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The physics of financial networks

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Abstract | As the total value of the global financial market outgrew the value of the real economy, financial institutions created a global web of interactions that embodies systemic risks. Understanding these networks requires new theoretical approaches and new tools for quantitative analysis. Statistical physics contributed significantly to this challenge by developing new metrics and models for the study of financial network structure, dynamics, and stability and instability. In this Review, we introduce network representations originating from different financial relationships, including direct interactions such as loans, similarities such as co-ownership and higher-order relations such as contracts involving several parties (for example, credit default swaps) or multilayer connections (possibly extending to the real economy). We then review models of financial contagion capturing the diffusion and impact of shocks across each of these systems. We also discuss different notions of 'equilibrium' in economics and statistical physics, and how they lead to maximum entropy ensembles of graphs, providing tools for financial network inference and the identification of early-warning signals of system-wide instabilities.

Statistical physics — the mathematical study of the relationship between microscopic and macroscopic properties of physical systems composed by many parts — has found a role in recent decades in investigating such relationships in social and economic systems[1–5]. This Review focuses on the study of complex networks and their applications to economics and finance[6–12]. Such studies are inherently interdisciplinary, positioned at the frontier of graph theory, statistical physics of networks and financial economics. The units of observation (that is, financial actors) belong, by their nature, to the domain of economics and finance, as do areas of applications such as the analysis of corporate influence and systemic risk.

In the field of financial networks, the application of physics to social systems has successfully led to results and impact[13–19]. Statistical tools and analytical models have helped to characterize financial risk by accounting for the complexity and the interconnectedness of the financial system. The key contributions to this endeavour are demonstrated by the adoption of concepts and metrics by practitioners and policymakers in the financial sector[20–22] and by scholars in the economics profession[23–25]. The aim of this Review is to present the main research questions and results, and the future avenues of research in this field.

Modelling the financial system as a network is a precondition to understanding and managing a wide range of phenomena that are relevant not just to finance professionals or economists but also to researchers in many other disciplines, as well as ordinary citizens, public agencies and governments[26,27]. This view is widely recognized today and reflected in the policy actions and discourse of financial authorities in both the USA[28] and the EU[29]. Indeed, network effects of various kinds had a key role in the 2007–2008 financial crisis[30], the impact of which persists after more than a decade[31].

The discipline of financial networks has filled a scientific gap by showing how many important phenomena in the financial system can be understood in terms of the interactions between financial actors. For example, if the price of a certain asset plummets, it affects not only those actors who have invested in that asset but also those who have invested in the obligations of those actors. Because of the existence of intricate chains of contracts and feedback mechanisms, the resulting effects can be much larger than the initial shocks. As in other domains of complex systems, the emergence of system-level instabilities can only be understood from the interplay of the network structure (for instance, closed chains) and key properties of links and nodes (such as properties pertaining to risk propagation and financial leverage)[32,33] (see BOX 1). Traditional economic models have described the financial system either as an aggregate entity or as a collection of actors in isolation, failing to provide an appropriate description of these mechanisms and their implications for society[34].

In this Review, we first define several types of financial networks (arising from direct, indirect or

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Key points

- Modelling the financial system as a network is crucial to capture the complex interactions between financial institutions.
- Such a network is naturally a time-dependent multiplex, because relationships between financial institutions are of many different kinds and keep changing.
- Models of financial contagion enable understanding of how shocks propagate from one financial institution to another.
- Missing information on financial networks can be 'reconstructed' using maximum entropy approaches borrowed from statistical mechanics.
- Techniques based on financial networks have been adopted by practitioners and policy institutions.

Bipartite networks
Networks in which nodes are of two different types, say A and B (for instance, companies and directors), and links exist only between nodes of different type (for instance, a director being connected to a company if they sit on the board of that company).

Multiplex networks
Collections of networks (also called layers) with the same set of nodes, but with different links. In this context, multiplex networks are used to represent different kinds of linkages between financial institutions.

Assets
Items on the balance sheet of an institution that have a positive economic value because they generate present or future income.

Small-world
A network structure characterized by a large clustering coefficient and a small average shortest path length.

higher-order interactions) and characterize the structure of static snapshots of each of these networks. We then review dynamic processes taking place on financial networks, focusing on financial contagion, first along bilateral links in unipartite networks, then through common neighbouring nodes on bipartite networks and lastly on multiplex networks. Next, we review the stream of works constructing statistical ensembles of financial networks compatible with the observed data. This construction entails a notion of empirically testable thermodynamic equilibrium for financial networks that we discuss in relation to the notion of economic equilibrium. Finally, although the study of complex systems across many fields has benefited from the availability of rich datasets, disaggregated data on financial networks are often not available owing to confidentiality issues. We discuss how statistical network ensembles allow to tackle this problem by estimating the structure of financial networks from partial information and identify early-warning signals of instabilities through changes in network structure.

Network structure

The financial system can be viewed as a set of interrelated economic agents, such as retail and investment banks, insurance companies, investment funds, central banks and supervisors, fintech companies, non-financial firms and households. Relationships between those agents are often formalized by contracts, such as loans (for instance, between two banks, or from a bank to a firm, or from a bank to a household), reciprocal ownerships or insurance policies. But relationships can be also implicit, such as investments in common assets. It is, therefore, natural to represent the financial system as a network in which nodes represent economic agents and edges represent the relationships between them. Between a pair of agents, there are typically several kinds of relationships that change over time. Hence, the most realistic representation of the financial system is a temporal multiplex network. However, in many cases, one focuses on individual processes, the timescale of which is much shorter than the timescale over which those relationships change. This simplification makes it possible to represent the financial system as a single-layer static network. Representing the financial system as a network enables explicit modelling of the propagation of shocks between agents. The importance of such mechanisms was especially clear as the 2007–2008 financial crisis unfolded. The failure of some financial institutions threatened to bring down other institutions exposed to them: for instance, the failure of Lehman Brothers caused the Reserve Primary Fund to 'break the buck', which, in turn, led to a run on money market mutual funds, while AIG (American International Group) was rescued by the government to prevent losses that could have led to the default of its counterparties[30].

Single-layer networks. Although economic networks comprise several types of relations, such as credit lending or supply of goods and services, the network of ownership best reflects the relations of power[35,36] between economic and financial actors. Through chains of ownership, shareholders have a means to influence, intentionally or not, the activities of firms owned directly and indirectly. Thus, one stream of research has investigated the structure of ownership networks and the implications of such structure. Ownership networks display small-world properties[37–39], are scale-free[40] and exhibit different concentration properties in different countries[41]. The global ownership network has a bow-tie structure with a concentrated core of financial companies[42] and a community structure that reflects geopolitical blocks[43]. The embedding in the geographical space explains several of its properties[44]. Its power structure appears to be very resilient, even to dramatic events like the 2007–2008 financial crisis[45].

Another stream of work has focused on the structure of networks of credit contracts between financial institutions. One of the first studies[46] found that the Austrian interbank network displayed power laws for both weight and degree distributions, an emerging community structure that mirrors sectors, a clustering coefficient smaller than other real-world networks and a small average path length. Shortly after, a study of interbank payments over the Fedwire Funds Service[47] also found power laws for both weight and degree distributions, along with a high clustering coefficient and degree disassortativity. These early works paved the way to a stream of works covering Switzerland[48], Italy[49–52], the USA[53,54], Belgium[55], Brazil[56,57], Japan[58], the Netherlands[59], Colombia[60], Germany[61] and Mexico[62], among others. Overall, the works on structure have highlighted in

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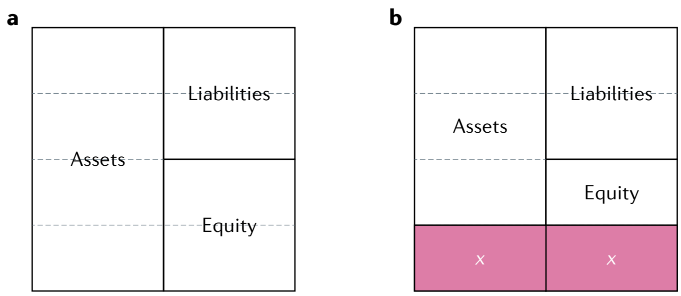
Box 1 | **Leverage**

Investors are said to be leveraged when they borrow money to invest. For instance, we use leverage when we get a mortgage to buy a house. If we put a capital of £40,000 as a down payment and we borrow £160,000 to buy a house worth £200,000, then our leverage is equal to 5: the value of our assets (the house) divided by our capital. Leverage is related to risk, because it amplifies our gains and losses. If the value of the house increases to £220,000, we could sell it, pay back our debt (let us assume for simplicity there is no interest rate) and we would have gained £20,000. We see, then, that an increase of 10% in the value of the house translates into an increase of 50% of our initial capital. The same is, however, true if the house is devalued: a devaluation of 10% would lead to a 50% reduction of our initial capital (from £40,000 to £20,000). More generally, if our leverage is equal to λ, a 1% change in the value of the house translates into a λ% change of our capital. The same applies to all leveraged investors. Leverage λ is defined in general for any investor or institution as the ratio between assets and equity. The figure shows the stylized representation of a balance sheet of an investor with $\lambda = 2$. When the assets are devalued by 25% (right part of the figure), the equity lost is 50%, equal to the asset devaluation multiplied by leverage: the higher leverage, the higher the amplification of losses, the higher the risk of the investor.

