EE 417 Introduction to Computer Vision

Point, Local and Global Operators

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Sabancı Universitesi

Image Processing

Mapping of an image into another image

Often as *image preprocessing*, i.e. prior to image analysis

Input

Output

edge →

Here: Input image mapped into an image showing high-frequency information only

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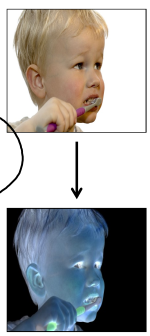
What types of image transformations can we do?

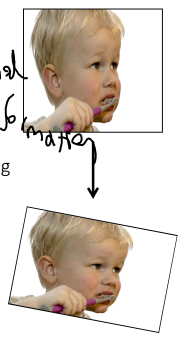
Filtering



changes pixel *values*

Spatial transformation

Warping

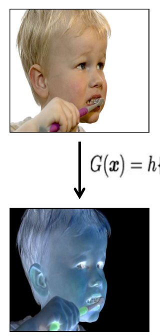


changes pixel *locations*

What types of image transformations can we do?

Filtering

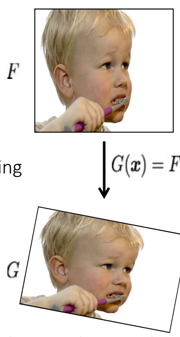
$G(x) = h\{F(x)\}$



changes *range* of image function

Warping

$G(x) = F(h\{x\})$

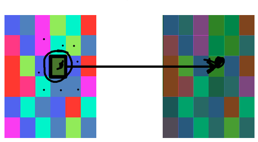


changes *domain* of image function

What types of image filtering can we do?

