*Project 1:*

# Application of User-Equilibrium Model, System Optimal Model, and Pricing Scheme in Sioux Falls Network, South Dakota

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## 1. Assignment Overview

This project aims to demonstrate the application of user equilibrium (UE), system optimal (SO), and the pricing scheme in a specific network. There are four objectives in this project. First, we exercise the arterial highway network of Sioux Falls, South Dakota (Figure.1), to determine the user equilibrium flow distribution. Second, using the result, we also evaluate the congested road inside the network, where the ratio of $\frac{v}{c} > 0.9$. In this analysis, we also discuss how this UE flow is distributed compared to the SO flow. Next, we propose an effective congestion pricing scheme for the network according to the UE analysis. In this task, we use the Marginal Cost Pricing scheme to provide an effective congestion pricing scheme, which will be proved to be the first-best tolling scheme for the network as the fourth objective. The network of Sioux Falls, South Dakota, is presented in Figure 1. The network parameter and the demand matrix are provided in Table.1 and Table.2.

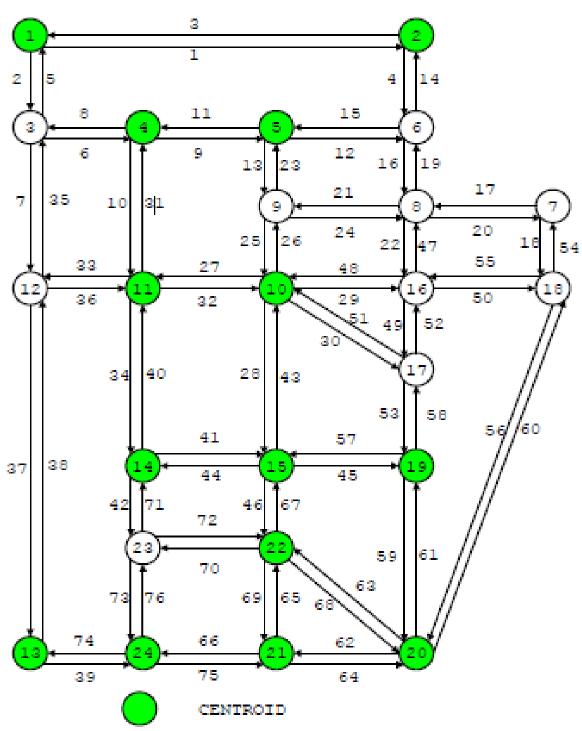


Figure 1 Highway network of Sioux Falls

Table 1 Link capacities and free flow travel times for the Sioux Falls network

| Link Number | $t_a^0$ (minutes) | $c_a$ (10^3 veh/hr) | Link Number | $t_a^0$ (minutes) | $c_a$ (10^3 veh/hr) |
|---|---|---|---|---|---|
| 1 | 3.6 | 6.02 | 39 | 2.4 | 10.18 |
| 2 | 2.4 | 9.01 | 40 | 2.4 | 9.75 |
| 3 | 3.6 | 12.02 | 41 | 3 | 10.26 |
| 4 | 3 | 15.92 | 42 | 2.4 | 9.85 |
| 5 | 2.4 | 46.81 | 43 | 3.6 | 27.02 |
| 6 | 2.4 | 34.22 | 44 | 3 | 10.26 |
| 7 | 2.4 | 46.81 | 45 | 2.4 | 9.64 |
| 8 | 2.4 | 25.82 | 46 | 2.4 | 20.63 |
| 9 | 1.2 | 28.25 | 47 | 3 | 10.09 |
| 10 | 3.6 | 9.04 | 48 | 3 | 10.27 |
| 11 | 1.2 | 46.85 | 49 | 1.2 | 10.46 |
| 12 | 2.4 | 13.86 | 50 | 1.8 | 39.36 |
| 13 | 3 | 10.52 | 51 | 4.2 | 9.99 |
| 14 | 3 | 9.92 | 52 | 1.2 | 10.46 |
| 15 | 2.4 | 9.9 | 53 | 1.2 | 9.65 |
| 16 | 1.2 | 21.62 | 54 | 1.2 | 46.81 |
| 17 | 1.8 | 15.68 | 55 | 1.8 | 39.36 |
| 18 | 1.2 | 46.81 | 56 | 2.4 | 8.11 |
| 19 | 1.2 | 9.8 | 57 | 2.4 | 4.42 |
| 20 | 1.8 | 15.68 | 58 | 1.2 | 9.65 |
| 21 | 2 | 10.1 | 59 | 2.4 | 10.01 |
| 22 | 3 | 10.09 | 60 | 2.4 | 8.11 |
| 23 | 3 | 20 | 61 | 2.4 | 6.05 |
| 24 | 2 | 10.1 | 62 | 3.6 | 10.12 |
| 25 | 1.8 | 27.83 | 63 | 3 | 10.15 |
| 26 | 1.8 | 27.83 | 64 | 3.6 | 10.12 |
| 27 | 3 | 20 | 65 | 1.2 | 10.46 |
| 28 | 3.6 | 27.02 | 66 | 1.8 | 9.77 |
| 29 | 3 | 10.27 | 67 | 2.4 | 20.63 |
| 30 | 4.2 | 9.99 | 68 | 3 | 10.15 |
| 31 | 3.6 | 9.82 | 69 | 1.2 | 10.46 |
| 32 | 3 | 20 | 70 | 2.4 | 10 |
| 33 | 3.6 | 9.82 | 71 | 2.4 | 9.85 |
| 34 | 2.4 | 9.75 | 72 | 2.4 | 10 |
| 35 | 2.4 | 46.81 | 73 | 1.2 | 10.16 |
| 36 | 3.6 | 9.82 | 74 | 2.4 | 11.38 |
| 37 | 1.8 | 51.8 | 75 | 1.8 | 9.77 |
| 38 | 1.8 | 51.8 | 76 | 1.2 | 10.16 |

Table 2 Origin-Destination matrix of the Sioux Falls transportation network in South Dakota

| Origin Nodes \ Destination Nodes | 1 | 2 | 4 | 5 | 10 | 11 | 13 | 14 | 15 | 19 | 20 | 21 | 22 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1320 | 1320 | 1320 | 1080 | 1100 | 1250 | 990 | 950 | 900 | 590 | 590 | 770 | 740 |
| 2 | 1320 | 0 | 1250 | 1300 | 1100 | 1120 | 900 | 950 | 940 | 1300 | 590 | 680 | 670 | 590 |
| 4 | 1320 | 1250 | 0 | 1320 | 1080 | 1070 | 950 | 900 | 840 | 800 | 1620 | 640 | 590 | 800 |
| 5 | 1320 | 1300 | 1320 | 0 | 1130 | 970 | 910 | 880 | 810 | 730 | 800 | 810 | 940 | 590 |
| 10 | 1080 | 1100 | 1080 | 1130 | 0 | 1330 | 900 | 990 | 1320 | 1170 | 950 | 900 | 970 | 590 |
| 11 | 1100 | 1120 | 1070 | 970 | 1330 | 0 | 940 | 1320 | 1110 | 950 | 740 | 610 | 1100 | 1050 |
| 13 | 1250 | 900 | 950 | 910 | 900 | 940 | 0 | 870 | 860 | 680 | 590 | 620 | 670 | 1320 |
| 14 | 990 | 950 | 900 | 880 | 990 | 1320 | 870 | 0 | 1320 | 1130 | 950 | 870 | 900 | 1130 |
| 15 | 950 | 940 | 840 | 810 | 1320 | 1110 | 860 | 1320 | 0 | 1320 | 1270 | 1140 | 1320 | 910 |
| 19 | 900 | 1300 | 800 | 730 | 1170 | 950 | 680 | 1130 | 1320 | 0 | 1320 | 1110 | 1100 | 800 |
| 20 | 590 | 590 | 1620 | 800 | 950 | 740 | 590 | 980 | 1270 | 1320 | 0 | 1320 | 1320 | 610 |
| 21 | 590 | 680 | 640 | 810 | 900 | 610 | 620 | 870 | 1140 | 1110 | 1320 | 0 | 1320 | 1320 |
| 22 | 770 | 670 | 590 | 940 | 970 | 1100 | 670 | 900 | 1320 | 1100 | 1320 | 1320 | 0 | 1130 |
| 24 | 740 | 590 | 800 | 590 | 590 | 1050 | 1320 | 1130 | 910 | 800 | 610 | 1320 | 1130 | 0 |

