complex-exponentials

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0.1 Applied EMT

0.1.1 Dennies Bor

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# Question 1
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First, getting B field at z:

$$\oint B(\vec{z}) \cdot d\vec{l} = \mu_0 I_{enclosed}$$

$$B_1 l + B_1 l = \mu_0 I$$

$$2B_1 l = \mu_0 I$$

$$B(z) = \frac{\mu_0 I}{2l}$$

For inductance directly from flux:

$$\Phi_m = BA = \frac{\mu_0 I}{2l}(hw)$$

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 hw}{2l}$$

Using energy density method to verify:

$$\mu_0 L I^2 = \int B^2 dv$$

$$\mu_0 L I^2 = (\frac{\mu_0 I}{2l})^2 (l \times h \times w)$$

$$\mu_0 L I^2 = \frac{\mu_0^2 I^2 hw}{4l}$$

$$L = \frac{\mu_0 hw}{4l}$$

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# Question 2
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The magnetic flux Φ through a surface is defined as:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{A}$$

where $d\mathbf{A}$ along the length of the cabl and perpendicular to $\mathbf{B}$:

$$dA = l ds$$

The flux through the region between a and b is:

$$\Phi = \int_a^b B(s)lds$$

Substituting $B(s) = \frac{\mu_0 I}{2\pi s}$:

$$\Phi = \int_a^b \frac{\mu_0 I}{2\pi s}lds$$

$$\Phi = \frac{\mu_0 Il}{2\pi}\ln(\frac{b}{a})$$

For inductance L ### 1. Using computed Φ

$$L = \frac{\Phi}{I}$$

Substituting Φ:

$$L = \frac{\mu_0 l}{2\pi}\ln(\frac{b}{a})$$

0.1.2 2. Verify with energy density

$$\mu_0 LI^2 = \int B^2 dv$$

Substitute $B = \frac{\mu_0 I}{2\pi s}$ and $dv = l, ds, d\theta$ (volume = length $\times$ area element in polar):

$$\mu_0 LI^2 = \int_0^{2\pi}\int_a^b (\frac{\mu_0 I}{2\pi s})^2 l, ds, d\theta$$

Now evaluate s integral:

$$\mu_0 LI^2 = \frac{\mu_0^2 I^2 l}{2\pi}[-\frac{1}{s}]_a^b$$

$$\mu_0 LI^2 = \frac{\mu_0^2 I^2 l}{2\pi}(\frac{1}{a} - \frac{1}{b})$$

Solve for L:

$$L = \frac{\mu_0 l}{2\pi}(\frac{1}{a} - \frac{1}{b})$$

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[ ]: # Question 3
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From Euler's identity of postive and negative phases

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

For sin(+):

$$\sin(\alpha+\beta) = \frac{e^{i(\alpha+\beta)} - e^{-i(\alpha+\beta)}}{2i}$$

Expand:

$$\sin(\alpha+\beta) = \frac{e^{i\alpha}e^{i\beta} - e^{-i\alpha}e^{-i\beta}}{2i}$$

$$= \frac{[\cos(\alpha) + i\sin(\alpha)][\cos(\beta) + i\sin(\beta)] - [\cos(\alpha) - i\sin(\alpha)][\cos(\beta) - i\sin(\beta)]}{2i}$$

Multiply brackets:

$$= \frac{[\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) + i(\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta))] - [\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) - i(\sin(\alpha)\cos(\beta}{2i}$$

Simplify:

$$= \frac{2i[\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)]}{2i}$$

$\therefore$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

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[5]: ## Question 4
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$$V(z,t) = \cos(\omega t - \beta z) + a\cos(\omega t + \beta z)$$

Write each cosine in terms of exponentials:

$$\cos(\omega t - \beta z) = \frac{e^{j(\omega t - \beta z)} + e^{-j(\omega t - \beta z)}}{2}$$

$$\cos(\omega t + \beta z) = \frac{e^{j(\omega t + \beta z)} + e^{-j(\omega t + \beta z)}}{2}$$

Substituting:

$$V(z,t) = \frac{e^{j(\omega t - \beta z)} + e^{-j(\omega t - \beta z)}}{2} + a\frac{e^{j(\omega t + \beta z)} + e^{-j(\omega t + \beta z)}}{2}$$

Expand:

$$V(z,t) = \frac{e^{j\omega t}e^{-j\beta z} + e^{-j\omega t}e^{j\beta z}}{2} + a\frac{e^{j\omega t}e^{j\beta z} + e^{-j\omega t}e^{-j\beta z}}{2}$$

