Discrete Optimization: Homework #6, Ex. #5

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March 30, 2018

We have to solve the following Linear Program with the Simplex Algorithm:

$$c = \begin{pmatrix} 6 \\ 9 \\ 2 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 3 & 3 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} -4 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad x_0^* = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

1 Step 1

$$B_0 = \{1, 2, 6\} \quad A_{B_0} = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_0 = \begin{pmatrix} 6 \\ -9 \\ 5 \end{pmatrix} \Rightarrow d_0 = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} \Rightarrow K = \{3, 4\}$$

The min, attained in $k = 4$,is equal to $\frac{1}{3}$. Index 2 is replaced by index 4.

2 Step 2

$$B_1 = \{1, 4, 6\} \quad A_{B_1} = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} \quad x_1^* = \begin{pmatrix} 0 \\ -\frac{4}{3} \\ 0 \end{pmatrix} \quad d_1 = \begin{pmatrix} 0 \\ \frac{1}{3} \\ -1 \end{pmatrix}$$

$$\Rightarrow K = \{3, 5\}$$

The min, attained in $k = 3$,is equal to $\frac{5}{2}$. Index 6 is replaced by index 3.

3 Step 3

$$B_2 = \{1, 3, 4\} \quad A_{B_2} = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 3 & -1 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow \lambda_2 = \begin{pmatrix} 5/2 \\ 1/2 \\ 2 \end{pmatrix}$$

$\Rightarrow B_2$ is optimal and $x_2^* = \begin{pmatrix} 0 \\ -1/2 \\ -5/2 \end{pmatrix}$. The function value with x^* is $\frac{-19}{2}$.