag{38}$$

Objective component 3 minimized the difference between a target volume goal for silo s g^s and the volume of silo s at the start of time period t. This objective aimed to retain a set amount of volume in each silo *s* at the start of time period *t*. This objective component helped regulate flow in the model and maintain a steady working operation at the plant.

$$\text{Min} \sum_{t \in T} \sum_{i \in S_s} w_{i,t}^{\text{p}} \tag{39}$$

Objective component 4 minimized the amount of purchased water $w_{i,t}{}^{\text{p}}$ that was used to clean the silos. Ensuring that the priority of water used to clean the silo was the recycled water from the water silos when available and purchased water only after this if required.

3.5 Rolling Horizon Algorithm

An adaptation of the rolling horizon algorithm was created to ensure a plan could be made over a couple day period instead of only 36 hours. The algorithm worked by locking the machine decision variables in the last 12-hour time period into the first 12 hours of the next model, then re-solving for the rest of the 24-hour period. This was adapted to give more insight to the site on how a week's schedule may look for the plant.

The VBA Macro tool was used to record two procedures to ensure the correct data was used going forward. The first being that the carryover volumes for the silos were adjusted to be the volume of the silo in the 24th hour. The second macro recorded the decision variables being moved from the last 12-hour time period into the first 12-hour time period before the rest of the decision cells were cleared. Also recorded in the second macros were the milk delivery volumes from the first 12 hours being swapped with the last 12-hour time period to ensure the following day had the correct milk delivery set. These two macros were added to the spreadsheet as buttons for ease of access where the user would first click the reset carryover volumes button and then click the reset decision variables button. Julia code was written to lock these decision cells in the model by checking whether they weren't empty and locking them by setting a hard constraint to the value in the spreadsheet if so.

Figure 4 shows how the adapted rolling horizon algorithm worked to solve a set of solutions for a scheduling horizon. The first iteration solves for a model, which is the first prediction horizon. Every iteration from here, until the scheduling horizon has been completed, takes the solution in the control horizon (the last 12-hour period of the prediction horizon) and locks it in the next solutions prediction horizon as the first 12-hour period. In figure 4 it can be seen how the rolling horizon algorithm solves a set of solutions for a scheduling horizon of 7 days.

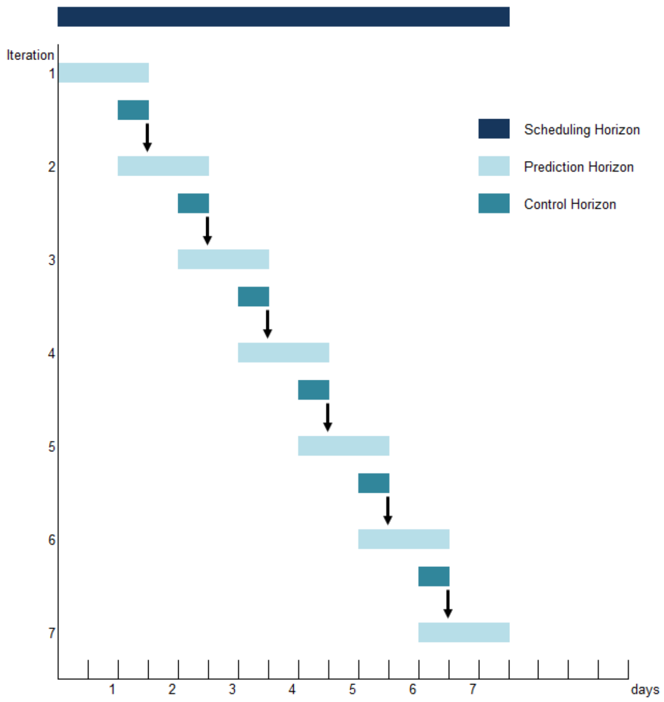

Figure 4 - The Implemented Rolling Horizon Algorithm

4 Results

The model was solved using different objectives, for the same starting silo volumes, and milk delivery timeframe. A template results spreadsheet was created with all the relevant features of the model to show the recommended machine decision variables over the 36-hour time period and the varying silo volume over this time. A variety of different solvers, including Gurobi, HiGHs, and CDC, were also compared to verify which solver was the best for this problem. History was also considered to generate a week's worth of scheduling for the model so that workers may see how schedules may look for the plant and the fluctuating silo levels over time.

4.1 Model Results

When finding solutions for the model, a consistent set of termination rules were set to ensure decent solutions, or if this was not possible, then a solution within a set timeframe was found, if any. Runs were terminated when a MIP gap of less than 5% was achieved, between the optimal LP and incumbent solutions, as this is indicative of a decent solution and would allow solve times to decrease. If this was not possible in a timely manner, a run was also terminated after 100 seconds. The first objective tested used the 3rd objective component.

The initial silo volume for silo s was set as the target volume goal g^s, and the start of the 24th time period was the volume optimized for silo s.

$$\text{Min} \sum_{s \in S} |v_0^s - v_{24}^s|$$

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178
Silo 1 Volume	216	214	153	91	65	98	191	285	378	472	530	375	220	65	38	85	84	107
Machine 1a	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c
Machine 1b	0	1	1	1	1	1	1	1	c	c	c	0	0	0	1	1	1	1
Machine 1c	1	1	1	c	c	c	0	0	0	1	1	1	1	1	1	1	1	c
Machine 1d	1	1	1	1	c	c	c	0	0	0	1	1	1	1	1	1	1	c
Silo 2 Volume	347	394	442	458	421	474	528	582	636	576	570	563	557	551	599	646	640	549
Machine 2	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1
Water	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
Water Silo 1	9	19	28	37	47	47	47	47	47	56	0	0	0	0	0	0	0	0
Silo 3a Volume	168	183	244	304	365	332	254	176	98	113	129	144	160	175	191	207	222	238
Machine 3a	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1
Machine 3b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Silo 3b Volume	45	58	70	30	42	16	16	16	16	29	15	27	13	26	38	50	10	23
Truck 1	0	0	0	2	0	1	0	0	0	0	1	0	1	0	0	0	2	0
Silo 4 Volume	429	452	409	366	323	280	222	164	105	47	71	94	118	142	83	107	131	154
Machine 4a	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c
Machine 4b	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1
Silo 5 volume	0	0	0	0	0	98	278	278	278	278	278	196	114	32	0	0	0	98
Machine 5	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	c
Water Silo 2	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	166	210	205	178	152	81	10	21	31	113	195	227	238	226	237	210	184	113
Machine 6	c	c	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1
Silo 7 Volume	-65	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41
Machine 7a	1	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1
machine 7b	1	1	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1
Water Silo 3	41	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	25	12	12	12	12	23	10	21	32	19	30	30	5	5	16	28	39	50
Truck 2	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0

Time	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	203	242	414	334	274	214	309	368	368	367	427	486	546	605	700	640	640	640
Machine 1a	c	0	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0
Machine 1b	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c
Machine 1c	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	c	0	0
Machine 1d	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 2 Volume	458	511	619	528	436	345	285	279	273	213	153	93	178	232	286	286	286	286
Machine 2	1	c	c	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0
Water	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
Water Silo 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	253	175	97	113	128	144	159	208	257	305	354	402	357	312	267	222	177	177
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c
Machine 3b	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0
Silo 3b Volume	35	35	35	21	33	46	58	70	82	95	107	119	119	119	119	119	119	119
Truck 1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	178	202	225	249	273	394	418	387	355	324	293	261	230	198	167	234	300	300
Machine 4a	c	0	0	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c
Machine 4b	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	0
Silo 5 volume	196	294	294	212	130	48	0	0	0	0	0	0	98	196	294	294	294	294
Machine 5	c	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0
Water Silo 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	113	113	144	155	166	117	90	64	38	82	127	171	171	171	171	171	171	171
Machine 6	c	c	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0
Silo 7 Volume	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0	0
Machine 7a	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0
machine 7b	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	0
Water Silo 3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	61	72	23	23	23	10	21	32	43	6	17	17	17	17	17	17	17	17
Truck 2	0	0	2	0	0	1	0	0	0	2	0	0	0	0	0	0	0	0

