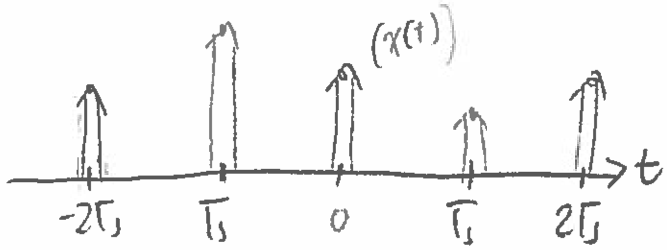
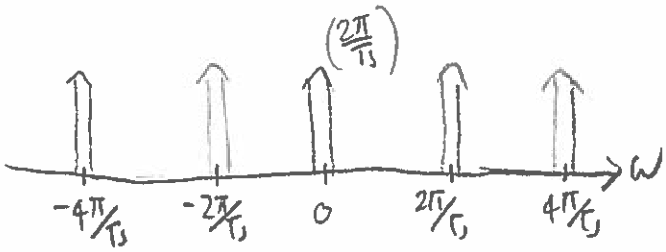
Dennis Chen PS08

1a. Sketch $x_p(t)$
(sampled original signal)

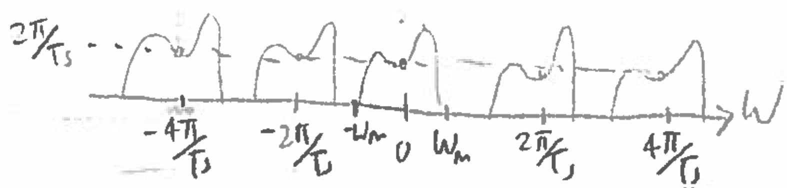


b. Sketch $P(w)$



$\sqrt{\frac{1}{LC} - \frac{1}{2}\frac{R}{L}}$

c. Sketch $X_p(w)$

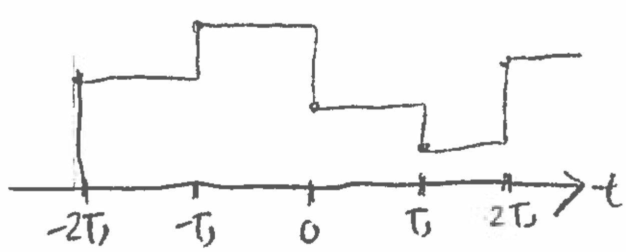


d. $W_m \leq \frac{\pi}{T_s}$ to keep all information.

e. To recover $x(t)$ from $x_p(t)$, bandpass $x_p(t)$ b/w $-W_m$ & W_m, & then scale by $\frac{T_s}{2\pi}$ to get $x(t)$.

g. given $z(t)$, sketch $x_z(t) = x_p * z(t)$

holds last sampled value until the next one

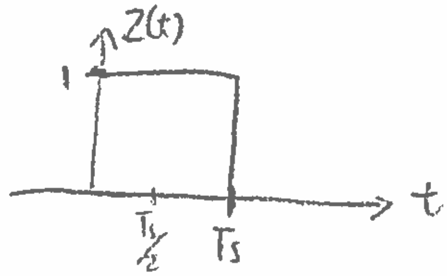


h. Sketch $X_z(w)$

$1 - \cos(-wT_s) - j\sin(-wT_s)$

$z(t) = 1$ when $0 \le t \le T_s$, 0 otherwise $\quad Z(w) = \int_{-\infty}^{\infty} z(t) e^{-jwt} dt \qquad e^{j(-wT_s)} = \cos(-wT_s) + j\sin(-wT_s)$

$= \int_0^{T_s} e^{-jwt} dt \qquad e^{i\theta} = \cos\theta + i\sin\theta$



$= \left[\frac{e^{-jwt}}{-jw} \right]_0^{T_s} = \frac{e^{-jwT_s}}{-jw} - \frac{1}{-jw}$