## 2. METHOD

In this work, we use Frank-Wolf Algorithm to solve the UE and SO model as a numerical search. The algorithm is coded in Jupyter (See Appendix). For the algorithm, we define two main functions, i.e., the All-or-Nothing algorithm for determining descent direction and the Bisection algorithm for the step size. The All-or-Nothing algorithm is constructed mainly using the Bellman-Ford method in Networkx packages. For the Bisection algorithm, we construct the iteration directly using looping and logic features. In all of Frank-Wolf Algorithm iteration processes, we set the tolerance $\varepsilon = 0.005$ for consistency and assuring that each discussion and evaluation between UE, SO, and pricing scheme results are comparable in terms of its variance.

## 3. OBJECTIVE 1 & 2: USER EQUILIBRIUM AND CONGESTED ROADS

### 3.1 PROBLEM FORMULATION

This section focuses on the first two objectives, i.e., determining the user equilibrium flow and recognizing the congested roads. The User Equilibrium Formulation is formulated as following:

<u>Parameter and Decision Variable</u>

Given a network $G = (N, A)$ with $N$ as the set of nodes and $A$ is the set of arcs, let $OD$ become the set of origin-destination with demand $q^{rs}$ for each pair for origin node $r$ to destination node $s$. For every arc $(i, j) \in A$ in the network, we define parameters $t_{0,ij}$ as the free flow travel time, and $ca_{ij}$ as the capacity of that arc. Each node $i \in N$ is associated with its *forward* and the *reverse star* as the set of arcs that are outgoing from $i$ or incoming to $i$, respectively. Mathematically, these two sets are defined as $\delta_i += \{j \in N: (i, j) \in A\}$ and $\delta_i -= \{j \in N: (j, i) \in A\}$. Then, we define $t_{ij}$ as a variable that captures the time it takes to traverse arc $(i, j)$. This time is calculated as per the BPR function as $t_{ij} = t_{0,ij}\left(1 + 0.15\left(\frac{v_{ij}}{u_{ij}}\right)^4\right)$. For the decision variable, we define $x_{ij}^{rs}$ as the total flow on arc $(i, j)$ moving from origin node $r$ to destination node $s$. Lastly, we define $x_{ij}$ as the total flow on arc $(i, j)$.

<u>User Equilibrium Problem</u>

$$\min\left(\sum_{\forall(i,j)\in A}\int_0^{x_{ij}} t_{ij}(w)dw\right) = \min\left(\sum_{\forall(i,j)\in A} t_{o,ij}\left(x_{ij} + \frac{0.03}{\left(ca_{ij}\right)^4}x_{ij}^5\right)\right) \tag{1}$$

$s.t.$

$$x_{ij} = \sum_{\forall(r,s)\in OD} x_{ij}^{rs}\ \forall(ij) \in A \tag{2}$$

$$\sum_{\forall j\in\delta(i)+} x_{ij}^{rs} - \sum_{\forall j\in\delta(i)-} x_{ij}^{rs} = \begin{cases} q^{rs} & \forall i = r \\ q^{rs} & \forall i = s \quad \forall(r,s) \in OD \\ 0\ otherwise & \end{cases} \tag{3}$$

$$x_{ij}^{rs} \geq 0\ \forall(i,j) \in A\ \forall(r,s) \in OD \tag{4}$$

The equation (1) represent the objective function of the User Equilibrium Problem. The equation (2) defines the total flow on the arcs. The equation (3) is the flow conservation constraints. Then, equation (4) indicates nonnegativity on the flow.

For the Sioux Falls Network, we have 24 nodes, 76 arcs, and196 origin-destination pairs. The complete flow conservation equation of UE is Appendix.

In this project, we solve the UE formulation using Frank-Wolf Algorithm, as describe as follow:

**Frank-Wolf Algorithm**

| | |
|---|---|
| *0* | Initialization. Perform all-or-nothing assignment using $t_{ij} = t_{ij}(0)\ \forall\ (ij) \in A$. Set counter $n \coloneqq 1$ |
| *1* | Update. Set $t_{ij}^n = t_{ij}\left(x_{ij}^n\right)\ \forall\ (ij) \in A$ |
| *2* | Direct finding. Perform all-or-nothing assignment based on $t_{ij}^n\ \forall\ (ij) \in A$ to give flow $y_{ij}^n$ |
| *3* | Line Search. Use Bisection to find $0 \le \alpha \le 1$ that solves $$\sum_{(ij)\in A} t_{0ij}\left[\left(y_{ij}^n - x_{ij}^n\right) + \frac{0.15\left(y_{ij}^n - x_{ij}^n\right)\left(x_{ij}^n + \alpha\left(y_{ij}^n - x_{ij}^n\right)\right)^4}{\left(ca_{ij}\right)^4}\right] = 0$$ |
| *4* | Move. Set $$x_{ij}^{n+1} = x_{ij}^n + \alpha\left(y_{ij}^n - x_{ij}^n\right)\forall(ij) \in A$$ |
| *5* | Convergence Test. If: $$\frac{\sqrt{\sum_{ij\in A}\left(x_{ij}^n - x_{ij}^{n+1}\right)^2}}{\sum_{ij\in A} x_{ij}^n} < \varepsilon, where\ we\ define\ \varepsilon = 0.0005$$ , stop; otherwise $n \coloneqq n + 1$ |

### 3.2 Result and Discussion

The result of user equilibrium flow distribution is presented in Table 3. There are 9 congested roads in the network according to user equilibrium flow distribution (See Figure 2). The congested arcs are Link 2, 14, 19, 34, 39, 40, 57, 66, 74, and 75; where the biggest congestion occurred on Link 66 with $\frac{v}{c} = 1.189$. In contrast, there are two links that are underused in the network, i.e., Link 50 and 55 with $\frac{v}{C} = 0.043$.

The huge difference on flow distribution across the network indicates the "selfish" behavior in user equilibrium formulation to allow driver to choose the route to minimize driver travel time. Also, we notice that all congested roads are within two adjacent central nodes and/or in the central of the network. This condition tells us that being connected to a central node shows how the road will likely to choose to traverse across the central nodes. These experiments illustrate the real condition of the network. We could consider central node as set of important business district or attraction in a region. Regardless the position of these districts located and as the demand attracted to these nodes are significant, the direct connecting road be chosen as it provides a direct connectivity. Also, for the centrality, we observe that as the road become more central, it is likely that most of the path options from origin-destination will pass the central road. This also illustrate the reality condition in transportation network, where main road always located and span in the center of the city are usually congested because these arcs are critical center arc that provide connectivity to all perimeter nodes in the region. Equally interesting, the flow distribution showed that Link 50 and 55 are underused. Two can address this condition that those links are in the perimeter/border of the network and there are few direct connections to the central nodes.

