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Q1) Optimize the following using K-map

a) (10) f(A,B,C,D)=A'B'+B'C+AC'+AD+ACD

b) (10) $f(X,Y,Z,T) = \sum(0,1,2,3,4,6,8,10,12,14)$

c) (10) $f(X,Y,Z,T) = \prod(0,1,2,3,4,6,8,10,12,14)$

a)

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	1	1	1	0
10	1	1	1	1

A'B' → 0000, 0001, 0010, 0011

B'C → 0010, 0011, 1010, 1011

AC' → 1000, 1001, 1100, 1101

AD → 1001, 1011, 1101, 1111

ACD → 1011, 1111

$= B' + AC' + AD$

$= B' + A(C'+D)$ ad-hoc

b)

XY \ ZT	00	01	11	10
00	1	1	1	1
01	1			1
11	1			1
10	1			1

$= T' + X'Y'$

c) $f(X,Y,Z,T) = \prod(0,1,2,3,4,6,8,10,12,14)$

$= \sum(5,7,9,11,13,15)$

XY \ ZT	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$= YT + XT$

$= T(X+Y)$

Q2) (20) Design a circuit with the following definition, using K-map approach:

- Input A: 3-bit unsigned number (0 <= A <= 5)
- Input B: 1-bit value
- Output C: 3-bit unsigned number (0 <= C <= 5)

If B = 1, C = (A + 2) % 6
else, C = (A - 2) % 6

	A_2	A_1	A_0	B	C_2	C_1	C_0
0	0	0	0	0	1	0	0
	0	0	0	1	0	1	0
1	0	0	1	0	1	0	1
	0	0	1	1	0	1	1
2	0	1	0	0	0	0	0
	0	1	0	1	1	0	0
3	0	1	1	0	0	0	1
	0	1	1	1	1	0	1
4	1	0	0	0	0	1	0
	1	0	0	1	0	0	0
5	1	0	1	0	0	1	1
	1	0	1	1	0	0	1
6	1	1	0	0	X	X	X
	1	1	0	1	X	X	X
7	1	1	1	0	X	X	X
	1	1	1	1	X	X	X

K-map for C_2:

$A_2A_1 \backslash A_0B$	00	01	11	10
00	1			1
01		1	1	
11	X	X	X	X
10	0	0	0	0

$= A_2'A_1'B' + A_1B$

K-map for C_1:

$A_2A_1 \backslash A_0B$	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	X	X	X	X
10	1	0	0	1

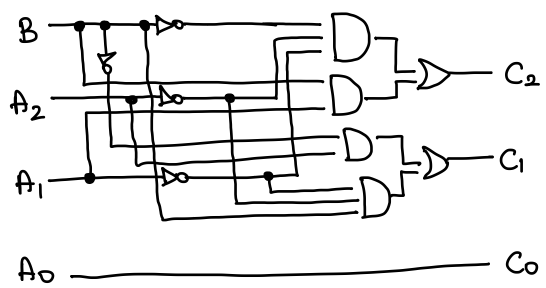
$= A_2B' + A_2'A_1'B$

K-map for C_0

$A_2A_1 \backslash A_0B$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	X	X	X	X
10	0	0	1	1

$= A_0$

Circuit:



Q3) (50)
Design a 2-bit signed/unsigned adder/subtractor circuit. Circuit will have a signed_unsigned input pin to determine the signed/unsigned operation and an adder_subtractor input to determine adder/subtractor operation. Draw the circuit diagram. Use the K-map approach, using only 4-value K-maps.

I assumed
S/U = 0 → signed
S/U = 1 → unsigned
A/S = 0 → adder
A/S = 1 → subtractor

A1	A0	B1	B0	S/U=0	A/S=0	S/U=0	A/S=1	S/U=1	A/S=0	S/U=1	A/S=1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	1	1	0	1	1	1
0	0	1	0	1	0	1	0	1	0	1	0
0	0	1	1	1	1	0	1	1	1	0	1

signed subtractor

00 − 01 = 00, + 11 = 11

00 − 10 = 00, + 10 = 10

00 − 11 = 00, + 01 = 01

01 → 10; 00 + 1 → 01

unsigned subtractor

$\cancel{0}\cancel{0}$ − 01 = 11

$\cancel{0}0$ − 10 = 10

00 − 11 = 01

A1	A0	B1	B0	S/U=0	A/S=0	S/U=0	A/S=1	S/U=1	A/S=0	S/U=1	A/S=1
0	1	0	0	0	1	0	1	0	1	0	1
0	1	0	1	1	0	0	0	1	0	0	0
0	1	1	0	1	1	1	1	1	1	1	1
0	1	1	1	0	0	1	0	0	0	1	0

01 + 00 = 01
01 + 01 = 10
01 + 10 = 11
01 + 11 = 00

01 − 00 = 01
01 − 01 = 00
01 − 10 = 01, + 10 = 11
01 − 11 = 01, + 01 = 10

01 + 01 = 10
01 + 10 = 11
01 + 11 = 00

$\cancel{0}1$ − 10 = 11
$\cancel{0}1$ − 11 = 10

A_1	A_0	B_1	B_0	S/U=0 A/S=0	S/U=0 A/S=1	S/U=1 A/S=0	S/U=1 A/S=1
1	0	0	0	1 0	1 0	1 0	1 0
1	0	0	1	1 1	0 1	1 1	0 1
1	0	1	0	0 0	0 0	0 0	0 0
1	0	1	1	0 1	1 1	0 1	1 1

