1 $1 \mid\mid \sum w_j C_j$

1.1 Alternative Formulation 1

The notation implies that we have a single machine and the objective is to minimize a weighted sum of the completion times.

Parameters:
l = $C_{max} - 1$

Decision variables:
$x_{jt} = 1$ if job j starts at time t, 0 otherwise

Model

$$\begin{aligned}
\min & \sum_{j=1}^{n} \sum_{t=0}^{l} x_{jt} \times (t + p_j) \times w_j \\
\text{s.t.} \quad & \sum_{t=0}^{l} x_{jt} = 1 \quad \forall j \\
& \sum_{j=1}^{n} \sum_{s=max\{0,t-p_j\}}^{max\{0,t-1\}} x_{js} \leq 1 \quad \forall t
\end{aligned} \tag{1}$$

1.2 Alternative Formulation 2

Decision variables:
$x_{jk} = 1$ if job j precedes job k, 0 otherwise

Note that the completion of time of job j is $C_j = \sum_{k=1}^{n} P_k \times x_{kj} + p_j$

Model

$$\begin{aligned}
\min & \sum_{j=1}^{n} [\sum_{k=1}^{n} x_{kj} \times p_j] \times w_j \\
\text{s.t.} \quad & x_{kj} + x_{jk} = 1 \quad \forall (j,k), j \neq k \\
& x_{jj} = 0 \quad \forall j \\
& x_{kj} + x_{lk} + x_{jl} \geq 1 \quad \forall (j,k,l), \quad j \neq k \neq l \\
& x_{jk} \in \{0,1\} \quad \forall (j,k)
\end{aligned} \tag{2}$$

2 Scheduling with custom scenario

2.1 Scenario

1. Multiple jobs with precedence constraints
2. Jobs belong to categories and can be ran together in parallel if they belong to the same category at no additional cost
3. Continuous time
4. Objective is to minimize makespan

2.2 Mathematical Model

Let there be n jobs and c categories. Let a_j be the parameter that denotes the category that job j belongs to. Let p_j be the duration of job j. Let z_{ij} be the binary parameter that denotes whether job i has to come before job j due to precedence constraints. Finally, let d_j be the due date of job j.

Decision variables:

1. C_{max}: makespan
2. s_j: start time of job j
3. e_j: end time of job j
4. y_{ij}: no overlap constraint

Model:

$$
\begin{aligned}
\min \quad & C_{max} \\
\text{s.t.} \quad & C_{max} \geq e_j \quad \forall j \\
& e_j = s_j + p_j \quad \forall j \\
& e_j \leq d_j \quad \forall j \\
& e_i \leq s_j \quad \forall (i,j) \quad s.t. \quad z_{ij} = 1 \\
& e_i \leq s_j + M \times y_{ij} \quad \forall (i,j) \quad s.t. \quad i < j, \quad z_{ij} = 0 \quad a_j = a_i \\
& e_i \leq s_j + M \times (1 - y_{ij}) \quad \forall (i,j) \quad s.t. \quad i < j, \quad z_{ij} = 0, \quad a_j = a_i \\
& y_{ij} \in \{0,1\} \\
& s_j \geq 0 \quad \forall j \\
& e_j \geq 0 \quad \forall j
\end{aligned} \tag{3}
$$

3 Modeling Extensions

3.1 Modeling the number of machine startups

Let x_t be the binary decision variable denoting whether the machine is active or not at time t. Let s_t be the binary decision variable denoting whether the machine is started up at time t. Then the following three inequalities derive s_t from x_t

$$x_t \geq s_t \tag{4}$$

$$1 - x_{t-1} \geq s_t \tag{5}$$

$$x_t - x_{t-1} \leq s_t \tag{6}$$