Figure 5 - Solution Spreadsheet for objective component 3

$$\text{Min} \sum_{t\in T}\sum_{i\in S_s} w_{i,t}^{\mathrm{p}}$$

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	216	369	463	556	590	623	596	570	543	577	610	490	370	250	318	520	614	672	708	687	859	839	779	779	934	968	943	917	892	866	841	815	850	850	850	850
Machine 1a	0	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	c	c	c	0	0
Machine 1b	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	c	c	c	1
Machine 1c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 1d	0	0	0	1	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	1
Silo 2 Volume	353	407	460	423	385	402	418	434	397	359	322	284	247	155	209	317	424	531	639	548	456	365	220	75	38	54	70	86	103	119	280	388	388	243	98	60
Machine 2	0	0	0	1	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	1	1	1	1	1	c	c	0	1	1	1
Water	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1
Water Silo 1	0	0	0	9	19	28	37	47	56	65	75	84	93	100	50	0	0	0	0	9	16	25	35	44	53	63	72	81	91	100	50	0	0	0	0	0
Silo 3a Volume	107	62	17	66	114	163	211	260	275	291	306	322	337	353	275	230	185	140	95	111	126	142	157	173	188	204	252	301	350	398	353	308	263	312	405	466
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c
Machine 3b	c	c	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	1
Silo 3b Volume	33	33	33	45	6	18	30	42	55	67	79	65	26	38	38	38	38	12	12	24	36	49	61	73	85	98	110	70	56	69	17	17	17	3	15	27
Truck 1	0	0	0	0	2	0	0	0	0	0	0	1	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2	1	0	2	0	0	1	0	0
Silo 4 Volume	374	342	311	280	346	413	480	448	472	496	519	543	567	590	614	582	649	716	782	904	1026	1147	1269	1293	1316	1340	1309	1277	1246	1215	1183	1152	1219	1285	1285	1340
Machine 4a	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 4b	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c
Silo 5 volume	98	196	196	114	32	0	0	0	0	0	0	0	98	196	196	114	32	0	0	0	0	0	0	0	98	196	196	114	32	0	0	0	0	0	0	0
Machine 5	0	0	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1
Water Silo 2	0	0	1	99	100	100	51	0	53	99	99	100	50	0	49	49	0	26	0	0	1	1	47	100	50	0	30	42	98	99	99	100	80	30	0	0
Silo 6 Volume	121	121	203	285	299	299	299	294	267	241	214	188	117	46	128	210	224	224	224	224	224	224	224	269	269	269	350	382	370	344	317	291	220	149	149	149
Machine 6	0	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c
Silo 7 Volume	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0
Machine 7a	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c
machine 7b	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
Water Silo 3	0	0	0	0	0	0	0	0	0	41	82	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	41	59	59	59	9	50	0
Silo 8 Volume	25	1	1	1	1	1	1	1	1	1	12	23	34	45	56	68	68	68	68	68	68	68	68	68	68	19	19	19	19	19	30	16	28	39	50	61
Truck 2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1	0	0	0	0

Note: The purchased water in results table 4, is only the amount of purchased water that was used for machines that could've used recycled water – which consists of machines 2, 4a, 4b, 7a and 7b.

Figure 6 - Solution Spreadsheet for objective component 4

Objective Used	Model Output									
	Solver	Incumbent Solution Objective	Explored Nodes	Objective Function Value	Optimality Gap (%)	Solve Time (seconds)	Feasible without slack & surplus variables	Recycled water used (m^3)	Purchased water used (m^3)	$\sum_{s\in S}\lvert v_0^s - v_{24}^s\rvert$ (m^3)
Objective component 3 – Target Volume	Gurobi	0	91,357	61.54	50.75	100	Yes	0	2,150	61.54
Objective component 4 - Wastewater	Gurobi	0	1,396	100	0	6.93	Yes	1,150	100	1,993

Table 4 - Results for single objective components 3 and 4

Table 4 shows that the model prioritizing objective component 4 had a better solve time at 6.93 seconds whereas the model prioritizing objective component 3 solved in 100 seconds. This correlates to the number of explored nodes, as the objective component 4 model only explored 1,396 nodes to the 91,357 of the objective component 3 model. The optimality gap of the model prioritizing objective component 3 was relatively high, with an optimality gap of 50.75%, whereas it was 0% for the model prioritizing objective component 4 – meaning no difference between the incumbent and optimal LP solution, and that an optimal solution was found. The incumbent solution objective was 0 for both models signalling that the model formulations were somewhat poor.

There was a noticeable difference in the amount of recycled water used for each model, with the objective component 4 model recycling 1,150 m^3 of water for cleaning. In contrast, the objective 3 component model didn't consider using recycled water. The objective component 4 model only had to purchase 100 m^3 of water to complete the cleaning, in comparison to the objective component 3 model that purchased 2,150 m^3 of water to do this. It seems that the model prioritizing objective component 4 wasn't regulating flow as much throughout the model (seen in the rising silo volumes) to ensure less cleaning with purchased water was done. The model prioritizing objective component 3 was better at regulating flow for the first 24 hours but seemed to also have some troubles after this.

4.2 Solver Results

Gurobi, HiGHs, and Cbc, were tested on the model to see what solver would give the best results in a 100 second timeframe. These solvers were experimented on two objective functions to give a wide perspective of solutions for different cases.

$$\text{Min} \sum_{t \in T} v_t^5 + \sum_{s \in S} |v_0^s - v_{24}^s|$$

	Model Output					
Solver	Incumbent Solution Objective	Explored Nodes	Objective Function Value	Optimality Gap (%)	Solve Time (seconds)	Feasible without slack & surplus variables
Gurobi	0	86,215	60.38	71.49	100.04	Yes
HiGHs	0	14,021	316.48	99.37	99.9	Yes
Cbc	0	7,305	263.83	100	99.02	Yes

Table 5 - Solver results using objective components 2 and 3 together

$$\text{Min} \sum_{t \in T} v_t^5 + \sum_{s \in S} |v_0^s - v_{24}^s| + \sum_{t \in T} \sum_{i \in S_s} w_{i,t}^{\mathrm{p}}$$

	Model Output					
Solver	Incumbent Solution Objective	Explored Nodes	Objective Function Value	Optimality Gap (%)	Solve Time (seconds)	Feasible without slack & surplus variables
Gurobi	0	5,299	273.17	34.89	100.04	Yes
HiGHs	0	995	592.68	99.66	100	Yes
Cbc	0	4,128	/	/	98.87	No

Table 6 - Solver results using objective components 2, 3, and 4 together

In the first experiment in table 5, all the solvers found a solution using the model. Gurobi found the best objective function value in comparison to HiGHs and Cbc, which was helped by exploring 6 and 12 times more nodes than HiGHS and Cbc, respectively, in the 100 second timeframe. The optimality gap was also lower and therefore better for Gurobi compared to the other solvers. The second experiment in table 6 also paints a similar picture with Gurobi exploring the most nodes and finding the best objective function value. Although it was able to explore more nodes than the HiGHS solver, the Cbc solver could not find a solution in the second experiment. In neither experiment did any solver find an incumbent solution objective, meaning the model formulations were somewhat poor for the models.

4.3 History Results

4.3.1 Initial History Run

The addition of history into the model allows the worker the ability to generate a weeks' worth of scheduling prior using the adapted rolling horizon algorithm. In the following example, objective components 1, 2 and 3 were minimized and 7 solutions generated over a 7-day period. The fourth objective component was not included in this objective for simplicity purposes, as well as it being seen in figure 6 that it wasn't regulating flow well throughout the plant which would cause problems through a 7 day history run where flow regulation was vital to good solutions being created.

Note: Objective component 3's target volume goal g^s for silo s was set as the initial silo volume v_0^s for silo s, and this did not change with every iteration of the model as it was set as a constant before the first run.

$$\text{Min} \sum_{t\in T}\sum_{s\in S}(\text{suplus}_t^s \text{ - slack}_t^s) + \text{Min} \sum_{t\in T} v_t^5 + \text{Min} \sum_{s\in S}|g^s - v_{24}^s|$$

	Model Output							
Day	Solver	Incumbent Solution Objective	Explored Nodes	Objective Function Value	Optimality Gap (%)	Solve Time (seconds)	Feasible without slack & surplus variables	$\sum_{s\in S}\lvert v_0^s - v_{24}^s\rvert$ (m^3)
1	Gurobi	20.785	61,169	155.34	50.35	100.02	Yes	155.34
2	Gurobi	9.855	659,305	87.64	0.3172	100.02	Yes	87.64
3	Gurobi	718.82	2,637	881.44	0	2.45	Yes	685.44
4	Gurobi	710.71	1	774.41	0	2.64	Yes	774.41
5	Gurobi	1,309.77	3,567	1,331.24	0	1.25	Yes	1331.24
6	Gurobi	1,760.34	46,795	2,040.37	0	11.20	Yes	1813.37
7	Gurobi	1,687.61	9,339	1,743.27	0	1.82	No	449.48

Table 7 - History results for initial objective

It is evident via table 7 that over the course of the 7-day period that the rise of the objective is quite large in the model. This drastic rise can also be seen in the absolute difference between target volume and actual volume, where the absolute differences between the target volume and actual volume in 24th hour for all silos *s* rose gradually over time with each iteration of the model. For the first 6 days it is clear that the objective is dominated by the 3rd objective component, and on the last it is dominated by the 2nd objective component. This is clear in Appendix A.7 where the cumulative volume in silo 5 throughout the schedule is quite large. The 1st objective component only comes into play on the 7th day when a feasible solution could not be found, and hence slack and surplus variables were added. The effect of this component is similar to that of the 3rd objective component on the 7th day. The optimality gap tends to decrease with each iteration of the rolling horizon algorithm, with days 3-7 having an optimality gap of 0%. This means that the model found an optimal solution for these days, which isn't quite what would be expected with the output high objective function values. Given that these values were considered "optimal" the model was revisited to find and inspect this issue.