$I_o(x_p,y_p)=I_n(x_p,y_p)$

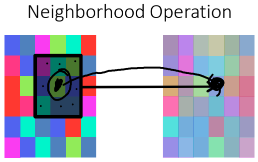
Point Operation



point processing

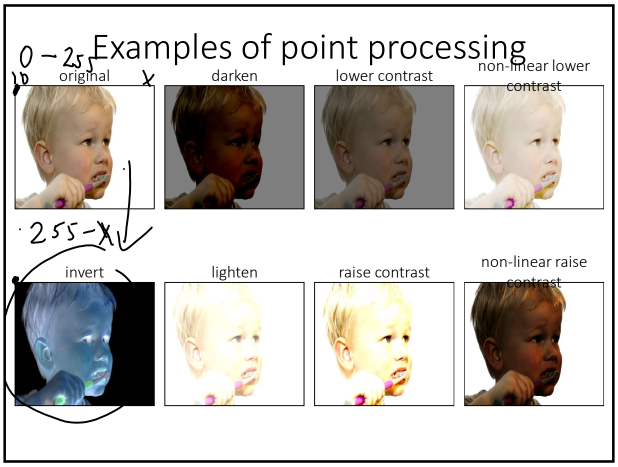
Local Operation

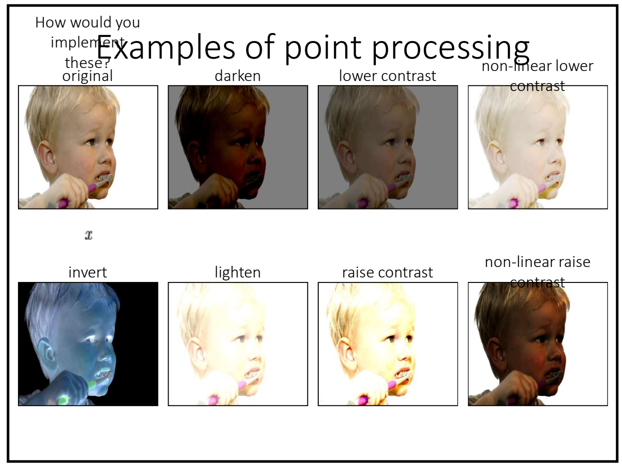
Neighborhood Operation

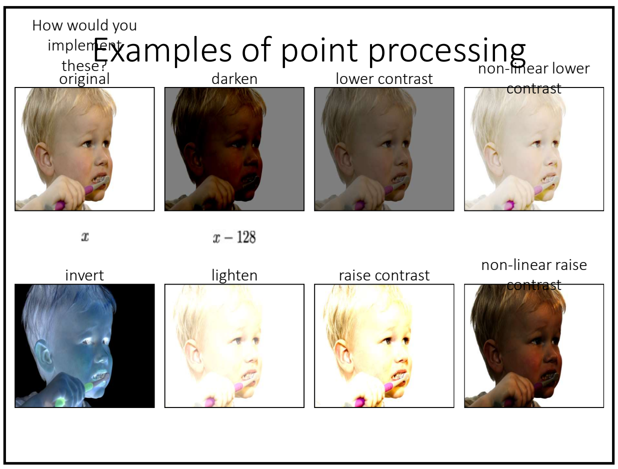


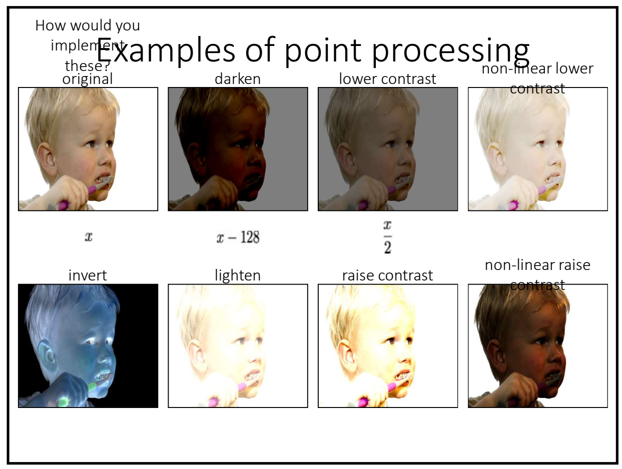
"filtering"

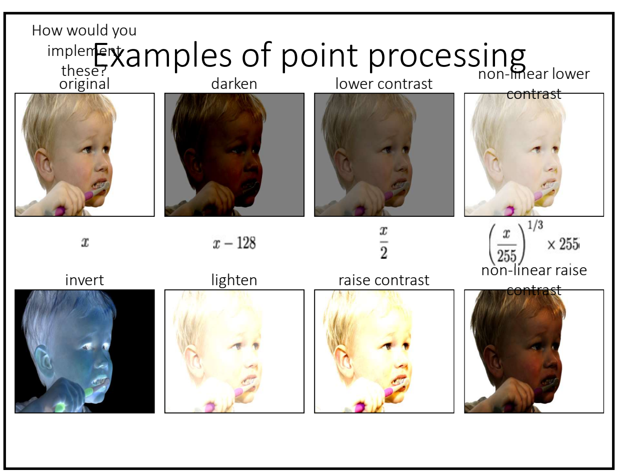
Point processing

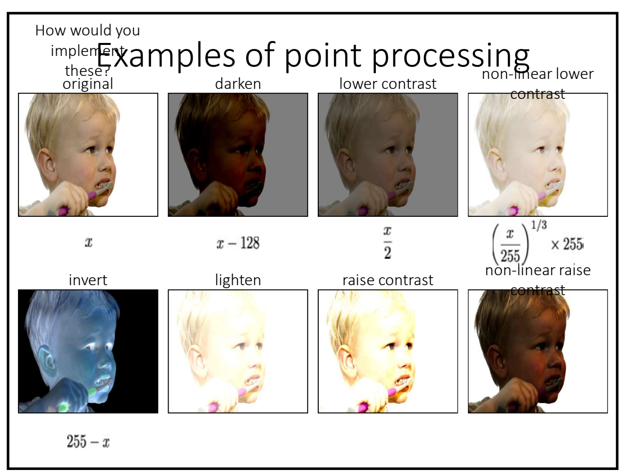

0 – 255
Examples of point processing
original
darken
lower contrast
non-linear lower contrast
x
255 – x
invert
lighten
raise contrast
non-linear raise contrast


How would you implement these?
Examples of point processing
original
darken
lower contrast
non-linear lower contrast
x
invert
lighten
raise contrast
non-linear raise contrast


How would you implement these?
Examples of point processing
original
darken
lower contrast
non-linear lower contrast
x
$x - 128$
invert
lighten
raise contrast
non-linear raise contrast


How would you implement these?
Examples of point processing
original
darken
lower contrast
non-linear lower contrast
x
$x - 128$
$\frac{x}{2}$
invert
lighten
raise contrast
non-linear raise contrast


How would you implement these?
Examples of point processing
original
darken
lower contrast
non-linear lower contrast
x
$x - 128$
$\frac{x}{2}$
$\left(\frac{x}{255}\right)^{1/3} \times 255$
invert
lighten
raise contrast
non-linear raise contrast

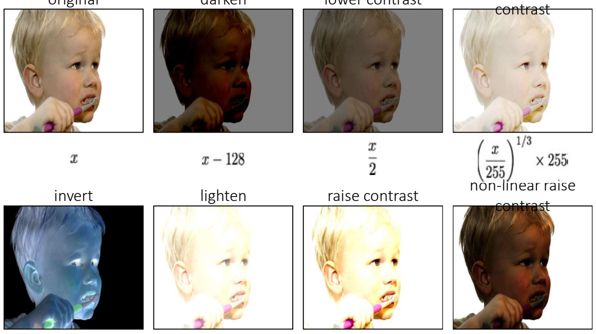

How would you implement these?
Examples of point processing
original
darken
lower contrast
non-linear lower contrast
x
$x - 128$
$\frac{x}{2}$
$\left(\frac{x}{255}\right)^{1/3} \times 255$
invert
lighten
raise contrast
non-linear raise contrast
$255 - x$