To summarize, the UE flow distribution shows that the formulation provide allow "selfish" behavior in the network to minimizing individual travel time. Precisely, the chosen and congested roads that selfishly chosen are the links that located in the center or provide direct connection to central nodes.

Table 3 User Equilibrium Flow Distribution Result

| Link | UE Link Total Flow (veh/hr) | Link Travel Time (minutes) | V/C | Congested | Link | UE Link Total Flow (veh/hr) | Link Travel Time (minutes) | V/C | Congested |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 4420.835 | 3.757 | 0.734 | - | **39** | 10330.000 | 2.782 | 1.015 | V |
| **2** | 11479.165 | 3.348 | 1.274 | V | **40** | 11019.599 | 2.986 | 1.130 | V |
| 3 | 4300.000 | 3.609 | 0.358 | - | 41 | 4849.369 | 3.023 | 0.473 | - |
| 4 | 11510.835 | 3.123 | 0.723 | - | 42 | 7567.932 | 2.526 | 0.768 | - |
| 5 | 11600.000 | 2.401 | 0.248 | - | 43 | 8540.401 | 3.605 | 0.316 | - |
| 6 | 7651.865 | 2.401 | 0.224 | - | 44 | 4089.599 | 3.011 | 0.399 | - |
| 7 | 11607.301 | 2.401 | 0.248 | - | 45 | 5870.535 | 2.450 | 0.609 | - |
| 8 | 8080.401 | 2.403 | 0.313 | - | 46 | 9540.000 | 2.416 | 0.462 | - |
| 9 | 14771.865 | 1.213 | 0.523 | - | 47 | 3730.000 | 3.008 | 0.370 | - |
| 10 | 6480.000 | 3.743 | 0.717 | - | 48 | 1690.000 | 3.000 | 0.165 | - |
| 11 | 15150.401 | 1.202 | 0.323 | - | 49 | 3730.000 | 1.203 | 0.357 | - |
| 12 | 8570.000 | 2.453 | 0.618 | - | 50 | 1690.000 | 1.800 | 0.043 | - |
| 13 | 6731.865 | 3.075 | 0.640 | - | 51 | 2120.000 | 4.201 | 0.212 | - |
| **14** | 11390.000 | 3.782 | 1.148 | V | 52 | 3730.000 | 1.203 | 0.357 | - |
| 15 | 8570.000 | 2.602 | 0.866 | - | 53 | 5850.000 | 1.224 | 0.606 | - |
| 16 | 10840.835 | 1.211 | 0.501 | - | 54 | 4950.000 | 1.200 | 0.106 | - |
| 17 | 4950.000 | 1.803 | 0.316 | - | 55 | 1690.000 | 1.800 | 0.043 | - |
| 18 | 4950.000 | 1.200 | 0.106 | - | 56 | 6640.000 | 2.562 | 0.819 | - |
| **19** | 10720.000 | 1.458 | 1.094 | V | **57** | 4744.404 | 2.877 | 1.073 | V |
| 20 | 4950.000 | 1.803 | 0.316 | - | 58 | 5850.000 | 1.224 | 0.606 | - |
| 21 | 2160.835 | 2.001 | 0.214 | - | 59 | 5682.068 | 2.437 | 0.568 | - |
| 22 | 3730.000 | 3.008 | 0.370 | - | 60 | 6640.000 | 2.562 | 0.819 | - |
| 23 | 7110.401 | 3.007 | 0.356 | - | 61 | 4555.937 | 2.515 | 0.753 | - |
| 24 | 2040.000 | 2.000 | 0.202 | - | 62 | 5790.000 | 3.658 | 0.572 | - |
| 25 | 8892.699 | 1.803 | 0.320 | - | 63 | 3045.596 | 3.004 | 0.300 | - |
| 26 | 9150.401 | 1.803 | 0.329 | - | 64 | 5241.534 | 3.639 | 0.518 | - |
| 27 | 4910.000 | 3.002 | 0.245 | - | 65 | 8894.346 | 1.294 | 0.850 | - |
| 28 | 8282.699 | 3.605 | 0.307 | - | **66** | 11611.740 | 2.352 | 1.189 | V |
| 29 | 1690.000 | 3.000 | 0.165 | - | 67 | 10164.063 | 2.421 | 0.493 | - |
| 30 | 2120.000 | 4.201 | 0.212 | - | 68 | 2437.932 | 3.002 | 0.240 | - |
| 31 | 6530.001 | 3.706 | 0.665 | - | 69 | 8451.740 | 1.279 | 0.808 | - |
| 32 | 4910.000 | 3.002 | 0.246 | - | 70 | 4218.260 | 2.411 | 0.422 | - |
| 33 | 4409.599 | 3.622 | 0.449 | - | 71 | 8100.000 | 2.565 | 0.822 | - |
| **34** | 11277.301 | 3.047 | 1.157 | V | 72 | 3792.053 | 2.408 | 0.379 | - |
| 35 | 11299.599 | 2.401 | 0.241 | - | 73 | 5178.260 | 1.212 | 0.510 | - |
| 36 | 4717.301 | 3.629 | 0.480 | - | **74** | 10330.000 | 2.644 | 0.908 | V |
| 37 | 10570.000 | 1.800 | 0.204 | - | **75** | 11505.879 | 2.317 | 1.178 | V |
| 38 | 10570.000 | 1.800 | 0.204 | - | 76 | 5284.121 | 1.213 | 0.520 | - |

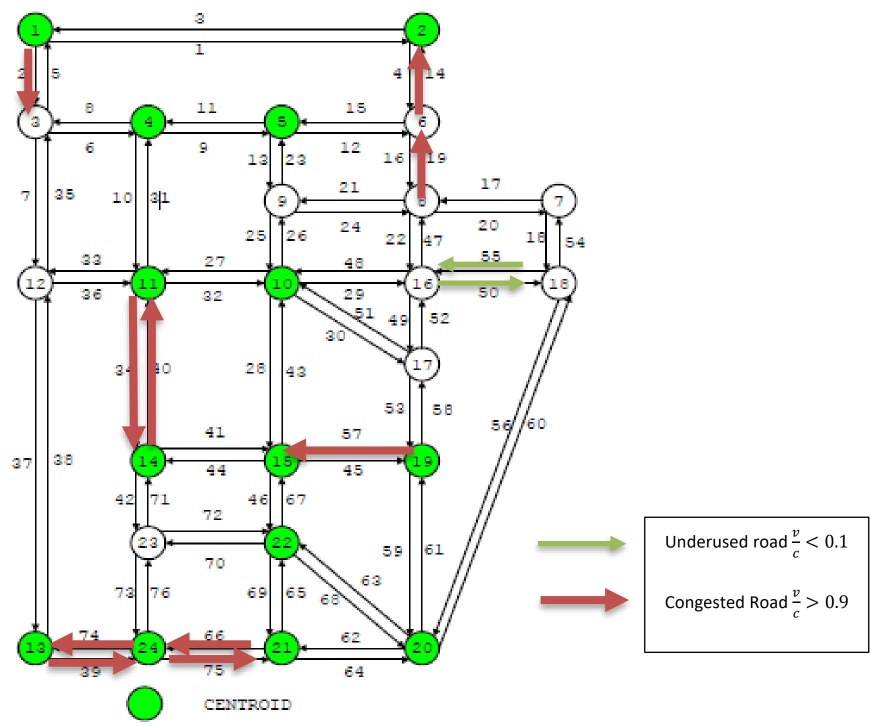


Figure 2 Congested road from UE Result on the highway network of Sioux Falls

## 4. Objective 3: Congestion Pricing Scheme

### 4.1 Problem Formulation

In this task, we use the Marginal Cost Pricing scheme to provide an effective congestion pricing scheme. The toll price $\tau_{ij}$ is defined using the System Optimal Problem and formulated as,