4.3.2 Revised History Run

The 3rd objective was adapted to ensure the 36th hour was now being optimized instead of the 24th hour which may have been causing some error in the initial history run. This is because the following 12 hours (after the 24th hour) were not being optimized by this objective and were the cells being locked into the first twelve hours of the next model. Therefore, a second batch of history results were solved for this change. The new objective was:

$$\text{Min} \sum_{t \in T} \sum_{s \in S} (\text{suplus}_t^s \text{ - slack}_t^s) + \text{Min} \sum_{t \in T} v_t^5 + \text{Min} \sum_{s \in S} |g^s - v_{36}^s|$$

	Model Output							
Day	Solver	Incumbent Solution Objective	Explored Nodes	Objective Function Value	Optimality Gap (%)	Solve Time (seconds)	Feasible without slack & surplus variables	$\sum_{s \in S} \lvert v_0^s - v_{36}^s \rvert$ (m^3)
1	Gurobi	0	42,376	97.95	38.76	100.02	Yes	96.95
2	Gurobi	13.58	15,639	59.41	0	17.55	Yes	58.41
3	Gurobi	488.40	64,249	681.99	0	31.63	Yes	220.99
4	Gurobi	160.43	489,927	201.35	4.71	100.00	Yes	201.35
5	Gurobi	98.00	300,024	136.89	1.0661	100.02	No	5.77
6	Gurobi	51.64	298,397	141.76	17.37	100.03	No	38.67
7	Gurobi	0	167,067	31.61	3.21	100.02	No	0

Table 8 - History results for revised objective

The results in table 8 show that the objective values stay fairly consistent with a slight rise on the third day, which is due to the effect of the second objective component. This objective then falls back to what would be expected and desired following this peak. The third objective component stays fairly consistent and doesn't rise over a 221 m^3 cumulative difference between the silo volumes and the target volumes over 7 days. This model does a better job of regulating flow throughout the model that we can generally see in Appendix B.1-7. The model requires the full 100 seconds as solve time for 5 out of the 7 schedules generated, which is expected with the explored nodes going beyond 10^5. The first four solution schedules were feasible without slack & surplus variables, whereas the last three were not and required surplus and slack variables in the silo constraints to complete the run. This indicates minor breaches to the silo volume constraints towards the back end of the rolling horizon algorithm. With generally good solutions being found (as shown by the respective objective functions aside from day 3), the optimality gaps for the runs suggest that the solutions are nearly optimal for most of the schedules created.

5 Discussion

5.1 Model Recommendation

Gurobi has proven to be the best solver to be used on the model. This was because in both the tests conducted, it found the best objective values and had the lowest optimality gaps from the list of solvers. This was to be expected by the research done and was backed up by the Gurobi Performance Benchmarks in [3].

From the model results, it can be seen that the model is working and generating suitable, feasible solutions in a timely manner for a range of different objectives and combinations of these objectives. However, it was noticed that the models with one objective gave results that skewed heavily in terms of prioritizing this and lacked in other considerations, such as regulating flow. The purpose of the different objectives was to provide the plant flexibility with what they wanted to prioritize, whether that be aiming for a target volume in the silos or minimizing the use of purchased water for cleaning. It was also found that combining these objectives could provide reasonable, timely, and more realistic solutions that considered the different aspirations of the plant. As with many machine scheduling problems, there is never a one-size fit all solution, instead, it is circumstantial to what the plant would like to achieve.

The recommended use of the model would mainly be in conjunction with the worker's pre-knowledge of the plant. For example, the model generates provisional schedules that the workers can review and adjust (if required) to suit their intentions for the day. Blindly following the model created could have drastic implications (as with any AI related problems) and should not be the intended use for the project. This is due to some of the assumptions that have been made that may cause the model to misrepresent some parts of the plant, which were less so evident in one-off uses but more apparent with history.

The rolling horizon algorithm was an added feature to show how the model developed the solution spreadsheet over a week. From the results, it can be seen that the revised objective provided a much better set of solutions for a week at the plant than the initial run. This can be seen where in comparison to the initial run, generally, the objectives are much smaller and more consistently values that would be expected. This highlights the importance of pre-knowledge at the plant being used to verify the model, as if the initial run was used at the plant, it would've caused problems.

It would also not be recommended to use the history feature for an extended period of time due to the error that is added with each solution spreadsheet created. This is seen vividly for the revised history run, where in order t o get feasible solutions for the 5th, 6th, and 7th days, slacks and surpluses must be added to the silo volume formulation to ensure feasibility. This indicates that a small breach of these constraints was allowed to ensure feasibility. This is because with every solution spreadsheet generated, the difference between the model and spreadsheet silo volumes rises, and the model no longer represents the situation at the plant as accurately by the end of the week. The recommended use of the rolling horizon algorithm would be to use it for a 5 days or as long as it produces feasible solutions up to the 5th day (using the recommended model use instructions above) before re-solving for an initial spreadsheet without using history. This would ensure that the effects of carryover error be mitigated over a longer period of time, and more consistent schedules are created with the freshly generated schedules made every 5 days.

5.2 Model Limitations

As mentioned, it would be unwise to blindly follow the solution spreadsheets generated by the model and instead would require a supervisor to verify with their pre-existing knowledge. This is because some assumptions have been made in the model, which may misestimate volume levels in some circumstances, especially over time using history. One assumption that may have such effect was assuming no flow can go from machines 4a and 4b into machine 5, and instead being directed through silo 5 before being able to go into machine 5. This in practice, is slightly different to how the model was formulated. The effect of this assumption was minimized by creating an objective function that minimized the flow in silo 5. This ensured that the turnover rate in the silo was relatively quick.

Another assumption made was that milk deliveries were reused at the plant. On every iteration of the rolling horizon algorithm, the milk delivery volumes from the first twelve hours were swapped with the back twelve hours (between 25-36) to ensure the following day had the correct milk delivery set. This limited the model and the history results as tests weren't conducted on any differing milk delivery volumes, which in reality could vary on a day-to-day basis depending on the plant's needs.

Similarly, another limitation to the model was that it was only tested on one set of initial carryover volumes. This limited the model as there weren't any tests done on input data sets to ensure feasibility when there were different inputs into the model. The history component does give a glimpse into using different inputs in the model, as every day there was a new set of inputs to the model. However, in saying this disaster scenarios weren't considered meaning that the model wasn't pushed to its limits. Hence, it is uncertain to what extent the model can be applied to, to still be able to give feasible solutions consistently.

A limitation of the target objective is that the objective is only setting a target volume for the 36th hour and has not considered solutions in between. This leads to the solutions providing optimality for the 36th hour and not throughout the whole model. The potential impact this could have had wasn't apparent in the revised history run due to the low target volumes which were set as constants using the initial volume in the silos. This regulated the flow in the model quite well and didn't show the potential effect of this objective. However, the impact of this may be more apparent if the model was used with different inputs which were larger. Hence, it would also be recommended to use smaller and more appropriate target volumes for all silos to regulate flow better.

5.3 Future Work

Future work to be done includes:

- Testing the model on disaster scenarios to see the extent that the model will work for.
- Using bi-objective functions on the model to optimize multiple aspects of the model.
- Adding cleaning constraints on all the machines that reflect the plant.
- Trialling a new horizon for the rolling horizon algorithm with the 13-24th hours being used as the following days first twelve hours.

