Discrete Optimization: Homework #7, Ex. #3

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We consider the following LP : $\max\{c^T x : Ax \leq b, b \in \mathbb{R}^n\}$ and assume that it is feasible and bounded. Thus, we know that there exists an optimal solution, say x^*. We also know that the dual of our LP : $\min\{b^T y : A^T y = c, y \geq 0\}$ is feasible and bounded by the Strong Duality theorem. That's why, we have that, for y^* an optimal solution of the dual LP : $c^T x^* = b^T y^*$ i.e. the optimal values coincide.

But x^* is optimal for the primal LP and y^* for the dual LP. So for all (x^*, y^*) satisfying the previous equation and the respective constraints of primal and dual LP, (x^*, y^*) will be optimal. In other words, this means that all (x, y) in the following Polyhedron P are optimal.

$$c^T x = b^T y$$
$$Ax \leq b$$
$$A^T y = c$$
$$y \geq 0$$

We have $P = \{\tilde{x} \in \mathbb{R}^2 n : \tilde{A}\tilde{x} \leq \tilde{b}\}$ with $\tilde{A} = \begin{pmatrix} c^T & -b^T \\ -c^T & b^T \\ A & 0 \\ 0 & A^T \\ 0 & -A^T \\ 0 & -I \end{pmatrix}$, $\tilde{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

and $\tilde{b} = \begin{pmatrix} 0 \\ 0 \\ b \\ c \\ -c \\ 0 \end{pmatrix}$.

Since we know that there exists the optimal solution (x^*, y^*) of the LP, the vector $w = \begin{pmatrix} x^* \\ y^* \end{pmatrix}$ is in P and is optimal because x^* and y^* respectively maximized and minimized their objective functions. So using the oracle algorithm on the Polyhedron P in a single call, we find an optimal solution of our LP.