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ECE 4554/5554 - Computer Vision

Homework 2:

Problem 1:

a) The 3 operation to be done are, $T(-x_0,-y_0)(P)$ then $R(P)$ and $T(x_0,y_0)(P)$ respectively where P is the point $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ in homogenous coordinates.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T(x_0,y_0)} \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R(\,)} \underbrace{\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T(-x_0,-y_0)} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x-x_0 \\ y-y_0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta.(x-x_0) - \sin\theta.(y-y_0) \\ \sin\theta.(x-x_0) + \cos\theta(y-y_0) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta.(x-x_0) - \sin\theta(y-y_0) + x_0 \\ \sin\theta.(x-x_0) + \cos\theta(y-y_0) + y_0 \\ 1 \end{bmatrix}$$

b) $(x_0, y_0) = (200, 150)$, $\theta = 15^\circ$

i) $(x, y) = (0, 0)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(15) \cdot (0-200) - \sin(15) \cdot (0-150) + 200 \\ \sin(15) \cdot (0-200) + \cos(15) \cdot (0-150) + 150 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 45.63 \\ -46.65 \\ 1 \end{bmatrix}$$ is the homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 45.63 \\ -46.65 \end{bmatrix}$$ is the inhomogeneous coordinates

ii) $(x, y) = (200, 150)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(15) \cdot (0) - \sin(15) \cdot 0 + 200 \\ \sin(15) \cdot (0) + \cos(15) \cdot 0 + 150 \\ 1 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \\ 1 \end{bmatrix}$$

is the homogeneous coordinates.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$ is the inhomogeneous coordinates

iii) $(x, y) = (210, 150)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(15) \cdot (10) - \sin(15) \cdot 0 + 200 \\ \sin(15) \cdot (10) + \cos(15) \cdot 0 + 150 \\ 1 \end{bmatrix} = \begin{bmatrix} 209.65 \\ 152.58 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 209.65 \\ 152.58 \end{bmatrix}$$

iv) $(x,y) = (400,300)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(15).(200) - \sin(15)(150) + 200 \\ \sin(15).200 + \cos(15)(150) + 150 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 354.36 \\ 346.65 \\ 1 \end{bmatrix} , \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 354.36 \\ 346.65 \end{bmatrix}$$

Problem 2 :

Convolution operation has associative property therefore, $(I * g) * h = I * (g * h)$. If we say $f = g * h$ we can do the same operation with $I * f$.

$g(0,0) = -1$, $g(1,0) = -2$, $g(2,0) = 0$,

$g(0,1) = -4$, $g(1,1) = 5$, $g(2,1) = 6$

$g(0,2) = 7$, $g(1,2) = 8$, $g(2,2) = 0$

$h(0,0) = 1$, $h(0,1) = 3$, $h(1,0) = 2$, $h(1,1) = 4$

$$f(m,n) = g(m,n) * h(m,n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} g(i,j) \cdot h(m-i, n-j)$$

$$f(m,n) = h(m,n) * g(m,n) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} h(i,j) \cdot g(m-i, n-j)$$

because of the commutative property.

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$$f(m,n) = h(0,0)\cdot g(m,n) + h(0,1)\cdot g(m,n-1) + h(1,0)\cdot g(m-1,n) + h(1,1)\cdot g(m-1,n-1)$$

for $m,n = (0,0)$

$$f(0,0) = h(0,0)\cdot g(0,0) + \overset{0}{\cancel{h(0,1)\cdot g(0,-1)}} + \overset{0}{\cancel{h(1,0)\cdot g(-1,0)}} + \overset{0}{\cancel{h(1,1)\cdot g(-1,-1)}}$$

$$= h(0,0)\cdot g(0,0) = 1 \times -1 = -1$$

for $m,n = (0,1)$

$$f(0,1) = h(0,0)\cdot g(0,1) + h(0,1)\cdot g(0,0) + h(1,0)\cdot \cancel{g(-1,1)} + h(1,1)\cdot \cancel{g(-1,0)}$$

$$= \underbrace{h(0,0)}_{1}\cdot \underbrace{g(0,1)}_{-4} + \underbrace{h(0,1)}_{3}\cdot \underbrace{g(0,0)}_{-1}$$

$$= -7$$

for $m,n = (0,2)$

$$f(0,2) = h(0,0)\cdot g(0,2) + h(0,1)\cdot g(0,1) + h(1,1)\cdot \cancel{g(-1,2)} + h(1,1)\cdot \cancel{g(-1,1)}$$