So far, we have considered an isolated investor, but the concept of leverage as an amplifier of losses can be generalized to the context of a network of interconnected balance sheets. For instance, when banks lend money to each other, the interbank assets of a bank correspond to the interbank liabilities of other banks. When a bank is under stress, the value of the interbank assets associated with its liabilities are devalued, which puts its creditors under stress, and so on. It can be shown that the propagation of shocks within the network is governed by a matrix, called the matrix of interbank leverage, the leading eigenvalue of which determines the level of endogenous amplification of exogenous shocks[32].



$$\lambda = \frac{A}{E} = 2 \qquad \frac{x}{E} = \frac{A}{E} \cdot \frac{x}{A} = \lambda \frac{x}{A} = 2\frac{x}{A}$$

national banking systems the existence of some stylized facts, that is, statistical features that are common across the different networks. These financial networks were found to be typically very sparse, with heavy-tailed degree distributions, high clustering and short average path length, and disassortative.

A few studies found that financial networks are characterized by a core–periphery structure. This means that a subset of institutions — the core — is tightly connected and a subset of institutions — the periphery — are loosely connected with each other and often connected to the core[63–67]. It has been shown that the core–periphery topology can emerge as the consequence of the imperfect competition for the benefits of intermediation[68]. However, the core–periphery topology is neither ubiquitous nor robust to null model comparisons[59] and, depending on the granularity of the data, other structures can better fit the data[69]. After the 2007–2008 crisis, the establishment of central clearing counterparties (CCPs) has meant that many bilateral contracts between clearing members were rerouted through a single institution (the CCP). It has been shown that central clearing often reduces systemic risk[70–74] but that it can also increase the demand for collateral[75], especially when the number of CCPs is large[72]. It has also been shown that current standards for default funds by clearing members might not be sufficient[76].

Community structure
A network characterization in which nodes can be grouped into sets such that each set of nodes is densely connected internally.

Path
On a network, a sequence of consecutive edges connecting a sequence of distinct nodes. The shortest path between two nodes is the path of minimal length connecting them.

Disassortativity
The tendency of nodes to be linked to other nodes with dissimilar degrees. Conversely, assortativity is the tendency for nodes to be linked to other nodes with similar degrees.

Co-occurrence networks. In several circumstances, financial entities are not necessarily related via 'direct' interactions (such as flows of money, holdings of shares or financial exposures) but via some form of co-occurrence, which may be indirect, such as commonality, similarity or correlation. For instance, two institutions may be related by the fact that their boards of directors share one or more members (so-called interlock relationship[77]), or that their portfolios share one or more assets (co-ownership/overlap[78]), or that their stock prices follow similar trends (price correlation[79]). Technically, these forms of co-occurrence can be represented via bipartite networks and their one-mode projections. A special type of co-occurrence network is a network with nodes that represent financial entities described by some empirical time series (for example, stocks traded in a financial market) and whose links are weighted by the measured correlation[79–81] or causality[82] (for instance, Granger causality) between the corresponding time series. This network can be regarded as a one-mode projection of the original set of multivariate time series in which the two types of nodes of a bipartite network represent stocks and time steps, respectively (FIG. 1a,b).

The analysis of these types of financial networks has shown that co-occurrence can reveal higher-order properties that are not immediately evident or predictable from the intrinsic properties of nodes. For instance, an analysis of the US corporate governance interlock network[77] revealed that the most influential directors do not necessarily serve on the boards of large companies. Similarly, the analysis of correlation-based networks in several financial markets has empirically identified groups of strongly correlated stocks[79,80,83,84] and credit default swaps (CDSs)[85] that are unpredictable from sector or geographical classification. Such data-driven clustering of assets can improve the performance of standard factor models for risk modelling and portfolio management[85]. In general, because shocks on portfolios can propagate to their owners (as we discuss in the section on dynamics), the existence of non-obvious groups of correlated financial assets can have important consequences for shock propagation.

Various characteristics of co-occurrence networks require special caution and can make their analysis more complicated than that of other types of networks. First, although other types of networks are typically sparse, one-mode projections obtained from empirical co-occurrence or correlation can be very dense and often do not contain zeros, in which case, they do not immediately result in a network. This property has led to the introduction of several filtering techniques aimed at sparsifying those matrices while retaining the 'strongest' connections.

Second, in the presence of heterogeneous entities, the same measured value of similarity (for instance,

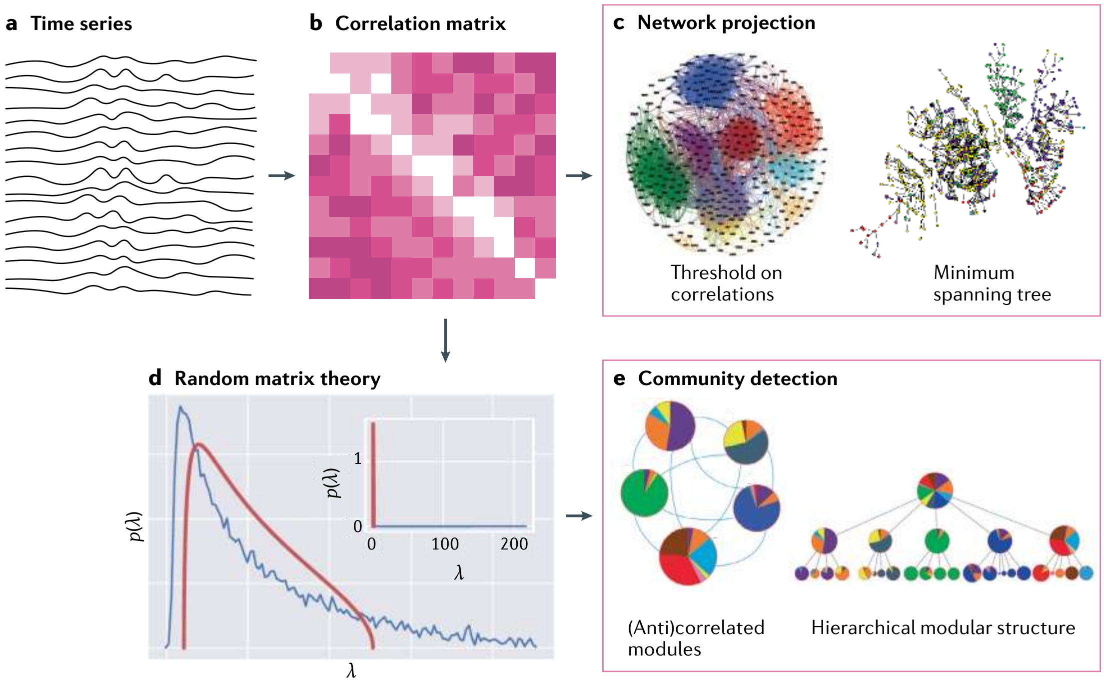


Fig. 1 | **Correlation-based networks from multivariate time series. a** | The original data consist of n time series extending over m time steps. **b** | The data are converted into an $n \times n$ correlation matrix **C**, where the entry c_{ij} is the correlation coefficient between the ith and the jth time series. **c–e** | The correlation matrix **C** can be used to produce different types of network structures on the original n objects, either by directly creating a network formed by links connecting pairs of nodes with correlation c_{ij} exceeding a given threshold[87] or by belonging to some imposed structure in an embedding geometry (such as minimum spanning tree[79] or maximally planar graph[80]) (part **c**), or by comparing the empirical distribution of eigenvalues of **C** (blue line) with the expected, so-called Marchenko–Pastur density (red line) exhibited by a random correlation matrix (Wishart ensemble in random matrix theory[93,94]) (part **d**) to filter out both noisy and global components[89–91] and, subsequently, identify (possibly hierarchical) communities of time series that are internally maximally correlated and mutually maximally anticorrelated[83–85] (part **e**). The networks in parts **c** and **e** are obtained starting from time series of stocks in the S&P 500 market (reproduced from REFS[83,87]) and different colours represent different sectors to which stocks belong: in all cases, the sectors are not predictive of the network structure, indicating that the network structure encodes higher-order information with respect to the standard classification. Part **c** adapted with permission from REFS[83,87]. Part **d** adapted with permission from REF.[85]. Part **e** adapted with permission from REF.[83].

correlation) might correspond to very different levels of statistical significance for distinct pairs of nodes. For this reason, simply imposing a common global threshold on all correlations is inadequate, and alternative filtering techniques that project the original correlation matrix onto minimum spanning trees[79], maximally planar graphs[80] or more general manifolds[86] have been introduced (FIG. 1c). These approaches found that financial entities belonging to the same nominal category can have very different connectivity properties (such as centrality, number and strength of relevant connections) in the network[87,88]. An open question is the theoretical justification for the choice of the embedding geometry wherein the network is constructed.

Third, in general, all entries of empirical similarity matrices tend to be shifted towards large values, as a result of an overall relatedness existing across all nodes, as, for example, a common market trend. This 'global' or 'market' mode[83,89–91] obscures the genuine dyadic dependencies that any network representation aims to portray.

