IBSL Unit Exam Review
Probability and Statistic

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanation. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g., if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working. Working may be continued below the lines, if necessary.

1. The letters of the word PROBABILITY are written on 11 cards as show below. (adapted from 1IB 2009)

P	**R**	**O**	**B**	**A**	**B**	**I**	**L**	**I**	**T**	**Y**

Two cards are drawn at random without replacement.
Let A be the even the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

a) Find $P(A)$. $= \boxed{\frac{1}{11}}$

b) Find $P(B|A)$. $= \dfrac{P(B\cap A)}{P(A)} = \dfrac{\frac{1}{11}\cdot\frac{2}{10}}{\frac{1}{11}} = \boxed{\frac{1}{5}}$

c) Find $P(A\cap B)$. $= P(A)\cdot P(B) = \frac{1}{11}\cdot\frac{1}{5} = \boxed{\frac{1}{55}}$

2. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X. (adapted from IB 2005)

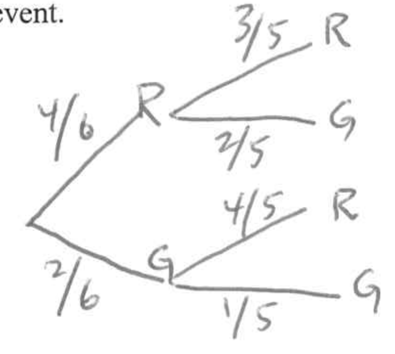
X	0	1	2
P(X = x)	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

a) Calculate E (X), the mean number of red balls chosen. [3 marks]

$E(X) = 0\left(\frac{3}{10}\right) + 1\left(\frac{6}{10}\right) + 2\left(\frac{1}{10}\right) = \boxed{\frac{8}{10} = .8}$

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

bi) Draw a tree diagram to represent the above information, including the probability of each event. [3 marks]



$\rightarrow RR = \frac{4}{6}\cdot\frac{3}{5} = \frac{12}{30}$

$\rightarrow GG = \frac{2}{6}\cdot\frac{1}{5} = \frac{2}{30}$

bii) Hence find the probability distribution for Y, where Y is the number of red balls chosen. [5 marks]

Y	0	1	2
$P(Y=y)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

c) Calculate the probability that two red balls are chosen. [5 marks]

$P(1 \text{ or } 6) = P(1) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

Bag A $\Rightarrow \frac{2}{6} \cdot \frac{1}{10} = \boxed{\frac{2}{60}} \Rightarrow P(A \cap RR)$

Bag B $\Rightarrow \frac{4}{6} \cdot \frac{12}{30} = \boxed{\frac{48}{180}} \Rightarrow P(B \cap RR)$

$\Rightarrow \frac{2}{60} + \frac{48}{180} = \boxed{\frac{3}{10} = .3}$

d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die. [3 marks]

$P(1 \text{ or } 6 \mid RR) \Rightarrow$ Since 1 or 6 come from Bag A $\Rightarrow P(A)$

$\therefore P(A \mid RR) = \frac{P(A \cap RR)}{P(RR)} = \frac{\frac{2}{60}}{3/10} = \boxed{\frac{1}{9}}$

3. In a game a player rolls a biased four-faced die. The probability of each possible score is shown below. (adapted from IB 2005)

Score	1	2	3	4
Probability	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{3}{8}$	x

a) Find the value of x. $\frac{1}{6} + \frac{2}{7} + \frac{3}{8} + X = 1 \Rightarrow \boxed{X = \frac{29}{168}}$ [2 marks]

b) Find E (X) $= 1\left(\frac{1}{6}\right) + 2\left(\frac{2}{7}\right) + 3\left(\frac{3}{8}\right) + 4\left(\frac{29}{168}\right)$ [3 marks]

$E(X) = \sum x f(x) = \boxed{\frac{143}{56} \approx 2.55}$

c) The die is rolled twice. Find the probability of obtaining two scores of 3. [2 marks]

$\frac{3}{8} \cdot \frac{3}{8} = \boxed{\frac{9}{64} \approx .141}$

4. The probability of obtaining heads on a biased coin is $\frac{1}{5}$. Simba tosses the coin three times. Find the probability of getting: (adapted from IB 2005)

a) Three heads. $\left(\frac{1}{5}\right)^3 = \boxed{\frac{1}{125} = .008}$ [2 marks]

b) Two heads and one tail. $\binom{3}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^1 = \boxed{\frac{12}{125} = .096}$ [3 marks]

Lizzie plays a game in which she tosses the coin 15 times.

c) Find the expected number of heads. $15\left(\frac{1}{5}\right) = \boxed{3}$ [2 marks]

d) Lizzie wins \$ 10 for each head obtained, and loses \$ 6 for each tail. Find her expected winnings. [3 marks]