6 Conclusion

A mixed integer linear program has been used to generate high quality plant schedules for a production plant in NZ. The intended use of the models was to generate these solutions schedules to be applied to the plant directly. Upon inspection, the recommended use of the models would be to act as a provisional schedule to the plant, being verified by a supervisor, before being implemented in practice. This is due to some of the assumptions that have been made, that over the course of a week period may cause the model to begin to misrepresent the silo volumes. A range of different objective functions were tested, such as minimizing the use of purchased wastewater for cleaning purposes, and the preservation of volume levels in the silos on a daily basis. The plant is given full permission to use these objectives as they see fit to determine the best schedules for the site. They are also given full permission to develop the prototype further as they wish.

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Appendix A

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	216	274	333	331	330	328	326	325	383	477	570	570	510	390	398	505	504	467	408	292	429	374	279	219	314	348	323	297	272	246	221	255	290	290	290	290
Machine 1a	1	1	1	1	1	1	1	0	0	c	c	c	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	0	0	0	0
Machine 1b	0	0	c	c	c	0	0	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	0	0	0	0	c	c
Machine 1c	1	1	1	1	1	1	1	1	0	c	c	c	0	0	1	1	1	1	1	1	1	1	0	0	0	0	c	c	c	0	0	0	0	0	0	1
Machine 1d	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	0	0	0	0
Silo 2 Volume	293	233	227	221	215	209	202	287	341	395	395	304	266	229	222	270	318	365	413	353	293	233	287	341	303	319	335	352	368	384	347	454	454	454	309	195
Machine 2	1	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	0	c	c	1	1
Water	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	1	1
Water Silo 1	9	19	28	37	47	56	65	65	65	65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	201	249	265	280	296	311	327	249	171	93	15	30	46	61	77	93	108	157	205	254	302	318	240	162	177	193	209	224	240	255	271	193	115	37	97	158
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Machine 3b	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Silo 3b Volume	45	58	70	82	94	107	119	119	119	93	93	105	117	130	142	154	166	179	191	151	111	124	98	46	58	70	82	95	107	119	131	131	131	131	144	156
Truck 1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2	2	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	374	342	366	390	413	437	461	484	508	630	751	873	897	838	780	722	663	550	436	323	210	151	273	395	516	638	580	521	463	405	346	288	230	171	46	3
Machine 4a	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	0
Machine 4b	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1	1
Water Silo 2	53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	166	160	134	107	81	54	28	1	46	46	46	46	40	51	62	73	84	95	105	187	269	301	230	159	88	17	28	38	120	202	284	316	327	337	348	322
Machine 6	0	f	1	1	1	1	1	1	0	c	c	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	f	1	1	1	1
Silo 7 Volume	0	0	61	57	53	49	45	41	-24	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49
Machine 7a	0	0	0	1	1	1	1	1	1	0	c	c	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	0	1	1	1
Machine 7b	0	0	0	0	1	1	1	1	1	1	0	c	c	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	0	c	c	0	1	1
Water Silo 3	0	0	0	41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	14	14	14	14	25	36	47	59	70	81	56	56	56	56	56	68	54	16	28	39	50	50	25	25	37	48	34	46	32	43	43	43	43	43	55	66
Truck 2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	2	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0

Figure A.1: Day 1 Spreadsheet Solution

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	155	155	155	155	155	155	155	155	155	0	0	0	0	0	128	262	214	178	156	99	232	40	0	0	0	153	153	153	153	153	153	153	153	153	153	0
Silo 1 Volume	314	348	323	297	272	246	221	255	290	290	290	290	255	100	73	180	239	262	263	182	354	334	274	214	94	92	151	209	268	326	384	443	596	750	903	903
Machine 1a	1	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Machine 1b	1	1	1	1	1	1	0	0	0	c	c	c	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
Machine 1c	0	0	c	c	c	0	0	0	0	0	0	1	1	1	1	1	1	1	0	c	c	c	0	0	1	1	1	1	1	1	1	0	0	0	0	0
Machine 1d	c	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 2 Volume	303	319	335	352	368	384	347	454	454	454	309	195	189	183	177	171	165	158	175	228	282	336	390	352	346	286	226	166	106	46	132	132	132	132	132	132
Machine 2	1	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
Water	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
Water Silo 1	9	19	28	37	47	56	65	65	65	65	75	84	93	91	91	91	100	100	100	100	100	100	100	100	37	37	37	37	37	0	0	0	0	0	0	0
Silo 3a Volume	177	193	209	224	240	255	271	193	115	37	97	158	218	279	294	310	326	341	357	279	201	123	90	150	211	271	332	347	396	444	399	354	309	264	219	219
Machine 3a	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	0	c	c	c	c	1	1	1	1	1	1	1	1	0
Machine 3b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
Silo 3b Volume	58	70	82	95	107	119	131	131	131	131	144	156	168	180	193	205	217	229	190	138	86	86	86	46	58	70	83	95	107	119	119	119	119	119	119	119
Truck 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	516	638	580	521	463	405	346	288	230	171	46	3	58	113	235	357	380	404	428	451	475	499	456	413	370	327	382	503	570	637	703	770	837	903	872	774
Machine 4a	c	c	1	1	1	1	1	1	1	1	1	0	c	c	c	c	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
Machine 4b	c	c	1	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0	0	0	1	1
Water Silo 2	0	0	98	99	99	99	99	99	99	99	99	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0
Silo 6 Volume	88	17	28	38	120	202	284	316	327	337	348	322	251	180	180	180	224	269	263	237	210	184	158	131	105	149	149	149	149	149	149	149	149	149	194	238
Machine 6	1	1	1	1	c	c	0	f	1	1	1	1	1	1	c	c	0	0	f	1	1	1	1	1	1	0	0	0	0	0	0	0	c	c	0	0
Silo 7 Volume	53	49	45	41	-24	0	0	0	61	57	53	49	45	41	-24	0	0	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0	0	0	0
Machine 7a	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0
Machine 7b	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0
Water Silo 3	41	82	100	100	100	100	100	100	100	41	59	59	59	59	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0
Silo 8 Volume	37	48	34	46	32	43	43	43	43	43	55	66	77	88	50	61	61	61	61	61	61	24	10	21	33	44	55	6	6	6	6	6	6	6	6	6
Truck 2	0	0	1	0	1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2	1	0	0	0	0	2	0	0	0	0	0	0	0	0

Figure A.2: Day 2 Spreadsheet Solution

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	94	92	151	209	268	326	384	443	596	750	903	903	903	808	781	828	827	790	731	615	632	552	492	492	647	801	896	955	1015	1074	1074	1073	1073	918	858	798
Machine 1a	1	1	1	1	1	1	1	c	c	c	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	c	c	c	0	0	0	0	0	0	0	0
Machine 1b	1	c	c	c	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	0	0	0
Machine 1c	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	0	1	1	1	1	1	1	1	0	c	c
Machine 1d	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	c	c	c	0	0	0	1	1	1	1	1	1	1
Silo 2 Volume	346	286	226	166	106	46	132	132	132	132	132	132	217	355	403	451	498	546	593	641	603	512	367	367	367	421	506	591	531	525	519	513	506	415	324	233
Machine 2	1	1	1	1	1	1	0	c	c	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	c	c	1	1	1	1	1	1	1	1
Water	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1
Water Silo 1	9	19	28	37	47	56	56	56	56	56	56	56	56	56	65	75	56	56	56	56	56	56	56	56	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	211	271	332	347	396	444	399	354	309	264	219	219	219	186	247	263	278	294	309	325	340	356	371	293	215	170	125	80	129	145	160	176	236	297	357	418
Machine 3a	c	c	c	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
Machine 3b	1	1	1	1	c	c	c	c	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1
Silo 3b Volume	58	70	83	95	107	119	119	119	119	119	119	119	93	41	2	14	26	12	25	37	49	61	22	22	22	22	22	22	34	46	58	71	83	95	107	120
Truck 1	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	0	0	1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	370	327	382	503	570	637	703	770	837	903	872	774	594	469	344	286	227	169	111	134	158	280	401	425	449	417	386	355	323	347	370	394	449	504	559	614
Machine 4a	1	1	0	0	0	0	0	0	c	c	c	c	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	0	0	0	0	0	0	0	0	0
Machine 4b	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	0	0	0	0
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	98	98	98	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	c	c	0	0	c	c	0	0	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	c	c	0	0
Water Silo 2	53	1	1	1	1	1	1	1	1	1	2	2	3	3	4	4	5	5	6	7	7	7	7	7	7	7	7	7	7	0	0	0	0	0	0	0
Silo 6 Volume	105	149	149	149	149	149	149	149	149	149	194	238	270	281	292	303	313	324	335	380	380	330	303	277	250	224	197	171	215	260	304	349	349	349	349	349
Machine 6	1	0	0	0	0	0	0	0	c	c	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0
Silo 7 Volume	41	-24	0	0	0	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0
Machine 7a	1	1	c	c	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	0
Machine 7b	1	1	1	c	c	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0
Water Silo 3	41	82	82	82	82	82	82	82	82	82	82	82	82	82	100	100	82	82	82	82	82	82	82	82	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	33	44	55	6	6	6	6	6	6	6	6	6	6	6	6	17	28	39	2	13	24	24	24	24	35	46	57	69	31	17	17	17	17	17	17	17
Truck 2	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	0

