T-61.3040 Statistical Signal Modeling Lendasse, Miche, Eirola

Exercise 6, October 20, 2011

1. Find the Padé approximation of second order to a signal $x(n)$ that is given by

$$x(n) = 2, 1, 0, -1, 0, 1, 0, -1, 0, 1, ...$$

i.e., $x(0) = 2$, $x(1) = 1$, $x(2) = 0$, and so on. In other words, fit the coefficients $b(0)$, $b(1)$, $b(2)$, $a(1)$, and $a(2)$ of an ARMA(2,2)-model so that its impulse response matches the first values of the sequence.

2. Consider linear prediction of the values of the signal $x(n)$ using the past values of the signal, i.e.

$$\hat{x}(n) = \sum_l a_l x(n - k_l).$$

The signal may be approximately periodic. 11 first values of its autocorrelation are

$$[r_x(0) \cdots r_x(10)] = [1.0, 0.4, 0.4, 0.3, 0.2, 0.9, 0.4, 0.4, 0.2, 0.1, 0.7].$$

(a) Consider the predictor

$$\hat{x}(n) = a_5 x(n-5) + a_{10} x(n-10)$$

that tries to take into account the periodicity of the signal. The error made by the predictor is $e(n) = x(n) - \hat{x}(n)$. The coefficients of the predictor are chosen so that the mean square error $E[e(n)e^*(n)]$ is minimised. What is this minimum error?

(b) Compare the error in (a) with the error when the predictor is

$$\hat{x}(n) = a_1 x(n-1) + a_2 x(n-2).$$

(c) Let's consider the predictors

$$\hat{x}(n) = a_N x(n - N)$$

where a_N and N are parameters of the predictor (N is one of the values $1, 2, \ldots, 10$). Determine the parameter values that minimise the modelling error and compare the error with the previous cases.

3. The autocorrelations of an AR(2) process $x(n)$ are assumed to have values

$$r_x(0) = 2, \quad r_x(1) = 1 - j \quad \text{and} \quad r_x(2) = \frac{2}{3} - j$$

where j is the imaginary unit. Using the Yule-Walker equations, find the parameters $a(1)$, $a(2)$ and $b(0)$ of the process.
NB: the inverse of a complex 2×2 matrix can be found using the same formula as in the real-valued case.

4. Assume that you observe a real valued process $x(n)$ and you estimate the following autocorrelations:

$$r_x(0) = 2,\ r_x(1) = 0.8,\ r_x(2) = -0.1$$

Find an second order FIR filter that approximately filters the process $x(n)$ into the process $y(n)$ whose power spectrum is $P_y(\exp(j\omega)) = 1$ at all ω.

5. (Bonus point exercise)

   (a) For an ARMA(1,1)-model, find an expression for the first two values, $h(0)$ and $h(1)$, of its impulse response in terms of the parameters $b(0)$, $b(1)$, and $a(1)$.

   (b) Using part (a) and the Yule-Walker equations, find the parameters for an ARMA(1,1)-process with the autocorrelation

$$r_x(k) = 0.1(0.8)^{|k|} + 0.1\delta(k).$$