$$\tau_{ij} = \widetilde{t_{ij}^{SO}} - t_{ij}^{SO} \tag{5}$$

where $t_{ij}^{SO}$ is the total travel time for link $(i,j) \in A$ and $\widetilde{t_{ij}^{SO}}$ is the total marginal travel time for link $(i,j) \in A$ that both are solved using SO formulation. The marginal travel time that constructed using BPR Function are adjusted and formulated as,

$$\widetilde{t_{ij}} = t_{0ij}\left(1 + 0.75\left(\frac{x_{ij}}{ca_{ij}}\right)^4\right) \tag{6}$$

To calculate it, first, we need to define the problem formulation of System Optimal Formulation, a following:

Parameter and Decision Variable

Given a network $G = (N, A)$ with $N$ as the set of nodes and $A$ is the set of arcs, let $OD$ become the set of origin-destination with demand $q^{rs}$ for each pair for origin node $r$ to destination node $s$. For every arc $(i,j) \in A$ in the network, we define parameters $t_{0,ij}$ as the free flow travel time, and $ca_{ij}$ as the capacity of that arc. Each node $i \in N$ is associated with its *forward* and the *reverse star* as the set of arcs that are outgoing from $i$ or incoming to $i$, respectively. Mathematically, these two sets are defined as $\delta_i+= \{j \in N: (i,j) \in A\}$ and $\delta_i-= \{j \in N: (j,i) \in A\}$. Then, we define $t_{ij}$ as a variable that captures the time it takes to traverse arc $(i,j)$. This time is calculated as per the BPR function as $t_{ij} = t_{0,ij}\left(1 + 0.15\left(\frac{v_{ij}}{u_{ij}}\right)^4\right)$. For the decision variable, we define $x_{ij}^{rs}$ as the total flow on arc $(i,j)$ moving from origin node $r$ to destination node $s$. Lastly, we define $x_{ij}$ as the total flow on arc $(i,j)$.

System Optimal Problem

$$\min\left(\sum_{\forall(i,j)\in A} x_{ij}t_{ij}\right) = \min\left(\sum_{\forall(i,j)\in A}\int_0^{x_{ij}} \widetilde{t_{ij}}(w)dw\right) = \min\left(\sum_{\forall(i,j)\in A} t_{o,ij}\left(x_{ij} + \frac{0.15}{\left(ca_{ij}\right)^4}x_{ij}^5\right)\right) \tag{7}$$

$s.t.$

$$x_{ij} = \sum_{\forall(r,s)\in OD} x_{ij}^{rs}\ \forall(ij)\in A \tag{8}$$

$$\sum_{\forall j\in\delta(i)+} x_{ij}^{rs} - \sum_{\forall j\in\delta(i)-} x_{ij}^{rs} = \begin{cases} q^{rs} & \forall i = r \\ q^{rs} & \forall i = s \quad \forall(r,s)\in OD \\ 0\ otherwise \end{cases} \tag{9}$$

$$x_{ij}^{rs} \geq 0\ \forall(i,j)\in A\ \forall(r,s)\in OD \tag{10}$$

The equation (7) represent the objective function of the System Optimal Problem. The equation (8) defines the total flow on the arcs. The equation (9) is the flow conservation constraints. Then, equation (10) indicates nonnegativity on the flow.

For the Sioux Falls Network, we have 24 nodes, 76 arcs, and196 origin-destination pairs. The complete flow conservation equation of SO is written in Appendix.

In this project, we solve the SO formulation using Frank-Wolf Algorithm, as describe as follow:

**Frank-Wolf Algorithm**

| | |
|---|---|
| *0* | Initialization. Perform all-or-nothing assignment using $\widetilde{t_{ij}} = \widetilde{t_{ij}}(0)\ \forall\ (ij)\in A$. Set counter $n := 1$ |
| *1* | Update. Set $\widetilde{t_{ij}^n} = \widetilde{t_{ij}}\left(x_{ij}^n\right)\ \forall\ (ij)\in A$ |
| *2* | Direct finding. Perform all-or-nothing assignment based on $t_{ij}^n\ \forall\ (ij)\in A$ to give flow $y_{ij}^n$ |
| *3* | Line Search. Use Bisection to find $0 \leq \alpha \leq 1$ that solves $$\sum_{(ij)\in A} t_{0ij}\left[\left(y_{ij}^n - x_{ij}^n\right) + \frac{0.75\left(y_{ij}^n - x_{ij}^n\right)\left(x_{ij}^n + \alpha\left(y_{ij}^n - x_{ij}^n\right)\right)^4}{\left(ca_{ij}\right)^4}\right] = 0$$ |
| *4* | Move. Set $$x_{ij}^{n+1} = x_{ij}^n + \alpha\left(y_{ij}^n - x_{ij}^n\right)\ \forall(ij)\in A$$ |
| *5* | Convergence Test. If: $$\frac{\sqrt{\sum_{ij\in A}\left(x_{ij}^n - x_{ij}^{n+1}\right)^2}}{\sum_{ij\in A} x_{ij}^n} < \varepsilon, where\ we\ define\ \varepsilon = 0.0005$$ , stop; otherwise $n := n+1$ |

### 4.2 RESULT

The result of system optimal flow distribution and the pricing are presented in Table 4 and Table 5. The marginal-cost pricing is proposed for all arcs in the network. The first-best scheme proof will be explained in Section 5. The comparison of SO and UE would be discussed on Section 6.

Table 4 System Optimal Flow Distribution and Marginal-cost Pricing Scheme (Link 1-38)

| Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) | Tilted Time (minutes) | Tolls (minutes) | Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) | Tilted Time (minutes) | Tolls (minutes) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 5477.770 | 3.970 | 5.533 | 1.563 | 20 | 4963.150 | 1.803 | 1.814 | 0.011 |
| 2 | 10431.544 | 3.047 | 5.561 | 2.515 | 21 | 3953.960 | 2.007 | 2.035 | 0.028 |
| 3 | 3099.501 | 3.602 | 3.612 | 0.010 | 22 | 3730.000 | 3.008 | 3.042 | 0.034 |
| 4 | 12558.456 | 3.174 | 3.888 | 0.714 | 23 | 9437.092 | 3.022 | 3.109 | 0.086 |
| 5 | 12809.813 | 2.402 | 2.410 | 0.008 | 24 | 2048.134 | 2.001 | 2.003 | 0.002 |
| 6 | 7389.878 | 2.401 | 2.404 | 0.003 | 25 | 10895.518 | 1.806 | 1.831 | 0.025 |
| 7 | 10798.703 | 2.401 | 2.405 | 0.004 | 26 | 11392.370 | 1.808 | 1.837 | 0.029 |
| 8 | 9423.134 | 2.406 | 2.432 | 0.026 | 27 | 6330.426 | 3.005 | 3.021 | 0.017 |
| 9 | 15043.433 | 1.214 | 1.271 | 0.057 | 28 | 10675.102 | 3.613 | 3.666 | 0.053 |
| 10 | 5272.629 | 3.662 | 3.930 | 0.268 | 29 | 1685.842 | 3.000 | 3.002 | 0.001 |
| 11 | 16461.046 | 1.203 | 1.214 | 0.011 | 30 | 2114.662 | 4.201 | 4.206 | 0.005 |
| 12 | 8545.974 | 2.452 | 2.660 | 0.208 | 31 | 5888.272 | 3.670 | 3.949 | 0.279 |
| 13 | 7034.414 | 3.090 | 3.435 | 0.345 | 32 | 5734.380 | 3.003 | 3.015 | 0.012 |
| 14 | 10180.187 | 3.499 | 5.509 | 2.010 | 33 | 3939.115 | 3.614 | 3.670 | 0.056 |
| 15 | 7560.909 | 2.522 | 3.056 | 0.534 | 34 | 8905.459 | 2.651 | 3.656 | 1.006 |
| 16 | 12554.255 | 1.220 | 1.302 | 0.081 | 35 | 11143.715 | 2.401 | 2.406 | 0.005 |
| 17 | 4621.400 | 1.802 | 1.810 | 0.008 | 36 | 3945.856 | 3.614 | 3.670 | 0.056 |
| 18 | 4963.150 | 1.200 | 1.200 | 0.000 | 37 | 10549.841 | 1.800 | 1.802 | 0.002 |
| 19 | 9190.921 | 1.339 | 1.919 | 0.580 | 38 | 10901.594 | 1.801 | 1.803 | 0.002 |

Table 5 System Optimal Flow Distribution and Marginal-cost Pricing Scheme (Link 39-76)

| Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) | Tilted Time (minutes) | Tolls (minutes) | Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) | Tilted Time (minutes) | Tolls (minutes) |
|---|---|---|---|---|---|---|---|---|---|
| 39 | 10305.785 | 2.778 | 4.290 | 1.512 | 58 | 5844.662 | 1.224 | 1.321 | 0.097 |
| 40 | 8918.316 | 2.652 | 3.664 | 1.012 | 59 | 6079.774 | 2.449 | 2.645 | 0.196 |
| 41 | 3748.689 | 3.008 | 3.041 | 0.033 | 60 | 6311.400 | 2.532 | 3.066 | 0.534 |
| 42 | 6549.004 | 2.470 | 2.752 | 0.282 | 61 | 3830.142 | 2.458 | 2.688 | 0.230 |
| 43 | 10648.084 | 3.613 | 3.665 | 0.052 | 62 | 6087.778 | 3.671 | 3.950 | 0.279 |
| 44 | 3209.163 | 3.004 | 3.023 | 0.018 | 63 | 4280.290 | 3.014 | 3.071 | 0.057 |
| 45 | 5802.945 | 2.447 | 2.637 | 0.189 | 64 | 5640.367 | 3.652 | 3.859 | 0.207 |
| 46 | 10180.641 | 2.421 | 2.507 | 0.086 | 65 | 6987.479 | 1.236 | 1.394 | 0.158 |
| 47 | 2614.243 | 3.002 | 3.011 | 0.009 | 66 | 8796.114 | 1.977 | 2.705 | 0.728 |
| 48 | 2322.480 | 3.001 | 3.006 | 0.005 | 67 | 11863.729 | 2.439 | 2.598 | 0.159 |
| 49 | 3730.000 | 1.203 | 1.215 | 0.012 | 68 | 2110.476 | 3.001 | 3.004 | 0.003 |
| 50 | 1685.842 | 1.800 | 1.800 | 0.000 | 69 | 6837.229 | 1.233 | 1.370 | 0.137 |
| 51 | 2597.939 | 4.203 | 4.212 | 0.009 | 70 | 6397.184 | 2.460 | 2.687 | 0.227 |
| 52 | 3246.723 | 1.202 | 1.209 | 0.008 | 71 | 7131.388 | 2.499 | 2.889 | 0.390 |
| 53 | 5844.662 | 1.224 | 1.321 | 0.097 | 72 | 5760.209 | 2.440 | 2.580 | 0.141 |
| 54 | 4621.400 | 1.200 | 1.200 | 0.000 | 73 | 6841.503 | 1.237 | 1.379 | 0.142 |
| 55 | 1690.000 | 1.800 | 1.800 | 0.000 | 74 | 10657.538 | 2.677 | 3.778 | 1.101 |
| 56 | 6648.992 | 2.563 | 3.213 | 0.651 | 75 | 8498.953 | 1.955 | 2.623 | 0.669 |
| 57 | 3553.313 | 2.550 | 3.152 | 0.602 | 76 | 6786.911 | 1.236 | 1.365 | 0.129 |

## 5. Objectives 4: First-Best Tolling Scheme Verification

### 5.1 Problem Formulation

In this section, we provide proof to show that Marginal-cost Pricing is one of the “first-best” toll scheme. By definition [1,2], the pricing scheme is considered as “first-best” tolls scheme if we could get SO result using tolled UE formulation. One can prove this by solving a Tolled UE Problem using the tolled travel time, formulated as,

$$\widehat{t_{ij}} = t_{0ij}\left(1 + 0.15\left(\frac{x_{ij}}{ca_{ij}}\right)^4\right) + \tau_{ij} \tag{11}$$

Then, using this adjustment, we could solve Tolled UE using UE formulation as formulated in Section 3 with adjustment as,

<u>Tolled User Equilibrium Problem</u>

$$\begin{aligned} &\min\left(\sum_{\forall(i,j)\in A}\int_0^{x_{ij}}\widehat{t_{ij}}(w)dw\right) \\ &= \min\left(\sum_{\forall(i,j)\in A} t_{o,ij}\left(x_{ij} + \frac{0.15}{\left(ca_{ij}\right)^4}x_{ij}^5\right) + \tau_{ij}x_{ij}\right) \end{aligned} \tag{12}$$

$s.t.$ (2), (3), (4)

The equation (12) represent the objective function of the Tolled User Equilibrium Problem. In this project, we solve the Tolled User Equilibrium formulation using Frank-Wolf Algorithm, as describe as follow:

**Frank-Wolf Algorithm**

| | |
|---|---|
| *0* | Initialization. Perform all-or-nothing assignment using $\widehat{t_{ij}} = \widehat{t_{ij}}(0)\ \forall\ (ij) \in A$.<br>Set counter $n \coloneqq 1$ |
| *1* | Update. Set $\widehat{t_{ij}^n} = \widehat{t_{ij}}(x_{ij}^n)\ \forall\ (ij) \in A$ |
| *2* | Direct finding. Perform all-or-nothing assignment based on $t_{ij}^n\ \forall\ (ij) \in A$ to give flow $y_{ij}^n$ |
| *3* | Line Search. Use Bisection to find $0 \le \alpha \le 1$ that solves<br>$$\sum_{(ij)\in A} t_{0ij}\left[(y_{ij}^n - x_{ij}^n) + \frac{0.75(y_{ij}^n - x_{ij}^n)\left(x_{ij}^n + \alpha(y_{ij}^n - x_{ij}^n)\right)^4}{(ca_{ij})^4}\right] + \tau_{ij}(y_{ij}^n - x_{ij}^n) = 0$$ |
| *4* | Move. Set<br>$$x_{ij}^{n+1} = x_{ij}^n + \alpha(y_{ij}^n - x_{ij}^n)\ \forall (ij) \in A$$ |
| *5* | Convergence Test. If:<br>$$\frac{\sqrt{\sum_{ij\in A}(x_{ij}^n - x_{ij}^{n+1})^2}}{\sum_{ij\in A} x_{ij}^n} < \varepsilon, where\ we\ define\ \varepsilon = 0.0005$$<br>, stop; otherwise $n \coloneqq n + 1$ |

### 5.2 Result and Discussions

Table 6 and Table 7 provide the comparison flow and travel time for Tolled UE and SO problem. According to the result, we obtain similar result of total travel time of the network T(Tolled UE) and T(SO) with difference 0.0003%. We also observe a consistent flow distribution between these two results, where the difference on the link flow only 2% in average and on the travel time only 1.5%. Therefore, Marginal-cost Pricing is proven as "first-best" toll scheme.

