1. The Challenge of Complex Systems

The aim of this book is to develop concepts and methods which allow us to deal with complex systems from a unifying point of view. The book is composed of two parts: The introductory chapter deals with complex systems in a qualitative fashion, while the rest of the book is devoted to quantitative methods. In Chap. 1 we shall present examples of complex systems and some typical approaches to dealing with them, for instance thermodynamics and synergetics. We shall discuss the concept of self-organization and, in particular, various aspects of information. The last section of this chapter gives an outline of our new theory, which may be viewed as a *macroscopic* approach to synergetics. In Chap. 2 a brief outline of the *microscopic* approach to synergetics is presented, while Chap. 3 provides the reader with an introduction to the maximum information entropy principle, which will play an important role in our book. Chapter 4 illustrates this principle by applying it to thermodynamics.

The remainder of the book will then be devoted to our quantitative method and its applications; detailed examples from physics and biology will be presented. Finally, it will be shown that an important approach in the field of pattern recognition is contained as a special case in our general theory so that indeed a remarkable unification in science is achieved. Readers who are not so much interested in a qualitative discussion may skip this introductory chapter and proceed directly to Chaps. 2, or 3 and 4, or 5, depending on their knowledge.

But now let us start with some basics.

1.1 What Are Complex Systems?

First of all we have to discuss what we understand by complex systems. In a naive way, we may describe them as systems which are composed of many parts, or elements, or components which may be of the same or of different kinds. The components or parts may be connected in a more or less complicated fashion. The various branches of science offer us numerous examples, some of which turn out to be rather simple whereas others may be called truly complex.

Let us start with some examples in physics. A gas is composed of very many molecules, say of 10^{22} in a cubic centimeter. The gas molecules fly around in quite an irregular fashion, whereby they suffer numerous collisions with each other (Fig.1.1). By contrast, in a crystal the atoms or molecules are well-arranged and undergo only slight vibrations (Fig.1.2). We may be interested in specific properties,

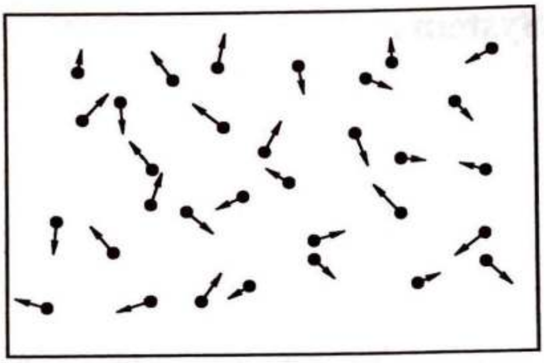


Fig. 1.1. Gas atoms moving in a box

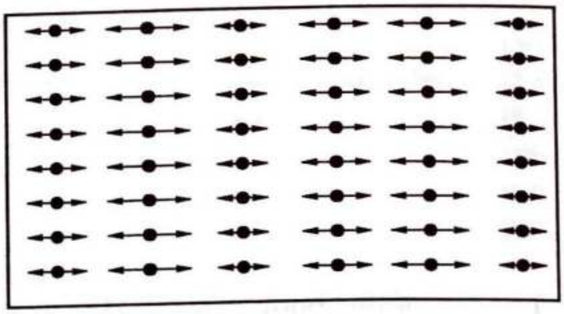


Fig. 1.2. Atoms in a crystal

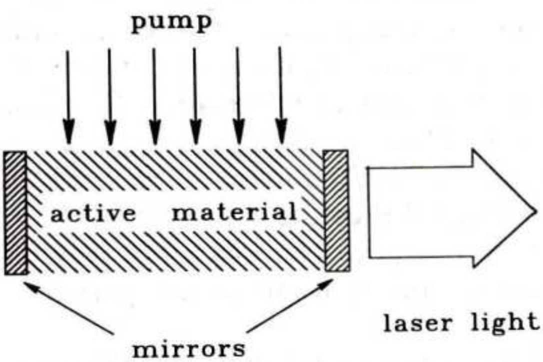


Fig. 1.3. Schematic drawing of a laser

such as the pressure or temperature of a gas or the compressibility of a crystal. Or we may consider these systems with a view to their serving a purpose, e.g. a gas like water vapor may be used in a steam engine, a crystal may be used as a conductor of electricity etc. Some physical systems are primarily devised to serve a purpose, e.g. a laser (Fig.1.3). This new light source is constructed to produce a specific type of light.

In chemistry we are again dealing with complex systems. In chemical reactions, very many molecules participate, and lead to the formation of new molecules. Biology abounds with complex systems. A cell is composed of a complicated cell membrane, a nucleus and cytoplasm, each of which contain many further components (Fig.1.4). In a cell between a dozen and some thousand metabolic processes may go on at the same time in a well-regulated fashion. Organs are composed of many cells which likewise cooperate in a well-regulated fashion. In turn organs serve specific purposes and cooperate within an animal. Animals themselves form animal societies (Fig.1.5). Probably the most complex system in the world is the human brain composed of 10^{10} or more nerve cells (Fig.1.6). Their cooperation allows us to recognize patterns, to speak, or to perform other mental functions.

In the engineering sciences we again have to deal with complex systems. Such systems may be machines, say an engine of an automobile, or whole factories, or power plants forming an interconnected network. Economy with its numerous

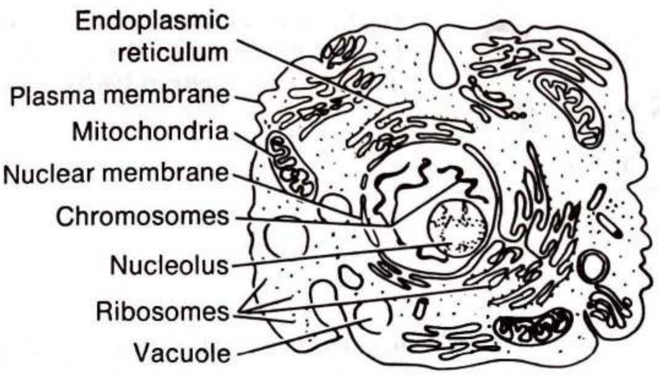
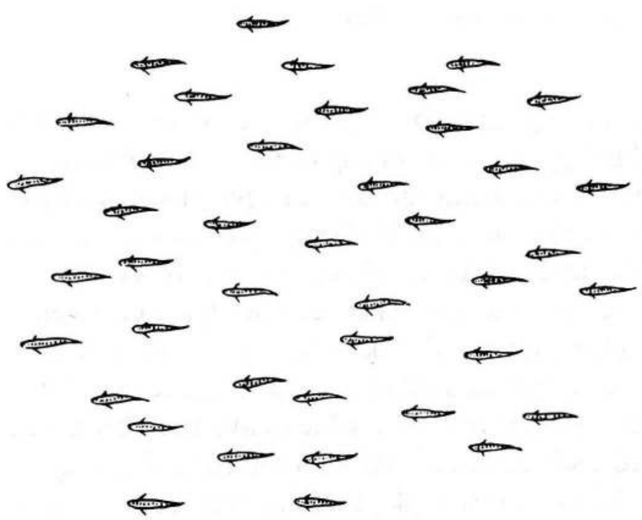


