Constructive representation theory and applications to causal structures Part IV: a taste of composition*

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*. Download software for MATLAB/Octave at github.com/replab/replab

Wreath product components

- The group $H = S_n$ permutes n components.

- Each component has symmetry group A + representation $\rho\colon A \to \mathcal{U}(d)$

- Wreath product group: $\quad W = S_n \wr A = S_n \ltimes (A_1 \times ... \times A_n)$.

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What should are natural representations of W?

- Imprimitive representation $\mathbb{R}^{nd} = \underbrace{\mathbb{R}^d \oplus \mathbb{R}^d \oplus \cdots \oplus \mathbb{R}^d}_{n \text{ times}}$

- Primitive representation $\mathbb{R}^{d^n} = \underbrace{\mathbb{R}^d \otimes \mathbb{R}^d \otimes \cdots \otimes \mathbb{R}^d}_{n \text{ times}}$

… with "what you expect" action.

Party with m measurement settings with k outcomes

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- Imprimitive representation

$$\mathbb{R}^{mk} = \underbrace{\mathbb{R}^k \oplus \mathbb{R}^k \oplus \cdots \oplus \mathbb{R}^k}_{m \text{ times}} \qquad \vec{P} = (\vec{P}(a|x=1), ..., \vec{P}(a|x=m))$$

- Primitive representation

$$\mathbb{R}^{k^m} = \underbrace{\mathbb{R}^k \otimes \mathbb{R}^k \otimes \cdots \otimes \mathbb{R}^k}_{m \text{ times}} \qquad P(a_{x=1} a_{x=2} ... a_{x=m})$$

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What should are natural representations of W?

- Imprimitive representation (conditional probability distributions)

$$\mathbb{R}^{mk} = \underbrace{\mathbb{R}^k \oplus \mathbb{R}^k \oplus \cdots \oplus \mathbb{R}^k}_{m \text{ times}} \qquad \vec{P} = (\vec{P}(a|x=1), ..., \vec{P}(a|x=m))$$

- Primitive representation (weights of deterministic strategies)

$$\mathbb{R}^{k^m} = \underbrace{\mathbb{R}^k \otimes \mathbb{R}^k \otimes \cdots \otimes \mathbb{R}^k}_{m \text{ times}} \qquad P(a_{x=1} a_{x=2} ... a_{x=m})$$

Scenario with n parties with m measurement settings with k outcomes

Wreath product group $S_n \wr S_m \wr S_k = S_n \ltimes ((S_m \wr S_k) \times \cdots \times (S_m \wr S_k))$.

Two levels (sic!) of wreath products

Scenario level $S_n \wr (S_m \wr S_k)$	Party level $S_m \wr S_k$	Final representation
Imprimitive	Imprimitive	List of POVM elements
Primitive	Imprimitive	$P(abc...\|xyz...)$
Primitive	Primitive	Deterministic local strategies

See https://github.com/replab/replab/blob/master/jupyter/
QCS_part4_TasteOfComposition.ipynb