However, there are several arcs that have significant difference in link flow, where the highest difference found in Link 51 with 15% and Link 52 with 12% difference. One possible explanation is that the link flows are not converged to the optimal solution as we only set the converge as $\varepsilon = 0.0005$. Increasing the convergence tolerance would provide optimal and comparable result.

Interestingly, the Link 51 and Link 52 are both not a central arc of the network. Both links are in the perimeter of the network and there are few direct connections to central nodes. One could possibly explain that, even though the tolling scheme try to shift the flow of "selfishness" to "optimal" condition, still we found lesser affect in improving the flow near perimeter because of the topology characteristics and how the flow generated and attracted to central nodes.

Table 6 Comparison of Tolled UE and SO Flow Distribution (Link 1-38)

| **T(TolledUE)** | 1,255,464.91 | min. | **T(SO)** | 1,255,461.74 | min. |
|---|---|---|---|---|---|
| Link | Tolled UE Link Total Flow (veh/hr) | Link Travel Time (minutes) | Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) |
| 1 | 5307.580 | 3.926 | 1 | 5477.770 | 3.970 |
| 2 | 10592.420 | 3.088 | 2 | 10431.544 | 3.047 |
| 3 | 3000.965 | 3.602 | 3 | 3099.501 | 3.602 |
| 4 | 12397.580 | 3.165 | 4 | 12558.456 | 3.174 |
| 5 | 12899.035 | 2.402 | 5 | 12809.813 | 2.402 |
| 6 | 7552.420 | 2.401 | 6 | 7389.878 | 2.401 |
| 7 | 10820.000 | 2.401 | 7 | 10798.703 | 2.401 |
| 8 | 9458.277 | 2.406 | 8 | 9423.134 | 2.406 |
| 9 | 15237.353 | 1.215 | 9 | 15043.433 | 1.214 |
| 10 | 5248.175 | 3.661 | 10 | 5272.629 | 3.662 |
| 11 | 16501.385 | 1.203 | 11 | 16461.046 | 1.203 |
| 12 | 8570.000 | 2.453 | 12 | 8545.974 | 2.452 |
| 13 | 7197.353 | 3.099 | 13 | 7034.414 | 3.090 |
| 14 | 10090.965 | 3.482 | 14 | 10180.187 | 3.499 |
| 15 | 7386.406 | 2.512 | 15 | 7560.909 | 2.522 |
| 16 | 12394.472 | 1.219 | 16 | 12554.255 | 1.220 |
| 17 | 4549.242 | 1.802 | 17 | 4621.400 | 1.802 |
| 18 | 4950.000 | 1.200 | 18 | 4963.150 | 1.200 |
| 19 | 8904.264 | 1.323 | 19 | 9190.921 | 1.339 |
| 20 | 4950.000 | 1.803 | 20 | 4963.150 | 1.803 |
| 21 | 3769.820 | 2.006 | 21 | 3953.960 | 2.007 |
| 22 | 3730.000 | 3.008 | 22 | 3730.000 | 3.008 |
| 23 | 9644.979 | 3.024 | 23 | 9437.092 | 3.022 |
| 24 | 2040.000 | 2.000 | 24 | 2048.134 | 2.001 |
| 25 | 10911.825 | 1.806 | 25 | 10895.518 | 1.806 |
| 26 | 11629.631 | 1.808 | 26 | 11392.370 | 1.808 |
| 27 | 6433.302 | 3.005 | 27 | 6330.426 | 3.005 |
| 28 | 10764.888 | 3.614 | 28 | 10675.102 | 3.613 |
| 29 | 1690.000 | 3.000 | 29 | 1685.842 | 3.000 |
| 30 | 2120.000 | 4.201 | 30 | 2114.662 | 4.201 |
| 31 | 5890.000 | 3.670 | 31 | 5888.272 | 3.670 |
| 32 | 5958.029 | 3.004 | 32 | 5734.380 | 3.003 |
| 33 | 3930.000 | 3.614 | 33 | 3939.115 | 3.614 |
| 34 | 8795.112 | 2.638 | 34 | 8905.459 | 2.651 |
| 35 | 11220.758 | 2.401 | 35 | 11143.715 | 2.401 |
| 36 | 3930.000 | 3.614 | 36 | 3945.856 | 3.614 |
| 37 | 10570.000 | 1.800 | 37 | 10549.841 | 1.800 |
| 38 | 10970.758 | 1.801 | 38 | 10901.594 | 1.801 |

Table 7 Comparison of Tolled UE and SO Flow Distribution (Link 39-76)

| **T(TolledUE)** | 1255464.905 | min. | **T(SO)** | 1255461.740 | min. |
|---|---|---|---|---|---|
| Link | Tolled UE Link Total Flow (veh/hr) | Link Travel Time (minutes) | Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) |
| 39 | 10330.000 | 2.782 | 39 | 10305.785 | 2.778 |
| 40 | 8961.664 | 2.657 | 40 | 8918.316 | 2.652 |
| 41 | 3418.079 | 3.006 | 41 | 3748.689 | 3.008 |
| 42 | 6598.959 | 2.473 | 42 | 6549.004 | 2.470 |
| 43 | 10598.336 | 3.613 | 43 | 10648.084 | 3.613 |
| 44 | 2766.200 | 3.002 | 44 | 3209.163 | 3.004 |
| 45 | 5785.226 | 2.447 | 45 | 5802.945 | 2.447 |
| 46 | 10116.508 | 2.421 | 46 | 10180.641 | 2.421 |
| 47 | 2370.369 | 3.001 | 47 | 2614.243 | 3.002 |
| 48 | 2188.615 | 3.001 | 48 | 2322.480 | 3.001 |
| 49 | 3730.000 | 1.203 | 49 | 3730.000 | 1.203 |
| 50 | 1690.000 | 1.800 | 50 | 1685.842 | 1.800 |
| 51 | 2981.017 | 4.205 | 51 | 2597.939 | 4.203 |
| 52 | 2868.983 | 1.201 | 52 | 3246.723 | 1.202 |
| 53 | 5850.000 | 1.224 | 53 | 5844.662 | 1.224 |
| 54 | 4549.242 | 1.200 | 54 | 4621.400 | 1.200 |
| 55 | 1690.000 | 1.800 | 55 | 1690.000 | 1.800 |
| 56 | 6640.000 | 2.562 | 56 | 6648.992 | 2.563 |
| 57 | 3530.132 | 2.546 | 57 | 3553.313 | 2.550 |
| 58 | 5850.000 | 1.224 | 58 | 5844.662 | 1.224 |
| 59 | 6074.533 | 2.449 | 59 | 6079.774 | 2.449 |
| 60 | 6239.242 | 2.526 | 60 | 6311.400 | 2.532 |
| 61 | 3819.439 | 2.457 | 61 | 3830.142 | 2.458 |
| 62 | 6141.478 | 3.673 | 62 | 6087.778 | 3.671 |
| 63 | 4309.148 | 3.015 | 63 | 4280.290 | 3.014 |
| 64 | 5677.374 | 3.653 | 64 | 5640.367 | 3.652 |
| 65 | 6571.425 | 1.228 | 65 | 6987.479 | 1.236 |
| 66 | 8855.839 | 1.982 | 66 | 8796.114 | 1.977 |
| 67 | 11553.171 | 2.435 | 67 | 11863.729 | 2.439 |
| 68 | 2087.400 | 3.001 | 68 | 2110.476 | 3.001 |
| 69 | 6886.621 | 1.234 | 69 | 6837.229 | 1.233 |
| 70 | 6722.309 | 2.474 | 70 | 6397.184 | 2.460 |
| 71 | 7447.390 | 2.518 | 71 | 7131.388 | 2.499 |
| 72 | 6252.421 | 2.455 | 72 | 5760.209 | 2.440 |
| 73 | 6792.658 | 1.236 | 73 | 6841.503 | 1.237 |
| 74 | 10730.758 | 2.685 | 74 | 10657.538 | 2.677 |
| 75 | 8076.538 | 1.926 | 75 | 8498.953 | 1.955 |
| 76 | 7171.202 | 1.245 | 76 | 6786.911 | 1.236 |