Fig. 1.4. A typical biological cell [from D.A. Anderson, R.J. Sobieski: *Introduction to Microbiology* (C.V. Mosby Company 1980)]



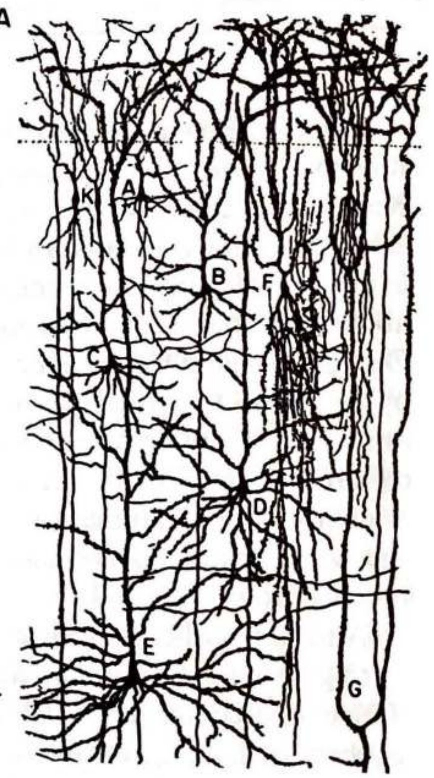


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Fig. 1.5. Shoal of fish [from B.L. Patridge: "Wie Fische zusammenhalten." © Spektrum der Wissenschaft (Aug. 1982)]

Fig. 1.6. Net of nerve cells [from G.C. Quarton, T. Melnechuck, F.O. Schmitt: *The Neuro*-sciences (The Rockefeller University Press, New York 1967)] ▶

participants, its flows of goods and money, its traffic, production, consumption and storage of goods provides us with another example of a complex system. Similarly, society with its various human activities and their political, religious, professional, or cultural habits is a further example of such a system. Computers are more and more conceived as complex systems. This is especially so with respect to computers of the so-called 5th generation, where knowledge processing will be replacing the number crunching of today's computers.

Systems may not only be complex as a result of being composed of so many parts but we may also speak of complex behavior. The various manifestations of human behavior may be very complex as is studied e.g. in psychology. But on the other hand, we also admire the high coordination of muscles in locomotion, breathing etc. (Fig. 1.7). Finally, modern science itself is a complex system as is quite evident from its enormous number of individual branches.

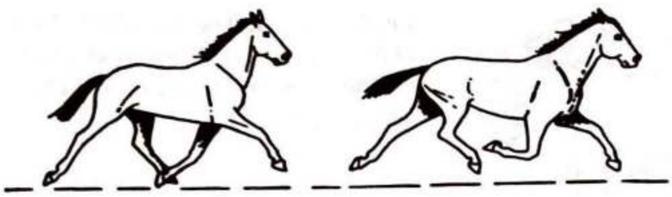


Fig. 1.7. Trotting horse [from E. Kolb: *Lehrbuch der Physiologie der Haustiere* (Fischer-Verlag, Stuttgart 1967)]

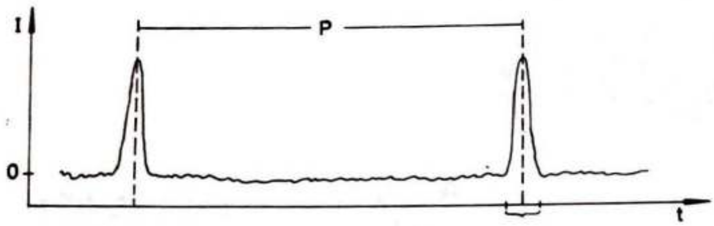


Fig. 1.8. Light pulses from a pulsar [from Weigert, Wendke: *Astronomie und Astrophysik* (Physik-Verlag, Weinheim 1982)]

We may now ask the question of why numerous systems are so complex and how they came into existence. In biology as well as in engineering, for instance, we readily see the need for complexity. These systems serve specific purposes and upon scrutinization we find that these purposes can be fulfilled only by a complex system composed of many parts which interact in a well-regulated fashion. When we talk about their coming into existence, we may distinguish between two types of systems: On the one hand we have man-made systems which have been designed and built by people so that these machines or constructs serve a specific purpose. On the other hand there are the very many systems in nature which have been produced by nature herself, or in other words, which have been self-organized. Here, quite evidently, the evolutionary vision, i.e. Darwinism, plays an important role in biology, where an attempt is made to understand, why and how more and more complex systems evolve.

After this rather superficial and sketchy survey of complex systems, let us now try to give a more rigorous definition. A modern definition is based on the concept of algebraic complexity. At least to some extent, systems can be described by a sequence of data, e.g. the fluctuating intensity of the light from stars (Fig.1.8), or the fever curve of a sick person where the data are represented by numbers. So, let us consider a string of numbers and let us try to define the complexity of such a string. When we think of specific examples, say of numbers like 1, 4, 9, 16, 25, 36, ..., we realize that such a string of data can be produced by a simple law, namely in this case by the law n^2 where n is an integer. Therefore, whenever a string of data is presented, we may ask whether there is a computer program and a set of initial data which then allow us to compute the whole set of data by means of this program. Of course, depending on the construction of the computer, one computer program may be longer than that of another.

Therefore, in order to be able to compare the length of programs, we must introduce a universal computer. Without going into details we may state that such a universal computer can be constructed, at least in a thought experiment as was shown by Turing (Fig.1.9). Therefore, we shall call such universal computer a Turing machine. The idea then is to try to compress the program and the initial set of data to a minimum. The minimum length of a program and of the initial data is a measure

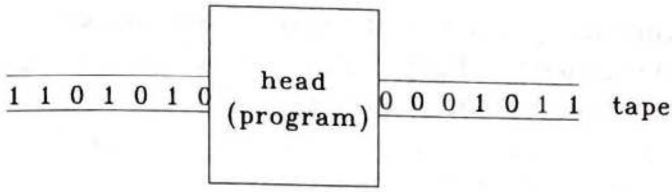


Fig. 1.9. Schematic of a Turing machine

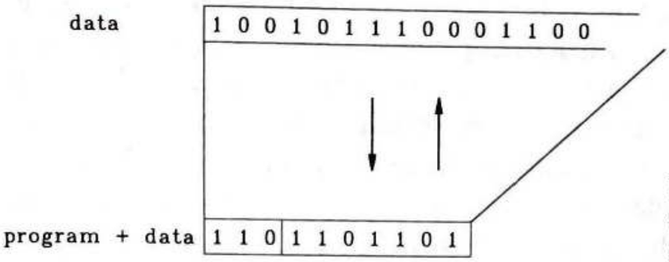


Fig. 1.10. Compression of a string of data into a minimal set of a program and data

of the algebraic degree of complexity (Fig.1.10). However, such a definition has a drawback. As can be shown by means of a famous theorem by Goedel, this problem of finding a minimum program and a minimum number of initial data cannot be solved in a universal fashion. In other words, there is no general algorithm available which could solve this problem. Rather we can develop such algorithms only in special cases. Indeed, occasionally one can construct shortcuts. Let us consider a gas. There, one might attempt to follow up the paths of the individual particles and their collisions and then derive the distribution function of the velocity of the individual particles. This problem has not been solved yet, when one starts from a microscopic description. Nevertheless, it has been possible in statistical mechanics to derive this distribution function, known as the Boltzman distribution, in a rather simple and elegant fashion without invoking the microscopic approach, but using the concept of entropy (see below). A number of similar examples can be formulated which show us that there exist shortcuts by which an originally very complicated problem can be solved in a rather direct fashion. Thus, we realize that the concept of complexity is a very subtle one. Indeed the main purpose of our book will be to provide such shortcuts from a unifying point of view which will then allow us to deal with complex systems.

