Constructive representation theory and applications to causal structures Part III: how does it work*

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*. Download software for MATLAB/Octave at github.com/replab/replab

How does it work?

Given ρ: $\quad G \longrightarrow \mathsf{U}(\mathbb{K}^d)$

$$X = X^\dagger \in \mathbb{K}^{d\times d}$$

Sample from Gaussian Ens.
(*well separated EV*)

$$\downarrow$$

$$\overline{X} = \frac{1}{|G|}\sum_g \rho_g \, X \, \rho_g^\dagger$$

Project in commutant alg.
1st sample, cost: $|G| d^2$

$$\downarrow$$

$$\overline{X} = U \, D \, U^\dagger$$

Eigenvalue decomposition

$$D = \Big(\underbrace{\lambda_1, \ldots, \lambda_1}_{d_1 \text{ times}}, \underbrace{\lambda_2, \ldots, \lambda_2}_{d_2 \text{ times}}, \ldots \Big)$$

Cost: d^3

Irrep dimension

$$\downarrow$$

$$U^\dagger \rho \, U = \begin{pmatrix} \rho^1 & & \\ & \rho^2 & \\ & & \cdots \end{pmatrix}$$

Postprocessing (2nd sample)
Multiplicities
Division alg. over $\mathbb{R}$

Reducing the cost of averaging

Consider the symmetric group S_4 acting on 4 elements. Then every element $g \in \mathsf{S}_4$ can uniquely be written as:

$$g = (c_4)^i (c_3)^j (c_2)^k, \qquad c_4 = (1,2,3,4), c_3 = (1,2,3), c_2 = (1,2),$$

with $i = 0,1,2,3$, $j = 0,1,2$ and $k = 0,1$.

$$\sum_g \boldsymbol{\rho}_g[X] = \sum_i \boldsymbol{\rho}_{(c_4)^i}\left[\sum_j \boldsymbol{\rho}_{(c_3)^j}\left[\sum_k \boldsymbol{\rho}_{(c_2)^k}[X]\right]\right].$$

Corresponds to the chain of subgroups $\mathsf{S}_4 \supset \mathsf{S}_3 \supset \mathsf{S}_2$ (not nec. normal).

$$\text{Cost:} \qquad |G| \mapsto \frac{|\mathsf{S}_4|}{|\mathsf{S}_3|} + \frac{|\mathsf{S}_3|}{|\mathsf{S}_2|} + \frac{|\mathsf{S}_2|}{1}, \qquad \text{for } \mathsf{S}_m\text{:} \quad m! \longrightarrow \frac{(m+1)m}{2}$$

Other groups: use chain of stabilizer subgroups (Sims 1970, 1971).

Later on...

– Replace exact averaging by Monte Carlo integration

  (+) works with compact groups (-) need to control numerical errors

– Accelerate eigendecomposition

RepLAB v2

– (Compact) group as an oracle "get sample from the Haar measure"
– Representation as an explicit map $G \to U(d)$

– Optimized special cases (monomial representations, representations of symmetric group, etc...)
– Control of numerical error
– More basic rep. theory primitives like restriction/induction