

Assigned: Nov. 9, 2019

Due :Nov. 18, 2020 by 11:00 a.m.

Assignment #1 : Linear Algebra Review

- (1) Express $x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ as a linear combination of $y_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $y_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.
- (2) Given the basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ for R^3 , which vectors can be removed and replaced by the vector $\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$, while still maintaining a basis? Illustrate this geometrically.
- (3)
- $$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$
- i. Determine a basis for the column span of matrix **A**? Justify your answer. Is the basis unique?
 - ii. Determine a basis for the row- span of matrix **A**? What is the dimension for the row-space of matrix **A**? Justify your answer.
 - iii. Determine the rank of matrix **A**? Justify your answer.
 - iv. Determine the nullity of matrix **A**? Justify your answer.
- (4) a) Determine a basis of **unit vectors** $\{\mathbf{v}_i\}_{i=1}^n$ in which matrix $\mathbf{C} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$ is diagonal
- b) Is this basis $\{\mathbf{v}_i\}_{i=1}^n$ orthonormal? Justify your answer
- c) Determine the inverse of the modal matrix $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n]$.
- (5) Determine a transformation, which first scales a rectangular object in x - y plane by a factor of 2 and 3 in x and y direction respectively, then it rotates the object by $\pi/3$ radians in counterclockwise direction and finally results in 5 and 10 units shifting in x and y directions, respectively.