A complex system may be considered from various points of view. For instance, we may treat a biological system at the macroscopic level by studying its behavior, or at an intermediate level by studying the functioning of its organs, or finally we could study the chemistry of DNA. The data to be collected often seem to be quite inexhaustible. In addition it is often impossible to decide which aspect to choose a priori, and we must instead undergo a learning process in order to know how to cope with a complex system.

1.2 How to Deal with Complex Systems

The more science becomes divided into specialized disciplines, the more important it becomes to find unifying principles. Since complex systems are ubiquitous, we are confronted with the challenge of finding unifying principles for dealing with such systems. In order to describe a complex system at a microscopic level, we need an enormous amount of data which eventually nobody, even not a society, is able to handle. Therefore, we have to introduce some sort of economy of data collecting or of thinking. In addition, we can hope to obtain deep insights when we find laws which can be applied to a variety of quite different complex systems.

When we look for universal laws it is wise to ask at which level we wish to formulate them; be it microscopic or macroscopic. Accordingly we may arrive at a quite different description of a system. For instance at a microscopic level, a gas is entirely disordered, whereas at the macroscopic level it appears to be practically homogeneous, i.e. structureless. In contrast, a crystal is well ordered at the microscopic level, whereas again at the macroscopic level it appears homogeneous. In biology we deal with a hierarchy of levels which range from the molecular level through that of cells and organs to that of the whole plant or animal. This choice of levels may be far too rough, and an appropriate choice of the level is by no means a trivial problem. In addition, "microscopic" and "macroscopic" become relative concepts. For instance a biomolecule may be considered as "macroscopic" as compared to its atomic constituents, whereas it is "microscopic" as compared to a cell. Incidentally, at each level we are confronted with a specific kind of organization or structure.

The method of modern western science can certainly be described as being analytical. By decomposing a system into its parts we try to understand the properties of the whole system. In a number of fields we may start from first principles which are laid down in fundamental laws. The field in which this trend is most pronounced is, of course, physics and in particular elementary particle physics. Usually it is understood that the parts and their properties are "objectively" given and that then one needs "merely" to deduce the properties of the total system from the properties of its parts. Two remarks are in place. First, strictly speaking, we in fact infer microscopic events from macroscopic data, and it is an interesting problem to check whether different microscopic models can lead to the same macroscopic set of data. Second, the analytic approach is based on the concept of reducibility, or in the extreme case on reductionism. But the more we are dealing with complex systems, the more we realize that reductionism has its own limitations. For example, knowing chemistry does not mean that we understand life. In fact, when we proceed from the microscopic to the macroscopic level, many new qualities of a system emerge which are not present at the microscopic level.

For instance, while a wave can be described by a wavelength and an amplitude, these concepts are alien to an individual particle such as an atom. What we need to understand is not the behavior of individual parts but rather their orchestration. In order to understand this orchestration, we may in many cases appeal to model systems in which specific traits of a complex system can be studied in detail. We shall discuss a number of model systems in Sect. 1.4. Another approach to dealing with complex systems is provided by a macroscopic description. For example, we do not describe a gas by listing all the individual coordinates of its atoms at each instant, but rather in terms of macroscopic quantities such as pressure and temperature. It is a remarkable fact that nature herself has provided us with means of measuring or sensing these quantities.

In order to deal with complex systems, we quite often still have to find adequate variables or relevant quantities to describe the properties of these systems. In all cases, a macroscopic description allows an enormous compression of information so that we are no more concerned with the individual microscopic data but rather with global properties. An important step in treating complex systems consists in establishing relations between various macroscopic quantities. These relations are a consequence of microscopic events which, however, are often unknown or only partially known. Examples of such relations are provided by thermodynamics where, for instance, the law relating pressure and temperature in a gas is formulated, and derived by statistical mechanics from microscopic laws. In general we have to guess the nature of the microscopic events which eventually lead to macroscopic data.

In this book we want to show how such guesses can be made for systems belonging to quite different disciplines. At the same time we shall see that at a sufficiently abstract level there exist profound analogies between the behavior of complex systems; or, in other words, complicated behavior can be realized on quite different substrates. Very often we recognize that the more complex a system is, the more it acquires traits of human behavior. Therefore, we are led, or possibly misled, into describing the behavior of complex systems in anthropomorphic terms. In the natural sciences it has become a tradition to try to exorcise anthropomorphisms as far as possible and to base all explanations and concepts on a more or less mechanistic point of view. We shall discuss this dilemma: mechanistic versus anthropomorphic in later sections of this chapter, in particular when we come to discuss information and the role of meaning and purpose.

Let me conclude this section with a general remark. Not so long ago it was more or less generally felt that a great discrepancy exists between, say physics or natural science on the one hand and humanistics on the other, the latter dealing with truly complex behavior and complex systems. Physics was for a long time revered because of its ability to predict events within an unlimited future. As we shall see, the more physics has to deal with complex systems, the more we realize that new concepts are needed. Some of the characteristics which were attributed to physics such as the capability of making precise predictions are losing their hold.

1.3 Model Systems

The great success of physics rests on its methodology. In this, complex systems are decomposed into specific parts whose behavior can be studied in a reproducible fashion, whereby only one or very few parameters are changed. Famous examples of this method are the experiments by Galileo on falling bodies, or Newton's analysis of the motion of the planets by means of considering only a system composed of the sun and one planet. Or in other words, he treated a one-, or at maximum, a two-body problem. This approach gave rise to Newtonian mechanics. From its formulation it was deduced, e.g. by Laplace, that Newtonian mechanics implies total predictability of the future, once the velocity and positions of the individual particles

Fig. 1.11

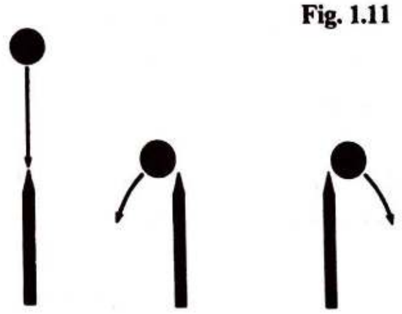


Fig. 1.12

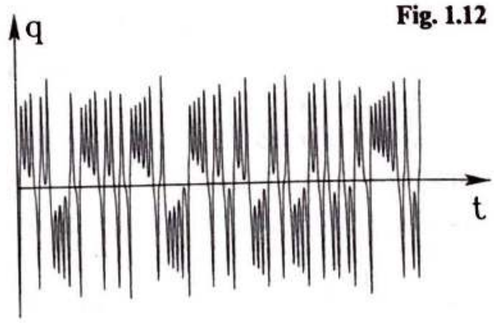


Fig. 1.11. Steel ball falling on a razor blade. Depending on its initial position, the ball is deflected along a wide trajectory to the left or to the right

Fig. 1.12. Time variation of a quantity in a chaotic system

of a system are known at an initial time. The concept of predictability has been shaken twice in modern physics. Quantum mechanics tells us that we are not able to measure the velocity and the position of a particle at the same time both with infinite precision and, therefore, that we are not able to make accurate predictions of the future path of a particle.

