Student Information

Full Name : Türkan Deniz ÖZkırıç
Id Number : 2521870

Answer 1

a)

$$Blue \longrightarrow 1 * \frac{1}{6} + 2 * \frac{1}{6} + 3 * \frac{1}{6} + 4 * \frac{1}{6} + 5 * \frac{1}{6} + 6 * \frac{1}{6} = \frac{7}{2} = 3.5$$

$$Yellow \longrightarrow 1 * \frac{3}{8} + 3 * \frac{3}{8} + 4 * \frac{1}{8} + 8 * \frac{1}{8} = \frac{24}{8} = 3$$

$$Red \longrightarrow 2 * \frac{5}{10} + 3 * \frac{2}{10} + 4 * \frac{2}{10} + 6 * \frac{1}{10} = \frac{30}{10} = 3$$

b)

Expected value of rolling one die from each color and summing them up:

$$E.V.(Blue) + E.V.(Yellow) + E.V.(Red)$$

$$= 3.5 + 3 + 3 = 9.5$$

Expected value of rolling three blue dice:

$$E.V.(Blue) + E.V.(Blue) + E.V.(Blue)$$

$$3.5 + 3.5 + 3.5 = 10.5$$

We choose to roll three blues because it is expected to yield a larger sum.

c)

Expected value of rolling one die from each color and summing them up would become:

$$E.V.(Blue) + E.V.(Yellow) + E.V.(Red)$$

$$= 3.5 + 8 + 3 = 14.5$$

Expected value of rolling three blue dice would still be 10.5. Therefore, we would choose to roll one die from each color because it would be expected to yield a larger sum.

d)

$$P(Red \cap 3) = \frac{1}{3} * \frac{2}{10} = \frac{1}{15}$$

$$P(X = 3) = P(Blue \cap 3) + P(Yellow \cap 3) + P(Red \cap 3) = \frac{1}{3} * \frac{1}{6} + \frac{1}{3} * \frac{3}{8} + \frac{1}{3} * \frac{2}{10}$$

$$\frac{1}{18} + \frac{1}{8} + \frac{1}{15} = \frac{89}{360} = 0.2472$$

$$\frac{P(R \cap 3)}{P(3)} = 0.26966$$

e)

There are three option for their sums to be 5:

- Blue=1 Yellow=4
- Blue=2 Yellow=3
- Blue=4 Yellow=1

$$P_B(1) * P_Y(4) + P_B(2) * P_Y(3) + P_B(4) * P_Y(1)$$

$$= \frac{1}{6} * \frac{1}{8} + \frac{1}{6} * \frac{3}{8} + \frac{1}{6} * \frac{3}{8} = \frac{7}{48} = 0.14583$$

Answer 2

a)

For company A, We have 80 distributions and each has a possibility 0.025 of offering discount. This is a binomial distribution with:

- success $\longrightarrow$ giving discount
- n = 80 (80 independent trials because 80 distributions)
- P(1) = 0.025 (possibility of success, aka a specific distribution offering discount)

Possibility of at least 4 successes equals to 1-(Possibility of at most 3 successes)

$$\begin{aligned} P(X \leq 3) = &P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= C(80,0) * \left(\frac{1}{40}\right)^0 * \left(\frac{39}{40}\right)^{80} + C(80,1) * \left(\frac{1}{40}\right)^1 * \left(\frac{39}{40}\right)^{79} + \\ &C(80,2) * \left(\frac{1}{40}\right)^2 * \left(\frac{39}{40}\right)^{78} + C(80,3) * \left(\frac{1}{40}\right)^3 * \left(\frac{39}{40}\right)^{77} \\ &= 0.131938 + 0.270642 + 0.274111 + 0.182741 \\ &= 0.859432 \end{aligned}$$

$$\begin{aligned} 1 - P(X \leq 3) &= 1 - 0.859432 \\ &= 0.140568 \end{aligned}$$

b)

P = Probability of getting a discount on a specific day, either from A or B
P = 1 -(not getting discount from neither A nor B)
Not getting a discount from A is Binomial distribution with n=80, P(1) = $\frac{1}{40}$, x=0 (so no distribution of A succeeds aka offers discount)

$$C(80,0) * \left(\frac{1}{40}\right)^0 * \left(\frac{39}{40}\right)^{80} = 0.131938$$

Above number is the possibility that no distribution of A gives a discount on a specific day.
Not getting a discount from B is Bernoulli distribution with $P(1) = 0.1$

$$P(0) = 0.9$$

Above number is the possibility that company B does not give a discount on a specific day.
If we multiply these two numbers we get $0.131938 * 0.9 = 0.118744$ which is the possibility that neither company gives discount. Therefore, the possibility that at least one of them will give discount is $1 - 0.118744 = 0.881256$.
The probability that first success(aka first discount) will be achieved in two trials(aka two days) is Geometric distribution with P(success) = 0.881256. We compute the possibility of reaching first success in 1 or 2 trials.

$$\begin{aligned} P(X \leq 2) &= F(2) \\ &= 1 - (1 - 0.881256)^2 \\ &= 0.98590 \end{aligned}$$

where F(2) is cdf.

Answer 3

```
results = randi(6, 3, 1000);
sumOfEachTrial = sum(results, 1);
sumOfEachTrialTot=sum(sumOfEachTrial);
fprintf('Avarage total when you roll three blues : %f \n',
   sumOfEachTrialTot/1000)

Y = [1,1,1,3,3,3,4,8];
R = [2,2,2,2,2,3,3,4,4,6];
resultsB = randi(6, 1, 1000);
resultsY = Y(randi(8, 1, 1000));
resultsR = R(randi(10, 1, 1000));
sumOfEachTrial2 = resultsB + resultsY + resultsR;
sumOfEachTrialTot2=sum(sumOfEachTrial2);
fprintf('Avarage total when you roll one of each color : %f \n',
   sumOfEachTrialTot2/1000)
```

```
Avarage total when you roll three blues : 10.546000

R =

    2     2     2     2     2     3     3     4     4     6

Avarage total when you roll one of each color : 9.485000
>> dr2
Avarage total when you roll three blues : 10.447000
Avarage total when you roll one of each color : 9.415000
>> dr2
Avarage total when you roll three blues : 10.345000
Avarage total when you roll one of each color : 9.680000
>>
```

Trials confirm our theoretical results at 1.b. Rolling 3 blues gives results close to 10.5 and rolling 3 different colors gives results close to 9.5 on average. It makes more sense to roll three blues.