Student Information

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Answer 1

a)

As stated in our book, (Chapter 3.3.1), expected (mean) value is calculated with the formula:

$$\sum_x xP(x)$$

With that information, we can calculate the expected values of dices as follows:
E(blue): 2 * 1/6 + 2 * 1/6 + 2 * 1/6 + 2 * 1/6 + 3 * 1/6 + 4 * 1/6 = 2/6 + 2/6 + 2/6 + 2/6 + 3/6 + 4/6 = 15/6 = 2.5
E(yellow): 1 * 1/6 + 1 * 1/6 + 2 * 1/6 + 2 * 1/6 + 3 * 1/6 + 3 * 1/6 = 1/6 + 1/6 + 2/6 + 2/6 + 3/6 + 3/6 = 12/6 = 2
E(red): 1 * 1/6 + 1 * 1/6 + 2 * 1/6 + 2 * 1/6 + 3 * 1/6 + 3 * 1/6 + 3 * 1/6 + 5 * 1/6 = 1/6 + 1/6 + 2/6 + 2/6 + 3/6 + 3/6 + 3/6 + 5/6 = 20/8 = 2.5

b)

As stated in the book, for any 2 random variables X and Y, E(X+Y) = E(X) + E(Y). By using this rule and expected values found for part(a), we can say that:
E(red+red+yellow) = E(red) + E(red) + E(yellow) = 2.5 + 2.5 + 2 = 7
E(yellow+yellow+blue) = E(yellow) + E(yellow) + E(blue) = 2 + 2 + 2.5 = 6.5

As expected value of 1st option is greater than expected value of second option, I would choose rolling 2 red and 1 yellow to maximize total value.

c)

If it is certain that blue's value will be 4, then we would take E(blue) as 4. Calculating the expected value of 2nd option again, we get:
E(yellow+yellow+blue) = E(yellow) + E(yellow) + E(blue) = 2 + 2 + 4 = 8
which has a greater expected value than 1st option right now. In this case, I would choose rolling 2 yellow and 1 blue.

d)

Let A be the event that the rolled die is red and B be the value of die is 3.
Here, what we need to find is conditional probability of A given B,
P{A | B} = $\dfrac{P\{A \cap B\}}{P\{B\}}$
P{A} = $\dfrac{1}{3}$ (As it is stated in the question that each color has equal probability in random choosing)
P{B} = $\dfrac{1}{3} * \dfrac{1}{6} + \dfrac{1}{3} * \dfrac{1}{3} + \dfrac{1}{3} * \dfrac{3}{8} = \dfrac{7}{24}$ (P(blue,3) + P(yellow,3) + P(red,3))
$P\{A \cap B\} = \dfrac{1}{3} * \dfrac{3}{8} = \dfrac{1}{8}$ (P(red,3))
P{A | B} = $\dfrac{P\{A \cap B\}}{P\{B\}} = \dfrac{\frac{1}{8}}{\frac{7}{24}} = \dfrac{3}{7}$

e)

Options that the total value will be 6:
1) red: 3, yellow: 3
2) red: 5, yellow: 1

For option 1, possibility of red dice's being 3 is 3/8 and yellow dice's being 3 is 1/3. Possibility of them taking these values together is 3/8 * 1/3 = 1/8.
With the same calculation logic, red could be 5 with the possibility 1/8 and yellow could be 1 with the possibility 1/3, which makes 1/8 * 1/3 = 1/24 together.
As we could get the value both with option 1 OR option 2, the possibility of two dices' total value is 6 is 1/24 + 1/8 = 1/6.

Answer 2

a)

By looking at 3rd entry in table, we see that P(0,2) = 0.17.

b)

As there are no such rows in given table, the probability, P(2,0) = 0.

c)

Let T denote possibility of Istanbul and Ankara's having 2 electric outages in total. Then, $P_T(2) = P\{X + Y = 2\} = P\{x = 0 \cap Y = 2\} + P\{x = 1 \cap Y = 1\} + P\{x = 2 \cap Y = 0\} = P(0,2) + P(1,1) + P(2,0) = 0.17 + 0.11 + 0 = 0.28$

d)

$P_A(1)$ = P(1,0) + P(1,1) + P(1,2) + P(1,3) = 0.6
(We sum all possibilities where Ankara has electric outage)

e)

Let A define electric outages in Ankara and I define the same for Istanbul. Then;
$P_A(0) = P(0,0) + P(0,1) + P(0,2) + P(0,3) = 0.4$
$P_A(1) = P(1,0) + P(1,1) + P(1,2) + P(1,3) = 0.6$
$P_I(0) = P(0,0) + P(1,0) = 0.2$
$P_I(1) = P(0,1) + P(1,1) = 0.24$
$P_I(2) = P(0,2) + P(1,2) = 0.39$
$P_I(3) = P(0,3) + P(1,3) = 0.17$

f)

We know that random variables X and Y are independent if
$P_{(X,Y)}(x, y) = P_X(x)P_Y(y)$
for all values of x and y (Definition 3.4)
By using this definition, let's check whether the electric outages in Ankara and Istanbul are independent or not, by checking whether their joint pmf factors into a product of marginal pmfs.

Let A define electric outages in Ankara and I define the same for Istanbul.
$P_{A,I}(0,0) = 0.08$ where $P_A(0)P_I(0) = 0.4 * 0.2 = 0.08$
$P_{A,I}(0,1) = 0.13$ where $P_A(0)P_I(1) = 0.4 * 0.24 = 0.096$
$P_{A,I}(0,2) = 0.17$ where $P_A(0)P_I(2) = 0.4 * 0.39 = 0.156$
$P_{A,I}(0,3) = 0.02$ where $P_A(0)P_I(3) = 0.4 * 0.17 = 0.068$
$P_{A,I}(1,0) = 0.12$ where $P_A(1)P_I(0) = 0.6 * 0.2 = 0.12$
$P_{A,I}(1,1) = 0.11$ where $P_A(1)P_I(1) = 0.6 * 0.24 = 0.144$
$P_{A,I}(1,2) = 0.22$ where $P_A(1)P_I(2) = 0.6 * 0.39 = 0.234$
$P_{A,I}(1,3) = 0.15$ where $P_A(1)P_I(3) = 0.6 * 0.17 = 0.102$

As seen, there are some pairs (a,i) violating the formula for independent random variables. As a result, we see that the electric outages in Ankara and Istanbul are dependent.