More recently, the theory of so-called deterministic chaos has shown that even in classical mechanics predictability cannot be guaranteed with absolute precision. Consider the following very simple example of a steel ball falling on a vertical razor blade (Fig.1.11). Depending on its precise position with respect to the razor blade, its trajectory may be deflected to the left or to the right. That means the future path of the particle, i.e. the steel ball, depends in a very sensitive fashion on the initial condition. A very small change of that condition may lead to quite a different path. Over the past years numerous examples in physics, chemistry, and biology have been found where such a sensitivity to initial conditions is present (Fig.1.12). But in spite of these remarks the general idea of finding suitable model systems for a complex system is still valid.

Here we wish to list just a few well-known examples of model systems. The light source laser has become a paradigm for the self-organization of coherent processes because in the laser the atoms interact in a well-regulated fashion so to produce the coherent laser wave (Fig.1.13). Another example for the self-organized formation of macroscopic structures is provided by fluids. For instance, when a fluid is heated from below, it may show specific spatial patterns such as vortices or honeycombs (Fig.1.14). Or when a fluid is heated more, it may show spatio-temporal patterns, e.g. oscillations of vortices. Chemical reactions may give rise to macroscopic patterns, e.g. chemical oscillations where a change of colour occurs periodically, for instance from red to blue to red etc. Other phenomena are spiral patterns or concentric waves (Fig.1.15). In biology the clear water animal hydra has become a model system for morphogenesis. When a hydra is cut into two parts, a new head is readily formed where there was only a foot left and vice versa, a foot is formed where only a head was left (Fig.1.16).

Detailed experiments may allow us to draw conclusions on the mechanism of this restoration on the basis of the concept of chemical fields which are formed by

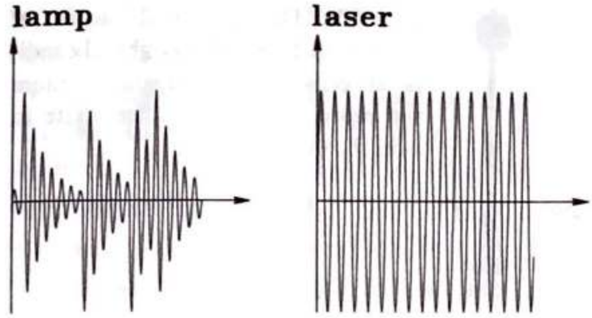


Fig. 1.13. The basic difference between the light from a lamp and from a laser. In both cases the electric field strength of the field amplitude is plotted versus time. On the left hand side the light from a lamp consists of individual uncorrelated wave tracks. On the right hand side in the laser the light wave consists of a single practically infinitely long sinusoidal wave

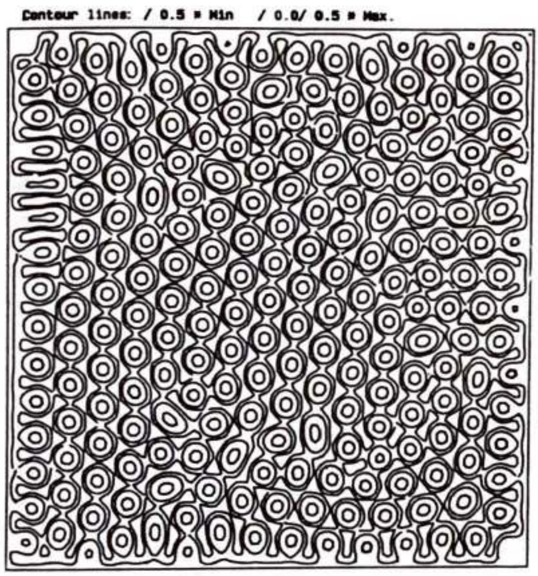


Fig. 1.14. A pattern in fluid dynamics

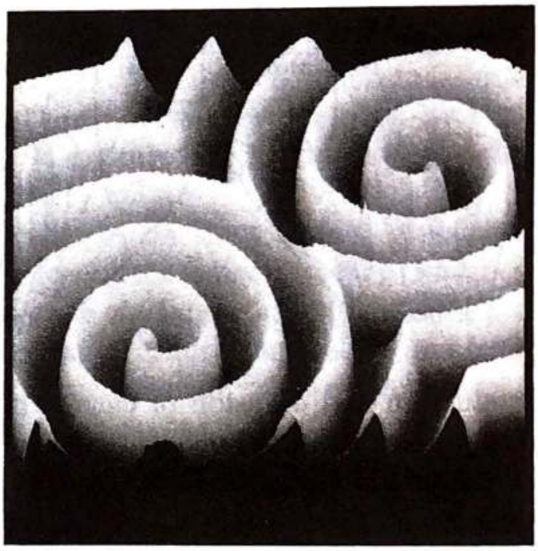


Fig. 1.15. Spiral waves in the Belousov-Zhabotinsky reaction [from S.C. Müller, T. Plesser, B. Hess (unpublished)]

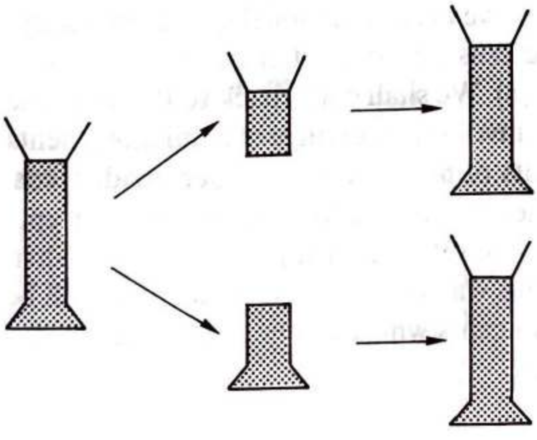


Fig. 1.16. An experiment on hydra reveals that in this species the information on the differentiation of cells cannot be laid down in the genes. From left to right: Intact hydra with head and tail is cut in the middle into two pieces. After a while the upper part regenerates by forming a tail, the lower part regenerates by forming a head