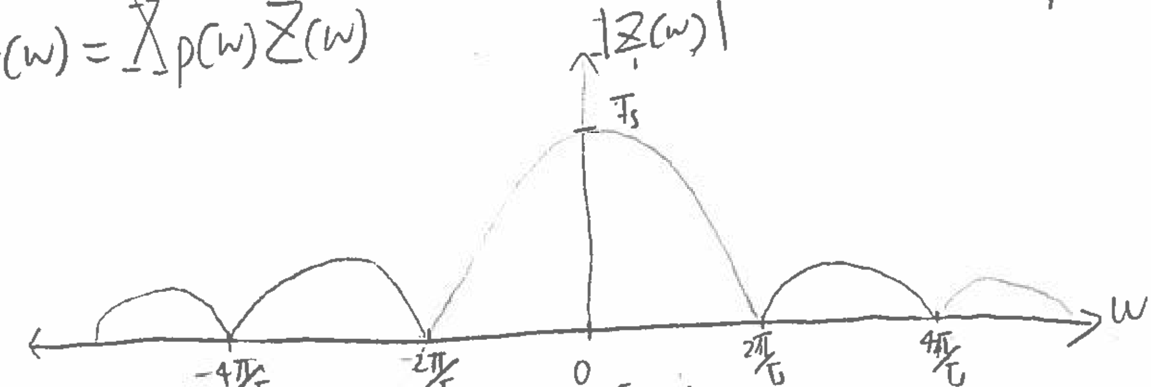
$\sin = 0$ at $w =$ $\quad x = 0$ or π

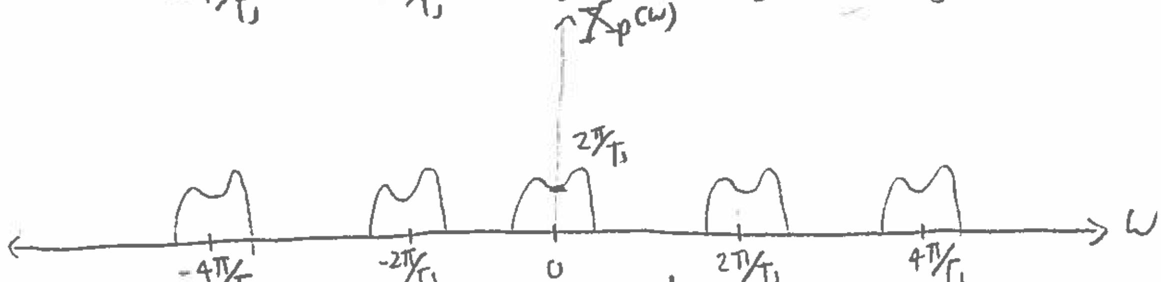
$= \frac{1 - e^{-jwT_s}}{jw}$

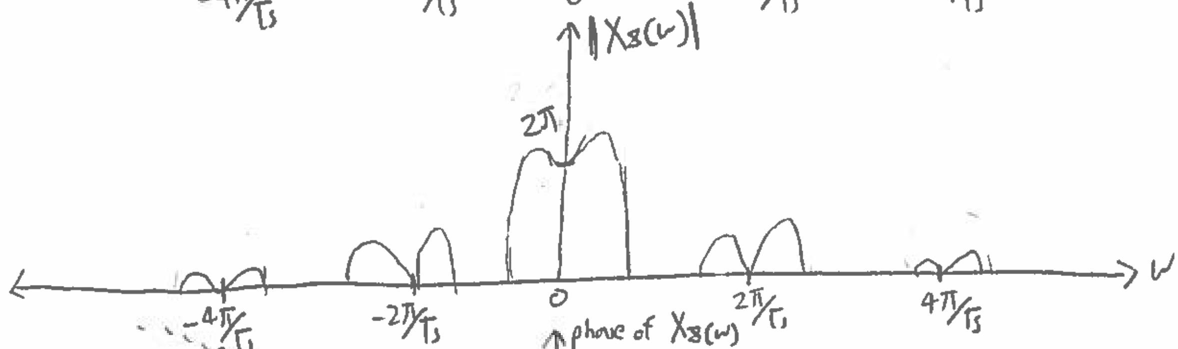
$x(t - t_0) \rightarrow e^{-jwt_0} X(w)$