Examples of point processing

How would you implement these?

original | darken | lower contrast | non-linear lower contrast

invert | lighten | raise contrast | non-linear raise contrast



x | $x - 128$ | $\frac{x}{2}$ | $\left(\frac{x}{255}\right)^{1/3} \times 255$

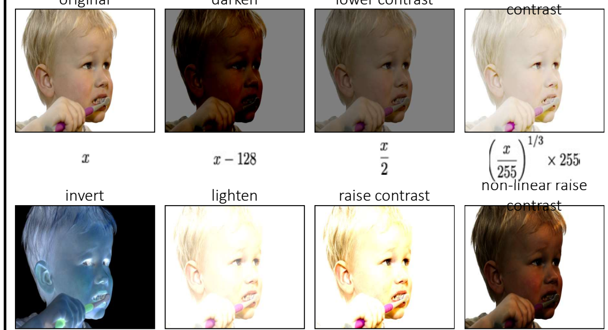
$255 - x$ | $x + 128$

Examples of point processing

How would you implement these?

original | darken | lower contrast | non-linear lower contrast

invert | lighten | raise contrast | non-linear raise contrast



x | $x - 128$ | $\frac{x}{2}$ | $\left(\frac{x}{255}\right)^{1/3} \times 255$

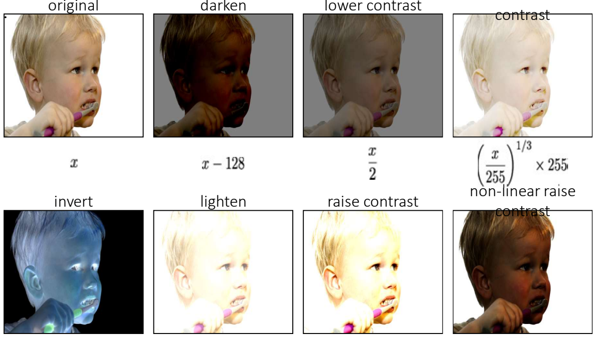
$255 - x$ | $x + 128$ | $x \times 2$

Examples of point processing

How would you implement these?

original | darken | lower contrast | non-linear lower contrast

invert | lighten | raise contrast | non-linear raise contrast



x | $x - 128$ | $\frac{x}{2}$ | $\left(\frac{x}{255}\right)^{1/3} \times 255$

$255 - x$ | $x + 128$ | $x \times 2$ | $\left(\frac{x}{255}\right)^{2} \times 255$

Many other types of point processing

camera output | image after stylistic tonemapping

[Bae et al., SIGGRAPH 2006]

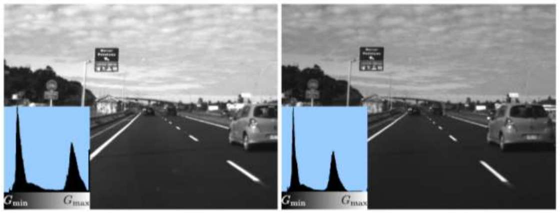
Many other types of point processing

Example: Noise Removal

Noise is unwanted data

For example, *challenges of outdoor image recording:*
Difficulties with lighting, motion blur, or sudden changes in scenes

Noise removal is one of the reasons for mapping a given image into a "better" image illustrating three examples of noise for vision-based driver-assistance



- Pair of stereo images taken time-synchronized but of different brightness; see the shown gray-level histograms
- **Goal**: Transform images such that resulting images are "as taken at uniform illumination"

- Blurring caused by rain and obstruction of view by a wiper
- **Goal:** Reduce impacts of rain and wiper when analysing this image (e.g.when calculating distance to car in front of the ego-vehicle); we could try to start with image sharpening for removing the blur

- Gaussian noise in a scene recorded at night
- **Goal:** Remove the Gaussian noise, enhance the image contrast

Point Operators

Time-efficient methods for image processing

Gradation Functions

Transform image I into image I_{new} of the same size:

Map value u at pixel location p in I by *gradation function* g onto value

$$v = g(u)$$

u
← p=(x,y)
v
← p (x,y)

at the same pixel location p in I_{new}

Change only depends on value u at location p, thus a *point operator* defined by gradation function g

Which Gradation Function?

Which function g was used to map the left into the right image?

X   255 − X

A historic photo of events in April 1960 at Korea University, Seoul, leading to an uprising against the autocratic ruler Syngman Rhee of South Korea.