$$= \overbrace{h(0,0)}^{1}\cdot \overbrace{g(0,2)}^{7} + \overbrace{h(0,1)}^{3}\cdot \overbrace{g(0,1)}^{-4}$$

$$= -5$$

$$f(0,3) = \cancel{h(0,0).g(0,3)} + h(0,1).g(0,2) +$$

$$\cancel{h(1,0).g(-1,3)} + \cancel{h(1,1).g(-1,2)}$$

$$= h(0,1).g(0,2) = 3 \times 7 = 21$$

$$f(0,4) = h(0,0).\cancel{g(0,4)} + h(0,1).\cancel{g(0,3)} +$$

$$h(1,0).\cancel{g(-1,4)} + h(1,1).\cancel{g(-1,3)}$$

$f(0,4)$ is out of bondories.

$$f(1,0) = h(0,0).g(1,0) + \cancel{h(0,1).g(1,-1)} +$$

$$h(1,0).g(0,0) + \cancel{h(1,1).g(0,-1)}$$

$$= \underbrace{h(0,0)}_{1}.\underbrace{g(1,0)}_{-2} + \underbrace{h(1,0)}_{2}.\underbrace{g(0,0)}_{-1}$$

$$= -4$$

$$f(1,1) = h(0,0).g(1,1) + h(0,1).g(1,0) +$$

$$h(1,0).g(0,1) + h(1,1).g(0,0)$$

$$= 1 \times 5 + 3 \times -2 + 2 \times -4 + 4 \times -1$$

$$= 5 - 6 - 8 - 4 = -13$$

$$f(1,2) = h(0,0) \cdot g(1,2) + h(0,1) \cdot g(1,1) + h(1,0) \cdot g(0,2) + h(1,1) \cdot g(0,1)$$

$$= +1 \times 8 + 3 \times 5 + 2 \times 7 + 4 \times -4$$
$$= 8 + 15 + 14 - 16 = 21$$

$$f(1,3) = \cancel{h(0,0) \cdot g(1,3)} + h(0,1) \cdot g(1,2) + \cancel{h(1,0) \cdot g(0,3)} + h(1,1) \cdot g(0,2)$$

$$= 3 \times 8 + 4 \times 7$$
$$= 52$$

$$f(1,4) = h(0,0) \cdot g(1,4) + h(0,1) \cdot g(1,3) + h(1,0) \cdot g(0,4) + h(1,1) \cdot g(0,3)$$

$\hookrightarrow f(1,4)$ is out of bandores.

$$f(2,0) = h(0,0) \cdot g(2,0) + \cancel{h(0,1) \cdot g(2,-1)} + h(1,0) \cdot g(1,0) + \cancel{h(1,1) \cdot g(1,-1)}$$

$$= 1 \times 0 + 2 \times -2 = -4$$

$$f(2,1) = h(0,0) \cdot g(2,1) + h(0,1) \cdot g(2,0) + h(1,0) \cdot g(1,1) + h(1,1) \cdot g(1,0)$$

$$= 1 \times 6 + \cancel{3 \times 0} + 2 \times 5 + 4 \times -2$$
$$= 6 + 10 - 8 = 8$$

$$f(2,2) = h(0,0).g(2,2) + h(0,1).g(2,1) + h(1,0).g(1,2) + h(1,1).g(1,1)$$

$$= 1\times 0 + 3\times 6 + 2\times 8 + 4\times 5$$

$$= 18 + 16 + 20 = 54$$

$$f(2,3) = h(0,0).\cancel{g(2,3)} + h(0,1).g(2,2) + \cancel{h(1,0).g(1,3)} + h(1,1).g(1,2)$$

$$= 3\times 0 + 4\times 8 = 32$$

$f(2,4)$ will be out of bardores.

$$f(3,0) = \cancel{h(0,0).g(3,0)} + h(0,1).\cancel{g(3,-1)} + h(1,0).g(2,0) + \cancel{h(1,1).g(2,-1)}$$

$$= 2\times 0 = 0$$

$$f(3,1) = h(0,0).\cancel{g(3,1)} + h(0,1).\cancel{g(3,0)} + h(1,0).g(2,1) + h(1,1).g(2,0)$$

$$= 2\times 6 + 4\times 0 = 12$$

$$f(3,2) = h(0,0).\cancel{g(3,2)} + h(0,1).\cancel{g(3,1)} + h(1,0).g(2,2) + h(1,1).g(2,1)$$

$$= 2\times 0 + 4\times 6 = 24$$

$$f(3,3) = h(0,0).\cancel{g(3,3)} + h(0,1).\cancel{g(3,2)} + h(1,0).g(2,3) + h(1,1)g(2,2) = 4\times 0 = 0$$

$f(3,4)$ will be out of bondories.