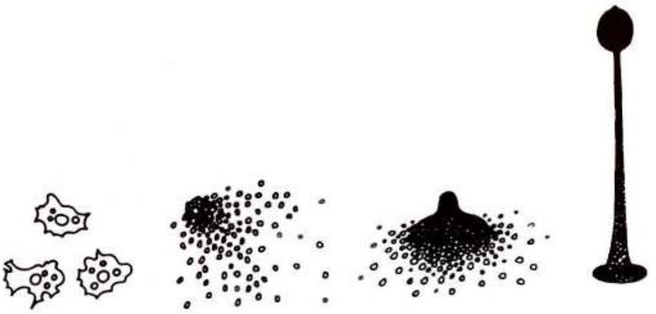


Fig. 1.17. Developmental stages of slime mold. From left to right: The individual cells assemble, aggregate more and more, and finally differentiate to form the mushroom

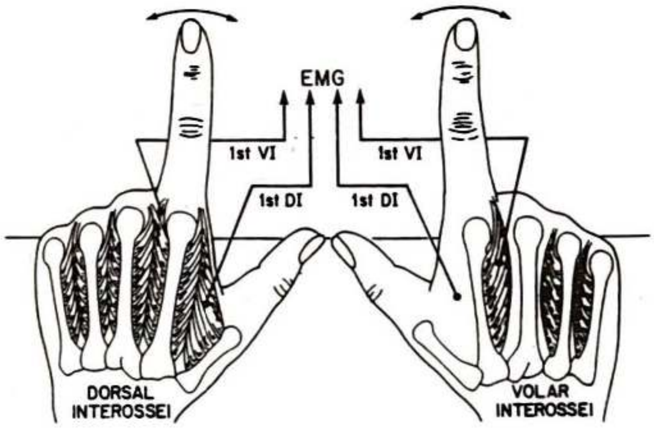


Fig. 1.18. Experimental setup to study involuntary changes of hand movements [from J.A.S. Kelso: "Dynamic Patterns in Complex, Biological Systems: Experiment and Synergetic Theory" (preprint)]

production and diffusion of chemicals. Another example of self-organization in morphogenesis is provided by slime mold (Fig.1.17). This little fungus usually exists in form of individual cells which live on a substrate. But then within the lifecycle of slime mold, these individual cells assemble at a point, differentiate and form the mushroom which then eventually spreads its spores, whereupon the life cycle starts again. Another model system is the squid axon used to study nerve conduction, or the well-known example of Drosophila in genetics where the giant chromosomes, the rapid multiplication rate and the possibility of causing mutations make this little animal an ideal object of study in this field.

More recently human hand movements have become a model system for studying the coordination of muscles and nerve cells and in particular the transitions between various kinds of movement (Fig.1.18). We shall come back to this example and to other examples later in the book. The involuntary change of hand movements is strongly reminiscent of the change of gaits of horses, cats or other quadrupeds. Quite generally speaking, these model systems allow us to develop new concepts which can first be checked against a variety of relatively simple systems and then later applied to truly complex systems. In this way our subsequent chapters will be devoted to the development of such new concepts whose applicability will then be illustrated by a number of explicit examples.

1.4 Self-Organization

As mentioned before, we may distinguish between man-made and self-organized systems. In our book we shall be concerned with self-organized systems. It may be mentioned, however, that the distinction between these two kinds of systems is not completely sharp. For instance humans may construct systems in such a way that by building in adequate constraints the system will be enabled to find its specific function in a self-organized fashion. A typical example mentioned before is the laser where the specific set-up of the laser by means of its mirrors allows the atoms to produce a specific kind of light. Quite evidently in the long run it will be desirable to construct computers which do programming in a self-organized fashion. = AI/OL

For what follows it will be useful to have a suitable definition of self-organization at hand. We shall say that a system is self-organizing if it acquires a spatial, temporal or functional structure without specific interference from the outside. By "specific" we mean that the structure or functioning is not impressed on the system, but that the system is acted upon from the outside in an nonspecific fashion. For instance the fluid which forms hexagons is heated from below in an entirely uniform fashion, and it acquires its specific structure by self-organization. In our book we shall mainly be concerned with a particular kind of self-organization, namely so-called nonequilibrium phase transitions.

As we know, systems in thermal equilibrium can show certain transitions between their states when we change a parameter, e.g. the temperature. For instance, when we heat ice it will melt and form a new state of a liquid, namely water. When we heat water up more and more, it will boil at a certain temperature and form vapor. Thus, the same microscopic elements, namely the individual molecules, may give rise to quite different macroscopic states which change abruptly from one state to another. At the same time new qualities emerge, for example ice has quite different mechanical properties to those of a gas.

In the following we shall be concerned with similar changes in the state of systems far from thermal equilibrium. Examples have been provided in Sect. 1.4, for instance by the liquid which forms a particular spatial pattern, by the laser which emits a coherent light wave, or by biological tissues which undergo a transition towards a differentiation leading to the formation of specialized organs.

1.5 Aiming at Universality

1.5.1 Thermodynamics

Thermodynamics is a field which allows us to deal with arbitrarily complex systems from a universal point of view. For instance we may ascribe temperature to a stone, to a car, to a painting, or to an animal. We further know that important properties of systems change when we change their temperature. Just think of melting of ice at the melting temperature, or of the importance of measuring the temperature of an ill person. However, this example illustrates at the same time that temperature alone is certainly not sufficient to characterize a car or a painting in many other respects. A stone, a car, a dress and a painting have the properties of being in thermal equilibrium. Such a state is reached when we leave a system entirely on its own, or when we couple it to another system which is in thermal equilibrium at a specific temperature.

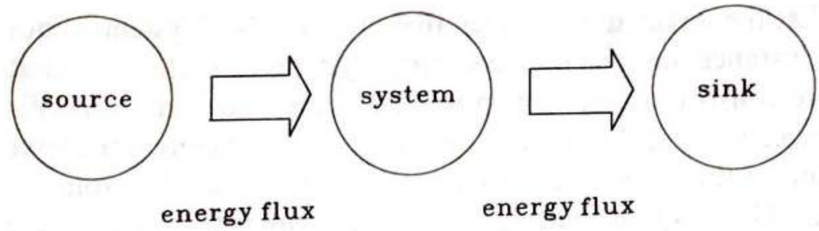


Fig. 1.19. Scheme of an open system which receives its energy from a source and dissipates the rest of the energy into a sink

Another important and even central concept of thermodynamics is that of entropy. Entropy is a concept which refers to systems in thermal equilibrium which can be characterized by a temperature T. The change of entropy is then given by the well-known formula $dS = dQ_{\text{rev}}/T$. Here, T is the absolute temperature and dQ_{rev} is the amount of heat which is reversibly added to or removed from the system. The general laws of thermodynamics are:

1) The first law which states that in a closed system energy is conserved, whereby energy may acquire various forms, such as the internal energy, work being done, or heat. So a typical form relating the changes dU, dA, dQ to one another reads

$$dU = dQ - dA \ . \tag{1.1}$$

2) The second law tells us that in a closed system entropy can never decrease, but can only increase until it reaches its maximum. As we shall see later, the conservation laws, e.g. for energy, together with the so-called maximum entropy principle, allow us to derive certain microscopic properties of a system from macroscopic data. For instance we may derive the velocity distribution function of a gas in a straight forward manner. In the present book we shall be practically exclusively concerned with *open systems* (Fig.1.19). These are systems which are maintained in their specific states by a continuous influx of energy and/or matter. As we shall see, traditional thermodynamics is not adequate for coping with these systems; instead we have to develop some new kind of thermodynamics which will be explained in detail in the following chapters.