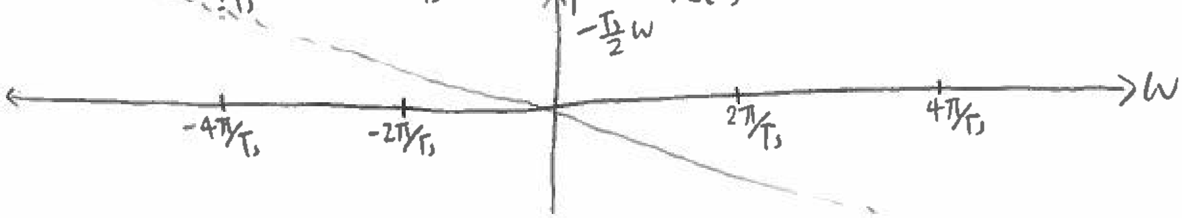
$Z(w) = e^{-jw\frac{T_s}{2}} \cdot T_s \operatorname{sinc}\left(\frac{wT_s}{2\pi}\right)$ → this is of form $Ae^{i\theta}$ w/ amplitude & phase

$X_z(w) = X_p(w) Z(w)$

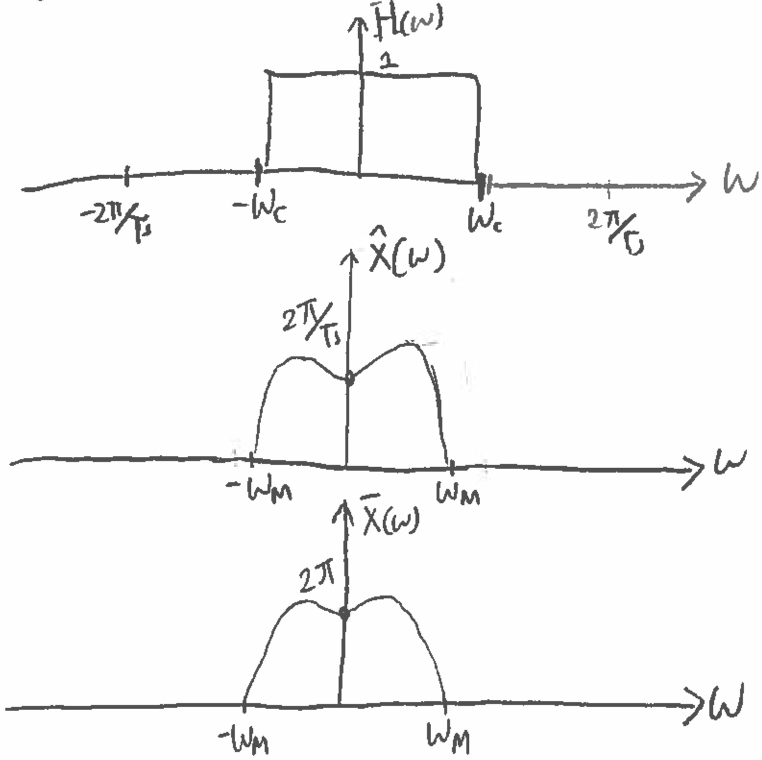








i. Sketch $\bar{X}(\omega) = X_z(\omega)H(\omega)$ & $\hat{X}(\omega) = X_p(\omega)H(\omega)$



j. How are $\bar{X}(\omega)$ & $\hat{X}(\omega)$ different?

