Problem Formulation for Multi-Objective Task Scheduling in Green Fog Computing

1. Problem Overview

Efficient task scheduling in fog computing involves balancing multiple objectives, including energy consumption, latency, resource utilization, and cost. Tasks with various requirements are allocated to fog nodes with limited resources, considering environmental sustainability.

2. Parameters and Notation

System Parameters

- N_T: Number of tasks.
- N_F: Number of fog nodes.

Decision Variable

- x_{ij}: Binary decision variable indicating whether task i is assigned to fog node j. Defined as:

$$x_{ij} = \begin{cases} 1 & \text{if task } i \text{ is assigned to fog node } j, \\ 0 & \text{otherwise.} \end{cases}$$

Task Attributes

Each task i ($i = 1, 2, \ldots, N_T$) has:

- R_i^{cpu}: CPU demand (cycles/second).
- R_i^{mem}: Memory demand (MB/GB).
- R_i^{bw}: Bandwidth demand (Mbps).
- D_i^{deadline}: Deadline (seconds).
- C_i^{data}: Data size to transfer to the fog node (MB/GB).
- T_i^{priority}: Priority level.

Fog Node Attributes

Each fog node j ($j = 1, 2, \ldots, N_F$) has:

- C_j^{cpu}: Total CPU capacity (cycles/second).
- C_j^{mem}: Total memory capacity (MB/GB).
- C_j^{bw}: Total bandwidth capacity (Mbps).
- P_j^{idle}: Idle power consumption (Watts).
- P_j^{active}: Active power consumption (Watts per unit resource used).
- F_j^{cost}: Per-unit usage cost.

Network Attributes

- B_{ij}: Available bandwidth between task i and fog node j (Mbps).
- L_{ij}^{comm}: Communication latency between task i and fog node j (seconds).

3. Objectives

a) Minimization of Energy Consumption (E)

$$E = \sum_j \left(P_j^{\text{idle}} + \sum_i x_{ij} \cdot R_i^{\text{cpu}} \cdot P_j^{\text{active}} \right).$$

b) Minimization of Task Latency (L)

$$L = \sum_{i,j} x_{ij} \left(L_{ij}^{\text{comm}} + \frac{R_i^{\text{cpu}}}{C_j^{\text{cpu}}} + \text{Queueing delay at } j \right).$$

c) Maximization of Resource Utilization (U)

$$U = \frac{\sum_j \sum_i x_{ij} \left(\frac{R_i^{\text{cpu}}}{C_j^{\text{cpu}}} + \frac{R_i^{\text{mem}}}{C_j^{\text{mem}}} + \frac{R_i^{\text{bw}}}{C_j^{\text{bw}}} \right)}{N_F}.$$

d) Minimization of Cost (C)

$$C = \sum_{i,j} x_{ij} \left(R_i^{\text{cpu}} \cdot F_j^{\text{cost}} + \frac{C_i^{\text{data}}}{B_{ij}} \cdot F_j^{\text{cost}} \right).$$

4. Constraints

a) Task Assignment Constraint

Each task must be assigned to exactly one fog node:

$$\sum_j x_{ij} = 1, \quad \forall i.$$

b) Resource Capacity Constraints

$$\sum_i x_{ij} \cdot R_i^{\text{cpu}} \leq C_j^{\text{cpu}}, \quad \forall j,$$

$$\sum_i x_{ij} \cdot R_i^{\text{mem}} \leq C_j^{\text{mem}}, \quad \forall j,$$

$$\sum_i x_{ij} \cdot R_i^{\text{bw}} \leq C_j^{\text{bw}}, \quad \forall j.$$

c) Deadline Constraints

$$\sum_j x_{ij} \cdot \left(L_{ij}^{\text{comm}} + \frac{R_i^{\text{cpu}}}{C_j^{\text{cpu}}} + \text{Queueing delay at } j \right) \leq D_i^{\text{deadline}}, \quad \forall i.$$

d) Bandwidth Constraints

$$\sum_i x_{ij} \cdot R_i^{\text{bw}} \leq B_{ij}, \quad \forall i, j.$$

5. Optimization Model

$$\text{Minimize: } \alpha_1 E + \alpha_2 L - \alpha_3 U + \alpha_4 C,$$

Subject to:

- Task Assignment Constraint.
- Resource Capacity Constraints.
- Deadline Constraints.
- Bandwidth Constraints.

Here, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the weight coefficients representing the relative importance of the objectives.