Figure A.3: Day 3 Spreadsheet Solution

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	155	155	155	155	155	155	155	155	155	0	0	0	0	0	128	262	214	178	156	99	232	40	0	0	0	153	153	153	153	153	153	153	153	153	153	0
Silo 1 Volume	647	801	896	955	1015	1074	1074	1073	1073	918	858	798	738	643	616	723	722	685	626	510	527	447	387	327	292	410	529	647	766	884	1002	1001	1034	1068	1101	981
Machine 1a	0	c	c	c	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	c	c	c	0	0	0	0	0	0	0	0	0	0
Machine 1b	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1
Machine 1c	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	0	c	c	c	0
Machine 1d	c	c	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1
Silo 2 Volume	367	421	506	591	531	525	519	513	506	415	324	233	173	167	160	353	546	593	641	688	651	560	468	355	386	417	449	480	511	543	682	644	607	569	532	494
Machine 2	0	0	c	c	1	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	0	c	c	0	0	0	0	1	1	1	1	1
Water	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1
Water Silo 1	0	0	0	0	9	19	28	37	47	56	65	75	84	93	75	75	75	75	75	75	75	75	75	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	215	170	125	80	129	145	160	176	236	297	357	418	433	449	464	386	308	324	340	355	371	386	402	450	405	360	315	282	249	216	183	244	304	365	425	441
Machine 3a	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	0	0	1
Machine 3b	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1
Silo 3b Volume	22	22	22	22	34	46	58	71	83	95	107	120	80	66	26	26	26	39	51	63	75	88	74	34	34	34	34	34	34	34	34	46	59	71	83	95
Truck 1	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	0	0	0	0	0	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	449	417	386	355	323	347	370	394	449	504	559	614	736	858	979	921	862	804	746	687	629	571	512	399	368	434	501	556	611	666	721	776	831	886	941	1063
Machine 4a	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
Machine 4b	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	1	1	1	1	1	1	c	c	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0	0
Water Silo 2	53	1	2	2	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0	0	0	0	0	0	0
Silo 6 Volume	250	224	197	171	215	260	304	349	349	349	349	349	299	228	157	168	179	189	200	282	364	396	407	417	391	320	249	178	178	178	178	178	178	178	178	178
Machine 6	1	1	1	1	c	c	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0
Silo 7 Volume	53	49	45	41	-24	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0
Machine 7a	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	0
Machine 7b	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	0	0	0
Water Silo 3	41	82	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	35	46	57	69	31	17	17	17	17	17	17	17	17	17	17	4	15	26	37	49	60	11	11	11	22	33	44	55	18	29	29	29	29	29	29	29
Truck 2	0	0	0	0	2	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	2	0	0	0	0	0	0	2	0	0	0	0	0	0	0

Figure A.4: Day 4 Spreadsheet Solution

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	292	410	529	647	766	884	1002	1001	1034	1068	1101	981	861	681	749	951	1070	1033	974	858	935	820	665	510	665	819	914	948	983	1017	1052	1086	1121	1001	1001	1001
Machine 1a	c	c	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	c	c	c	0	0	1	1	1	1	1	1	1	1	0	0	0
Machine 1b	c	c	c	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	0	0	0
Machine 1c	1	1	1	1	1	1	1	0	c	c	c	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	0	0	0
Machine 1d	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	0	0	c	c	c	0	0	0	0	0	0	0	0
Silo 2 Volume	386	417	449	480	511	543	682	644	607	569	532	494	510	419	473	558	605	653	701	694	688	682	676	531	531	585	692	655	617	580	542	505	467	322	322	322
Machine 2	0	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	1	c	c
Water	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	0
Water Silo 1	0	0	0	0	0	0	0	9	19	28	37	47	56	65	65	65	75	84	93	65	65	65	65	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	405	360	315	282	249	216	183	244	304	365	425	441	457	472	394	316	332	347	363	378	394	409	425	474	429	384	339	354	370	385	401	416	432	447	414	381
Machine 3a	1	1	1	0	c	c	c	c	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Machine 3b	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1
Silo 3b Volume	34	34	34	34	34	34	34	46	59	71	83	95	82	68	42	42	2	14	27	39	51	63	76	36	36	10	10	22	34	47	59	71	83	96	96	96
Truck 1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	2	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	368	434	501	556	611	666	721	776	831	886	941	1063	1184	1306	1248	1189	1131	1073	1014	956	898	839	781	668	734	801	868	989	1111	1233	1256	1280	1304	1327	1284	1241
Machine 4a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	1	1	1	1	1	1
Machine 4b	1	c	c	c	c	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	c	c	c	c	0	0	0	0	0
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	c	c	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0	1	1	1	1	1	1
Water Silo 2	53	53	53	53	53	53	53	53	53	53	53	53	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	391	320	249	178	178	178	178	178	178	178	178	178	128	57	68	79	89	100	111	193	275	306	317	328	257	186	115	44	44	44	89	133	178	222	267	311
Machine 6	1	1	1	1	c	c	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0
Silo 7 Volume	53	49	45	41	-24	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0
Machine 7a	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	0	c	c	0	0	0	0
Machine 7b	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	0	0	0
Water Silo 3	41	82	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	22	33	44	55	18	29	29	29	29	29	29	29	29	29	29	15	27	38	49	11	22	22	22	22	33	45	56	67	29	40	40	40	40	40	40	40
Truck 2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0

Figure A.5: Day 5 Spreadsheet Solution

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	155	155	155	155	155	155	155	155	155	0	0	0	0	0	128	262	214	178	156	99	232	40	0	0	0	153	153	153	153	153	153	153	153	153	153	0
Silo 1 Volume	665	819	914	948	983	1017	1052	1086	1121	1001	1001	1001	1001	941	1009	1091	1125	1123	1099	1018	1035	920	765	730	695	813	872	905	939	972	970	969	967	966	1024	989
Machine 1a	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	0	0	0	0	c	c	c	0	0	0	0	0	0	0	0	0
Machine 1b	c	c	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	0	0	0
Machine 1c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1
Machine 1d	0	c	c	c	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	c	c	c	1	1	1	1	1	1	1	1	0	0
Silo 2 Volume	531	585	692	655	617	580	542	505	467	322	322	322	376	429	591	607	623	639	656	703	697	691	577	464	495	580	687	650	612	606	600	594	588	528	559	591
Machine 2	c	c	0	1	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	1	0	0
Water	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	0
Water Silo 1	0	0	0	9	19	28	37	47	56	65	65	65	65	65	65	75	84	93	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	429	384	339	354	370	385	401	416	432	447	414	381	348	315	282	343	358	374	390	405	421	436	452	467	389	344	299	348	396	412	427	443	459	474	396	318
Machine 3a	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Machine 3b	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1
Silo 3b Volume	36	10	10	22	34	47	59	71	83	96	96	96	96	44	18	30	42	28	41	53	65	25	12	24	24	24	24	36	48	61	73	85	97	110	110	110
Truck 1	0	1	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0	1	0	0	0	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	734	801	868	989	1111	1233	1256	1280	1304	1327	1284	1241	1198	1155	1112	1167	1289	1313	1336	1278	1220	1161	1103	1045	986	873	842	810	877	999	1120	1242	1266	1289	1313	1337
Machine 4a	0	c	c	c	c	0	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1
Machine 4b	0	0	0	c	c	c	c	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	98	98	16	16	16	16	16	16	16	16	0	0	0	0	0	0	0	0	0
Machine 5	c	c	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1
Water Silo 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	257	186	115	44	44	44	89	133	178	222	267	311	306	279	253	182	111	40	51	133	214	246	257	268	279	290	270	244	244	244	244	244	288	333	378	422
Machine 6	1	1	1	1	c	c	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0
Silo 7 Volume	53	49	45	41	-24	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0
Machine 7a	1	1	1	1	1	0	c	c	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	0
Machine 7b	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	0	0	0
Water Silo 3	41	82	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	33	45	56	67	29	40	40	40	40	40	40	40	40	40	40	51	38	49	11	23	34	34	9	9	20	32	43	54	16	27	27	27	27	27	27	27
Truck 2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	0	0	1	0	0	0	0	0	2	0	0	0	0	0	0	0