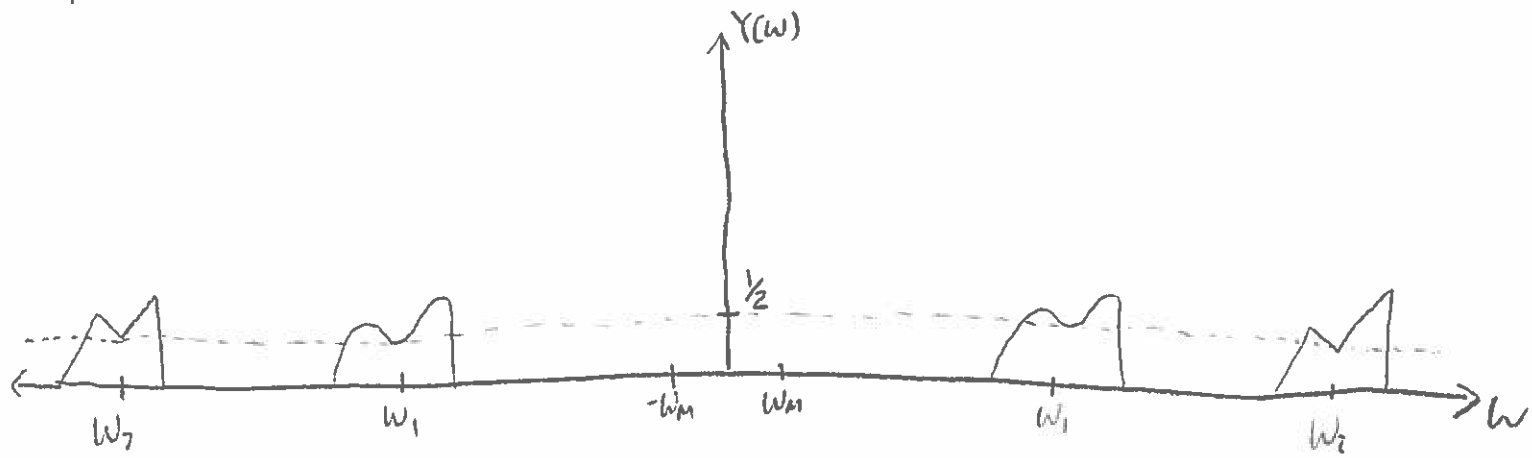
$\bar{X}(\omega)$ & $\hat{X}(\omega)$ have different amplitudes, & $\bar{X}(\omega)$ was multiplied by a part of a sinc, so it is not a scaled version of the original, it's frequencies decrease more quickly when you go from 0 to ω_M as compared to $\hat{X}(\omega)$. Another way of saying that is that it has a different envelope.

k. What is the ratio of $\bar{X}(\omega_M)$ to $\hat{X}(\omega_M)$ when $\omega_M = \frac{\pi}{T_s}$?