Finally, the measurement of correlation (or co-occurrence) networks is intrinsically prone to the curse of dimensionality. With n the number of time series and m the length of those time series, to measure with statistical robustness the $n(n-1)/2$ entries of a correlation or similarity matrix one needs $m \geq n$, that is, a sufficiently large number of temporal observations (or nodes in the other layer of the bipartite network) in the original data to avoid dependency and statistical noise. Unfortunately, increasing m for a given set of n nodes is often not possible in practice, for instance, because one would need to consider a time span so long that non-stationarities would unavoidably kick in, making the measured correlation unstable and not properly interpretable.

The above complications lead to the requirement of a comparison with a proper null hypothesis that controls simultaneously for node heterogeneity, for a possible 'obfuscating' global mode and for 'cursed' noisy measurements. An important caveat here is that, in co-occurrence networks, even the null model necessarily has mutually dependent links. This key difference with respect to single-layer networks arises from the fact that, if node i is positively correlated with (or co-occurring with) node j, which is, in turn, positively correlated with

One-mode projections
A one-mode projection of a bipartite network contains only nodes of one type (say A) and any two such nodes are connected to each other with an intensity proportional to the number of their common neighbours of the other type in the original bipartite network (for instance, two directors are connected by a link indicating the number of common boards on which they sit).

node k, then nodes i and k are typically also positively correlated. This 'metric' constraint survives also in the null hypothesis of random correlations, whereas it does not apply in the usual null models developed for single-layer networks. Therefore, naively using those null models introduces severe biases in the statistical analysis of co-occurrence networks.

Fortunately, the statistical physics literature has contributed adequate null models defined in terms of a random correlation matrix[83,89–92] (technically, a Wishart matrix[93]), the entries of which are automatically dependent on each other in the desired way. Indeed, random matrix theory[93,94] has become a key tool in the analysis of correlation matrices. A successful use of this theory is in comparing the spectra of a measured correlation matrix with random correlation matrices to select the empirically deviating eigenvalues, in order to construct the filtered (non-random) component of the measured matrix[83] (FIG. 1d). This filtered matrix enables the detection of patterns such as communities (FIG. 1e) and the identification, in a purely data-driven fashion, of empirical dependencies that, again, are unpredictable a priori from the nominal classification or taxonomy of nodes. Research in this direction is active[85,92,95,96] and more general matrix ensembles have been recently studied using notions from supersymmetry[92] to further refine the analytical characterization of the null hypothesis.

Multiplex and higher-order networks. The examples of financial networks discussed so far condense all the information about the relationships between a pair of financial institutions into one (possibly weighted) edge. This is often a useful abstraction, but, in reality, many of those relationships are more complex, and this complexity can have consequences for the propagation of risk.

For instance, pairs of financial entities can be connected by multiple types of relationships, each capturing a different 'layer' of interaction. Multiplex networks[97,98] provide a natural framework to describe such relationships. Empirical case studies of financial multiplexes include: credit and liquidity exposures in the UK interbank market[99]; payments and exposures in the Mexican banking system[62]; the Mexican interbank market[100]; the Italian interbank market[101]; correlation of returns in the stock and foreign exchange markets[102,103]; correlation of returns in the stock market and news sentiments[104]; Colombian financial institutions and market infrastructures[105]; the EU derivatives market[21]; the UK interest rate, foreign exchange and credit derivatives market[106]; and corporate networks[107]. Such studies of financial multiplexes have found that the network structure of different layers can be very different[62,99,101], that links in distinct layers do not have the same persistence[100,101] and that overlaps between different layers are not trivial[106,107]. Compared with simpler single-layer networks, multiplex networks can give rise to a richer phenomenology when dynamic processes, such as financial contagion, take place on them. Several generalizations of financial contagion to multiplex networks are discussed later in this Review.

Another fruitful area of investigation has been the network implied by the derivative market. Owing to data availability, most early studies have focused on CDSs, which are derivative contracts in which an institution offers to insure another institution over the default of a third institution. As such, they are an example of three-body interactions in financial networks, similar to models in physics[108]. CDSs should allow institutions to hedge their risks and provide a solid market valuation of the financial risk of different market players. Nevertheless, as shown in a series of studies[21,108–110], the contagion can propagate in the CDS market when CDS insurers absorb too much risk upon themselves[111]. Moreover, it has been shown[112] that, under certain circumstances, the presence of CDS contracts can make it impossible to determine which institutions are in default and that removing such ambiguity can be computationally unfeasible[113,114].

Dynamics of financial networks

Direct contagion via solvency and liquidity channels. In this section, we review models of financial contagion that focus on bilateral relationships between financial institutions (for brevity, referred to as 'banks' in the following), which are one of the most common examples of financial networks. Most models can be grouped under the general framework[30] illustrated in the top panel of FIG. 2. The idea is that with each bank are associated some dynamic state variables that represent key quantities in their balance sheet. Those variables are updated via dynamic equations that depend strongly on the relationships between banks, which are typically static.

The balance sheet that represents a bank consists of an asset side (things that generate income for the bank, such as loans extended to households, to other banks or to firms) and a liability side (claims of other economic agents towards that bank), such as customer deposits, funds borrowed from other banks or firms, bonds and shares issued. The balance sheet identity prescribes that the sum of assets of each bank is equal to the sum of its liabilities. Liabilities have different priorities (also known as seniorities). If a bank fails, its assets are liquidated and its liabilities are paid back, starting from those with a higher priority. The liability with the lowest priority is the equity, which corresponds to the residual claim of shareholders after all other liabilities have been paid back. Therefore, it measures the bank's net worth. Both assets and liabilities can be split according to the market to which they belong, in particular, it is customary to distinguish between interbank (or network) and external assets and liabilities. The interbank liabilities of bank i are the obligations that i has to other banks in the system, such as payments to be made imminently or loans, which correspond to payments to be made in the future. Similarly, the interbank assets of bank i are the obligations that other banks in the system have towards i. To each interbank liability (for instance, of i towards j), there corresponds an interbank asset (of j towards i). Hence, interbank assets and liabilities are equivalent representations of the obligations between pairs of banks. The network is built simply by associating to each bank one node and to each interbank liability (or asset) one link. Those networks are directed (obligations are not necessarily symmetrical), weighted (by the monetary

Filtered matrix
Matrix, for instance representing correlations, that has been statistically validated (or otherwise processed to eliminate the effects of noise) so that, ideally, only statistically significant information is retained.

Liquidity
Refers to the case in which the liquid assets (such as cash) of one institution are larger than its short-term liabilities (such as loans to be paid back overnight).

Liabilities
Items on the balance sheet of an institution that have a negative economic value because they represent debt to be repaid, potentially at different times (or maturities) in the future.

Equity
In accounting, equity is defined by the balance sheet identity as the difference between assets and liabilities. Therefore, it represents the net worth of the institution.

amount of the obligation) and without loops (banks do not make payments to themselves). For the sake of brevity, here, we have considered the case in which all obligations between each pair of banks are aggregated into one single interbank liability (or asset). More granular models based on multiplex networks can overcome this limitation, as discussed below.

The usual approach followed in modelling studies is to hit one or more banks with an exogenous shock, for example, by reducing the value of their external assets, and to propagate the shock across the network, not unlike an epidemic. Formally, this propagation corresponds to a dynamic process (triggered by an external shock) on the network that allows balance sheet variables to evolve. This approach is conceptually similar to the stress tests of the banking sector as implemented worldwide by regulators after the 2007–2008 financial crisis. However, those usually consider banks in isolation and neglect interactions between them. Therefore, the natural policy application of these models has been to incorporate network effects into more traditional stress testing models.

Although models differ in their implementation details, they describe only a handful of basic shock propagation mechanisms. One is liquidity contagion. In this case, the relevant balance sheet quantities are interbank liabilities, which represent payments to be delivered imminently, and liquid assets, which are a subset of external assets and consist of cash or assets that can readily be converted into cash. For example, let us imagine that all banks in FIG. 2 have one unit of cash and that their payment obligations (that is, the interbank liabilities) are as follows: $L_{13}=2$, $L_{21}=1$ and $L_{32}=2$. Payments happen in subsequent rounds. In the first round of payments, each bank relies only on their cash. Bank 1 pays one unit to bank 3, bank 2 pays one unit to bank 1 and bank 3 pays one unit to bank 2. Bank 2 has, therefore, paid its obligation in full. In the second round of payments, banks 1 and 3 can use the payments received in the first round to fully pay their obligations to banks 3 and 2,