Thermodynamics can be considered as a macroscopic phenomenological theory. Its foundations lie in statistical physics upon which we shall make a few comments in the next section.

1.5.2 Statistical Physics

In this field an attempt is made, in particular, to derive the phenomenological, macroscopic laws of thermodynamics by means of a microscopic theory. Such a microscopic theory may be provided by the Newtonian mechanics of the individual gas particles, or by quantum mechanics. By use of appropriate statistical averages the macroscopic quantities are then derived from the microscopic laws. A central concept is again entropy, S. According to Boltzmann, it is related to the number W

of the different microscopic states which give rise to the *same macroscopic* state of the system, by means of the law

$$S = k \ln W \tag{1.2}$$

where k is Boltzmann's constant. A crucial and not yet entirely solved problem is that of explaining why macroscopic phenomena may be irreversible while all fundamental laws are reversible. For instance the laws of Newtonian mechanics are invariant under the reversal of time, i.e. when we let a movie run backwards, all the processes shown there in reverse sequence are allowed in Newtonian mechanics. On the other hand it is quite evident that in macroscopic physics processes are irreversible. For instance when we have a gas container filled with gas molecules and we open a valve the gas will go to a second vessel and fill both vessels more or less homogeneously. The reverse process, i.e. that one vessel is emptied spontaneously and all the molecules return to the original vessel is never observed in nature.

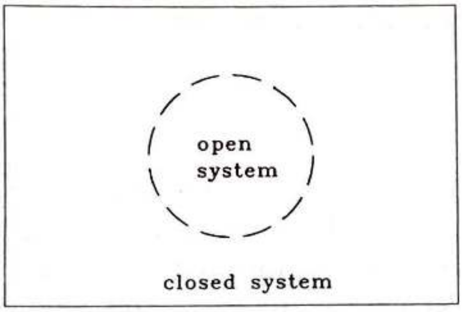
Despite the difficulty in rigorously deriving irreversibility, by means of statistical physics we can explain a number of the phenomena of irreversible thermodynamics, such as relaxation processes, heat conduction, diffusion of molecules, etc.

1.5.3 Synergetics

The third approach to formulating universal laws valid for complex systems is that of synergetics. In this field we study systems that can form spatial, temporal or functional structures by means of self-organization. In physics, synergetics deals with systems far from thermal equilibrium. Typical examples are fluids heated from below, or lasers. Systems from chemistry and biology can also be conceived as physical systems and can be treated again by synergetics. But synergetics deals also with other systems, such as those in economy or sociology. In synergetics we focus our attention on qualitative, macroscopic changes, whereby new structures or new functions occur. This restriction to qualitative, macroscopic changes is the price to be paid in order to find general principles.

We shall remind the reader of the main principles of synergetics in Chap. 2. There we shall see that in physics, synergetics starts from a microscopic formulation, for example from the microscopic equations of motion. In other cases such as biology or chemistry a mesoscopic approach may be appropriate where we start from suitable subsystems, for instance adequate properties of a total cell in biology. It is assumed that the system under consideration is subject to external constraints, such as a specific amount of energy being fed into the system. Then when this control parameter is changed, an instability may occur in which the system tends to a new state.

As is shown in synergetics, at such an instability point, in general just a few collective modes become unstable and serve as "order parameters" which describe the macroscopic pattern. At the same time these macroscopic variables, i.e. the order parameters, govern the behavior of the microscopic parts by the "slaving principle". In this way the occurrence of order parameters and their ability to enslave allows the system to find its own structure. When control parameters are changed over a



◀ **Fig. 1.20.** An open system embedded in a closed system

Fig. 1.21

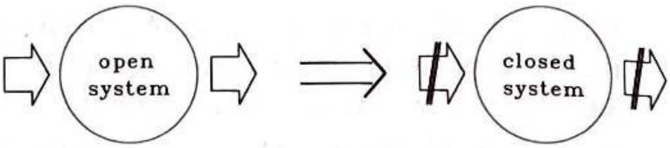


Fig. 1.21. A closed system may be considered as the limiting case of an open system into and out of which energy fluxes are cut

wide range, systems may run through a hierarchy of instabilities and accompanying structures.

Synergetics is very much an open-ended field in which we have made only the very first steps. In the past one or two decades it has been shown that the behavior of numerous systems is governed by the general laws of synergetics, and I am convinced that many more examples will be found in the future. On the other hand we must be aware of the possibility that still more laws and possibly still more general laws can be found.

As mentioned earlier, thermodynamics deals with systems in thermal equilibrium, whereas synergetics deals with systems far away from thermal equilibrium. But here quite a peculiar situation arises. On the one hand we can always embed an open system into a larger closed one (Fig.1.20). Earth for example, is an open system because it is fed with energy from the sun and it emits its energy during night into the universe. But taking the sun and, say, part of the universe as a whole system, we may consider the whole system as a closed one to which the laws of thermodynamics apply. In so far we see that the laws of synergetics must not be in contradiction to those of thermodynamics. But on the other hand any open system may be considered in the limiting case, where the energy or matter fluxes tend to zero so that eventually we deal with a closed system (Fig.1.21). Therefore, the general laws of thermodynamics must be obtainable as limiting cases of those of synergetics.

As the reader will notice this program is not yet finished but leaves space for future research. Until now, synergetics has started from the microscopic or mesoscopic level. In this book however, we shall attempt to present a second foundation of synergetics which we shall discuss in some detail in Chaps. 5–7 and then in greater detail in the following chapters. The starting point for this macroscopic approach is the concept of information and we shall deal with some of its most important aspects in the next section.

1.6 Information

The use of the word information is connected with considerable confusion. This is caused by the fact that the word information is used with many quite different meanings. In every day language, information is used in the sense of message or instruction. A letter, a television transmission or a telephone call all carry information. In the following we shall be concerned with the scientific use of the word information. We shall start with the concept of Shannon information where information is used without any meaning. Then we shall briefly study information with respect to messages and finally we shall be concerned with the problem of the self-creation of meaning.

Quite evidently, when dealing with physical systems, we wish to eliminate all kinds of anthropomorphisms because we wish to describe a physical system in an as objective manner as possible. But in biology too, this trend is quite obvious so that eventually we have a more or less physical or even mechanistic picture of biological systems. But strangely enough it appears with respect to the development of modern computers, e.g. those of the fifth generation, that we wish to reintroduce meaning, relevance, etc. Therefore, in this section we wish to discuss ways in which we can return from a concept of information from which meaning was exorcised to the act of self-creation of meaning.