Figure A.6: Day 6 Spreadsheet Solution

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	695	813	872	905	939	972	970	969	967	966	1024	989	954	894	902	984	983	946	887	771	788	673	578	543	698	852	1007	1101	1196	1230	1265	1239	1214	1034	914	794
Machine 1a	c	c	c	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	c	c	c	0	0	0	0	1	1	1	1	1	1
Machine 1b	c	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	1	1	1	1	1	1	1	1
Machine 1c	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	0	0	0	c	c	c	0	0	0	0	0	0	0
Machine 1d	c	c	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	0	0	0
Silo 2 Volume	495	580	687	650	612	606	600	594	588	528	559	591	499	462	478	526	573	621	668	716	710	650	536	391	391	391	300	209	171	134	150	166	182	290	397	505
Machine 2	c	c	0	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	0	0	0
Water	0	0	0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0
Water Silo 1	0	0	0	9	19	28	37	47	56	65	65	65	75	84	93	65	65	65	65	65	65	65	65	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	389	344	299	348	396	412	427	443	459	474	396	318	367	415	464	512	561	609	658	707	755	804	852	946	913	880	940	1001	1061	1122	1182	1243	1259	1181	1103	1025
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	c	c	c	c	1	1	1	1
Machine 3b	1	c	c	c	c	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
Silo 3b Volume	24	24	24	36	48	61	73	85	97	110	110	110	96	56	16	29	41	53	65	26	38	-2	10	23	23	23	35	47	59	72	84	96	108	108	108	108
Truck 1	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	0	0	0	0	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 4 Volume	986	873	842	810	877	999	1120	1242	1266	1289	1313	1337	1305	1192	1078	965	852	820	789	758	726	613	500	402	359	316	273	230	187	144	199	254	375	497	619	642
Machine 4a	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
Machine 4b	1	1	c	c	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	c	c	c	c	0	0	0	1
Silo 5 volume	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	98	196	196	114	114	114	32	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	0	0	0	c	c	1
Water Silo 2	98	1	2	2	2	2	2	2	3	3	4	4	5	5	6	7	7	8	8	8	8	8	8	8	8	8	8	8	8	0	0	0	0	0	0	0
Silo 6 Volume	279	290	270	244	244	244	244	244	288	333	378	422	417	427	438	449	460	433	362	362	444	476	487	498	486	459	433	406	451	496	496	446	375	304	233	206
Machine 6	1	1	1	1	c	c	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	0	c	c	f	1	1	1	1
Silo 7 Volume	53	49	45	41	-24	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49
Machine 7a	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	1	1	1
Machine 7b	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	0	0	c	c	1	1
Water Silo 3	41	82	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	20	32	43	54	16	27	27	27	27	27	27	27	27	27	27	38	50	61	23	34	21	21	21	21	32	43	54	65	27	39	39	14	14	14	25	12
Truck 2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	2	0	0	1	0	0	0	1

Figure A.7: Day 7 Spreadsheet Solution

Appendix B

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	216	249	223	196	170	143	116	90	123	217	370	310	250	190	258	400	434	432	468	412	489	374	219	184	304	363	363	397	432	406	381	355	390	330	270	210
Machine 1a	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	0	c	c	c	0	0	0	1	1	1	1	1	1	1	c	c	c	0	1
Machine 1b	0	1	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	0	1	1	1	1	1	1	1	1
Machine 1c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	c	c	c	0	0
Machine 1d	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	0
Silo 2 Volume	315	332	348	364	380	397	413	375	429	429	483	392	300	209	172	188	204	166	160	154	293	432	318	204	145	138	101	63	80	96	112	220	273	327	381	343
Machine 2	1	1	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	0	c	c	0	1
Water	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	1
Water Silo 1	9	19	28	37	47	56	65	75	75	75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	168	183	199	214	230	245	261	276	198	120	42	58	73	89	105	120	136	151	167	182	104	26	42	57	73	89	104	153	201	250	298	253	208	163	118	167
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Machine 3b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	0	0	0
Silo 3b Volume	45	58	44	56	68	81	93	105	105	53	53	65	78	90	102	114	127	139	99	59	59	59	72	32	44	56	69	81	93	53	66	66	14	14	14	26
Truck 1	0	0	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	2	2	0	0	0	2	0	0	0	0	0	2	0	0	2	0	0	0
Silo 4 Volume	527	468	410	352	293	235	177	118	60	2	25	147	269	390	512	634	575	517	459	400	342	284	225	167	108	230	352	418	485	552	618	685	752	638	525	412
Machine 4a	0	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	1	1	1
Machine 4b	0	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	1	1	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0	0	1	1	1
Water Silo 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	71	82	93	103	114	125	136	218	300	381	376	305	234	163	92	21	32	114	195	227	238	249	260	270	281	210	210	210	210	210	210	160	89	100	111	122
Machine 6	f	1	1	1	1	1	1	c	c	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	0	0	f	1	1	1	1
Silo 7 Volume	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	61	57	53	49
Machine 7a	0	0	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	1	1	1
Machine 7b	0	0	0	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	0	0	1	1
Water Silo 3	0	0	41	82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	14	14	14	25	36	23	34	45	7	7	7	7	7	19	30	41	52	63	74	74	74	74	37	48	59	70	81	92	43	43	43	43	43	43	6	17
Truck 2	0	0	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0	0	0	0	0	2	0

Figure B.1: Day 1 Spreadsheet Solution for updated objective

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	155	155	155	155	155	155	155	155	155	0	0	0	0	0	128	262	214	178	156	99	232	40	0	0	0	153	153	153	153	153	153	153	153	153	153	0
Silo 1 Volume	304	363	363	397	432	406	381	355	390	330	270	210	90	30	98	240	274	272	248	167	279	199	139	79	79	172	266	299	273	246	219	193	191	190	248	213
Machine 1a	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	1	1	1	1	1	1	1	c	c
Machine 1b	c	0	0	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	0	0	0
Machine 1c	1	1	c	c	c	0	0	c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
Machine 1d	c	1	1	1	1	1	1	1	c	c	c	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	0	0
Silo 2 Volume	145	138	101	63	80	96	112	220	273	327	381	343	252	161	123	139	156	172	188	151	113	22	76	76	129	183	146	162	178	194	210	204	198	283	315	346
Machine 2	1	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	1	c	c	0
Water	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0
Water Silo 1	9	19	28	37	47	56	65	65	65	65	65	75	84	93	91	91	91	100	100	100	100	100	37	37	37	37	37	37	0	0	0	0	0	0	0	0
Silo 3a Volume	73	89	104	153	201	250	298	253	208	163	118	167	182	198	213	229	245	260	276	291	340	388	343	298	253	208	257	305	321	336	352	368	383	305	227	149
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Machine 3b	1	1	1	0	c	c	c	c	0	0	0	0	1	1	1	1	1	1	1	1	0	c	c	c	c	0	0	0	1	1	1	1	1	1	1	1
Silo 3b Volume	44	56	69	81	93	53	66	66	14	14	14	26	38	50	11	23	9	21	8	20	32	44	44	44	44	44	57	69	29	41	54	66	26	26	26	26
Truck 1	0	0	0	0	0	2	0	0	2	0	0	0	0	0	2	0	1	0	1	0	0	0	0	0	0	0	0	0	2	0	0	0	2	0	0	0
Silo 4 Volume	108	230	352	418	485	552	618	685	752	638	525	412	353	295	237	178	120	62	3	125	192	258	325	392	458	525	493	462	486	509	533	557	498	440	382	405
Machine 4a	1	c	c	c	c	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	1	1	1	1	1	1	1	1	1	0
Machine 4b	1	c	c	c	c	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	0	1	1	1	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	c	c	0	0	c	c	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
Water Silo 2	98	98	98	98	98	98	98	98	98	76	77	77	78	78	79	5	5	6	7	7	7	7	7	7	7	7	7	7	0	0	0	0	0	0	0	0
Silo 6 Volume	281	210	210	210	210	210	210	160	89	100	111	122	133	143	225	307	339	350	360	289	218	147	76	76	76	76	121	166	160	134	107	81	92	102	113	158
Machine 6	1	1	c	c	0	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	0	c	c	0	0	f	1	1	1	1	1	1	c
Silo 7 Volume	45	41	-24	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	61	57	53	49	45	41	-24
Machine 7a	1	1	1	c	c	0	0	0	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	0	c	c	0	0	0	1	1	1	1	1	1
Machine 7b	1	1	1	1	c	c	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	0	c	c	0	0	0	1	1	1	1	1
Water Silo 3	41	82	100	100	100	100	100	100	100	41	59	59	59	59	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0
Silo 8 Volume	59	70	81	92	43	43	43	43	43	43	6	17	28	39	50	61	61	61	61	73	84	95	106	117	128	128	79	79	79	79	79	42	53	15	2	13
Truck 2	0	0	0	0	2	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	2	0	2	1	0