## 6. DISCUSSIONS

This section provide discussion about UE flow and SO flow. Table 8 show comparison of the total system performance between UE and SO. Table 9 shows the congested link using SO flow distribution.

According to Table 8, we found that the total system performance of SO is smaller than the system performance of UE flow, with price anarchy $\varphi = 1.01$. These results consistent with the routing behavior of UE to choose road to minimize each driver travel time and ignore the system performance. We observed list of congested road and underused road using UE flow previously in Section 3. This result indicate that UE flow distribution would provide a lower system performance compared to SO flow. In system optimal flow distribution, we try to give compromise among the driver to sacrifice their travel time to achieve the better system optimality. This project allows us to capture this behavior. Interestingly, the price of anarchy showed that there is an improvement of the system as we choose SO, still the improvement is not significant if we still choosing the UE.

Next, we discuss on how the flow redistribution affect the congested road. As we discussed in Section 3, the congested arcs on UE flow distribution are Link 2, 14, 19, 34, 39, 40, 57, 66, 74, and 75; where the biggest congestion occurred on Link 66 with $\frac{v}{c} = 1.189$. Table 9 shows that there are improvements of $\frac{v}{c}$ ratio for all arcs in average if we apply SO flow distribution. The final congested roads in SO flow are Link 1, 2, 14, 19, 34, 39, 40, 66, and 74. There are an improvement on $\frac{v}{c}$ of Link 2, 14, 19, 34, 39, 40, 66, and, 74. Significant improvement is that SO flow able to redistribute the flow in the network, so then, Link 75 and 57 become uncongested, and Link 66 flow is reduced significantly to $\frac{v}{c} = 0.9$. However, due to this redistribution, there are an increased flow in another roads and the most impacted one is Link 1 that becomes congested in SO while not in UE. Equally important, Link 50 and Link 55 are remained as underused arcs. One can still explain that Link 50 and Link 51 are in perimeter and have limited direct connectivity to the central nodes. Hence, the redistribution flow in SO does not provide significant effect to these links.

From this experiment, we observed that SO flow distribution would improve the congestion in a congested road by redistributing the flow to another road. However, as the consequences, we will obtain a reduced flow from congested roads, but we might having increased or new congested road due to this redistribution. Hence, the enforcing system optimal itself could not effectively solve the congestion in the network.

Table 8 Comparison of Total System Performance of UE and SO

| Total System Performance | |
|---|---|
| T(UE) min | T(SO) min |
| 1,270,894.80 | 1,255,461.74 |

Table 9 System Optimal Flow Distribution Result

| Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) | V/C | Congested | Link | SO Link Total Flow (veh/hr) | Link Travel Time (minutes) | V/C | Congested |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 5477.770 | 3.970 | 0.910 | V | **39** | 10305.785 | 2.778 | 1.012 | V |
| **2** | 10431.544 | 3.047 | 1.158 | V | **40** | 8918.316 | 2.652 | 0.915 | V |
| 3 | 3099.501 | 3.602 | 0.258 | - | 41 | 3748.689 | 3.008 | 0.365 | - |
| 4 | 12558.456 | 3.174 | 0.789 | - | 42 | 6549.004 | 2.470 | 0.665 | - |
| 5 | 12809.813 | 2.402 | 0.274 | - | 43 | 10648.084 | 3.613 | 0.394 | - |
| 6 | 7389.878 | 2.401 | 0.216 | - | 44 | 3209.163 | 3.004 | 0.313 | - |
| 7 | 10798.703 | 2.401 | 0.231 | - | 45 | 5802.945 | 2.447 | 0.602 | - |
| 8 | 9423.134 | 2.406 | 0.365 | - | 46 | 10180.641 | 2.421 | 0.493 | - |
| 9 | 15043.433 | 1.214 | 0.533 | - | 47 | 2614.243 | 3.002 | 0.259 | - |
| 10 | 5272.629 | 3.662 | 0.583 | - | 48 | 2322.480 | 3.001 | 0.226 | - |
| 11 | 16461.046 | 1.203 | 0.351 | - | 49 | 3730.000 | 1.203 | 0.357 | - |
| 12 | 8545.974 | 2.452 | 0.617 | - | **50** | 1685.842 | 1.800 | 0.043 | - |
| 13 | 7034.414 | 3.090 | 0.669 | - | 51 | 2597.939 | 4.203 | 0.260 | - |
| **14** | 10180.187 | 3.499 | 1.026 | V | 52 | 3246.723 | 1.202 | 0.310 | - |
| 15 | 7560.909 | 2.522 | 0.764 | - | 53 | 5844.662 | 1.224 | 0.606 | - |
| 16 | 12554.255 | 1.220 | 0.581 | - | 54 | 4621.400 | 1.200 | 0.099 | - |
| 17 | 4621.400 | 1.802 | 0.295 | - | **55** | 1690.000 | 1.800 | 0.043 | - |
| 18 | 4963.150 | 1.200 | 0.106 | - | 56 | 6648.992 | 2.563 | 0.820 | - |
| **19** | 9190.921 | 1.339 | 0.938 | V | **57** | 3553.313 | 2.550 | 0.804 | - |
| 20 | 4963.150 | 1.803 | 0.317 | - | 58 | 5844.662 | 1.224 | 0.606 | - |
| 21 | 3953.960 | 2.007 | 0.391 | - | 59 | 6079.774 | 2.449 | 0.607 | - |
| 22 | 3730.000 | 3.008 | 0.370 | - | 60 | 6311.400 | 2.532 | 0.778 | - |
| 23 | 9437.092 | 3.022 | 0.472 | - | 61 | 3830.142 | 2.458 | 0.633 | - |
| 24 | 2048.134 | 2.001 | 0.203 | - | 62 | 6087.778 | 3.671 | 0.602 | - |
| 25 | 10895.518 | 1.806 | 0.392 | - | 63 | 4280.290 | 3.014 | 0.422 | - |
| 26 | 11392.370 | 1.808 | 0.409 | - | 64 | 5640.367 | 3.652 | 0.557 | - |
| 27 | 6330.426 | 3.005 | 0.317 | - | 65 | 6987.479 | 1.236 | 0.668 | - |
| 28 | 10675.102 | 3.613 | 0.395 | - | **66** | 8796.114 | 1.977 | 0.900 | V |
| 29 | 1685.842 | 3.000 | 0.164 | - | 67 | 11863.729 | 2.439 | 0.575 | - |
| 30 | 2114.662 | 4.201 | 0.212 | - | 68 | 2110.476 | 3.001 | 0.208 | - |
| 31 | 5888.272 | 3.670 | 0.600 | - | 69 | 6837.229 | 1.233 | 0.654 | - |
| 32 | 5734.380 | 3.003 | 0.287 | - | 70 | 6397.184 | 2.460 | 0.640 | - |
| 33 | 3939.115 | 3.614 | 0.401 | - | 71 | 7131.388 | 2.499 | 0.724 | - |
| **34** | 8905.459 | 2.651 | 0.913 | V | 72 | 5760.209 | 2.440 | 0.576 | - |
| 35 | 11143.715 | 2.401 | 0.238 | - | 73 | 6841.503 | 1.237 | 0.673 | - |
| 36 | 3945.856 | 3.614 | 0.402 | - | **74** | 10657.538 | 2.677 | 0.937 | V |
| 37 | 10549.841 | 1.800 | 0.204 | - | **75** | 8498.953 | 1.955 | 0.870 | - |
| 38 | 10901.594 | 1.801 | 0.210 | - | 76 | 6786.911 | 1.236 | 0.668 | - |