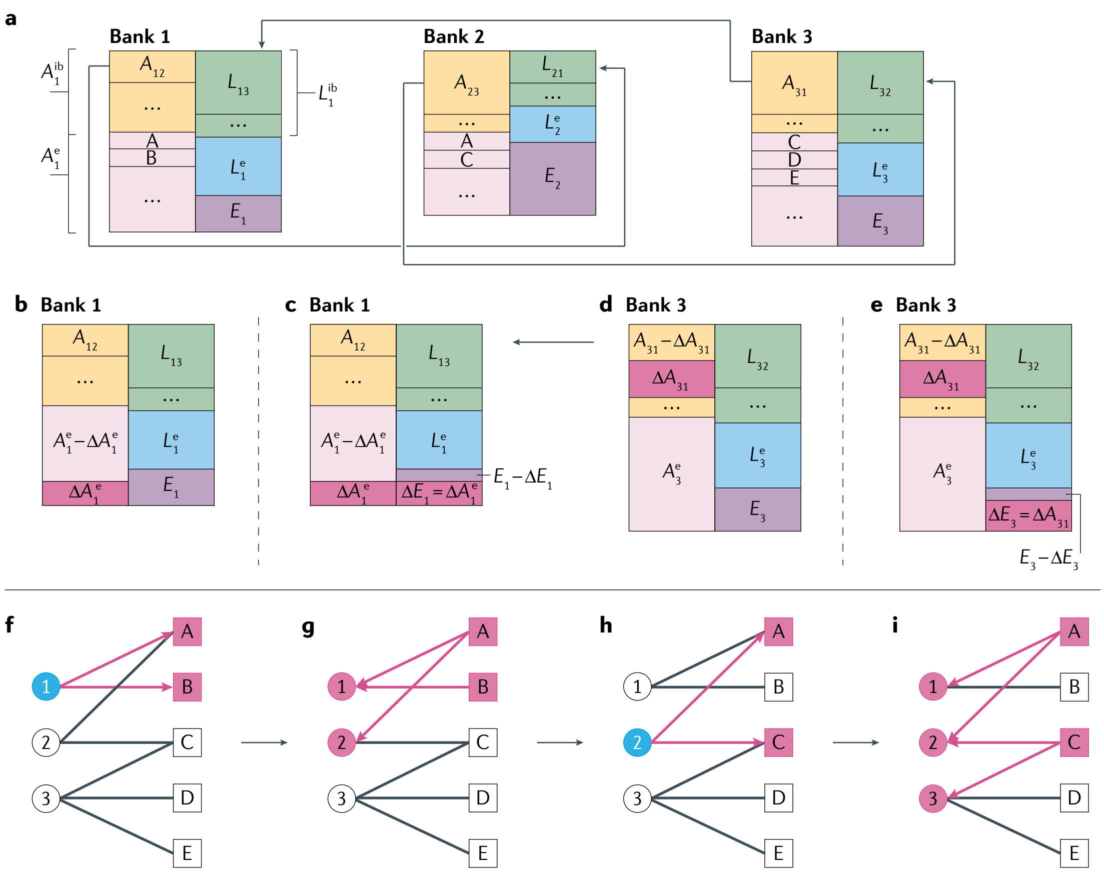


Fig. 2 | **Interbank networks and their dynamics. a** | A stylized interbank network consisting of three banks, each represented by its balance sheet. On the asset side is the interbank assets A_i^{ib}, further broken down in individual exposures (for instance, A_{12} is the exposure of bank 1 to bank 2), and external assets A_i^e (broken down as A, B,…). On the liability side are interbank liabilities L_i^{ib}, similarly broken down, external liabilities L_i^e and equity E_i. **b–e** | Solvency contagion via revaluation of interbank assets. An exogenous shock ΔA_1^e hits the external assets of bank 1 (part **b**) and is absorbed by bank 1's equity (part **c**). Because bank 3 is exposed to bank 1, it revaluates its interbank asset A_{31} (part **d**). The exact valuation method depends on the specific model. Finally, the reduction in bank 3's assets is absorbed by its equity (part **e**). **f–i** | Contagion via overlapping portfolios. Bank 1 sells assets A and B, for example, to meet its leverage target (part **f**). Doing so causes A and B to depreciate. Asset values of banks 1 and 2, which hold A and B, are reduced (part **g**). As a consequence, bank 2 now needs to deleverage and sells assets A and C (part **h**). Those assets depreciate and asset values of banks 1, 2 and 3 are reduced (part **i**).

respectively. This toy example mimics the iterative solution of the Eisenberg–Noe model[115], in which banks use both their liquid assets and the partial (proportional) payments received by other banks to meet their payment obligations. The Eisenberg–Noe model has been extended to the case in which banks that are not able to fully pay their obligations face bankruptcy costs[116], to the case in which payment obligations depend on other variables[117] and to the case in which banks do not make partial payments at all[118], as well as to continuous time[119]. In the Eisenberg–Noe model, contagion spreads when there are banks that would have been able to meet their own obligations if they had received their incoming payments. However, if some of those payments were not (or were only partially) delivered, the banks are not able to fully deliver their own payments, potentially putting banks on the receiving end of their payments in the same situation. The initial shock is normally an unforeseen payment obligation, such as a margin call due to price movements in the derivative markets[71,120,121].

Another shock propagation mechanism is solvency contagion, which occurs when the insolvency or the reduction in creditworthiness of a bank has an effect on its creditors. The simplest form of solvency contagion is known as contagion on default. In this case, as long as bank i's equity is larger than zero, i's creditors take their interbank assets towards i at face value. However, when bank i's equity becomes smaller than or equal to zero, i's creditors write off their interbank assets towards i because they do not expect to be fully paid back. (Negative equity is a common sufficient condition for insolvency or default. However, resolution frameworks put in place after the 2007–2008 financial crisis imply that banks can be wound down when they fail to comply with regulatory requirements, even though their equity is positive.)

In the most conservative case, i's creditors set the value of the corresponding interbank assets to zero, as they expect to recover nothing from the defaulted bank[122]. In general, they discount their interbank assets by a coefficient between zero and one known as recovery rate, such as in REF.[123]. When j, one of i's creditors, writes off its interbank assets, the total value of j's assets is reduced and, via the balance sheet identity, so is the total value of j's liabilities. Because it has the lowest priority, in the first instance, it is j's equity that must absorb the losses. However, if j's equity is not large enough, it will default too, thereby, triggering write-offs by its own creditors. The spreading of defaults across the financial network is known as a default cascade or a domino effect.

For example, let us imagine that, for banks in FIG. 2, we have $A_{31} > E_3$ and $A_{23} < E_2$ (where A_{ij} is the exposure of bank i to bank j and E_i is the equity of bank i) and that the recovery rate is equal to zero. If the initial shock to bank 1's equity is large enough to make bank 1 default, that is, $E_1 \leq 0$, then bank 3 will fully write off its interbank asset A_{31} and, because the corresponding loss is larger than its equity, it will default as well, that is, $E_3 < 0$. Therefore, bank 2 will also fully write off its interbank asset A_{23}, but because its equity is larger than the corresponding loss, it will not default.

Solvency
Solvency refers to the case in which the assets of one institution are larger than its liabilities and, therefore, its equity is positive.

These models are mathematically similar to linear threshold models and are, therefore, amenable to analytical treatment[13], for example, to derive the size of the cascade of defaults[124,125]. In more general models, write-offs are triggered not only by defaults but also by increases in probabilities of default. This is the approach followed by the family of DebtRank models[15,126,127], by empirical models[128] or by valuation models[129–132] (see the middle panel of FIG. 2). The mechanism of these general models mimics the accounting requirement of marking assets to market, which has been a large source of losses during the 2007–2008 financial crisis[133]. Interestingly, it has been shown[131] that several contagion models (such as the aforementioned Eisenberg–Noe, the contagion on default and DebtRank) are special cases of a more general valuation model. Solvency contagion is the contagion channel that has been probed empirically the most. The risk of systemic events has been found to be generally small[123,129,134–136] or at least to have sharply decreased since the 2007–2008 financial crisis[132]. Nevertheless, systemic events can be severe[129] and risks appear to be heavily concentrated in a few key institutions[122,128], which are not necessarily the largest ones. This finding points to the important role played by network structure[137] and challenges the 'too big to fail' paradigm[15,122].

Another type of contagion is funding contagion, which occurs when banks that have previously lent to bank i decide not to renew their loans once they expire[138,139]. Similarly to solvency contagion, the decision can be triggered by a change in the creditworthiness of bank i. Reference[140] reports a model that integrates solvency and funding contagion.

Models of bilateral exposures have also been used to investigate the relationship between the underlying topology of the network and its stability. Early works[141,142], following standard economic theory, show that more diversified (and, therefore, more interconnected) networks are more resilient, as shocks are dispersed across more banks. However, it has been shown[143,144] that the relationship between systemic risk and diversification is non-monotonic, in the presence of mechanisms that can amplify losses (for example, creditors' reactions). Moreover a more interconnected network is more resilient for small shocks and less resilient for large shocks[145], in line with the intuition that financial networks might be 'robust-yet-fragile'[27]. Similarly, it has been shown[146] that, although the probability of widespread contagion can be small, systemic events can be severe. It has been shown[24,147] that diversification does not have a monotonous effect on the extent of default cascades. In addition, the role of the topology is crucial[148,149], with no single network architecture being superior to the others[150]. In REF.[32], the instability of the contagion dynamics (see also REF.[151]) is linked to the presence of specific topological structures (unstable cycles), which are likely to appear in a more diversified network. A different approach is followed by REFS[152,153], in which the relationship between interconnectedness and resilience is investigated with minimal information about the network structure.

Yet another stream of the literature looks at assessing and designing (optimal) policies. For example, it has been shown[154] that limits on exposures often, but not always, reduce systemic risk and a toolkit[136] has been developed to test the impact of bail-ins. Several studies focus on the impact on public finances: it has been shown[155] that resolution frameworks can be effective in reducing bail-out, centrality-based bail-outs have been investigated[156] and conditions for optimal bail-outs have been determined[157–159]. Closely related are the studies on optimal repair strategies[160] and on the controllability of financial networks[161,162], aimed at reducing systemic risk, by approaches such as targeted taxes[163], requiring banks to disclose their systemic impact[164] or explicitly optimizing the exposures[165].