Figure B.2: Day 2 Spreadsheet Solution for updated objective

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	79	172	266	299	273	246	219	193	191	190	248	213	178	83	56	198	292	350	291	175	252	197	102	7	7	101	196	290	385	384	384	443	503	408	313	218
Machine 1a	c	c	c	1	1	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	0	0
Machine 1b	c	c	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	0	0	0
Machine 1c	0	0	0	0	0	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	0	c	c	c	1	1	1	1	1	1	1	0
Machine 1d	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	0	c	c	c	0	0	0	0	0	1	1	1	1	1	1	1	0
Silo 2 Volume	129	183	146	162	178	194	210	204	198	283	315	346	286	280	242	205	167	215	262	401	486	571	657	650	559	468	377	285	279	273	213	298	383	468	553	408
Machine 2	c	0	1	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	1	0	0	c	c	1	1	1	1	1	1	1	1	c	c	0	0	1
Water	0	0	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	1
Water Silo 1	0	0	9	19	28	37	47	56	65	65	65	65	75	84	93	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	253	208	257	305	321	336	352	368	383	305	227	149	198	246	295	343	359	374	390	312	234	156	78	127	175	224	272	321	369	385	400	322	244	166	88	149
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c
Machine 3b	c	c	c	c	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	0	0	c	c	c	c	1	1	1	1	1	1	1
Silo 3b Volume	44	44	57	69	29	41	54	66	26	26	26	26	38	51	63	75	87	100	60	60	60	60	60	72	84	97	83	95	107	120	132	132	132	80	80	40
Truck 1	0	0	0	0	2	0	0	0	2	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2	0	2
Silo 4 Volume	458	525	493	462	486	509	533	557	498	440	382	405	374	343	311	280	304	327	449	571	692	814	936	1002	1069	1136	1022	909	796	737	679	621	562	504	446	403
Machine 4a	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0
Machine 4b	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	c	c	c	c	0	1	1	1	1	1	1	1	1	1	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	98	196	196	114	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	0	0	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1
Water Silo 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 6 Volume	76	76	121	166	160	134	107	81	92	102	113	158	158	158	240	321	380	425	425	425	375	304	233	162	91	20	31	113	194	226	237	248	259	270	280	254
Machine 6	c	c	0	0	f	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1
Silo 7 Volume	-24	0	0	0	0	61	57	53	49	45	41	-24	0	0	0	0	0	0	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41
Machine 7a	0	c	c	0	0	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1
Machine 7b	1	0	c	c	0	0	0	1	1	1	1	1	1	c	c	0	0	0	0	0	0	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1
Water Silo 3	0	0	0	0	0	0	41	82	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Silo 8 Volume	128	128	79	79	79	79	79	42	53	15	2	13	24	24	24	24	24	24	24	24	24	24	24	35	46	57	69	80	42	42	42	17	29	40	2	13
Truck 2	0	0	2	0	0	0	0	2	0	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	1	0	0	2	0

Figure B.3: Day 3 Spreadsheet Solution for updated objective

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	155	155	155	155	155	155	155	155	155	0	0	0	0	0	128	262	214	178	156	99	232	40	0	0	0	153	153	153	153	153	153	153	153	153	153	0
Silo 1 Volume	7	101	196	290	385	384	384	443	503	408	313	218	218	218	286	453	572	595	596	540	617	562	407	287	167	260	354	412	411	349	347	346	344	343	341	221
Machine 1a	c	c	c	0	0	0	0	0	c	c	c	0	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c
Machine 1b	1	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1	1	0	c	c	c	0	0	0	0	1	1	1	1	1	1	1	0
Machine 1c	0	c	c	c	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	1	c	c
Machine 1d	c	c	0	0	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
Silo 2 Volume	559	468	377	285	279	273	213	298	383	468	553	408	263	172	112	52	46	40	179	318	403	542	504	467	375	284	224	218	266	259	253	247	241	380	487	342
Machine 2	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	0	0	c	c	1	1	1	1	1	1	1	1	1	1	1	c	c	1
Water	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1
Water Silo 1	9	19	28	37	47	56	65	65	65	65	65	75	84	93	91	91	91	100	100	100	100	100	100	100	37	37	37	37	37	0	0	0	0	0	0	0
Silo 3a Volume	175	224	272	321	369	385	400	322	244	166	88	149	209	270	331	346	362	377	299	221	143	65	81	96	112	127	143	158	174	190	205	221	236	191	146	195
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Machine 3b	0	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
Silo 3b Volume	84	97	83	95	107	120	132	132	132	80	80	40	0	13	25	37	49	10	10	10	10	10	22	34	46	7	19	31	43	56	68	54	66	14	14	27
Truck 1	0	0	1	0	0	0	0	0	0	2	0	2	2	0	0	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	1	0	2	0	0
Silo 4 Volume	###	1136	###	909	796	737	679	621	562	504	446	403	458	513	568	689	713	655	596	538	480	421	363	304	246	270	391	513	635	756	780	804	745	632	519	405
Machine 4a	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	1	1	1	1
Machine 4b	c	0	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	c	c	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1
Water Silo 2	0	0	98	62	62	63	63	64	64	65	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	66	0	0	0	0	0	0	0
Silo 6 Volume	91	20	31	113	194	226	237	248	259	270	280	254	254	254	204	133	106	117	128	139	150	232	313	345	356	330	259	188	117	46	90	135	166	177	188	199
Machine 6	1	1	1	c	c	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	f	1	1	1
Silo 7 Volume	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	61	57	53
Machine 7a	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	0	c	c	1	1
Machine 7b	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	0	c	c	1
Water Silo 3	41	82	100	100	100	100	100	41	59	59	59	59	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0
Silo 8 Volume	46	57	69	80	42	42	42	17	29	40	2	13	24	35	35	35	35	46	9	20	31	42	53	53	53	53	64	76	38	49	11	22	22	22	22	9
Truck 2	0	0	0	0	2	0	0	1	0	0	2	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0	1

Figure B.4: Day 4 Spreadsheet Solution for updated objective

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	167	260	354	412	411	349	347	346	344	343	341	221	221	161	194	301	360	323	264	148	225	145	85	25	180	334	489	583	678	712	747	721	696	516	336	216
Machine 1a	c	c	c	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1	1	0
Machine 1b	c	c	0	0	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	0	0	c	c	c	0	0	1	1	1	1	1	1
Machine 1c	c	0	1	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 1d	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1	1	c	c	c	0	0	0	0	1	1	1	1	1	1	1	1	0	0
Silo 2 Volume	375	284	224	218	266	259	253	247	241	380	487	342	251	191	185	179	226	274	321	315	278	187	240	240	240	240	294	203	165	128	144	160	176	193	300	354
Machine 2	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	1	0	0
Water	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	0	1
Water Silo 1	9	19	28	37	47	56	65	75	84	84	84	93	91	91	91	91	91	100	100	100	100	100	100	100	56	56	56	56	56	0	0	0	0	0	0	0
Silo 3a Volume	112	127	143	158	174	190	205	221	236	191	146	195	243	292	340	356	372	387	403	418	434	449	371	293	215	137	59	75	90	106	121	137	153	213	180	147
Machine 3a	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c
Machine 3b	1	1	1	1	1	1	1	1	1	0	c	c	c	c	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Silo 3b Volume	46	7	19	31	43	56	68	54	66	14	14	27	13	25	37	50	62	74	34	-5	7	19	19	19	19	19	19	31	44	4	16	28	41	53	53	27
Truck 1	0	2	0	0	0	0	0	1	0	2	0	0	1	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	1
Silo 4 Volume	246	270	391	513	635	756	780	804	745	632	519	405	292	179	65	7	31	152	274	396	517	541	483	424	366	308	249	191	133	74	98	220	341	396	451	408
Machine 4a	1	c	c	c	c	0	0	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0
Machine 4b	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	98	98	98	98	98	98	16	16	16	16	16	16	16	16	16	0	0	0	0	0
Machine 5	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1
Water Silo 2	98	1	1	1	1	1	2	2	3	3	4	4	5	5	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	0	0	0	0	0	0
Silo 6 Volume	356	330	259	188	117	46	90	135	166	177	188	199	210	221	231	313	313	313	263	192	121	132	143	154	164	246	328	410	442	453	433	362	291	220	149	194
Machine 6	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	f	1	1	1	1	1	1	0	c	c	f	1	1	1	1	1	1	0
Silo 7 Volume	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49	45	41	-24
Machine 7a	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1
Machine 7b	c	c	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1
Water Silo 3	0	41	82	100	100	100	59	59	59	59	59	59	59	59	59	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0
Silo 8 Volume	53	53	64	76	38	49	11	22	22	22	22	9	20	31	42	54	65	65	65	65	65	27	38	49	11	23	34	34	34	34	34	45	56	43	5	16
Truck 2	0	0	0	0	2	0	2	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	0	0	1	2	0