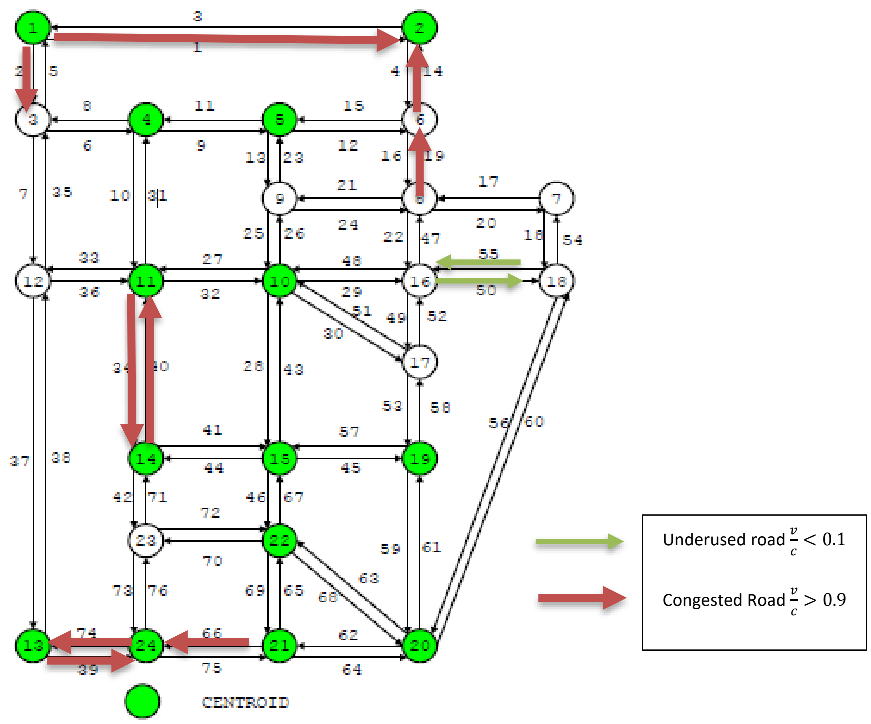


Figure 3 Congested road from SO Result on the highway network of Sioux Falls

## 7. Summary

Form this project, we conclude several remarks shown in the experiment tasks:

1. There are different flow distributions between UE and SO. The differences are led due to different the characteristics of problem formulation between UE and SO. UE are known for "selfish" flow assignment, while SO is achieving "system optimal" but letting some driver to sacrifice their travel time.
2. We showed that in system performance, total system time of UE flow are greater than total system time of SO flow, where T(UE) > T(SO)
3. We observed that SO flow distribution would improve the congestion in a congested road by redistributing the flow to another road. As the consequences, we will obtain a reduced flow from congested roads, but we might having increased or new congested road due to this redistribution.
4. In this project, we demonstrate how to obtain the Marginal-cost Pricing. Then, we prove that the Marginal-cost Pricing scheme is one of "first-best" tolling scheme using comparison of SO flow and Tolled UE flow.
5. From all the discussion and exploration, we find that topology characteristic is important in understanding on how the flow is distributed. Specifically, the result shows the important of direct connectivity and centrality of the arcs to explain how the flow travel within the network.

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## APPENDIX

### I. Flow-Conservation Constraints for UE, SO, and Tolled UE

$x_1 + x_2 - x_3 - x_5 = 0$

$x_3 + x_4 - x_1 - x_{14} = 0$

$x_5 + x_6 + x_7 - x_2 - x_8 - x_{35} = 0$

$x_8 + x_9 + x_{10} - x_6 - x_{11} - x_{31} = 0$

$x_{11} + x_{12} + x_{13} - x_9 - x_{12} - x_{23} = 0$

$x_5 + x_6 + x_7 - x_2 - x_8 - x_{35} = 0$

$x_{17} + x_{18} - x_{20} - x_{54} = 0$

$x_{19} + x_{20} + x_{21} + x_{22} - x_{16} - x_{17} - x_{24} - x_{47} = 0$

$x_{23} + x_{24} + x_{25} - x_{13} - x_{21} - x_{26} = 0$

$x_{26} + x_{27} + x_{28} + x_{29} + x_{30} - x_{25} - x_{32} - x_{43} - x_{48} - x_{51} = 0$

$x_{31} + x_{32} + x_{33} + x_{34} - x_{10} - x_{27} - x_{36} - x_{40} = 0$

$x_{35} + x_{36} + x_{37} - x_7 - x_{33} - x_{38} = 0$

$x_{38} + x_{39} - x_{37} - x_{74} = 0$

$x_{40} + x_{41} + x_{42} - x_{34} - x_{44} - x_{71} = 0$

$x_{43} + x_{44} + x_{45} + x_{46} - x_{28} - x_{41} - x_{57} - x_{67} = 0$

$x_{47} + x_{48} + x_{49} + x_{50} - x_{22} - x_{29} - x_{52} - x_{55} = 0$

$x_{51} + x_{52} + x_{53} - x_{30} - x_{49} - x_{58} = 0$

$x_{54} + x_{55} + x_{56} - x_{18} - x_{50} - x_{60} = 0$

$x_{57} + x_{58} + x_{59} - x_{45} - x_{53} - x_{61} = 0$

$x_{60} + x_{61} + x_{62} + x_{63} - x_{56} - x_{59} - x_{64} - x_{68} = 0$

$x_{64} + x_{65} + x_{66} - x_{62} - x_{69} - x_{75} = 0$

$x_{67} + x_{68} + x_{69} + x_{70} - x_{46} - x_{63} - x_{65} - x_{72} = 0$

$x_{71} + x_{72} + x_{73} - x_{42} - x_{70} - x_{76} = 0$

$x_{74} + x_{75} + x_{76} - x_{39} - x_{66} - x_{73} = 0$

### II. Jupyter Code of UE

### III. Jupyter Code of SO

### IV. Jupyter Code of Tolled UE