1.6.1 Shannon Information: Meaning Exorcised

We shall discuss the concept of Shannon information in detail in Chap. 3, but in order to have a sound basis for our present discussion we shall elucidate the concept of Shannon information by means of some examples. When we toss a coin we have two possible outcomes. Or when we throw a die we have six possible outcomes. In the case of the coin we have two kinds of information, head or tail; in the case of the die we have the information that one of the numbers from one to six has appeared. Similarly, we may have answers "yes" or "no", etc. The concept of Shannon information refers simply to the number of possibilities, Z, which in the case of a coin are two, in the case of a die are six. As we shall see later, a proper measure for information is not the number Z itself but rather its logarithm where usually the logarithm to base 2 is taken, i.e. information is defined by

$$I = \log_2 Z \ . \tag{1.3}$$

This definition can be cast into another form which we shall come across time and again in this book. Consider for example a language such as English. We may label its letters $a, b, c, \ldots$ by the numbers $j = 1, 2, \ldots$ i.e. $a \to 1, b \to 2$ etc. Then we may count the frequencies N_j of occurrence of these letters in a particular book or in a library perhaps. We define the relative frequency of a letter labeled by j as

$$p_j = \frac{N_j}{N} \tag{1.4}$$

where N is the total number of letters counted, $N = \sum N_j$. Then the average information per letter contained in that book (or library) is given by

$$i = -\sum_j p_j \log_2 p_j \ . \tag{1.5}$$

For a derivation of this formula see Chap. 3. Shannon used his concept to study the capacity of a communication channel to transfer information even under the impact of noise. Two features of the Shannon information are of importance in what follows. 1) Shannon information is not related to any meaning. So concepts such as meaningful or meaningless, purposeful etc. are not present. 2) Shannon information refers to closed systems. There is only a fixed reservoir of messages, whose number is Z.

1.6.2 Effects of Information

In this section we wish to introduce a new approach which is a step towards a concept of information which includes semantics. We are led to the basic idea by the observation that we can only attribute a meaning to a message if the response of the receiver is taken into account. In this way we are led to the concept of "relative importance" of messages which we want to demonstrate in the following.

Let us consider a set of messages each of which is specified by a string of numbers. The central problem consists in modeling the receiver. We do this by invoking modern concepts of dynamic systems theory or, still more generally, by concepts of synergetics. We model the receiver as a dynamic system. Though we shall describe such systems mathematically in the next chapter, for our present purpose a few general remarks will suffice. We consider a system, e.g. a gas, a biological cell, or an economy, whose states can be characterized at the microscopic, mesoscopic or macroscopic level by a set of quantities, q, which we shall label by an index j, i.e. q_j. In the course of time, the q_j's may change. We may lump the q_j's together into a state vector $\boldsymbol{q}(t) = [q_1(t), q_2(t), \ldots, q_N(t)]$. The time evolution of $\boldsymbol{q}$, i.e. the dynamics of the system, is then determined by differential equations of the form

$$\frac{d\boldsymbol{q}}{dt} = \boldsymbol{N}(\boldsymbol{q}, \alpha) + \boldsymbol{F}(t) \tag{1.6}$$

where $\boldsymbol{N}$ is the deterministic part and $\boldsymbol{F}$ represents fluctuating forces. All we need to know, for the moment, is the following: If there are no fluctuating forces, once the value of $\boldsymbol{q}$ at an initial time is set, and the so-called control parameters α are fixed, then the future course of $\boldsymbol{q}$ is determined uniquely. In the course of time, $\boldsymbol{q}$ will approach an attractor. To visualize a simple example of such an attractor consider a miniature landscape with hills and valleys modeled by paper (Fig.1.22,23). Fixing α means a specific choice of the landscape, in which a ball may slide under the action of gravity (and under a frictional force). Fixing $\boldsymbol{q}$ at an initial time means placing the ball initially at a specific position, for instance on the slope of a hill (Fig.1.22). From there it will slide down until it arrives at the bottom of the valley: this is then an attractor. As the experts know, dynamic systems may possess also other kinds of attractors, e.g. limit cycles, where the system performs an indefinite oscillation, or still more complicated are attractors such as "chaotic attractors". In the following, it will be sufficient to visualize our concepts by considering the attractor as the bottom of a valley (a so-called fixed point). When fluctuations $\boldsymbol{F}$ are present, the ball may jump from one attractor to another (Fig.1.24).

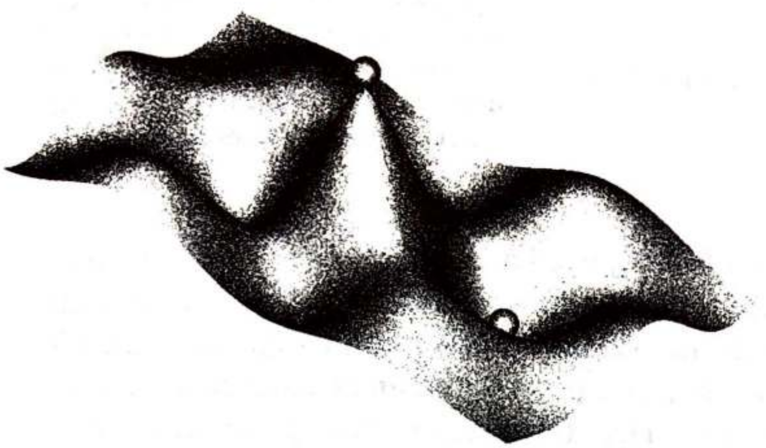


Fig. 1.22. Visualization of a dynamical system with fixed point attractors by means of a miniature landscape formed of deformed paper

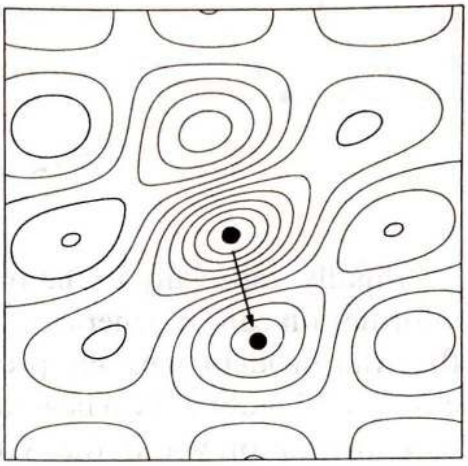


Fig. 1.23. Isobases belonging to the landscape of Fig. 1.22

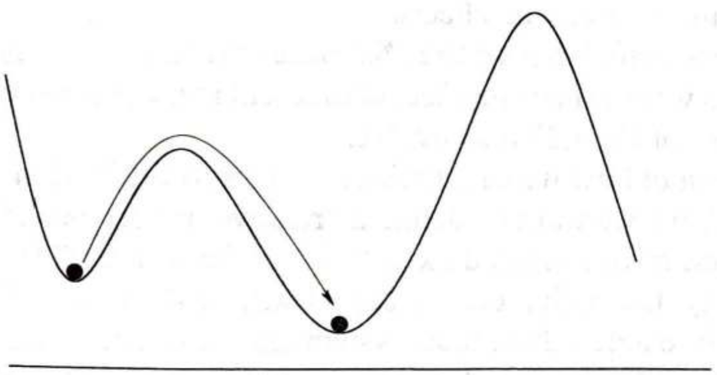


Fig. 1.24. Illustration of the jumping of a system between two fixed point attractors

After these preparatory remarks let us return to our original problem, namely to attribute a meaning to a message. We assume that the receipt of a message by the system means that the parameters α and the initial value of $\boldsymbol{q}$ are set by the message. For the time being we shall assume that these parameters are then uniquely fixed. An extension of the theory to an incomplete message is straightforward (see below). We first ignore the role of fluctuations. We assume that before the message arrives the system has been in an attractor which we shall call the neutral state. The attractor may be a resting state i.e. a fixed point, but it could equally well be a limit cycle, a torus or a strange attractor, or a type of attractor still to be discovered by dynamic systems theory. We shall call this attractor $\boldsymbol{q}_0$. After the message has been received and the parameters α and the initial value $\boldsymbol{q}$ are newly set, in principle two things may happen. Let us assume that we are allowed to wait for a certain measuring time so that the dynamic system can be assumed to be in one of its possible attractors. Then either the message has left the system in the $\boldsymbol{q}_0$ state. In such a case the message is evidently useless or meaningless.