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$$f = \begin{bmatrix} -1 & -7 & -5 & 21 \\ -4 & -13 & 21 & 52 \\ -4 & 8 & 54 & 32 \\ 0 & 12 & 24 & 0 \end{bmatrix}$$

Problem 3:

Laplacian filter is, $\Delta^2 f = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$

where $\frac{\delta^2 f}{\delta x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$ and

where $\frac{\delta^2 f}{\delta y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$

therefore, $\Delta^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$

therefore for $f(x,y)$:

$f(x-1,y-1)$	$f(x,y-1)$	$f(x+1,y-1)$
$f(x-1,y)$	$f(x,y)$	$f(x+1,y)$
$f(x-1,y+1)$	$f(x,y+1)$	$f(x+1,y+1)$

×

0	1	0
1	-4	1
0	1	0

doing crosscorrelation between these 2 images gives the equation for $\Delta^2 f$. Therefore this particular kernel is used for vertical & horizontal Laplacian.

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by making use of fspecial function from MATLAB, we know that Laplacian filter is created as follows,

$$\nabla^2 \approx \frac{4}{\alpha+1}\begin{bmatrix} \frac{\alpha}{4} & \frac{1-\alpha}{4} & \frac{\alpha}{4} \\ 1-\alpha/4 & -1 & \frac{1-\alpha}{4} \\ \frac{\alpha}{4} & \frac{1-\alpha}{4} & \alpha/4 \end{bmatrix}$$

$$\approx \begin{bmatrix} \frac{\alpha}{\alpha+1} & \frac{1-\alpha}{\alpha+1} & \frac{\alpha}{\alpha+1} \\ \frac{1-\alpha}{\alpha+1} & \frac{-4}{\alpha+1} & \frac{1-\alpha}{\alpha+1} \\ \frac{\alpha}{\alpha+1} & \frac{1-\alpha}{\alpha+1} & \frac{\alpha}{\alpha+1} \end{bmatrix}$$ which can be divided

into following parts as follows,

$$\underbrace{\frac{1-\alpha}{\alpha+1}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{we already know this Laplacian filter for vertical \& horizontal directions}} + \underbrace{\frac{\alpha}{1+\alpha}\begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{\text{this filter is for 2nd derivatives of diagonal of the image.}}$$

trying $\alpha = 0.2$ which is the default value of fspecial function we have,

$$\frac{0.8}{1.2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \frac{0.2}{1.2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2/12 & 8/12 & 2/12 \\ 8/12 & -40/12 & 8/12 \\ 2/12 & 8/12 & 2/12 \end{bmatrix} \rightarrow$$ multiplying this by 12 and dividing by 2 gives the kernel $$\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

Hence we can say $\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$ is the

weighted combination of vertical & horizontal Laplacian and the diagonal Laplacian.

Problem 4:

For function $f(x,y)$, 1^{st} order Taylor series expansion for the point $(x+u, y+v)$ is given as:

$$f(x+u, y+v) \approx f(x,y) + f_x(u,v).u + f_y(u,v).v$$

where $f_x(u,v).u = u.\frac{\partial f}{\partial x}$ and $f_y(u,v).v = v.\frac{\partial f}{\partial y}$

therefore we can rewrite the equation as: Proof this. This is not given by

(1) $$f(x+u, y+v) \approx f(x,y) + u.\frac{\partial f}{\partial x} + v.\frac{\partial f}{\partial y}$$

we know: $\nabla f(x,y) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$ vector.

$$\begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u.\partial f/\partial x \\ v.\partial f/\partial y \end{bmatrix}$$

therefore

$$f(x,y) + \nabla f.\begin{bmatrix} u \\ v \end{bmatrix} = f(x,y) + u.\frac{\partial f}{\partial x} + v.\frac{\partial f}{\partial y}$$

which is the same with equation 1.