Indirect contagion via overlapping portfolios. Shocks can propagate between banks even if they are not directly connected through a contract. This happens if they are indirectly connected through some co-occurrence relationship (as discussed above), for instance, when they invest in common assets. If one institution is in trouble, it may sell some of its assets. Doing so causes the devaluation of those assets and, therefore, losses for the other banks that had invested in them. This devaluation may cause these banks to sell their assets in turn, and so on. Although this type of contagion is mediated by the market through price, interactions can still be modelled as a network of overlapping portfolios. This network is bipartite, with two types of nodes representing banks and assets, and links connecting banks to the assets in their portfolio (see the section on co-occurrence networks and the bottom panel of FIG. 2). In the figure, bank 1 is not directly exposed to bank 2, yet, contagion from bank 1 to bank 2 can occur because they both invest in asset A. The same figure is useful to introduce the concept of indirect exposure, that is, the fact that, through the network of overlapping portfolios, banks may be effectively (and unknowingly) exposed to assets they are not investing in. For instance, although bank 1 has no direct exposure to asset C, it is indirectly exposed to it through the overlap between its portfolio and that of bank 2.

As in the case of direct exposures, the goal is to understand how the properties of the network of overlapping portfolios affects its stability, and under what conditions the system is able to either absorb or amplify exogenous shocks[78,166–170]. To model the dynamics of shock propagation on this network, one needs to specify how banks react to losses in their portfolios (for instance, how they readjust their portfolios to manage risk) and how asset prices react to the trading activity of banks. The response of prices to liquidation is typically implemented by means of a market impact function[171] that links the liquidation volume of an asset to its price: the more an asset is being liquidated, the greater its devaluation. Most of the literature considers market impact functions that are linear in returns or log-returns, but consideration has also been given to more complex forms that account for the fact that, when an asset is largely devalued, other investors would step in, attempting to buy the asset at a cheap price[170].

Concerning the dynamics of banks, the simplest choice is that of a linear threshold model[172], in which a bank is passive as long as its losses remain below a given threshold (typically chosen to be equal to its equity), and liquidates its entire portfolio otherwise[78,166]. Under this assumption, the dynamics can be approximated by a multi-type branching process and it is possible to derive analytical results for the case of random networks[166]. When the branching process is supercritical, even a small exogenous shock can propagate throughout the network. Using this approximation, it is possible to identify regions in the parameter space — typical parameters are the average degree of banks and assets in the network, strength of market impact, leverage — where cascades of defaults occur, and to show that increasing diversification, which reduces the risk of individual institutions, does not necessarily increase systemic stability[166]. In fact, as in the case of counterparty default contagion[146], the probability of observing a cascade of defaults is non-monotonic with respect to the average diversification of banks (their average degree in the network).

The study of random networks is important from the theoretical point of view, but the ultimate goal of contagion models is to characterize the stability of realistic systems. For instance, stress testing has been performed using numerical simulations on the network of overlapping portfolios between US commercial banks in 2007 (REF.[78]). These simulations reveal the existence of phase-transition-like phenomena between stable and unstable regimes. Comparing the banks that the model predicts should default with the list of actual defaults observed between 2008 and 2011 shows that the model identifies defaults significantly better than a random classifier does, and it identifies commercial real estate loans as the likely trigger of the observed defaults. The capability of network models of contagion due to overlapping portfolios to correctly identify defaults associated with the crisis is further confirmed in REF.[173], in which the analysis of REF.[78] is extended to a wider range of behavioural assumptions for the response of banks to the devaluation of their assets. The model of REF.[78] has also been used to study the Japanese banking crisis of the late 1990s[174,175], and, more recently, to test potential strategies to mitigate the occurrence of cascades of defaults[176]. Reference[176] considers, in particular, an application to the bipartite network of European banks and sovereign bonds, and shows that the stability of the system can be improved by protecting a small fraction of nodes.

Although the assumption of a passive investor is a useful benchmark against which to assess the effect of active risk management — and it may be realistic during a fast-developing crisis in which banks do not have time to react before they default — in practice, banks would react to changing market conditions by actively rebalancing their portfolios. This happens because of internal risk management procedures or because of regulatory constraints that need to be satisfied[177]. Active risk management is typically modelled by means of leverage-targeting dynamics[166–169]. In these models, banks that experience losses liquidate a fraction of their investment in an attempt to keep their leverage constant. In fact, it can be shown that leverage targeting is the optimal strategy of an investor that tries to maximize its expected return on equity while being

Value-at-risk
(VaR). Risk measure defined as a (typically) large quantile of the probability distribution of losses. For example, when the quantile is 0.99 and the distribution of losses is over a time horizon of 1 year, it is interpreted as the loss that occurs once every 100 years.

Expected shortfall
(ES). Risk measure defined as the mean loss exceeding a (typically) large quantile of the probability distribution of losses. It is always larger than the value-at-risk at the same quantile.

subject to a value-at-risk (VaR) or expected shortfall (ES) constraint[178].

A dynamic in between thresholding and leverage targeting has also been considered[170], in which banks do not react until their losses exceed a given threshold, after which they target leverage. Using this dynamic, a stress testing exercise on a network of overlapping portfolios between European banks was performed, considering the overlap between their investment in sovereign and corporate bonds of different countries, and the amount of indirect exposures induced by the network was computed. This analysis shows, for example, that, in spite of having small investments in mortgage of foreign countries, banks are effectively exposed to foreign real estate markets via the network of overlapping portfolios. For instance, the indirect exposures of banks of northern European countries to the real estate market of southern European ones are estimated to be, on average, almost twice as much as their nominal exposure. Although these estimates depend on many assumptions on the model and its calibration, they, nonetheless, provide a good argument that network effects can be significant. Moreover, by introducing two indicators of exposures to indirect contagion due to overlapping portfolios, it has also been shown[179] that a bank's size does not necessarily determine its systemic importance.

Contagion due to overlapping portfolios also affects financial institutions other than banks. In the past few years, studies have focused on funds that also actively manage their portfolios by liquidating assets in times of distress. For instance, REFS[180,181] provide empirical characterizations of the network of overlapping portfolios between funds, and REFS[182–184] introduce stress testing frameworks to study the stability of US mutual funds, European investment funds and German funds. A study of US mutual funds shows that the vulnerability of the network is large compared with the benchmark of random networks in which the degrees of nodes are preserved[185].

Most of the work so far has focused on specific sectors of the financial system (for instance, focusing on relationships between banks or between funds), but some effort has been directed towards the development of system-wide stress testing frameworks that also account for the existence of portfolio overlaps between different sectors[186,187]. Reference[186] presents a system-wide stress testing framework of the European financial system. The paper considers different types of contagion mechanisms and different risk management constraints, thereby allowing for the modelling of different institutions such as banks, hedge funds, investment funds and insurers. The models are built using granular data for the banking sector and a set of representative agents for non-bank institutions, and it is shown that accounting for non-banks in the models leads to increased shock amplification. The importance of accounting for the existence of portfolio overlaps between different sectors is confirmed in REF.[187], which reports a more granular model of the UK financial system in which each individual non-bank institution is modelled explicitly, rather than through a representative agent.

Contagion on multiplex networks. As mentioned in the section on multiplex and higher-order networks, granular models in which exposures are disaggregated, either by maturity[188], seniority[130] or asset class[189], can be represented by multiplex networks. Similarly, as described above, there are different contagion channels through which stress can propagate between financial institutions. Because each channel can be represented as a network, a complete characterization of financial contagion should consider multiple contagion channels simultaneously on a multiplex network. In principle, risks associated with different layers could either offset or reinforce each other. Indeed, the stability properties of the system depend on the interplay of the contagion processes across layers, which can differ in nature depending on the type of layer. By looking at a single layer at a time, one can fail to detect instability and fail to identify the possible contagion channels.

It has been shown[190] that, when two layers are coupled weakly, systemic risk is smaller in a multiplex network than in the aggregated single-layer network. Moreover, the sharp phase transition in the size of the cascade is more pronounced in the multiplex network. It has also been shown[191] that mixing debts of different seniority levels makes the system more stable. Reference[192] reports on a multiplex representation of the Mexican banking system between 2007 and 2013. Crucially, it is shown that focusing on a single layer can underestimate the total systemic risk by up to 90% and that risks generated by individual layers cannot be simply summed but interact with each other in a non-linear fashion. A similar result is found in REF.[193], which reports an agent-based model of the multiplex interbank network for large EU banks. Reference[194] outlines the possible interplay between layers corresponding to short-term funding, assets and collateral flows, clarifying how risk propagates from one layer to another using the case of Bear Stearns during the 2007–2008 financial crisis.

Some early works on financial contagion due to counterparty risk[13,146,147,195] consider the effects of fire sales by assuming the existence of one asset that is common to all banks and is liquidated when banks default, but their focus remains on understanding how the topology of the interbank exposures network affects its stability. More recently, by using data on direct interbank exposures between Austrian banks and also by assuming the existence of a common asset between banks, it has been suggested[196] that the interaction between contagion channels can significantly contribute to aggregate losses. These findings are confirmed in REF.[197], which reports using detailed data on direct exposures and overlapping portfolios between Mexican banks. Similarly, a system-wide stress test for the European financial system has been used to show[186] that interacting contagion channels can lead to five times more bank failures than the same channels acting in isolation.

