

Question 2:

in each iteration each data point is assigned to its closest center, therefore for t^{th} iteration:

$$\text{cost} (C_1^{(t+1)}, \dots, C_k^{(t+1)}; z_1^{(t)}, \dots, z_k^{(t)}) \leq \\ \text{cost} (C_1^{(t)}, \dots, C_k^{(t)}; z_1^{(t)}, \dots, z_k^{(t)})$$

then each cluster is re-centered at its mean

hence

$$\text{cost} (C_1^{(t+1)}, \dots, C_k^{(t+1)}; z_1^{(t+1)}, \dots, z_k^{(t+1)}) \leq \\ \text{cost} (C_1^{(t+1)}, \dots, C_k^{(t+1)}; z_1^{(t)}, \dots, z_k^{(t)}).$$