$v = g(u) = 255 - u$

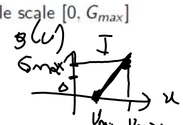
Linear Scaling

Not a histogram transform, but another example for a point operator

Image I has positive histogram values in interval $[u_{min}, u_{max}]$

Goal: Map values used in I linearly onto the whole scale $[0, G_{max}]$

$$u_{min} = \min\{I(x,y) : (x,y) \in \Omega\}$$
$$u_{max} = \max\{I(x,y) : (x,y) \in \Omega\}$$



Definition of gradation function:

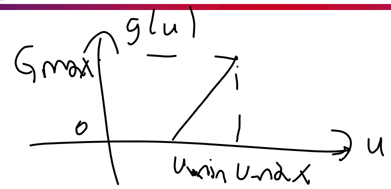
$$a = -u_{min} \quad \text{and} \quad b = \frac{G_{max}}{u_{max} - u_{min}}$$
$$g(u) = b(u + a)$$

$g(u) = ?$

Results:

Pixels having value u_{min} in I now have value 0 in I_{new}

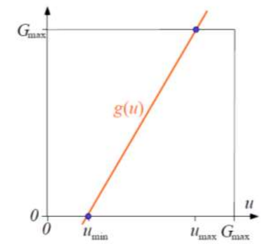
Pixels having value u_{max} in I now have value G_{max} in I_{new}



$$g(u) = \frac{G_{max}}{U_{max} - U_{min}} (u - U_{min})$$

Linear Scaling: Graph

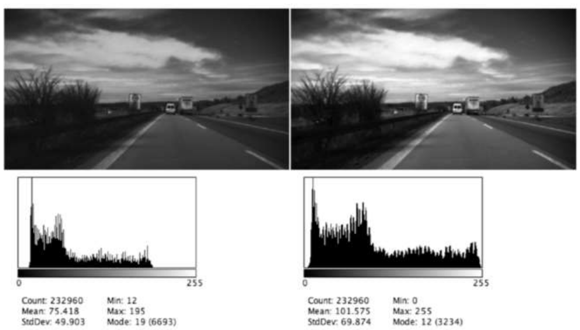
Graph of gradation function for linear scaling



Defined by being incident with points $(u_{\min}, 0)$ and $(u_{\max}, G_{\max})$: Derive the formula as given on the previous page

Linear Scaling: Example

Left: $u_{\min} = 12$ and $u_{\max} = 195$ *Right:* $u_{\min} = 0$ and $u_{\max} = 255$



Histogram Transforms

Histogram transforms are examples of point operators

Histogram Transform $I \longrightarrow g(u) \longrightarrow I_{output}$

If the goal is that I_{new} satisfies some properties defined in terms of its histogram then we speak about a *histogram transform*

Example: Histogram Equalization

Goal: Transform I such that all gray levels appear equally often in I_{new}

Ideal goal: For all $u \in \{0, 1, \ldots, G_{max}\}$

$$H_{I_{new}}(u) = \frac{N_{cols} N_{rows}}{G_{max} + 1} = \frac{|\Omega|}{G_{max} + 1} = const$$

Unfortunately: Not possible in general due to having digital images (and not continuous functions)

The Raging Bull Example

Left: Input image `RagingBull` with histogram
Right: The same image after histogram equalization

Example: A binary image would remain binary after histogram equalisation

Histogram Equalization: How To Do

Given:

$N_{cols} \times N_{rows}$ scalar image I with absolute frequencies $H_I(u)$, $0 \leq u \leq G_{max}$

Transform I into I_{new} of the same size by applying a gradation function

Use the following gradation function g $\in [0,1]$

$$g(u) = c_I(u) \cdot G_{max}$$

where c_I is the relative cumulative frequency function

Recall: What exactly is the definition of c_I?

Figure on previous page illustrates this histogram-equalization transform

In the probability theory, if X is a continuous Random variable with pdf $f_X(x)$ and cumulative distributive function $F_X(x) = P(X \le x)$

$y = F_X(x)$

uniform

Conditional Scaling

Not a histogram transform, but another example for a point operator

Goal: Map image J into image J_{new} such that J_{new} has the same mean and variance as image I

Definition of gradation function:

$$a = \mu_J \cdot \frac{\sigma_I}{\sigma_J} - \mu_I \quad \text{and} \quad b = \frac{\sigma_J}{\sigma_I}$$

$$g(u) = b(u + a)$$

Calculation of J_{new}: Replace value u at p in J by $v = g(u)$

Result: $\mu_{J_{new}} = \mu_I$ and $\sigma_{J_{new}} = \sigma_I$

Might solve noise issue (1) on Page 6 **if** brightness difference between both images is uniformly defined

Conditional Scaling: Example

Image I | Image J

Conditional scaling of J | Equalized J

Histogram Equalization

An image histogram is a graphic representation of the frequency counts of all allowable pixel intensities. Figure 2.1 shows a sample of an image with a skewed histogram.