$3 \cdot 10 = \$30$

$12 \cdot 6 = -72$

$\boxed{-\$42}$

5. A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches. (adapted from 2007 IB)

a) Find the mean number of defective switches in the sample. [2 marks]

$E(x) = np = \mu \Rightarrow 100(.04) = \boxed{4}$

b) Find the probability that there are exactly six defective switches in the sample. [2 marks]

$\binom{100}{6}(.04)^6(.96)^{94} = .10523$

OR binompdf$(100, .04, 6)$

c) Find the probability that there is at least one defective switch in the sample. [3 marks]

$P(x \geq 1) = 1 - P(x \leq 0) = 1 - \text{binomcdf}(100, .04, 0)$

OR $1 - \binom{100}{0}(.04)^0(.96)^{100} \approx \boxed{.9831}$

6. A and B are events such that P(A) = 0.4 and P(B) = 0.3. (adapted from 1995 IB)

a) If A and B are mutually exclusive events, find $P(A \cup B)$. $= P(A) + P(B)$ [2 Marks]

$= .4 + .3 = \boxed{.7}$

b) If A and B are independent events, find $P(A \cup B)$. [2 marks]

$P(A \cap B) = P(A) \cdot P(B) = (.4)(.3) = .12 \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .4 + .3 - .12$

$= \boxed{.58}$

c) If A and B are independent events, find $P(A|B)$. [2 marks]

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = \frac{.12}{.3} = \boxed{.4}$$

7. Consider the expansion of the expression $(x^3 - 3x)^6$. (adapted from 2007 IB)

a) Write down the number of terms in this expansion. $\boxed{7}$ [1 mark]

b) Find the term in x^{12}. $\binom{6}{3}(x^3)^3(-3x)^3 = 20(x^9)(-27x^3)$ [3 marks]

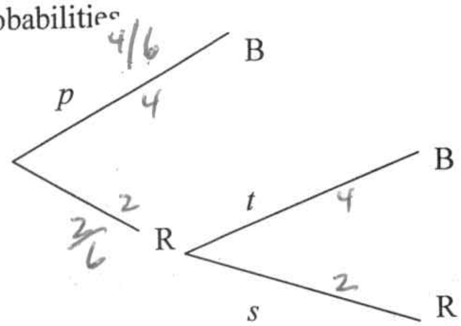
$= \boxed{-540x^{12}}$

8. A **six-sided** die has four blue faces and 2 red faces. The die is rolled. Let B the event of a blue face lands down, and R be the event a red face lands down. Write down: (adapted from 2008 IB)

a) P(B). $= 4/6$ [1 mark]

b) P(R). $= 2/6$ [1 mark]

c) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where *p, s, t* are probabilities.



$P = P(B) = \frac{4}{6}$

$t = P(R) \cdot P(B) = \frac{2}{6} \cdot \frac{4}{6} = \frac{8}{36}$

$s = P(R) \cdot P(R) = \frac{2}{6} \cdot \frac{2}{6} = \frac{4}{36}$

Find the value of *p*, of *s*, and of *t*. [2 marks]

Mojojojo plays a game where he rolls the die. If a blue face lands down, he scores 4 and is finished. If the red face lands down, he scores 2 and rolls one more time. Let X be the total score obtained.

d) Find P (X = 4). $= \frac{4}{6} + \frac{2}{6} \cdot \frac{2}{6} = \boxed{\frac{7}{9}}$ [1 mark]

e) Find P (X = 6). $= \frac{2}{6} \cdot \frac{4}{6} = \boxed{\frac{8}{36}}$ [2 marks]

f) Construct a probability distribution table for X. [3 marks]

x	4	6
$P(X=x)$	$\frac{7}{9}$	$\frac{8}{36}$

g) Calculate the expected value for X. [2 marks]

$E(X) = 4\left(\frac{7}{9}\right) + 6\left(\frac{8}{36}\right) = \boxed{\frac{40}{9} \approx 4.44}$

h) If the total score is 6, Mojojojo wins \$ 20. If the total score is 4 Mojojojo gets nothing. Mojojojo plays the game twice. Find the probability that he wins exactly \$ 20. [4 marks]

$2 \cdot P(X=4) \cdot P(X=6)$ $\Rightarrow$ Think:

1st Roll is a score of 4 and 2nd roll score of 6 $\Rightarrow$ win \$20

1st Roll is a score of 6 and 2nd roll score of 4 $\Rightarrow$ win \$20

$= 2\left(\frac{7}{9}\right)\left(\frac{8}{36}\right)$

$\approx .346 = \frac{28}{81}$