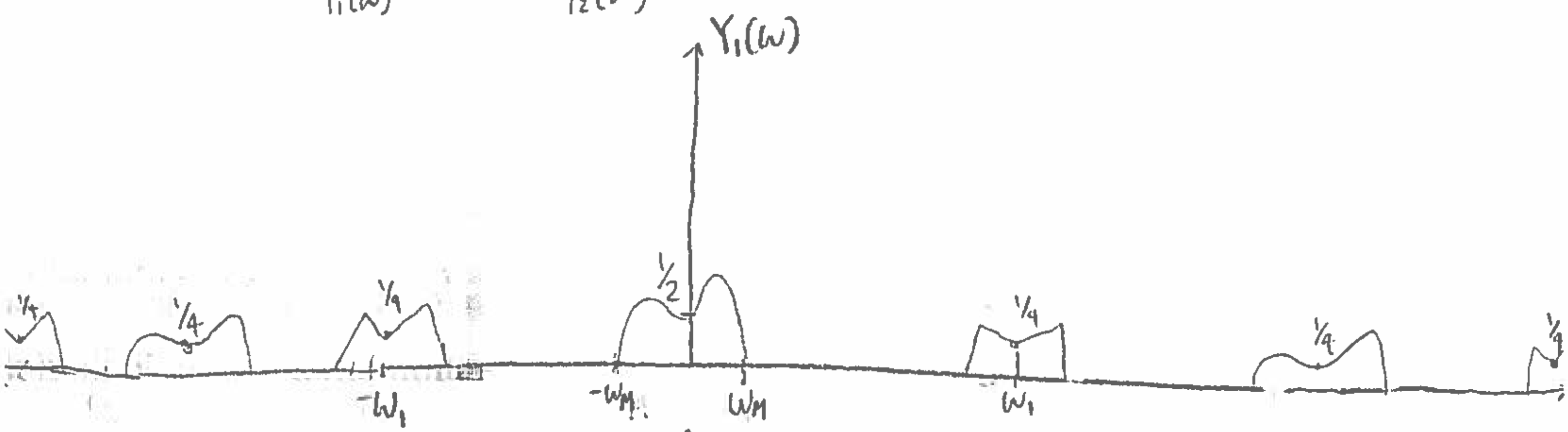
$$\left|Z\left(\frac{\pi}{T_s}\right)\right| = T_s \,\text{sinc}\left(\frac{1}{2}\right) \rightarrow T_s \cdot \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = T_s \cdot \frac{2}{\pi} = \frac{2T_s}{\pi}$$

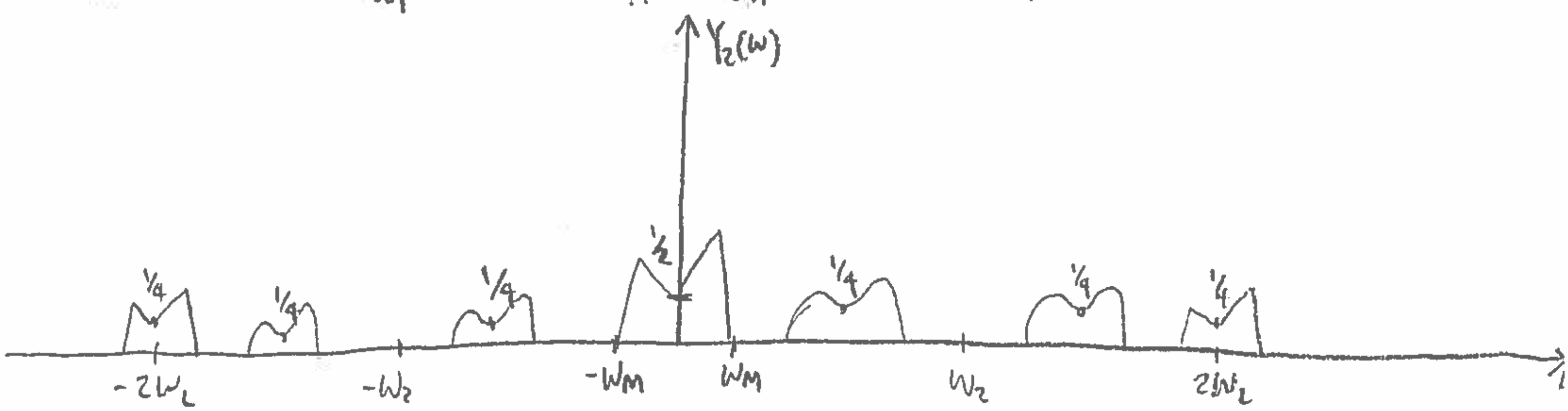
The ratio is $\frac{2T_s}{\pi}$, since you get $\bar{X}(\omega)$ effectively by multiplying $Z(\omega)$ by $X_p(\omega)$ & filtering b/w ω_c & $-\omega_c$.

