Charkha Protocol Examples

The Charkha white paper captures the Charkha protocol as a collection of formulas. These formulas are abstract; to help implementors understand how the formulas work, this document contains worked examples of calculations using them. Specifically, it covers the interest rate model and liquidation.

Interest Rate Model

First, let's understand the interest rate model. Let's create a brand-new money market for KDA in which one user has pledged 10,000 KDA as a supply and another has borrowed 6,000 KDA, leaving 4,000 KDA available to borrow.

$Cash_{KDA} = 4,000$
$Borrows_{KDA} = 6,000$
$Reserves_{KDA} = 0$

All markets have a set of constants used for interest rate calculations, which are subject to change according to the governance mechanism. Markets begin with these values:

$BaseRate_{KDA} = 0.025$
$Multiplier_{KDA} = 0.2$
$ReserveFactor_{KDA} = 0.01$

All markets begin with a 50:1 exchange rate for cTokens and an interest rate index of 1:

$ExchangeRate_{KDA,0} = 50$
$Index_{KDA,0} = 1$

This means that the initial supplying user has 200 cKDA, which is the total supply of cTokens for this market.

$cTokenSupply_{KDA} = 200$

We have our initial market! Now, the first transaction: a new user borrows 3,000 KDA. How does this affect the market? Let's work through it.

1. Prior to the Transaction

When a new transaction occurs we begin by compounding interest over the prior period, which does not include the transaction about to occur. To calculate interest we should begin with utilization. At the time of the first borrow the market has 60% utilization:

$$U_{KDA} = Borrows_{KDA}/(Cash_{KDA} + Borrows_{KDA})$$
$$U_{KDA} = 6,000/(4,000 + 6,000)$$
$$U_{KDA} = 0.6$$

Therefore, the borrow interest rate is 14.5% over the prior period:

$$BorrowInterestRate_{KDA} = BaseRate_{KDA} + U_{KDA} * Multiplier_{KDA}$$
$$BorrowInterestRate_{KDA} = 0.025 + 0.6 * 0.2$$
$$BorrowInterestRate_{KDA} = 0.145$$

And we can derive that our supply interest rate is 8.6%:

$$SupplyInterestRate_{KDA} = BorrowInterestRate_{KDA} * U_{KDA} * (1 - ReserveFactor_{KDA})$$
$$SupplyInterestRate_{KDA} = 0.145 * 0.6 * (1 - 0.01)$$
$$SupplyInterestRate_{KDA} = 0.08613$$

We can use the borrow interest rate to record compounded interest in our interest rate index. We'll assume that every transaction has the duration of one block on a Chainweb chain, which is about 30 seconds and therefore about 1,051,920 per year:

$$Index_{KDA,1} = Index_{KDA,0} * (1 + 0.145 * (1/1051920))$$
$$Index_{KDA,1} = 1 * 1.000000138$$
$$Index_{KDA,1} = 1.000000138$$

We can also update our total borrowing balance to reflect interest accrued:

$$Borrows_{KDA,1} = Borrows_{KDA,0} * (1 + 0.145 * (1/1051920))$$
$$Borrows_{KDA,1} = 6,000 * 1.000000138$$
$$Borrows_{KDA,1} = 6,000.000828$$

And a portion of the accrued interest is retained for the platform reserves:

$$Reserves_{KDA,1} = Reserves_{KDA,0} + Borrows_{KDA,0} * (0.145 * (1/1051920) * 0.01)$$
$$Reserves_{KDA,1} = 0 + 6,000 * 0.000000001$$
$$Reserves_{KDA,1} = 0.000006$$

And our exchange rate reflects the compounded interest:

$$ExchangeRate_{KDA} = (Cash_{KDA} + Borrows_{KDA} - Reserves_{KDA})/cTokenSupply_{KDA}$$
$$ExchangeRate_{KDA} = (4,000 + 6,000.000828 - 0.000006)/200$$
$$ExchangeRate_{KDA} = 50.00000411$$

If we had supplied the initial 10,000 KDA, then according to the interest rates we would now have 10,000.000822 KDA after 30 seconds.