Statistical physics of financial networks

Both the structural and the dynamic approaches discussed above are based on a static snapshot of the network or a temporal sequence of snapshots over which the same analyses are repeated. As such, they take as

Box 2 | **Maximum entropy ensembles of networks**

According to the principle of maximum entropy, the unbiased probability distribution $P(\mathbf{G})$ of a graph configuration $\mathbf{G}$ is found by imposing a set of structural properties ($\mathbf{C}$) chosen as constraints and maximizing the uncertainty about everything else[198]. The constraints $\mathbf{C}$ can be a set of properties $\mathbf{C}(\mathbf{G}^*)$ measured on a specific real-world network $\mathbf{G}^*$, in which case the distribution $P(\mathbf{G})$ generates an ensemble of randomized counterparts of $\mathbf{G}^*$. This ensemble can serve as a null model for $\mathbf{G}^*$, which is useful to detect empirical deviations of $\mathbf{G}^*$ — if the topology of the latter is completely known — from its randomized counterpart. Alternatively, the ensemble is an unbiased 'best guess' for the structure of $\mathbf{G}^*$ from partial information, which is useful to statistically reconstruct $\mathbf{G}^*$ — if the topology of the latter is not completely known — from the available properties $\mathbf{C}(\mathbf{G}^*)$. This construction is illustrated in FIG. 3.

Quantitatively, $P(\mathbf{G})$ is found by maximizing Shannon entropy, which is the functional

$$S[P] = -\sum_{\mathbf{G}\in\Omega} P(\mathbf{G})\ln P(\mathbf{G}), \tag{1}$$

where Ω is a certain set of graphs (for instance, all graphs with the same number N of nodes as $\mathbf{G}^*$), subject to the normalization condition $\sum_{\mathbf{G}\in\Omega}P(\mathbf{G})=1$ and to the condition that the chosen constraints $\mathbf{C}$ must be realized[213]. As in traditional statistical physics, the latter condition can be enforced either as a hard constraint on each realization (the microcanonical ensemble), that is, $\mathbf{C}(\mathbf{G})=\mathbf{C}(\mathbf{G}^*)$ for each allowed $\mathbf{G}$, or as a soft constraint on the ensemble average (the canonical ensemble), that is, $\sum_{\mathbf{G}\in\Omega}P(\mathbf{G})\mathbf{C}(\mathbf{G})=\mathbf{C}(\mathbf{G}^*)$. In the microcanonical ensemble, the maximum entropy probability is uniform over the configurations that realize the hard constraint. In the canonical ensemble, the probability takes the parametric form

$$P(\mathbf{G}|\boldsymbol{\theta}) = \frac{e^{-H(\mathbf{G},\boldsymbol{\theta})}}{Z(\boldsymbol{\theta})}, \quad \forall\ \mathbf{G}\in\Omega \tag{2}$$

where $H(\mathbf{G},\boldsymbol{\theta})=\boldsymbol{\theta}\cdot\mathbf{C}(\mathbf{G})$ is the Hamiltonian (a linear combination of the enforced properties), $\boldsymbol{\theta}$ is the set of Lagrange multipliers associated with the constraints and $Z(\boldsymbol{\theta})=\sum_{\mathbf{G}\in\Omega}e^{-H(\mathbf{G},\boldsymbol{\theta})}$ is the partition function. Importantly, $P(\mathbf{G}|\boldsymbol{\theta})$ depends on $\mathbf{G}$ only through the vector of enforced quantities $\mathbf{C}(\mathbf{G})$, which is the sufficient statistics. When $\mathbf{G}^*$ is only partially observed and $\mathbf{C}(\mathbf{G}^*)$ is the available information about it, constructing the maximum entropy distribution $P(\mathbf{G}|\boldsymbol{\theta})$ provides a statistical physics route to network reconstruction from partial information. This canonical ensemble coincides with the so-called exponential random graph model[198,210,211,262] and ensures that $P(\mathbf{G}|\boldsymbol{\theta})$ is the least biased towards the properties that are not enforced through the constraints[213].

Note that Eq. (2) only specifies the functional form of the probability distribution defining the canonical ensemble, while leaving the Lagrange multipliers (numerically) undetermined. Although it is possible to draw the Lagrange multipliers from some ad hoc probability density function to induce archetypal toy models of networks (such as homogeneous graphs, scale-free networks or block models)[210], in the analysis of real-world networks, it is crucial to fit these parameters to the actual network. To do so, the maximum likelihood principle prescribes maximizing the function[198,211]

$$\mathcal{L}(\boldsymbol{\theta}) = \ln P(\mathbf{G}^*|\boldsymbol{\theta}) = -H(\mathbf{G}^*,\boldsymbol{\theta}) - \ln Z(\boldsymbol{\theta}) \tag{3}$$

with respect to $\boldsymbol{\theta}$. This maximization retrieves the specific values $\boldsymbol{\theta}^*$ that ensure $\sum_{\mathbf{G}\in\Omega}P(\mathbf{G}|\boldsymbol{\theta}^*)\mathbf{C}(\mathbf{G})=\mathbf{C}(\mathbf{G}^*)$, implying that the ensemble average of each constraint matches its empirical value as desired.

input all the microscopic details of a specific network configuration — that is, the exact position and magnitude of all links. However, as in the analysis of all large systems, one may take a statistical physics perspective and wonder whether not all the microscopic details are relevant in order to produce the observed structural and dynamic patterns, in other words, whether it is sufficient to specify only certain 'key' network features and let the rest of the architecture follow from these. From a data science perspective, this question is equivalent to wondering whether modelling a specific network configuration (for instance, an interbank network observed at a given time and/or in a given geographical location) suffers from the problem of overfitting. In other words, are the results of such a model still useful when different configurations of the same type of network, for instance, at a different time or location, are considered?

Constraints
Quantities representing the structural properties either to be enforced in the network reconstruction process or to be discounted in the network validation process.

This problem can be addressed via the introduction of statistical ensembles of network configurations that have certain features in common with the empirical network, but are otherwise random[198]. This methodology is illustrated in BOX 2 and FIG. 3. Technically, such ensembles are constructed by looking for the probability distribution of graphs (defined over the allowed configurations) that maximizes an entropy functional, subject to a set of constraints that represent the key topological properties that one wants to enforce. This statistical physics construction effectively produces an energy function, or Hamiltonian, which is a linear combination of the specified constraints. Different configurations that have the same value of the constraints have the same 'energy' and occur with the same probability in the ensemble.

This procedure naturally defines a notion of networks at thermodynamic (or statistical) equilibrium: if a real-world network is consistent with the ensemble specified by a certain set of constraints, then those constraints capture robust or conserved properties (like the total energy in physics) in the real-world network[199]. Importantly, it has been found that the constraints that replicate several empirical structural properties (therefore, suggesting a consistency between real networks and equilibrium ensembles) are local, that is, they include the degrees (and possibly the strengths) of all nodes[198,200]. If only global properties are enforced (such as the total number of links or the total weight of all connections), the resulting networks are completely homogeneous and very different from the observed ones.

It is important to stress that, in general, this notion of thermodynamic equilibrium is not related to that of economic equilibrium, which is, instead, based on matching demand and supply (market clearing) and usually entails the maximization of some postulated utility function for each financial institution[201–203]. However, a connection between the two notions can be established[204] by considering that, if the observed network was the outcome of economic equilibrium, then all alternative configurations that matched the same supply and demand levels of all nodes would be equally viable. Indeed, according to Walrasian theory in economics, agents in an exchange economy care only about final allocations and are indifferent with respect to different market configurations that realize the same allocations. So, if the real network is at economic equilibrium, then any other configuration that realizes the same supply and demand constraints of all nodes should also be at economic equilibrium. Therefore, the maximum entropy ensemble constructed using the supply and demand of each node as constraints should provide a thermodynamic construction of Walrasian equilibrium[204]. Interestingly, this theoretical expectation leads to the identification of local (node-specific) network properties as candidates for the constraints that characterize Walrasian equilibrium, consistent with the aforementioned empirical result that local properties are the effective constraints to use in the maximum entropy construction.

Thus, in a certain sense, the notion of economic equilibrium should, in principle, try to explain the realized values of the supply and demand constraints observed in real-world financial networks, whereas the notion of thermodynamic equilibrium should, in principle, try to explain, given those values, the typical network properties arising from the multiplicity of market configurations consistent with (Walrasian) economic equilibrium. With these considerations in mind, we discuss the possible applications of statistical ensembles of financial networks to network reconstruction and pattern detection.