4	1	3	2
3	1	1	1
0	1	5	2
1	1	2	2

input image

4

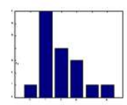


Figure 2.1 A sample image with a skewed histogram (poor intensity distribution)

u	H(u)	C(u)
0	1	1
1	7	8
2	4	12
3	2	14
4	1	15
5	1	16

$c(u) = \frac{C(u)}{16}$

relative cumulative frequency function

1. Compute the histogram of the image
2. Calculate the normalized sum of histogram
3. Transform the input image to an output image

u $\quad v = g(u) = c(u) \cdot G_{max}$

intensity	sum	normalized sum
0	1	1/16*5=0.31255 ≅ 0
1	8	2.5 = 8/16·5 ≅ 3
2	12	3.75 ≅ 4
3	14	4.375 → 4
4	15	4.6875 → 5
5	16	5.0 → 5

5	3	4	4
4	3	3	3
0	3	5	4
3	3	4	4

output image

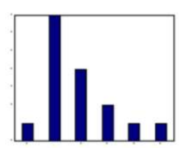


Figure 2.2 A histogram-equalized sample with histogram and normalized sum

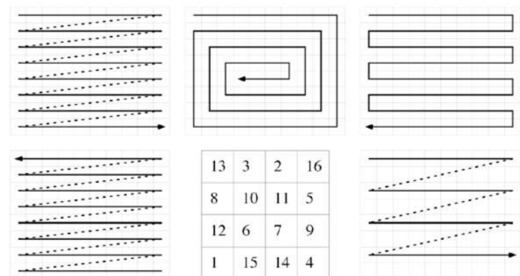
General Concept of a Local Operator

Map input image I into a new image J:

1. Given: $N_{cols} \times N_{rows}$ image I
2. Sliding window W_p of size $(2k+1) \times (2k+1)$
3. Reference point p at the center of the window
4. Reference point p moves into all possible pixel locations of I
5. At these locations we perform a *local operation*
6. Result of the operation defines new value at p

Moves of Reference Point p of Window W_p through I

Defined by a scan; upper left shows default scan:



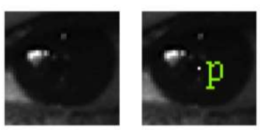
Two Examples: Local Mean and Maximum

Example: Local operation is mean $J(p) = \mu_{W_p(I)}$; for $p = (x, y)$ we have

$$\mu_{W_p(I)} = \frac{1}{(2k+1)^2} \cdot \sum_{i=-k}^{+k} \sum_{j=-k}^{+k} I(x+i, y+j)$$

Example: Local operation is the maximum; for $p = (x, y)$ we have

$$J(p) = \max\{I(x+i, y+j) : -k \le i \le k \wedge -k \le j \le k\}$$



Maximum in 41×41 input window (left) copied into p on the right
What happens if input window (i.e. p) moves one pixel to the right?

Border-pixel Strategy

Needed for windows centered at p and not completely contained in I

There is no general rule:

Define meaningful operation depending on the given local operation

Top, left: Original image. *Right*: Local maximum for $k = 3$

Linear Operators and Convolution

Linear local operators are a class of local operators

A linear local operator is defined by a *convolution* of I with a *filter kernel*

Reference point $p = (x, y)$, weights $w_{i,j} \in \mathbb{R}$, scaling factor $S > 0$:

$$J(p) = I * W(p) = \frac{1}{S} \sum_{i=-k}^{+k} \sum_{j=-k}^{+k} w_{i,j} \cdot I(x+i, y+j)$$

Array of $(2k+1) \times (2k+1)$ weights and S define the filter kernel

Example: All $w_{i,j} > 0$ and $S = \sum_{i=-k}^{+k} \sum_{j=-k}^{+k} w_{i,j}$

Common Way for Filter Kernel Representation

$$\begin{array}{|c|c|c|}\hline w_{11} & w_{12} & w_{13} \\ \hline w_{21} & w_{22} & w_{23} \\ \hline w_{31} & w_{32} & w_{33} \\ \hline \end{array} /S \qquad \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} /1 \qquad \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} /9$$

Left: General representation for a 3×3 filter kernel
Middle: Filter kernel for approximating derivation in x-direction, $S = 1$
Right: Filter kernel of a 3×3 box filter, $S = 9$

Box Filter

Local mean on Page 23 is a linear local operator known as *box filter*

All weights equal to 1, and $S = (2k+1)^2$ is the sum of all weights

Local Operators: Summary

1. Operations are limited to windows, typically of size $(2k+1) \times (2k+1)$
2. Window moves through the image following a selected scan order
3. Operation in the window is the same at all locations
4. Pixels close to the border of the image: case by case decisions
5. Results are either
   1. replacing values at reference points in input image I (*sequential local operator*); new values propagate like a "wave" over original values; windows of the local operator contain original data as well as already-processed pixel values
   2. written into a second array, leaving the original image unaltered this way (*parallel local operator*); this kind of a local operator can be implemented on parallel hardware

