$$\langle HF|a_a^\dagger a_b^\dagger a_d a_c|HF\rangle = \langle HF|h_a h_b h_d^\dagger h_c^\dagger|HF\rangle \tag{1}$$

$$= \langle HF|\overbrace{h_a h_b h_d^\dagger h_c^\dagger}|HF\rangle + \langle HF|\overbrace{h_a h_b h_d^\dagger h_c^\dagger}|HF\rangle \tag{2}$$

$$= (-1)\delta_{ad}\Theta_{\epsilon_F\epsilon_a}\delta_{bc}\Theta_{\epsilon_F\epsilon_b} + \delta_{ac}\Theta_{\epsilon_F\epsilon_a}\delta_{bd}\Theta_{\epsilon_F\epsilon_b} \tag{3}$$