Networks at equilibrium and reconstruction. The approaches discussed above assume that the presence and magnitude of relationships between financial institutions is known. Unfortunately, owing to confidentiality issues, that information is often only accessible to regulators. Even then, regulators only have a partial view of the financial network, typically limited to their jurisdiction. Therefore, there are problems relating to the

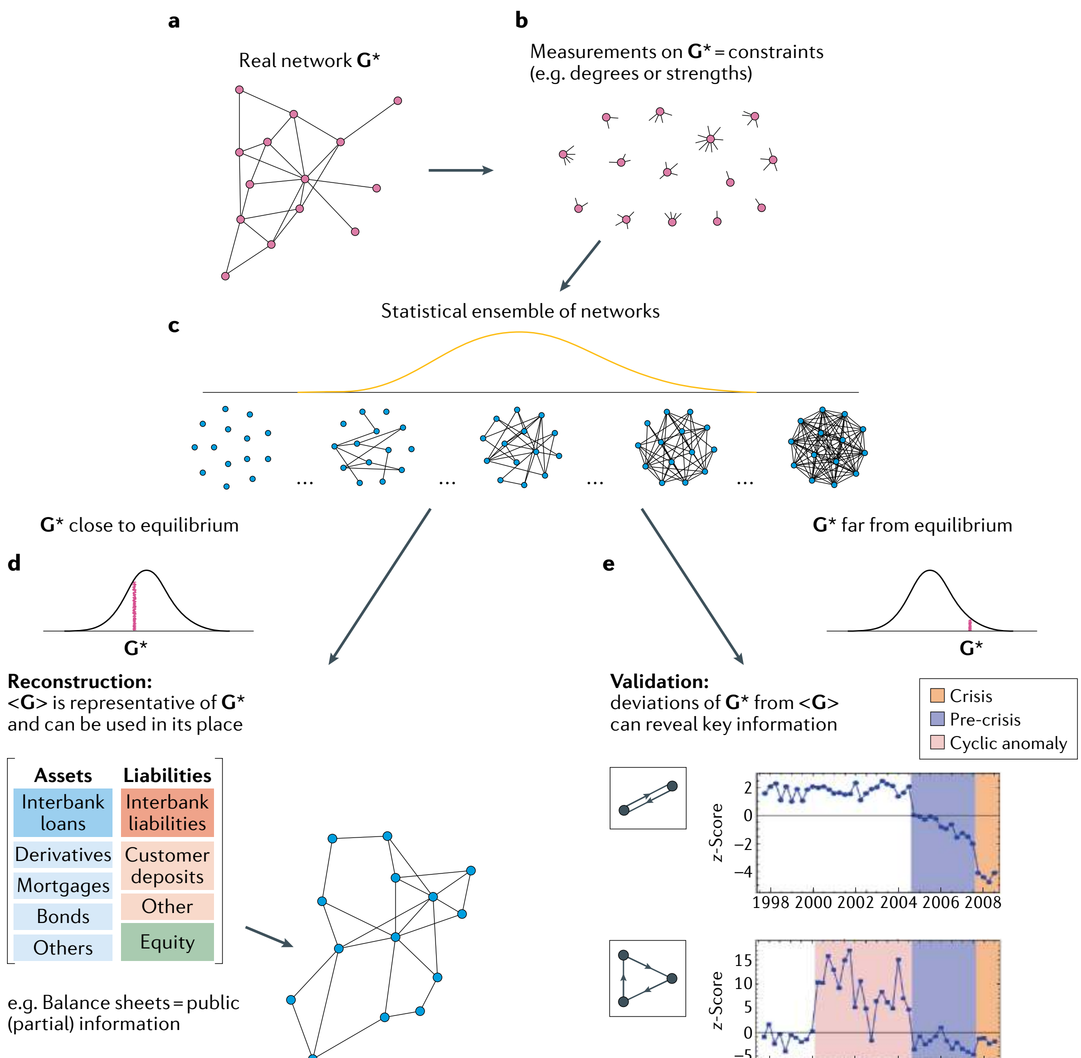


Fig. 3 | **Construction of statistical ensembles of financial networks and application to network reconstruction and pattern detection.** Starting from a real-world network **G*** (part **a**), a set **C**(**G***) of structural properties are chosen as constraints — for instance, the degrees and/or strengths of all nodes (part **b**). Then, a canonical ensemble of networks is constructed by calculating the probability distribution $P(\mathbf{G}|\boldsymbol{\theta})$ that maximizes the Shannon entropy under the chosen constraints, and the parameters $\boldsymbol{\theta}^*$ that maximize the likelihood[198,211] (part **c**). This construction ensures that the expected values of the constraints match the empirical ones. The ensemble can be used as a method for network reconstruction if **C**(**G***) is the only information available about the original network **G*** and the latter is believed to be at the thermodynamic equilibrium induced by the chosen constraints[205], for example (part **d**). Alternatively, the ensemble serves as a null model to detect empirical deviations of **G*** from equilibrium, for instance, systematic changes in the occurrence of small subgraphs (dyads or triads) that may even act as early-warning signals of major transitions in network structure[199] (part **e**). In the example shown, changes in the statistical significance of cycles of order 2 and 3 in the Dutch interbank network turn out to be early-warning signals of the 2008 crisis[59]. Figure adapted with permission from REFS[59,127].

Shannon entropy
Functional quantifying the amount of uncertainty associated with a probability distribution (see BOX 2). Its maximum is attained for a uniform distribution.

observability of financial networks and the reproducibility of the results obtained on a specific network.

The problem of missing data in complex networks is very general and has led to the birth of a research field known as network reconstruction. Because of the relevance of the problem, many reconstruction algorithms have been proposed in the context of financial (mainly interbank) networks[205] (see TABLE 1 for a list of the most relevant techniques). Roughly, these methods can be classified as either deterministic or probabilistic, depending on the result of the reconstruction procedure. Deterministic methods include the popular MaxEnt[134] (this approach should not be confused with the maximum entropy ensembles described above, as it implements a conceptually different optimization procedure) and iterative proportional fitting[135,206] algorithms. These methods produce a single instance of reconstructed network. Although apparently more intuitive, they suffer from the limitation of assigning zero probability to any other network configuration, including (almost certainly) the true unobserved one[207]. The same limitation affects methods that combine a probabilistic approach for estimating the network topology (that is, for determining the presence of links) with a deterministic recipe for estimating link weights[208,209].

Probabilistic methods overcome this limitation by generating an ensemble of reconstructed networks, each with its own probability to be the true network. This class of methods includes the network reconstruction methods rooted in the maximum entropy ensembles described above[205,210,211] (see BOX 2). From an information-theoretical perspective, the maximum entropy approach to inference minimizes the unsupported assumptions about the true distribution of the unobserved (that is, confidential) data[212,213] and states that the probability distribution that best describes the state of knowledge about a system is the one with largest entropy, constrained to satisfy the available information on the system itself[12,210]. Analogously to the application in statistical mechanics, one can derive the probability distribution of the unobserved individual exposures constrained by the observed total exposures — typically the aggregate interbank lending (assets) and borrowing (liabilities) of each bank. Uncertainty maximization is carried out by maximizing the Shannon entropy, and the available information is included as constraints in the optimization procedure. The underlying rationale is that of obtaining reconstructed networks with properties that are a consequence of the imposed constraints. In other words, this approach avoids making assumptions that are not supported by the available information and that would otherwise bias the entire estimation procedure. Maximum entropy inference is, thus, maximally 'indifferent' towards the network properties that are not accessible.

The first entropy-based algorithms were based on the assumption that the constraints concerning the binary and the weighted network structure jointly determine the reconstruction output. This approach is taken in the enhanced configuration model, which simultaneously constrains the degrees and the strengths of nodes[200,214]. However, the inaccessibility of empirical degrees (in other words, number of lenders or borrowers of each bank) makes these methods inapplicable for reconstructing interbank networks. This difficulty has led to the introduction of two-step algorithms[17,215] that, in

Table 1 | **Overview of the network reconstruction methods that can be found in the literature**

Name	ME	Density	Category	Brief description	Ref.
MaxEnt	✓	Dense	Deterministic	Maximizes Shannon entropy on network entries by constraining marginals	134,253
IPF	✓	Tunable	Deterministic	Minimizes the KL divergence from MaxEnt	254
Copula approach	×	Dense	Deterministic	Generates a network via a copula function of the marginals	255
MECAPM	✓	Dense	Probabilistic	Constrains matrix entries to match, on average, MaxEnt values	256
Drehmann and Tarashev	✓	Tunable	Probabilistic	Randomly perturbs the MaxEnt reconstruction	257
Mastromatteo et al.	✓	Tunable	Probabilistic	Explores the space of network structures with the message-passing algorithm	258
Moussa	✓	Tunable	Probabilistic	Implements IPF on non-trivial topologies	259
Fitness-induced ERG	✓	Tunable	Probabilistic	Uses the fitness ansatz to inform an ERG model	17,215
Gandy and Veraart	×	Tunable	Probabilistic	Implements an adjustable Bayesian reconstruction	209
Montagna and Lux	×	Tunable	Probabilistic	Assumes ad hoc connection probabilities depending on marginals	260
Hałaj and Kok	×	Sparse	Probabilistic	Uses external information to define a (geographical) probability map	261
Minimum-density	×	Sparse	Probabilistic	Minimizes the network density while satisfying the marginals	208

'ME' indicates whether the method is based on maximum entropy, 'Density' denotes the density of the reconstructed network and 'Category' is either deterministic or probabilistic, depending on whether the method generates a single network instance or an ensemble. ERG, exponential random graph; IPF, iterative proportional fitting; KL, Kullback–Leibler; MECAPM, maximum entropy capital asset pricing model.

order to overcome the lack of binary information, perform a preliminary estimation of the node degrees using the fitness model[216]. This idea of estimating the weighted network structure conditional on the preliminary estimation of the binary structure is properly formalized by maximization of the conditional Shannon entropy, using as constraints the weighted available information and as prior information the topological structure of the network — be it empirical or inferred[207].

