T-61.3040 Statistical Signal Modeling Lendasse, Miche, Eirola

Exercise 10, December 1, 2011

1. Determine the autocorrelation functions of the signals

$$\begin{aligned} x(t) &= 0.1\sin(0.5t + \phi_x) \\ y(t) &= 0.2\sin(0.2t + \phi_y) \\ z(t) &= x(t) + y(t) \end{aligned}$$

   when the phase angles ϕ_x ja ϕ_y are independent random variables that are evenly distributed on the interval $[0, 2\pi]$.

2. For the complex sinusoidal signal $x(t) = A\exp(j\omega t) + v(t)$ it is known that $r_x(0) = 2$ and $r_x(1) = j$, and that $v(t)$ is white noise.

   Determine the signal power $|A|^2$, the frequency ω of the signal, and the variance of the noise $v(t)$.

3. $x(t)$ is a real-valued signal. Assume that the autocorrelations $r_x(0) = 1$, $r_x(1) = c$, and $r_x(2) = 0$ are known.

   (a) Estimate the power spectrum using the maximum entropy method (MEM).

   (b) What is the power spectrum if you assume that $r_x(k) = 0$ for all $k > 2$?

4. For the signal in the previous problem with autocorrelations $r_x(0) = 1$, $r_x(1) = c$, $r_x(2) = 0$, calculate an estimate of the power spectrum using the minimum variance method.

   Hint: The method requires a matrix inversion which could be calculated by hand using the Levinson recursion, but here you can use some software – e.g., Wolfram Alpha – to find the inverse.

5. (Bonus point exercise)

   The process

$$x(n) = \sum_{i=1}^{p} A_i e^{j\omega_i n} + v(n)$$

   contains p sinusoids with frequencies $\omega_k = 2\pi k/M$. The noise $v(n)$ is white. Calculate the eigenvalues of the $M \times M$ autocorrelation matrix $\mathbf{R}_x$ of the process $x(n)$. Calculate also the inner products $\mathbf{v}_i^H \mathbf{v}_k$ where $\mathbf{v}_i$ is the eigenvector of $\mathbf{R}_x$ corresponding to the eigenvalue λ_i.

   Hint: the sum formula $\sum_{l=0}^{M-1} q^l = \frac{1-q^M}{1-q}$ for the geometric series is helpful when calculating the inner product.