Collect $e^{j\omega t}$ and $e^{-j\omega t}$ terms:

$$V(z,t) = \frac{e^{j\omega t}(e^{-j\beta z} + ae^{j\beta z}) + e^{-j\omega t}(e^{j\beta z} + ae^{-j\beta z})}{2}$$

First bracket expansion:

$$e^{j\omega t}(e^{-j\beta z} + ae^{j\beta z}) = [\cos(\omega t) + j\sin(\omega t)][\cos(\beta z) - j\sin(\beta z) + a(\cos(\beta z) + j\sin(\beta z))]$$

$$= [\cos(\omega t) + j\sin(\omega t)][\cos(\beta z)(1+a) + j\sin(\beta z)(a-1)]$$

$$= \cos(\omega t)\cos(\beta z)(1+a) + j\cos(\omega t)\sin(\beta z)(a-1) + j\sin(\omega t)\cos(\beta z)(1+a) - \sin(\omega t)\sin(\beta z)(a-1)$$

Second bracket expansion:

$$e^{-j\omega t}(e^{j\beta z} + ae^{-j\beta z}) = [\cos(\omega t) - j\sin(\omega t)][\cos(\beta z)(1+a) + j\sin(\beta z)(1-a)]$$

$$= \cos(\omega t)\cos(\beta z)(1+a) + j\cos(\omega t)\sin(\beta z)(1-a) - j\sin(\omega t)\cos(\beta z)(1+a) + \sin(\omega t)\sin(\beta z)(1-a)$$

Sum both and divide by 2:

$$V(z,t) = \frac{1}{2}[2\cos(\omega t)\cos(\beta z)(1+a) + 2\sin(\omega t)\sin(\beta z)(1-a)]$$
$$= (1+a)\cos(\omega t)\cos(\beta z) + (1-a)\sin(\omega t)\sin(\beta z)$$

Therefore:

$$A = 1 + a$$
$$B = 1 - a$$

[3]:
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## Qestion 4
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0.2 Using Euler Identity

$$A_1\cos(\theta+\delta_1) + A_2\cos(\theta+\delta_2)$$

In exponential form:

$$A_1 \mathrm{Re} e^{j(\theta+\delta_1)} + A_2 \mathrm{Re} e^{j(\theta+\delta_2)}$$
$$= \mathrm{Re} A_1 e^{j\theta} e^{j\delta_1} + A_2 e^{j\theta} e^{j\delta_2}$$
$$= \mathrm{Re} e^{j\theta}(A_1 e^{j\delta_1} + A_2 e^{j\delta_2})$$

Let $A_1 e^{j\delta_1} + A_2 e^{j\delta_2} = a + jb$, and by expanding leads to

$$a = A_1\cos(\delta_1) + A_2\cos(\delta_2)$$

$$b = A_1\sin(\delta_1) + A_2\sin(\delta_2)$$

Then:

$$= \mathrm{Re} e^{j\theta}(a+jb)$$
$$= \mathrm{Re}(a+jb)(\cos(\theta) + j\sin(\theta))$$
$$= a\cos(\theta) - b\sin(\theta)$$
$$= \sqrt{a^2+b^2}\cos(\theta + \arctan(b/a))$$

Where:

$$A = \sqrt{a^2+b^2}$$
$$\delta = \arctan(b/a)$$

0.3 2. Using Trig Identies

Original equation:

$$A_1 \cos(\theta + \delta_1) + A_2 \cos(\theta + \delta_2)$$

Apply identity to each term:

$$A_1[\cos(\theta)\cos(\delta_1) - \sin(\theta)\sin(\delta_1)] + A_2[\cos(\theta)\cos(\delta_2) - \sin(\theta)\sin(\delta_2)]$$

Group $\cos(\theta) and \sin(\theta)$ terms:

$$\cos(\theta)[A_1 \cos(\delta_1) + A_2 \cos(\delta_2)] - \sin(\theta)[A_1 \sin(\delta_1) + A_2 \sin(\delta_2)]$$

Let:

$$a = A_1 \cos(\delta_1) + A_2 \cos(\delta_2)$$
$$b = A_1 \sin(\delta_1) + A_2 \sin(\delta_2)$$

Then:

$$= a \cos(\theta) - b \sin(\theta)$$
$$= \sqrt{a^2 + b^2} \cos(\theta + \arctan(b/a))$$

Where:

$$A = \sqrt{a^2 + b^2}$$
$$\delta = \arctan(b/a)$$

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# End
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