Belonging to this class of two-step maximum entropy models, the density-corrected gravity method has been found to systematically outperform competing reconstruction methods by generating networks similar to the empirical ones in terms of topological and systemic risk properties. This evidence comes from four independent 'horse races' or comparative studies, carried out by researchers in academia and central banks[217–220]. One key ingredient for the effectiveness of the method is the ability to reproduce the density of the network to be reconstructed. However, such density can also be tuned, thus allowing the method to generate extreme scenarios of very dense or very sparse networks, analogously to fully connected[134] and minimum density[208] methods. However, as discussed in the previous section, it is not clear what the relationship is between density and stability of a financial network, hence, identifying the best-case and worst-case network structure a priori is not possible in general.

For a maximum entropy method, the agreement between the reconstructed network and empirical data requires certain conditions, as highlighted by the information-theoretical formulation of statistical mechanics. First, the network to be reconstructed is close to the average (equilibrium) configuration of the canonical ensemble defined by the imposed constraints (see FIG. 3 for a schematic of this concept). Second, the network evolution, if supposed to be driven by the evolution of the constraints themselves, is quasi-stationary[199]. The consequence is that the network at hand is characterized by smooth structural changes rather than abrupt transitions. Whereas smooth changes characterizing quasi-equilibrium networks can generally be controlled for, it is not possible to do so in the case of abrupt transitions, which characterize non-stationary networks.

As illustrative examples, we consider two systems, an economic and a financial one. The first example is the International Trade Network (ITN), the nodes of which represent world economies and links represent export relationships between them[221–226]. Many properties of the ITN change considerably over time (for instance, the total number of nodes doubled from 1950 to 2000 (REF.[227]), as countries gained independence), hence, this network is an ideal test bench for its (out-of-) equilibrium character. Indeed, for each time slice of the data, the binary structure of the ITN is accurately reproduced by maximum entropy models[199]. This happens despite the imposed constraints varying considerably across the time span of the dataset (presumably because of exogenous effects, such as the creation or recognition of new countries): deviations from the model expectations are bounded and systematic, thus making the ITN a quasi-equilibrium network. The second example is the Dutch Interbank Network (DIN)[59]. Unlike the ITN, the DIN is compatible with maximum entropy models only during certain time periods (in particular, far away from financial crises), hinting at its out-of-equilibrium character. It is, therefore, natural to question the usefulness of the maximum entropy formalism in the case of non-stationary networks. The next section is devoted to this question.

Networks out of equilibrium and validation. As clarified in the previous section, achieving a successful reconstruction requires a network to be at the (information-theoretical or, equivalently, thermodynamic) equilibrium implied by the imposed constraints. When full information on the empirical network is available, the entropy-based framework can be used to build null models and to check whether an empirical network is compatible with them. Unlike the reconstruction task, in which the constraints are defined by the available information, in this case, the imposed constraints are assumed to be the only explanatory variables for the network at hand (this is precisely the null hypothesis of any maximum entropy model). Therefore, if the empirical network is described accurately by the null model, the null hypothesis it embodies cannot be rejected. When this does not happen, it means that the chosen constraints do not lead to an exhaustive description of the empirical network (FIG. 3). In this sense, the empirical network is 'out of equilibrium'.

An example may be useful to clarify the discussion. We consider again the DIN[59], for which full information is available. For this network, the fraction of reciprocated links (that is, the number of bank pairs that lend money to each other) drops dramatically at the onset of the 2007–2008 financial crisis. Such a measure could be considered as a proxy of how much banks (do not) trust each other and hedge against each other's perceived risk by creating contracts pointing in opposite directions. In a sense, the change in the trend pointed to the erosion of such trust during the crisis. The picture changes by using an entropy-based null model (defined by constraining the number of borrowers and lenders of each bank) to highlight the properties that are out of equilibrium or, in other words, not compatible with the model itself. Significant deviations of the empirical reciprocity from the null model are observed in correspondence with the crisis — but also in the preceding 4 years. This result indicates that the system was already experiencing a decreasing phase, in terms of trust between banks, a few years before the crisis. This pattern emerges only after comparison with an entropy-based null model (FIG. 3). Such patterns can be considered as early-warning signals: before a drastic change in the structure of the network, it is still possible to detect smooth changes, which can be highlighted by observing the disagreement between a proper null model and the real system. Similar patterns were observed for other quantities, such as the cyclic motif (three banks involved in a cycling lending pattern), whose presence increases even before the pre-crisis period (FIG. 3), becoming statistically significant before the crisis hits.

Density
The fraction of possible connections that are actually realized in a network.

The same framework has been used to analyse the patterns of common asset holdings by financial institutions[228], with the idea that the portfolio overlaps that are not compatible with an entropy-based null model are carrying the highest risk for liquidation in fire sales. In this case, the null model is built by constraining the diversification of portfolios and the number of investors of each security, in order to account for the heterogeneity of actors in the system[229,230]. The analysis reveals that portfolio similarity significantly increases long before the 2007–2008 financial crisis, and peaked at its onset, in a way not compatible with the null model. In other words, the properties of the system are not explainable just by looking at the heterogeneity of portfolios and securities: the system is strongly out of equilibrium and the observation of significant portfolio overlaps is carrying extra information.

Null models are routinely used as benchmarks to extract significant information for a variety of systems. The entropy-based null models discussed above are characterized by soft constraints. That is, the constraints are satisfied on average over a suitably specified ensemble of networks. Alternatively, one can build null models characterized by hard constraints. In this case, the corresponding ensemble contains only the networks that individually satisfy the constraints. Null models with soft constraints correspond to the canonical ensemble, which is used to describe a physical system in which the average energy is specified. By contrast, null models with hard constraints correspond to the microcanonical ensemble, which is used to describe a physical system in which the energy is specified and does not fluctuate. As in traditional statistical physics, microcanonical models are typically much harder to approach analytically and, thus, it is often necessary to resort to fixed-point approximations or numerical sampling[97,231–239] (see also REF.[12] for more details).

Conclusions and perspectives

In the aftermath of the 2007–2008 financial crisis, the policy community and academia became widely aware that understanding and managing risk in the financial system required modelling it in terms of financial networks. In particular, recognition of the importance of network effects has led to key conceptual developments in policy. Microprudential regulation (in the finance policy jargon, an approach to regulation focusing on banks individually) has since been complemented by macroprudential regulation, which looks at the financial system as a whole (that is, as a network of financial institutions[154,240,241]) and seeks to limit the impact of financial shocks to the real economy. The latter approach recognizes that interconnectedness (modelled through networks) can have procyclical impact (that is, serve as positive feedback) on asset prices or, in other words, it can amplify risk.

Over the past decade, financial network models have been increasingly used by institutions, including: the European Central Bank to assess systemic risk[20]; the European Systemic Risk Board to characterize the derivative market[21,109] and the network of insurers[242]; the Office of Financial Research[194]; and the Bank of England to capture feedback mechanisms in stress tests[22] that arise from solvency contagion[243,244], funding contagion or overlapping portfolios[244]. Furthermore, the Bank for International Settlements (the institution coordinating banking regulation worldwide) has included interconnectedness among the criteria used to identify systemically important banks[245].

We now briefly discuss some of the key open challenges. First, despite increasing efforts to build models with multiple channels of financial contagion (as discussed above), there are general aspects that have not been sufficiently explored. For example, it is often unclear how to integrate different contagion channels that operate at the same timescale. Furthermore, models with both amplifying and dampening mechanisms tend to be less amenable to analytical treatment.

Second, whereas many models consider the relationships between financial actors as static, in reality, such relationships might change, sometimes suddenly. In a sense, most models make the implicit assumption that the timescale over which relationships change is much longer than the characteristic timescale of the model dynamics. Further empirical research is needed to establish when this assumption holds. When it is not the case, one needs to develop models that also account for how those relationships might form[246–248] and dissolve.

Third, many models are calibrated using a very small fraction of the vast data that financial markets generate daily. Indeed, regulatory data are often reported quarterly or annually, allowing only for the analysis of temporal snapshots that could be too far apart to detect rapid buildups of risk. We expect this issue to be partially alleviated by the increased availability of transaction-level datasets that capture market activity at the most granular timescale. Crucially, most datasets cover only individual jurisdictions, meaning that a comprehensive analysis of the global financial network remains out of reach.

Fourth, financial networks often include only one kind of financial institution (typically banks) and are decoupled from the rest of the economy. However, different kinds of financial institutions can play different roles in financial markets, which can only be accounted for in system-wide models with heterogeneous institutions[186]. The interactions between the financial system and the rest of the economy create a two-way feedback loop. Although there have been attempts to model those feedbacks with bank–firm networks[249–251], a more systematic approach would require embedding financial networks in a fully fledged macroeconomic model.

Finally, there is growing awareness of climate-related financial risks and of the key role of financial network models for climate stress tests[252]. A transition to a low-carbon economy can have implications for financial stability, if it is delayed and occurs in a disorderly manner. In fact, a substantial devaluation of carbon-intensive assets — which become 'stranded' — can impact the balance sheet of institutions holding those assets. Therefore, mapping the network of exposures of financial institutions to different sectors can help to identify and mitigate those risks.

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