Tex file link

1 Question 6.2

1.1 Initial Conditions

At $t = 0$:

$$\vec{r}_0 = (x_0, 0, 0)$$
$$\vec{v}_0 = (v_{x0}, 0, 0)$$
$$\theta = 0 \text{ (particle starts on x-axis)}$$

1.2 Magnetic Field

The dipolar magnetic field is given by:

$$\vec{B} = \frac{B_0}{r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$$

At $\theta = 0$, $\vec{B}$ points in the z-direction:

$$\vec{B} = \frac{B_0}{x_0^3}(2\hat{z})$$

1.3 Lorentz Force Equation

The Lorentz force is:

$$\vec{F} = q\vec{v} \times \vec{B}$$

At $t = 0$, the cross-product gives:

$$\vec{v} \times \vec{B} = (v_{x0}, 0, 0) \times (0, 0, \frac{2B_0}{x_0^3}) = (0, -\frac{2B_0 v_{x0}}{x_0^3}, 0)$$

This force is in the negative y-direction.

1.4 Forward Euler Method

For the numerical update, using the Forward Euler method:

$$v_x(t + \Delta t) = v_x(t) + \frac{q}{m}[\vec{v}(t) \times \vec{B}(\vec{r}(t))]_x \Delta t$$
$$v_y(t + \Delta t) = v_y(t) + \frac{q}{m}[\vec{v}(t) \times \vec{B}(\vec{r}(t))]_y \Delta t$$
$$x(t + \Delta t) = x(t) + v_x(t)\Delta t$$
$$y(t + \Delta t) = y(t) + v_y(t)\Delta t$$

1.5 Solution for $t = \Delta t$

For $t = \Delta t$:

$$v_x(\Delta t) = v_{x0}$$
$$v_y(\Delta t) = 0 + \frac{q}{m}\left(-\frac{2B_0 v_{x0}}{x_0^3}\right)\Delta t$$
$$x(\Delta t) = x_0 + v_{x0}\Delta t$$
$$y(\Delta t) = 0$$

1.6 Solution for $t = 2\Delta t$

For $t = 2\Delta t$:

$$
\begin{aligned}
v_x(2\Delta t) &= v_{x0} + \frac{q}{m}\left(v_y(\Delta t)\cdot\left(-\frac{2B_0}{x_0^3}\right)\right)\Delta t \\
v_y(2\Delta t) &= v_y(\Delta t) + \frac{q}{m}\frac{2B_0 v_{x0}}{x_0^3}\Delta t \\
&= \frac{q}{m}\frac{4B_0 v_{x0}}{x_0^3}\Delta t \\
x(2\Delta t) &= x_0 + 2v_{x0}\Delta t \\
y(2\Delta t) &= \frac{q}{m}\frac{2B_0 v_{x0}}{x_0^3}\Delta t^2
\end{aligned}
$$

2 Ampere's Law

The figure shows the cross-section of two large square planes separated by a distance t. A surface current $\mathbf{K_2} = K_0\hat{x}$ flows on the bottom plane (out of the page), and the top plane has a surface current $\mathbf{K_1} = -K_0\hat{x}$ (into the page). Here, K_0 has units of Ampere per meter.
We aim to find the magnetic field using Ampere's law for all regions along z. The planes are infinite in the x and z directions, and the problem is symmetric about $z = 0$.

2.1 Solution

(a) Symmetry considerations:

- Since the current is flowing in the x-direction, the magnetic field will be in the z-direction.
- The field depends only on z, as the planes are infinite in the x and z directions.
- The problem is symmetric about the $z = 0$ plane, so the magnetic field must reflect this symmetry.

Therefore, the magnetic field can be written as:

$$\mathbf{B}(z) = B_z(z)\hat{z}$$

(b) Applying Ampere's law: Consider an Amperian loop in the y-z plane with width w. We apply Ampere's law to calculate the magnetic field:

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

where I_{enc} is the enclosed current.

(c) Calculating the magnetic field: - The magnetic field due to a single current sheet is given by:

$$B_z = \pm\frac{\mu_0 K_0}{2}$$

- Now, we examine the contributions from each plane for different regions along z:

- For $z > t/2$: - The top plane ($\mathbf{K_1} = -K_0\hat{x}$) contributes:

  $$B_{z,1}(z > t/2) = -\frac{\mu_0 K_0}{2}$$

  - The bottom plane ($\mathbf{K_2} = K_0\hat{x}$) contributes:

  $$B_{z,2}(z > t/2) = +\frac{\mu_0 K_0}{2}$$

- Net field:

$$B_z(z > t/2) = B_{z,1} + B_{z,2} = 0$$

- For $-t/2 < z < t/2$: - The top plane contributes:

$$B_{z,1}(z) = +\frac{\mu_0 K_0}{2}$$

- The bottom plane contributes:

$$B_{z,2}(z) = +\frac{\mu_0 K_0}{2}$$

- Net field:

$$B_z(-t/2 < z < t/2) = \frac{\mu_0 K_0}{2} + \frac{\mu_0 K_0}{2} = \mu_0 K_0$$

- For $z < -t/2$: - The top plane contributes:

$$B_{z,1}(z < -t/2) = +\frac{\mu_0 K_0}{2}$$

- The bottom plane contributes:

$$B_{z,2}(z < -t/2) = -\frac{\mu_0 K_0}{2}$$

- Net field:

$$B_z(z < -t/2) = B_{z,1} + B_{z,2} = 0$$

(d) Justification:

- For $z > t/2$: The contributions from the top plane $(-\mu_0 K_0/2)$ and the bottom plane $(+\mu_0 K_0/2)$ cancel each other out, resulting in zero net magnetic field.
- For $-t/2 < z < t/2$: Both planes contribute equally with $\mu_0 K_0/2$, so the total field is $\mu_0 K_0$.
- For $z < -t/2$: The contributions from the top plane $(+\mu_0 K_0/2)$ and the bottom plane $(-\mu_0 K_0/2)$ cancel out, resulting in zero net field.