Point, Local, or Global Operator

Cases for $(2k+1) \times (2k+1)$ window size:

(1) $k = 0$, i.e. window is just a single pixel: A *point operator*

Example: Operators defined by a gradation function

(2) k such that whole image is covered by the window: A *global operator*

(3) If not global operator then a local operator (including point operators)

Examples: Local mean (box filter), local maximum

Linear shift-invariant image filtering

Linear shift-invariant image filtering

- Replace each pixel by a *linear* combination of its neighbors (and possibly itself).
- The combination is determined by the filter's *kernel*.
- The same kernel is *shifted* to all pixel locations so that all pixels use the same linear combination of their neighbors.

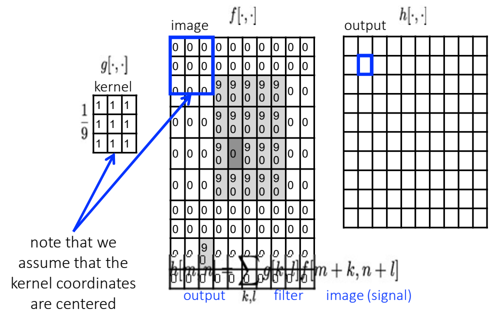
Example: the box filter

- also known as the 2D rect (not rekt) filter
- also known as the square mean filter

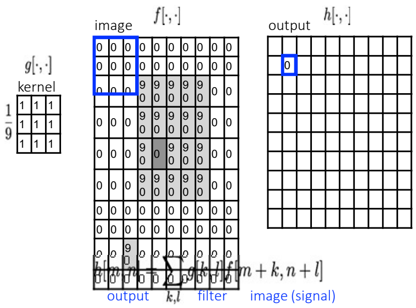
kern
el $g[\cdot,\cdot] = \frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$

- replaces pixel with local average
- has smoothing (blurring) effect

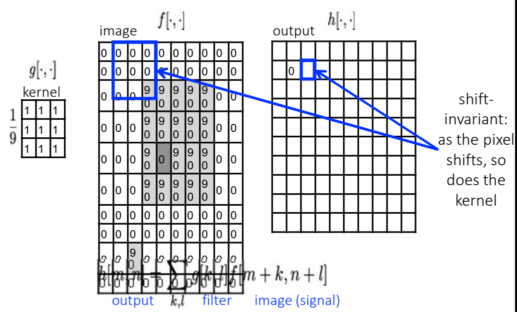
Let’s run the box filter



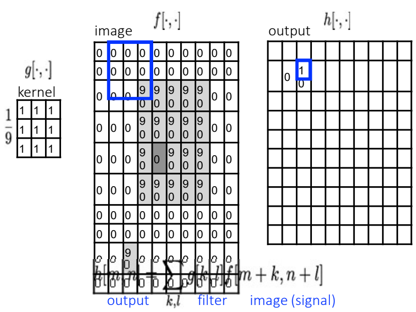
Let’s run the box filter



Let’s run the box filter



Let’s run the box filter



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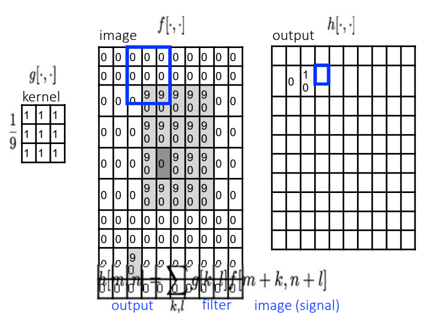

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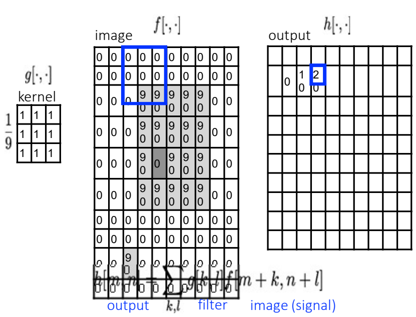

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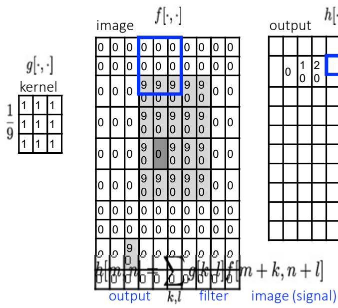

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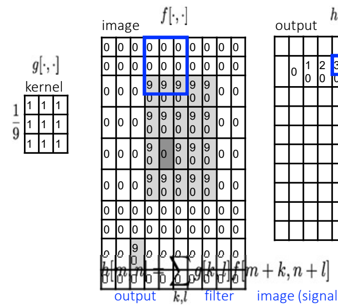