2a. $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$, sketch $Y(\omega)$



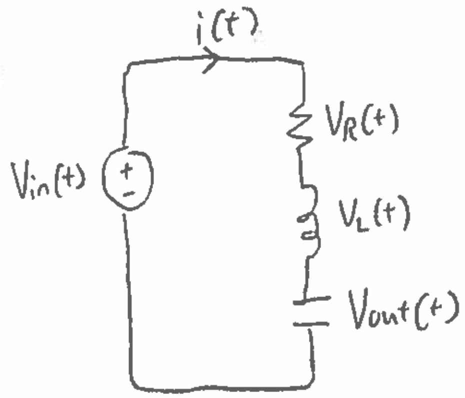
b. sketch FFT of $\underbrace{y(t)\cos(\omega_1 t)}_{Y_1(\omega)}$ & $\underbrace{y(t)\cos(\omega_2 t)}_{Y_2(\omega)}$





c. To recover $x_1(t)$ from $y(t)$, multiply $y(t)$ by $\cos(\omega_1 t)$, lowpass filter w/ a cutoff at ω_M, & multiply the amplitude of the signal by 2. To recover $x_2(t)$, repeat the same process but multiply by $\cos(\omega_2 t)$ instead.

3.



$$i(t) = C\frac{d}{dt}V_{out}(t)$$

$$V_L(t) = L\frac{d}{dt}i(t) = CL\frac{d^2}{dt^2}V_{out}(t)$$

$$V_R(t) = R\,i(t) = RC\frac{d}{dt}V_{out}(t)$$

a. Write a diffeq relating Vout & Vin.

$$V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$$

$$\boxed{V_{in}(t) = RC\frac{d}{dt}V_{out}(t) + LC\frac{d^2}{dt^2}V_{out}(t) + V_{out}(t)}$$

b. Find frequency response $H(\omega)$ of the system.

$$V_{in}(\omega) = j\omega RC\,V_{out}(\omega) + (j\omega)^2 LC\,V_{out}(\omega) + V_{out}(\omega)$$

$$V_{in}(\omega) = V_{out}(\omega)\left[j\omega RC + j^2\omega^2 LC + 1\right]$$

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{1}{j\omega RC + j^2\omega^2 LC + 1} = \boxed{\frac{1}{j\omega RC - \omega^2 LC + 1}}$$

$$\frac{(1-\omega^2 LC) - \omega RC j}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$

c. Find $|H(\omega)|$

$$|H(\omega)| = \frac{1}{|j\omega RC + 1 - \omega^2 LC|} = \boxed{\frac{1}{\sqrt{\omega^2(RC)^2 + (1-\omega^2 LC)^2}}}$$

$$(1-\omega^2 LC)^2 = 1 - 2\omega^2 LC + \omega^4 (LC)^2$$

d. Maximize $|H(\omega)|$

minimize $\omega^2(RC)^2 + (1-\omega^2 LC)^2 \rightarrow \omega^2(RC)^2 + 1 - 2\omega^2 LC + \omega^4(LC)^2$

when is derivative zero?