2. After the Transaction

With these values updated, we can move on to process our transaction. A new user has borrowed 1,000 KDA. How does that change our interest rates? First, let's establish our cash and borrows again, adding in the new 3,000 KDA borrow. Remember, the cash is the total supplied amount minus what's been borrowed.

$$Cash_{KDA} = 999.999172$$
$$Borrows_{KDA} = 9,000.000828$$

We can use this to determine that the new utilization ratio for KDA is 90%, an increase of 30%:

$$U_{KDA} = Borrows_{KDA}/(Cash_{KDA} + Borrows_{KDA})$$
$$U_{KDA} = 9,000.000828/(999.999172 + 9,000.000828)$$
$$U_{KDA} = 0.9$$

This raises the borrow interest rate from 14.5% to 20.5%, an increase of 6%:

$$BorrowInterestRate_{KDA} = BaseRate_{KDA} + U_{KDA} * Multiplier_{KDA}$$
$$BorrowInterestRate_{KDA} = 0.025 + 0.9 * 0.2$$
$$BorrowInterestRate_{KDA} = 0.205$$

The supply interest rate rises even faster, jumping from 8.6% to 18.3%, an increase of nearly 10%:

$$SupplyInterestRate_{KDA} = BorrowInterestRate_{KDA} * U_{KDA} * (1 - ReserveFactor_{KDA})$$
$$SupplyInterestRate_{KDA} = 0.205 * 0.9 * (1 - 0.01)$$
$$SupplyInterestRate_{KDA} = 0.182655$$

These rates will be applied on the next protocol transaction.

Liquidation

Next, let's understand what happens when a user exceeds their borrowing capacity and becomes eligible for liquidation. For the sake of these example calculations we will ignore the effects of interest, assuming that all events happen in the span of a few blocks and interest is negligible; in the real world, interest can have significant effects.

First, the user deposits 5 KETH at a price of $1,200 per ETH. Let's say that the Kadena-wrapped Ethereum market has the following exchange rate and collateral factors:

$$ExchangeRate_{KETH} = 50$$
$$CollateralFactor_{KETH} = 0.8$$

When the user deposits their 5 KETH they therefore receive 0.1 cKETH. We can use this to determine their borrowing capacity:

$$BorrowingCapacity_{KETH} = cTokenBalance_{KETH} * CollateralFactor_{KETH}$$
$$BorrowingCapacity_{KETH} = 0.1 * 0.8$$
$$BorrowingCapacity_{KETH} = 0.08$$

Therefore, they can borrow against up to 0.08 cKETH from their KETH collateral. We can normalize this value so it can be used to judge how much the user can borrow in other markets:

$$NormBorrowingCapacity_{KETH} = BorrowingCapacity_{KETH} * ExchangeRate_{KETH} * Price_{KETH}$$
$$NormBorrowingCapacity_{KETH} = 0.08 * 50 * 1200$$
$$NormBorrowingCapacity_{KETH} = 4800$$

Initially, the user can borrow up to $4,800 USD worth of other currencies, though their borrowing capacity will grow as they earn interest on their cKETH and will fluctuate with the price of KETH.

Next, the user borrows $4,000 USD worth of KDA from the protocol at a price of $1 per KDA. Their borrowing capacity remains $4,800 USD but they have now used $4,000 USD of it. Immediately after borrowing their KDA, the token experiences a sudden increase in value to $1.25 per KDA.

Uh oh! Suddenly, this user has borrowed $4,000 USD worth of KDA, but it has increased in value to $5,000 USD worth of KDA! They have exceeded their borrowing capacity and are eligible for liquidation. Let's see what happens.

First, a liquidator can pull the trigger to liquidate 25% of the user's KDA borrow in exchange for the equivalent amount of cKETH, plus an additional 5% fee. They can therefore send $1,250 worth of KDA to the protocol and receive $1312.50 worth of the user's cKETH. At $1200/KETH and a 50:1 exchange rate, that means the liquidator receives 0.022 cKETH from the user.

Now what happens? The user's borrow has been reduced from $5,000 USD to $3750. Their collateral has been reduced from 0.1 to 0.079 cKETH, which means their new borrowing capacity is $3,792.

$$NormBorrowingCapacity_{KETH} = (0.079 * 0.8) * 50 * 1200$$
$$NormBorrowingCapacity_{KETH} = 3792$$

They can narrowly avoid further liquidation, but they're not far from another liquidation event.