Student Information

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Answer 1

a)

As it takes between 60 and 180 minutes for a student to complete homework and distribution is uniform, pdf will be defined as:

$$f(x) = \begin{cases} \frac{1}{120}, & \text{if } 60 \leq x \leq 180 \\ 0, & \text{otherwise} \end{cases}$$

where 120 is derived from 180-60, with using definition from Chapter 4.2.1 of the book.

b)

$$E(X) = \frac{60 + 180}{2} = 120$$

$$Var(X) = \frac{(60 - 180)^2}{2} = 7200$$

$$Std(X) = \sqrt{Var(X)} \approx 84.8528$$

Values are calculated by using formulas for uniform distribution, on page 82 of the book.

c)

By using definition 4.1 from book, we can calculate the probability that X is between 90 and 120 minutes as:

$$P\{90 <= X <= 120\} = \int_{90}^{120} f(x)dx = \int_{90}^{120} \frac{1}{120}dx$$

$$= \frac{x}{120}|_{90}^{120} = \frac{120}{120} - \frac{90}{120} = \frac{1}{4}$$

d)

As it is given that the student always takes more than 120 minutes, this becomes conditional probability. Let Sn denote probability of student S to finish homework in *more than* n minutes. Then this conditional probability could be written as:
$P\{S150|S120\}$
By using conditional probability formula (2.7 on page 27) from the book, we know that:

$$P\{S150|S120\} = \frac{P\{S150 \cap S120\}}{P\{S120\}}$$

$P\{S150 \cap S120\} = P\{S150\}$, because if a student finishes the homework in more than 150 minutes, we intuitively know that (s)he finished the homework also more than in 120 minutes (S120 is a subset of S150).
We need to calculate $P\{S120\}$ and $P\{S150\}$, which could be calculated as:

$$P\{S120\} = \int_{120}^{\infty} \frac{1}{120} dx = \int_{120}^{180} \frac{1}{120} dx = \frac{x}{120}|_{120}^{180} = \frac{1}{2}$$

$$P\{S150\} = \int_{150}^{\infty} \frac{1}{150} dx = \int_{150}^{180} \frac{1}{120} dx = \frac{x}{150}|_{150}^{180} = \frac{1}{4}$$

Here, I need to explain 2 points:

- The reason why I replaced ∞ with 180 in both integrals is that whenever $X > 180$, the pdf will become 0 but not $\frac{1}{120}$. Therefore, we do not need to calculate any $X > 180$, as will be 0.
- The reason why we take integral to calculate $P\{Sn\}$ comes from definition of integrals. For continuous functions, we need to calculate area under pdf to find possibility, which results in taking integral. For $P\{Sn\}$, as it says that *finish homework in more than n minutes*, lower bound of the integral becomes n, and as the upper bound is not stated, it becomes ∞.

Hence,

$$P\{S150|S120\} = \frac{P\{S150\}}{P\{S120\}} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Answer 2

If we took this as Binomial distribution, we would say that n = 500, p = 0.02

a)

By using (4.19) from book;
Exp(X) = $\mu = np = 10$,
Std(X) = $\sqrt{np(1-p)} = \sqrt{10 * 0.98} \approx 3.13$

b)

$P\{CP < 8\} = P\{CP < 7.5\} = P\{\frac{CP-10}{3.13} < \frac{7.5-10}{3.13}\} = P\{Z < -0.799\} = \Phi(-0.799) \approx 0.212$

- Note that $P\{CP > n\}$ denotes that the possibility being more than n people supporting CP.
- Note that I used table A4 from book to obtain value of $\Phi(-0.799)$.
- Note that wtih the information that I retrieved from the section *Continuity correction* of the book (pg. 94), I moved the boundary 0.5 units. As CP supporters must be integer, ¡8 will be equivalent with ¡7. but not ¡8.5 as we should not include x=8 situation. This shift is applied also for parts c and d with the same logic.

c)

$P\{CP > 15\} = P\{CP > 15.5\} = P\{\frac{CP-10}{3.13} > \frac{15.5-10}{3.13}\} = P\{Z > 1.757\} = \Phi(1.757) \approx 0.96$

Note that I used table A4 from book to obtain value of $\Phi(1.757)$.

d)

$P\{7 \leq CP \leq 14\} = P\{6.5 \leq CP \leq 14.5\} = P\{\frac{6.5-10}{3.13} \leq \frac{CP-10}{3.13} \leq \frac{14.5-10}{3.13}\} = P\{-1.118 \leq Z \leq 1.438\} \approx \Phi(1.438) - \Phi(-1.118) = 0.9251 - 0.1562 = 0.7689$

Note that I used table A4 from book to obtain values of $\Phi(n)$.

Answer 3

E(X) $= \dfrac{1}{\lambda} = 1$ year
To calculate F(x), I have used formula (4.2) from book (pg. 82)

a)

As we know that a hit to building has occurred today, there will not be a strike within one year means that *a strike will happen again in more than or equal to one year.*

$$P\{T \geq 1\} = F(1) = 1 - e^{-\lambda} = 1 - e^{-1} = 1 - 0.367879 \approx 0.632$$

b)

As building has not been hit in one year and we are asked to calculate probability it won't be hit within one year, we are simply asked to calculate the probability that *a strike will happen at least more than 2 years.*

$$P\{T \geq 2\} = F(2) = 1 - e^{-2*\lambda} = 1 - e^{-2} = 1 - 0.13533 \approx 0.8647$$