$$\omega^4(LC)^2 - 2\omega^2 LC + \omega^2(RC)^2 + 1$$

$$4(LC)^2\omega^3 - 4LC\omega + 2(RC)^2\omega = 0$$

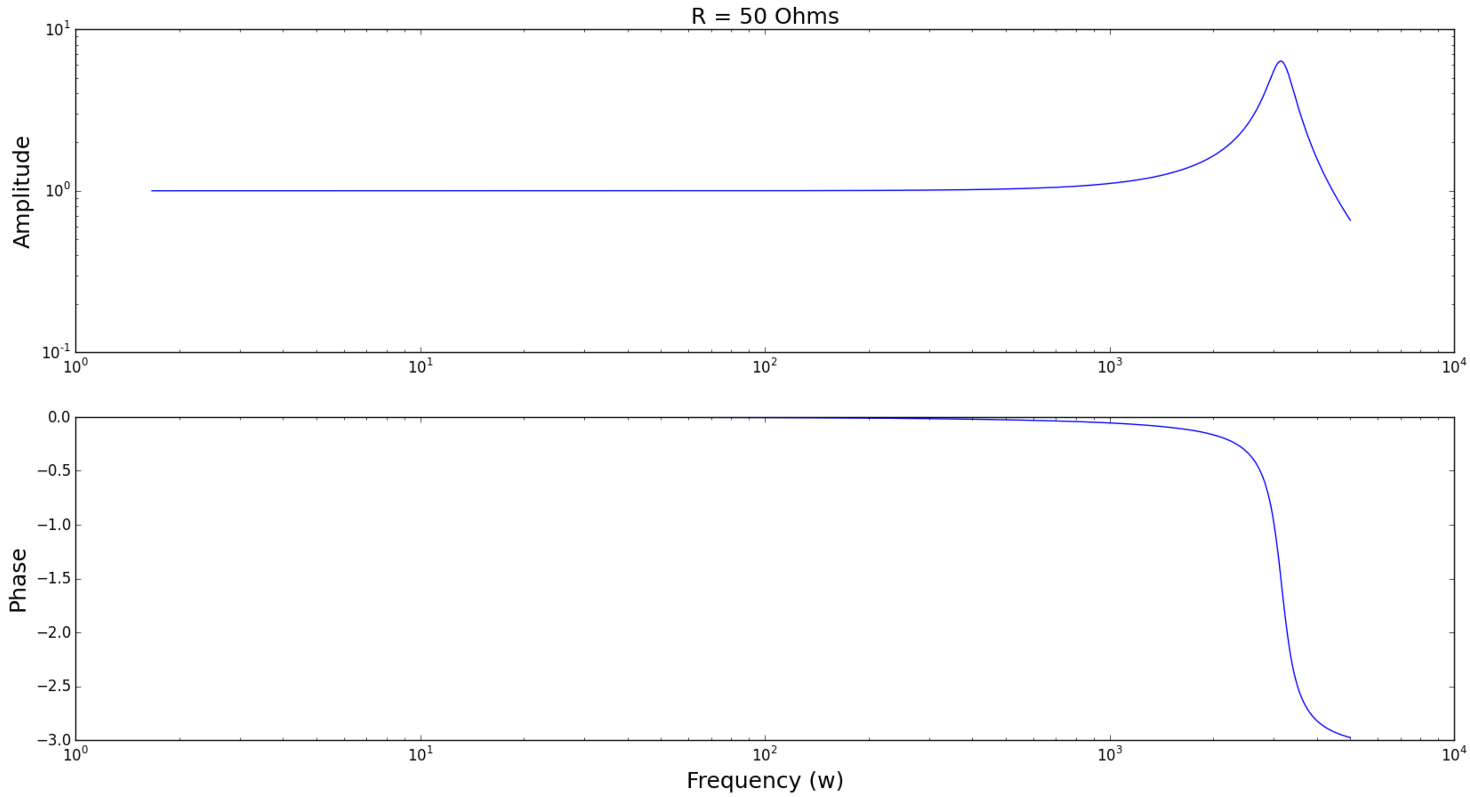
$$\omega\left[4(LC)^2\omega^2 - 4LC + 2(RC)^2\right] = 0$$