10 + 10 = *00

10 + 11 = *01

10 − 01 = 10; 10 + 11 = *01

10 − 10 = 10; 01 + 1 = 10; 10 + 10 = *00

10 − 11 = 10; 10 + 01 = 11

10 − 00 = 10

$\cancel{1}\cancel{0}^2$ − 01 = 01

10 − 10 = 00

$\cancel{1}\cancel{0}^2$ − 11 = 11

A_1	A_0	B_1	B_0	S/U=0 A/S=0	S/U=0 A/S=1	S/U=1 A/S=0	S/U=1 A/S=1
1	1	0	0	1 1	1 1	1 1	1 1
1	1	0	1	0 0	1 0	0 0	1 0
1	1	1	0	0 1	0 1	0 1	0 1
1	1	1	1	1 0	0 0	1 0	0 0

1

11 + 01 = *00

11 + 10 = *01

11 + 11 = *10

11 − 01 = 11; 10 + 1 = 11; 11 + 11 = 010

11 − 10 = 11; 01 + 1 = 10; 11 + 10 = *01

11 − 11 = 11; 00 + 1 = 01; 11 + 01 = *00

11 − 00 = 11

11 − 01 = 10

11 − 10 = 01

11 − 11 = 00

Kmaps for signed addition & unsigned addition (same results)

C_1:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	0	0	1	1
01	0	1	0	1
11	1	0	1	0
10	1	1	0	0

$$= A_1'A_0'B_1 + A_1'B_1B_0' + A_1'A_0B_1'B_0 + A_1B_1'B_0' + A_1A_0'B_1' + A_1A_0B_1B_0$$

C_0:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

$$= A_0'B_0 + A_0B_0' = A_0 \oplus B_0$$

Kmaps for signed & unsigned subtraction:

C_1:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	0	1	0	1
01	0	0	1	1
11	1	1	0	0
10	1	0	1	0

$$= A_1'A_0'B_1'B_0 + A_1'B_1B_0' + A_1'A_0B_1 + A_1A_0B_1' + A_1B_1'B_0' + A_1A_0'B_1B_0$$

C_0:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

$$= A_0'B_0 + A_0B_0' = A_0 \oplus B_0$$

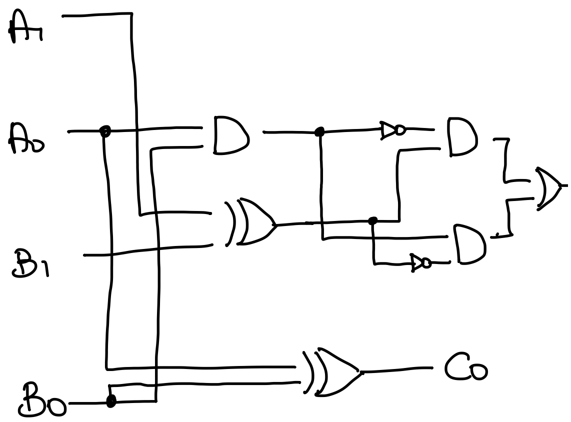
Circuit for addition:

$$C_1 = A_1'B_1(A_0' + B_0') + A_1B_1'(B_0' + A_0') + A_0B_0(A_1'B_1' + A_1B_1)$$

$$= (A_0' + B_0')(A_1'B_1 + A_1B_1') + (A_0B_0)(A_1'B_1' + A_1B_1)$$

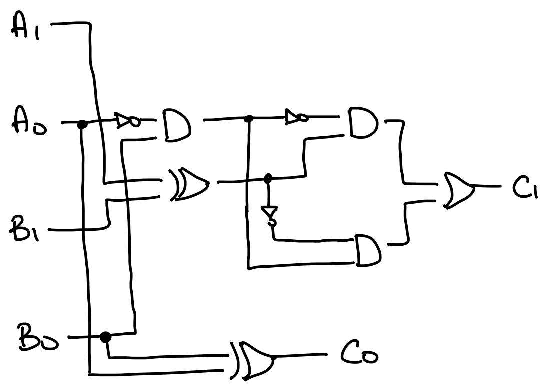
$$= (A_0' + B_0')(A_1 \oplus B_1) + (A_0B_0)(A_1 \oplus B_1)'$$

$$= (A_0B_0)'(A_1 \oplus B_1) + (A_0B_0)(A_1 \oplus B_1)'$$



Circuit for subtraction

$$C_1 = A_1'A_0'B_1'B_0 + A_1A_0'B_1B_0 + A_1'B_1B_0' + A_1'A_0B_1 + A_1A_0B_1' + A_1B_1'B_0'$$



$$= A_1'B_1(B_0'+A_0) + A_1B_1'(A_0+B_0')$$
$$A_0'B_0(A_1'B_1'+A_1B_1)$$

$$= (A_0+B_0')(A_1'B_1+A_1B_1') +$$
$$(A_0'B_0)(A_1 \oplus B_1)'$$

$$= (A_0'B_0)'(A_1 \oplus B_1) +$$
$$(A_0'B_0)(A_1 \oplus B_1)'$$

$$C_0 = A_0 \oplus B_0$$

Together: