Reading recommendations

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First session: "device-independence"

For the study of causal relations, see the work of Judea Pearl [11].

A good review of the "quantum information" viewpoint: nonlocality [1]. For a more mathematical description of the objects involved, see the thesis of Stefano Pironio [12] (a paper exploring this in depth, with me as coauthor, is coming soon under the title "Algebraic and geometric properties of local transformations").

The main paper about inflation is [16]; the paper is pretty long and systematic. Start with Examples 1 and 2, which correspond to the example in the lecture and problem session respectively (they are on page 9 for the version 4 on the arXiv). Those examples are not "robust", in the sense that they detect incompatibility only for distributions that have the same pattern of "zero" coefficients.

In the lab session GitHub repository, the "CutInflation" project implements a way to test compatibility in a robust fashion using linear programming; while the approach is numerical there, no additional theoretical notions are needed. The inflation paper [16] however goes in much more detail about deriving inequalities, etc…

We discussed briefly on the difference between classical and quantum resources, and the impact on the inflation technique (quantum resources, or resources beyond quantum cannot be copied). For the case of inflations *compatible* with quantum (or beyond quantum) resources, see the very readable paper [4].

The generality of the inflation technique was proved in [10]. Several papers about the precise generalization of the inflation technique to quantum resources are coming soon (with Elie Wolfe, Miguel Navascues and myself as coauthors). Some of those papers will combine the NPA/ "sum-of-squares"/"moment-based relaxations" technique with inflation

A lot of scenarios remain unexplored, and can be studied without using involved tools.

Sum-of-squares/moment-based relaxations/NPA

For the second session, the main papers are [9] and [2], along with the short letter [8]. I encourage the readers to skip the convergence proofs which are quite technical – they are not necessary for the validity of the technique (and the computation for a degree $d \to \infty$ is anyway not possible).

There is an example of the computation of the CHSH bound in the GitHub repository "CHSH_nonsymmetric" which should be pretty straightforward (given notions of Matlab/YALMIP). YALMIP[5] has pretty good tutorials online.

For larger problems, writing the moment matrices by hand becomes tedious. While there exists several toolboxes (SOSTOOLS, GloptiPoly, SparsePOP) for the commutative case (i.e. the Lasserre/Parrilo hierarchy), there are fewer tools for the noncommutative case which interests us.

NCSOSTOOLS is targeted at a slightly different audience. Right now, I recommend NCPOL2SDPA which is quite accessible. I'm writing a library called SymDPoly, see the accompanying paper [13] which gives hints at how to implement those hierarchies, including their symmetrization.

(All the software package names can be Googled).

Representation theory

A good introduction is given in the first two chapters of the book by Jean-Pierre Serre [14] (the character theory part can be skipped at first reading). Complement it with the paper by Gatermann&Parrilo [3] (skip however Section 5 and after, as they do apply only to the commutative case).

RepLAB should enable the reader to play with examples. Its documentation should expand in the next months.

For the interested reader, the technique used by RepLAB is described in the Supplemental Material of [15], and is inspired by [7, 6].

Bibliography

[1] Nicolas Brunner, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner. Bell nonlocality. *Rev. Mod. Phys.*, 86(2):419–478, apr 2014.

[2] A.C. Doherty, Yeong-Cherng Liang, B. Toner, and S. Wehner. The Quantum Moment Problem and Bounds on Entangled Multi-Prover Games. In *23rd Annual IEEE Conference on Computational Complexity, 2008. CCC '08*, pages 199–210. Jun 2008.

[3] Karin Gatermann and Pablo A. Parrilo. Symmetry groups, semidefinite programs, and sums of squares. *Journal of Pure and Applied Algebra*, 192(1⟨textendash|⟩3):95–128, sep 2004.

[4] Nicolas Gisin, Jean-Daniel Bancal, Yu Cai, Armin Tavakoli, Emmanuel Zambrini Cruzeiro, Sandu Popescu, and Nicolas Brunner. Constraints on nonlocality in networks from no-signaling and independence. *ArXiv190606495 Quant-Ph*, jun 2019.

[5] J. Lofberg. YALMIP : a toolbox for modeling and optimization in MATLAB. In *2004 IEEE International Conference on Robotics and Automation (IEEE Cat. No.04CH37508)*, pages 284–289. Sep 2004.

[6] Takanori Maehara and Kazuo Murota. A numerical algorithm for block-diagonal decomposition of matrix *-algebras with general irreducible components. *Japan J. Indust. Appl. Math.*, 27(2):263–293, sep 2010.

[7] Kazuo Murota, Yoshihiro Kanno, Masakazu Kojima, and Sadayoshi Kojima. A numerical algorithm for block-diagonal decomposition of matrix $${*}$$-algebras with application to semidefinite programming. *Japan J. Indust. Appl. Math.*, 27(1):125–160, jun 2010.

[8] Miguel Navascués, Stefano Pironio, and Antonio Acín. Bounding the Set of Quantum Correlations. *Phys. Rev. Lett.*, 98(1):10401, jan 2007.

[9] Miguel Navascués, Stefano Pironio, and Antonio Acín. A convergent hierarchy of semidefinite programs characterizing the set of quantum correlations. *New J. Phys.*, 10(7):73013, jul 2008.

[10] Miguel Navascues and Elie Wolfe. The inflation technique solves completely the classical inference problem. *ArXiv170706476 Quant-Ph Stat*, jul 2017.

[11] Judea Pearl and Dana Mackenzie. *The Book of Why: The New Science of Cause and Effect*. Basic Books, New York, 1 edition edition, may 2018.

[12] Stefano Pironio and Serge Massar. *Aspects of Quantum Non-Locality*. PhD thesis, Université Libre de Bruxelles, 2004.

[13] Denis Rosset. SymDPoly: symmetry-adapted moment relaxations for noncommutative polynomial optimization. *ArXiv180809598 Quant-Ph*, aug 2018.

[14] Jean-Pierre Serre. *Linear Representations of Finite Groups*. Graduate Texts in Mathematics. Springer, 1977.

[15] Armin Tavakoli, Denis Rosset, and Marc-Olivier Renou. Enabling computation of correlation bounds for finite-dimensional quantum systems via symmetrisation. Aug 2018.

[16] Elie Wolfe, Robert W. Spekkens, and Tobias Fritz. The Inflation Technique for Causal Inference with Latent Variables. *ArXiv160900672 Quant-Ph Stat*, sep 2016.