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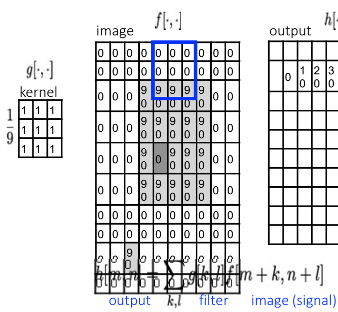

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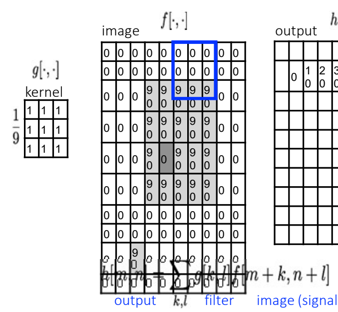
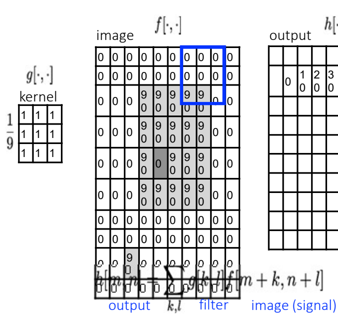
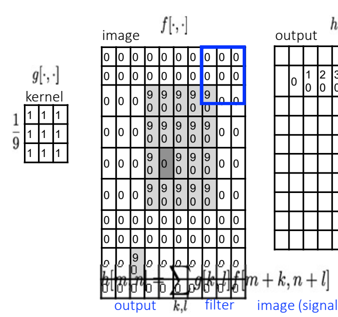


Let’s run the box filter



Let’s run the box filter



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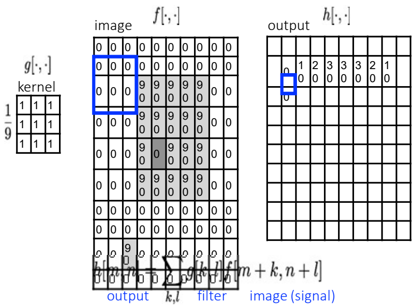

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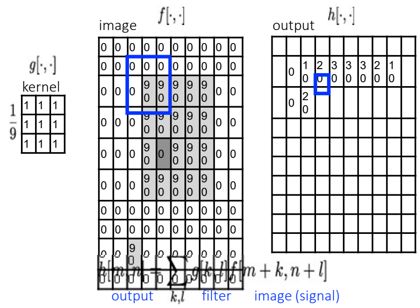

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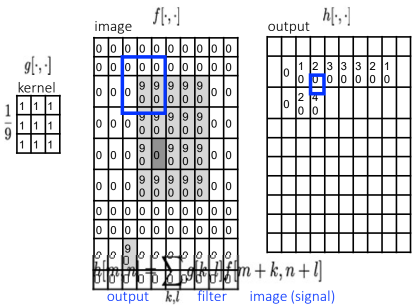

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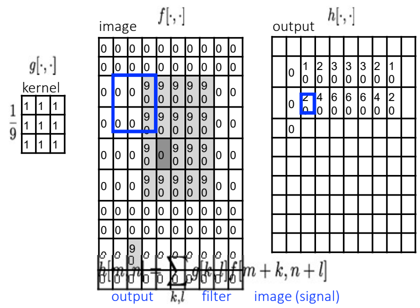

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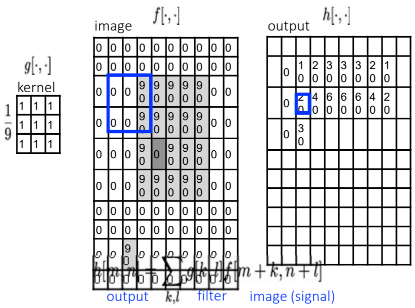
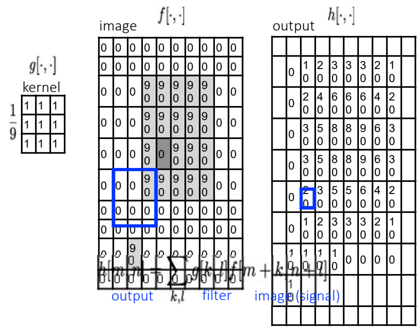
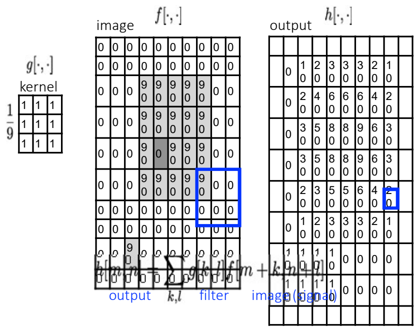


