$$\langle HF|a_a^\dagger a_b^\dagger a_c^\dagger a_d a_f a_e|HF\rangle \;=\; \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle \tag{1}$$

$$= \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle \;+\; \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle$$

$$+ \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle \;+\; \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle$$

$$+ \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle \;+\; \langle HF|h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger|HF\rangle$$

$$\tag{2}$$

$$= (-1)\delta_{ad}\Theta_{\epsilon_F \epsilon_a}\delta_{bf}\Theta_{\epsilon_F \epsilon_b}\delta_{ce}\Theta_{\epsilon_F \epsilon_c} \;+\; \delta_{ad}\Theta_{\epsilon_F \epsilon_a}\delta_{be}\Theta_{\epsilon_F \epsilon_b}\delta_{cf}\Theta_{\epsilon_F \epsilon_c}$$

$$+ \delta_{af}\Theta_{\epsilon_F \epsilon_a}\delta_{bd}\Theta_{\epsilon_F \epsilon_b}\delta_{ce}\Theta_{\epsilon_F \epsilon_c} \;+\; (-1)\delta_{af}\Theta_{\epsilon_F \epsilon_a}\delta_{be}\Theta_{\epsilon_F \epsilon_b}\delta_{cd}\Theta_{\epsilon_F \epsilon_c}$$

$$+ \delta_{ae}\Theta_{\epsilon_F \epsilon_a}\delta_{bf}\Theta_{\epsilon_F \epsilon_b}\delta_{cd}\Theta_{\epsilon_F \epsilon_c} \;+\; (-1)\delta_{ae}\Theta_{\epsilon_F \epsilon_a}\delta_{bd}\Theta_{\epsilon_F \epsilon_b}\delta_{cf}\Theta_{\epsilon_F \epsilon_c}$$

$$\tag{3}$$