# Discrete Optimization: Homework #12, Ex. #4

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May 25, 2018

Given a graph $G = (V = \{v_1, ..., v_n\}, E)$, we can obtain an associated bipartite graph $G' = (V', E')$ with $V' = A \cup B$, $|A| = |B| = |V|$, $(a_i, b_j) \in E'$ if and only if $(v_i, v_j) \in E$. Then one can find a 2-matching in $G$ if and only if one can find a perfect matching in $G'$, which can be done in polynomial time. If $M'$ is a perfect matching of $G'$, then $\forall e \in M'$, $e = (a_i, b_j)$ or $e = (b_j, a_i)$ and we can add $(v_i, v_j)$ to the 2-Matching $M$ in $G$.

By König-Hall's Theorem, we can only find a perfect matching in $G'$ if $|N(S)| \geq |S| \quad \forall S \subseteq A$.