Figure B.5: Day 5 Spreadsheet Solution for updated objective

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	155	155	155	155	155	155	155	155	155	0	0	0	0	0	128	262	214	178	156	99	232	40	0	0	0	153	153	153	153	153	153	153	153	153	153	0
Silo 1 Volume	180	334	489	583	678	712	747	721	696	516	336	216	156	61	154	261	320	343	344	288	365	285	225	225	225	378	497	615	614	552	490	429	367	306	279	219
Machine 1a	c	c	0	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	1	0	c	c	c	0	1	1	1	1	1	1	1	0	0
Machine 1b	0	c	c	c	0	0	1	1	1	1	1	1	1	0	c	c	c	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
Machine 1c	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	c	c	c	0	0	1	1	1	1	1	1	1	1	0	0	0
Machine 1d	0	0	1	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	0	c	c	c	0	0	1	1	1	1	1	1	1	c	c
Silo 2 Volume	240	240	294	203	165	128	144	160	176	193	300	354	294	180	174	168	162	156	149	143	106	160	160	160	160	191	222	361	409	456	504	551	599	615	524	379
Machine 2	c	c	0	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	0	c	c	0	0	0	0	1	1	1	1	1	1	1	1
Water	0	0	0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
Water Silo 1	0	0	0	9	19	28	37	47	56	65	65	65	75	84	93	65	65	65	65	65	65	65	65	65	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	215	137	59	75	90	106	121	137	153	213	180	147	208	268	284	299	315	330	346	361	377	344	311	278	245	167	89	11	27	42	58	73	89	104	120	135
Machine 3a	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	1
Machine 3b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Silo 3b Volume	19	19	19	31	44	4	16	28	41	53	53	27	13	25	38	50	62	74	87	47	59	59	59	59	33	33	7	7	19	32	44	56	68	55	67	27
Truck 1	0	0	0	0	0	2	0	0	0	0	0	1	1	0	0	0	0	0	0	2	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	2
Silo 4 Volume	366	308	249	191	133	74	98	220	341	396	451	408	365	322	346	370	393	417	441	464	488	543	598	653	610	552	493	435	377	318	260	202	143	167	289	410
Machine 4a	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0
Machine 4b	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	0	0
Silo 5 volume	16	16	16	16	16	16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	1	1	1	1	0	0
Water Silo 2	98	1	2	2	3	3	4	4	4	4	4	4	5	5	6	7	7	8	8	9	9	9	9	9	9	9	9	9	9	0	0	0	0	0	0	0
Silo 6 Volume	164	246	328	410	442	453	433	362	291	220	149	194	238	233	207	180	154	127	101	74	119	119	119	69	42	53	64	75	86	96	178	260	292	265	194	123
Machine 6	1	0	c	c	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	0	f	1	1	1	1	1	1	c	c	f	1	1	1
Silo 7 Volume	41	-24	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49	45	41	-24	0	0	61	57	53
Machine 7a	1	1	0	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	0	c	c	0	1	1	1	1	1	1	0	c	c	1	1
Machine 7b	1	1	1	0	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	c	c	0	1
Water Silo 3	41	82	82	82	82	82	41	59	59	59	59	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0
Silo 8 Volume	11	23	34	34	34	34	34	45	56	43	5	16	27	27	27	27	38	50	61	72	83	94	94	94	94	94	81	68	79	41	3	14	14	14	14	25
Truck 2	2	0	0	0	0	0	0	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	2	2	0	0	0	0	0

Figure B.6: Day 6 Spreadsheet Solution for updated objective

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Milk delivery	0	153	153	153	153	153	153	153	153	153	153	0	0	0	128	262	214	178	156	99	232	40	0	0	155	155	155	155	155	155	155	155	155	0	0	0
Silo 1 Volume	225	378	497	615	614	552	490	429	367	306	279	219	219	219	252	359	358	321	262	146	163	83	23	23	118	212	247	281	316	350	385	479	514	454	334	214
Machine 1a	c	c	0	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	1	1	1	1	1	1	1	c	c	c	1	1	1
Machine 1b	c	c	0	0	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	0	c	c	c	0	0	0	0	0	1	1	1	1	1
Machine 1c	0	1	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	0	0	0	0	c	c	c	0	0	0	0	0	0	0	0	0
Machine 1d	c	c	c	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	c	c	c	0	1	1	1	1	1	1	1	c	c	c	1
Silo 2 Volume	160	191	222	361	409	456	504	551	599	615	524	379	234	174	313	505	553	601	648	696	658	567	422	331	239	202	164	127	89	197	251	358	412	374	337	353
Machine 2	0	0	0	0	1	1	1	1	1	1	1	1	1	1	c	c	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	0	0	1	1	1
Water	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1
Water Silo 1	0	0	0	0	9	19	28	37	47	56	65	75	84	93	75	75	75	75	75	75	75	75	75	75	0	0	0	0	0	0	0	0	0	0	0	0
Silo 3a Volume	245	167	89	11	27	42	58	73	89	104	120	135	151	167	89	11	26	42	57	73	88	104	119	135	150	166	182	230	279	234	189	144	66	81	97	157
Machine 3a	c	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
Machine 3b	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1
Silo 3b Volume	33	33	7	7	19	32	44	56	68	55	67	27	39	52	52	52	64	76	88	101	113	125	137	150	136	96	108	69	29	29	29	29	29	15	27	40
Truck 1	1	0	1	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	2	2	0	0	0	0	1	0	0
Silo 4 Volume	610	552	493	435	377	318	260	202	143	167	289	410	532	555	497	439	380	322	264	205	147	89	30	152	274	395	517	584	552	521	490	458	482	506	529	404
Machine 4a	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	1	c	c	c	c	0	1	1	1	1	1	1	1	1
Machine 4b	c	1	1	1	1	1	1	1	1	1	c	c	c	c	1	1	1	1	1	1	1	1	1	c	c	c	c	0	0	0	0	0	0	0	0	1
Silo 5 volume	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Machine 5	1	1	1	1	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	1	1	1	1	c	c	0	0	0	1	1	1	1	1	1	1	1
Water Silo 2	53	1	2	2	3	3	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0	0	0	0	0	0	0
Silo 6 Volume	42	53	64	75	86	96	178	260	292	265	194	123	52	26	37	119	200	232	243	254	265	276	286	215	215	215	215	165	139	112	86	60	33	7	51	133
Machine 6	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	c	c	f	1	1	1	1	1	1	0	c	c	f	1	1	1	1	1	1	0	0
Silo 7 Volume	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	61	57	53	49	45	41	-24	0	0	0	61	57	53	49	45	41	-24	0
Machine 7a	0	1	1	1	1	1	1	0	c	c	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1	0
Machine 7b	0	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	1	1	1	1	1	1	c	c	0	0	1	1	1	1	1	1
Water Silo 3	0	41	82	100	100	100	59	59	59	59	59	59	59	59	59	100	100	100	100	100	100	100	100	100	100	100	100	100	100	0	0	0	0	0	0	0
Silo 8 Volume	94	94	81	68	79	41	3	14	14	14	14	25	36	23	34	45	57	57	57	57	68	30	17	28	39	50	1	1	1	1	12	23	35	21	8	19
Truck 2	0	0	1	1	0	2	2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	2	1	0	0	0	2	0	0	0	0	0	0	1	1	0

Figure B.7: Day 7 Spreadsheet Solution for updated objective