

$$\langle HF | a_a^\dagger a_b^\dagger a_c^\dagger a_d a_f a_e | HF \rangle = \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle \quad (1)$$

$$\begin{aligned}
&= \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle + \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle \\
&+ \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle + \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle \\
&+ \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle + \langle HF | h_a h_b h_c h_d^\dagger h_f^\dagger h_e^\dagger | HF \rangle
\end{aligned} \quad (2)$$

$$\begin{aligned}
&= (-1) \delta_{ad} \Theta_{\epsilon_F \epsilon_a} \delta_{bf} \Theta_{\epsilon_F \epsilon_b} \delta_{ce} \Theta_{\epsilon_F \epsilon_c} + \delta_{ad} \Theta_{\epsilon_F \epsilon_a} \delta_{be} \Theta_{\epsilon_F \epsilon_b} \delta_{cf} \Theta_{\epsilon_F \epsilon_c} \\
&+ \delta_{af} \Theta_{\epsilon_F \epsilon_a} \delta_{bd} \Theta_{\epsilon_F \epsilon_b} \delta_{ce} \Theta_{\epsilon_F \epsilon_c} + (-1) \delta_{af} \Theta_{\epsilon_F \epsilon_a} \delta_{be} \Theta_{\epsilon_F \epsilon_b} \delta_{cd} \Theta_{\epsilon_F \epsilon_c} \\
&+ \delta_{ae} \Theta_{\epsilon_F \epsilon_a} \delta_{bf} \Theta_{\epsilon_F \epsilon_b} \delta_{cd} \Theta_{\epsilon_F \epsilon_c} + (-1) \delta_{ae} \Theta_{\epsilon_F \epsilon_a} \delta_{bd} \Theta_{\epsilon_F \epsilon_b} \delta_{cf} \Theta_{\epsilon_F \epsilon_c}
\end{aligned} \quad (3)$$