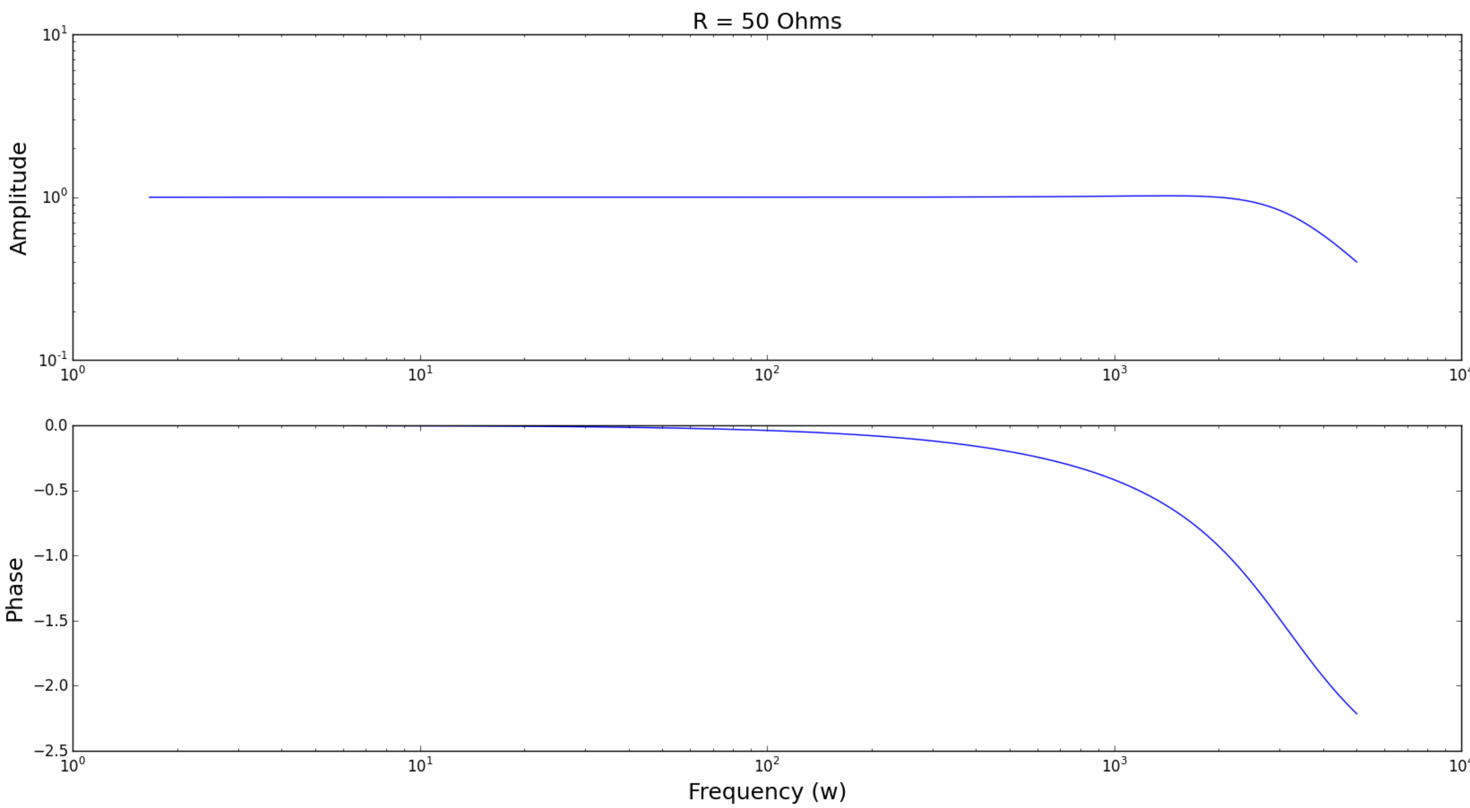
3d continued. $w\left[4L^2C^2w^2 - 4LC + 2(RC)^2\right] = 0$

$$w^2 = \frac{4LC - 2(RC)^2}{4(LC)^2} = \frac{4LC}{4(LC)^2} - \frac{2R^2C^2}{4L^2C^2}$$

$$w^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\boxed{w = \pm\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \quad \text{to maximize } |H(w)|}$$


R = 50 Ohms
Amplitude
Phase
Frequency (w)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