The other case is that the system goes into a new attractor. We first assume that this attractor is uniquely determined by the incident message. Clearly, different messages can give rise to the same attractor. In this case we will speak of redundancy of the messages.

Fig. 1.25. A message can reach two different attractors by means of internal fluctuations of the system by a process depicted in Fig.1.24. In this way two attractors become accessible

Finally, especially in the realm of biology it has been a puzzle until now how information can be generated. This can be easily visualized, however, if we assume that the incident message produces the situation depicted in Fig.1.25, which is clearly ambiguous. Two new stable points (or attractors) can be realized depending on a fluctuation within the system itself. Here the incident message contains information in the ordinary sense of the word, which is ambiguous and the ambiguity is resolved by a fluctuation of the system. Loosely speaking, the original information is doubled because now two attractors become available. In the case of biology these fluctuations are realized in the form of mutations. In the realm of physics however, we would speak of symmetry breaking effects.

Taking all these different processes together we may list the elementary schemes shown in Fig.1.26. Of course, when we consider the effect of different messages, more complicated schemes such as those of Fig.1.27 may evolve.

We shall now treat the question of how we can attribute values to the incident messages or, more precisely speaking, we want to define a "relative importance of the messages". To this end we first have to introduce a "relative importance" for the individual attractors. In reality, the individual attractors will be the origin of new messages which are then put into a new dynamical system and we can continue this process ad infinitum. However, for practical purposes, we have to cut the hierarchical sequence at a certain level and at this level we have to attribute values of the relative importance to the individual attractors. Since our procedure can already be clearly demonstrated if we have a one-step process, let us consider this process in detail.

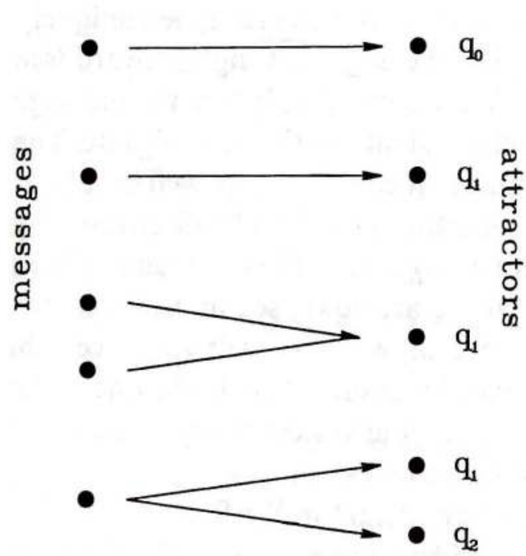


Fig. 1.26. Various possibilities for a message to reach attractors

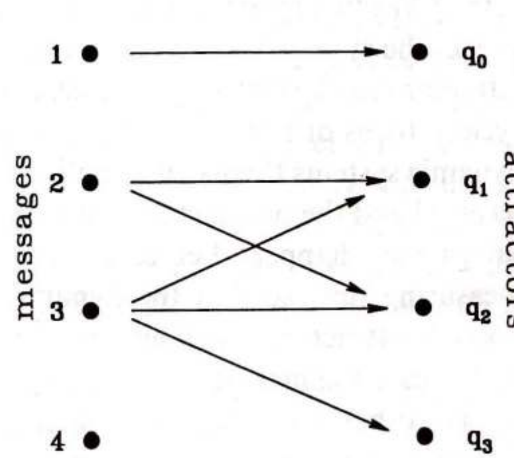


Fig. 1.27. Another example of how messages can reach attractors

Let us attribute a "relative importance" to the individual attractors where the attractor 0 with q_0 has the value 0, while the other attractors may have values $0 \leq p_j' \leq 1$, which we normalize to

$$\sum_j p_j' = 1 \ . \tag{1.7}$$

The assignment of p_j' depends on the task that the dynamic system has to perform. We may think of a specific task which can be performed just by a single attractor or we may think of an ensemble of tasks whose execution is of a given relative importance. Clearly the relative importance of the messages p_j does not only depend on the dynamic system but also on the tasks it must perform. The question is now: What are the values p_j of the incident messages? To this end we consider the links between a message and the attractor into which the dynamical system is driven after receipt of this message. If an attractor k (including the 0 attractor) is reached after receipt of the message j we attribute to this process the matrix element $M_{jk} = 1$ (or $= 0$). If we allow for internal fluctuations of a system, a single message can drive the system via fluctuations into several different attractors which may occur with branching rates M_{jk} with $\sum_k M_{jk} = 1$. We define the "relative importance" p_j by

$$p_j = \sum_k L_{jk} p_k' = \sum_k \frac{M_{jk}}{\sum_{j'} M_{j'k} + \varepsilon} p_k' \ , \tag{1.8}$$

where we let $\varepsilon \to 0$. (This is to ensure that the ratio remains determined even if the denominator and nominator vanish simultaneously.) We first assume that for any $p_k' \neq 0$ at least one $M_{jk} \neq 0$. One readily convinces oneself that p_j is normalized which can be shown by the steps

$$\sum_j p_j = \sum_{kj} \frac{M_{jk}}{\sum_{j'} M_{j'k} + \varepsilon} p_k' \tag{1.9}$$

$$= \sum_k \left(\sum_j \frac{M_{jk}}{\sum_{j'} M_{j'k} + \varepsilon} \right) p_k' \tag{1.10}$$

$$= \sum_k p_k' = 1 \tag{1.11}$$

where the bracket in (1.10) is equal to 1.

Now consider the case where for some k-values, for which $p_k' \neq 0$, all $M_{jk} = 0$. In this case in the sums over k in (1.9) and (1.10) some coefficients of $p_k' \neq 0$ vanish and, since $\sum_k p_k' = 1$, we obtain $\sum_j p_j < 1$. If this inequality holds, we shall speak of an *information deficiency*.

In a more abstract way we may adopt the left hand side of (1.8) as a basic definition where we assume

$$\sum_j L_{jk} \leq 1 \ , \tag{1.12}$$

where the equality sign holds only when there is no information deficiency.