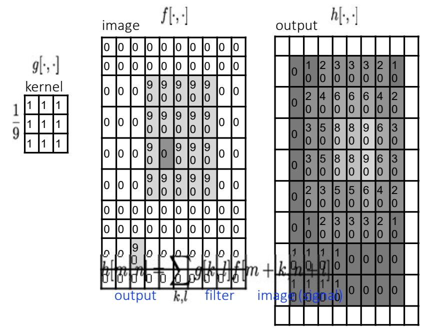
Let’s run the box filter



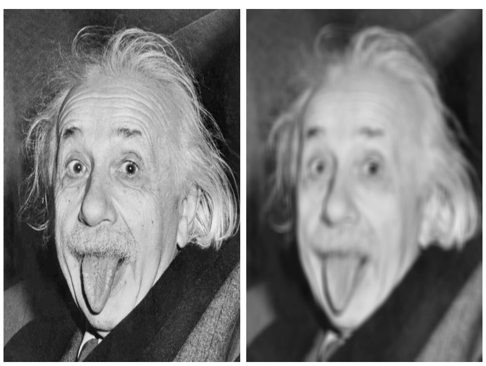
Let's run the box filter



… and the result is



Some more realistic examples



Some more realistic examples

Some more realistic examples

Convolution

Convolution for 1D continuous signals

Definition of filtering as convolution:

notice the flip

$$(f * g)(x) = \int_{-\infty}^{\infty} f(y) g(x - y) dy$$

filtered signal; filter; input signal

Convolution for 1D continuous signals

Definition of filtering as convolution:

notice the flip

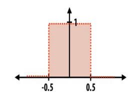
$$(f * g)(x) = \int_{-\infty}^{\infty} f(y) g(x - y) dy$$

filtered signal; filter; input signal

Consider the box filter example:

1D continuous box filter

$$f(x) = \begin{cases} 1 & |x| \leq 0.5 \\ 0 & otherwise \end{cases}$$



filtering output is a blurred version of g

$$(f * g)(x) = \int_{-0.5}^{0.5} g(x - y) dy$$

Convolution for 2D discrete signals

Definition of filtering as convolution:

notice the flip

$$(f * g)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j) I(x - i, y - j)$$

filtered image; filter; input image

Convolution for 2D discrete signals

Definition of filtering as convolution:

notice the flip

$$(f * g)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j) I(x - i, y - j)$$

filtered image; filter; input image

If the filt $f(i,j)$ is non-zero c $-1 \leq i, j \leq 1$, then

$$(f * g)(x, y) = \sum_{i,j=-1}^{1} f(i, j) I(x - i, y - j)$$

The kernel we saw earlier is the 3x3 matrix representation of $f(i,j)$.

Convolution vs correlation

Definition of discrete 2D convolution:

notice the flip

$$(f * g)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j) I(x - i, y - j)$$

Definition of discrete 2D correlation:

notice the lack of a flip

$$(f * g)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j) I(x + i, y + j)$$

- Most of the time won't matter, because our kernels will be symmetric.
- Will be important when we discuss frequency-domain filtering (lectures 5-6).

Separable filters

A 2D filter is separable if it can be written as the product of a "column" and a "row".

example: box filter

1	1	1
1	1	1
1	1	1

=

1
1
1

column

*

1	1	1

row

What is the rank of this filter matrix?

Separable filters

A 2D filter is separable if it can be written as the product of a "column" and a "row".

example: box filter

$$\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix} * \begin{bmatrix}1&1&1\end{bmatrix}$$

column row

Why is this important?

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2D convolution with a separable filter is equivalent to two 1D convolutions (with the "column" and "row" filters).

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2D convolution with a separable filter is equivalent to two 1D convolutions (with the "column" and "row" filters).

If the image has M x M pixels and the filter kernel has size N x N:
- What is the cost of convolution with a non-separable filter?

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  $M^2 \times N^2$
- What is the cost of convolution with a separable filter?

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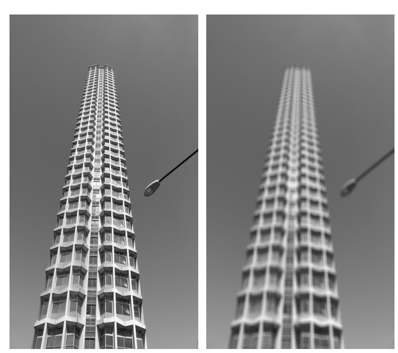
2D convolution with a separable filter is equivalent to two 1D convolutions (with the "column" and "row" filters).

If the image has M x M pixels and the filter kernel has size N x N:
- What is the cost of convolution with a non-separable filter?
  $M^2 \times N^2$
- What is the cost of convolution with a separable filter?
  $2 \times N \times M^2$

A few more filters



original 3x3 box filter

do you see any problems in this image?