# CS 180 Homework 3

February 21, 2015

## 1 Rectangles

(a) Design an $O(nlogn)$ algorithm that finds the outline of the rectangles.

```
Let I be the array of rectangle arrays
```

(b) Design an O(n) algorithm that, given an outline, finds a rectangle of maximal area that fits within the outline. Implement your algorithm with a single left-to-right scan through the outline data.

```
Let O be the array representing the outline
```

## 2 Interview Questions

(a) You are given a 3-pint container and a 5-pint container, and as much water as you want. Specify a sequence of filling and emptying steps that leave the containers holding exactly 7 pints of water.

(b) There are many problems like the one above; it is from the 1916 Stanford-Binet IQ test. A common interview question uses 3, 5, 4. Design an algorithm that, given three small integers like these as input, finds a sequence with the minimum number of steps.

(c) You are given an array $A$ of $n$ integers, and another integer $z$, and you want to determine whether the array contains two elements $a$ and $b$ such that $a + b = z$.

i. Give an algorithm that uses a min-heap and a max-heap to determine this in time $O(nlogn)$.

ii. Give an algorithm that runs in time $O(n)$, assuming that $A$ is given to you in sorted order.

(d) You are given an array $A$ of $n$ integers (possibly negative) and you want to determine whether the array contains three elements $a$, $b$, and $c$ such that $a + b + c = 0$. Give an algorithm that solves this problem in $O(n^2)$ time.

(e) You are given an array of size $n$ containing every number in $0, 1, 2, ..., n$ except for one. Give an algorithm to find the missing number in time $O(n)$, using only 1 memory cell that has $\lceil 2 \log_2 n \rceil$ bits. (For example, when $n = 50000$, the cell has 32 bits, and can represent numbers from 0 to $2^{32} - 1$.)

# 3 Optimal Submatrix

(a) Find a maximal positive rectangular submatrix - i.e., a submatrix containing only positive values that has the most elements.

(b) Find a maximum sum rectangular submatrix - i.e., a submatrix whose elements have maximal sum. (Hint: This is a generalization of the 'maxsum' problem discussed at the start of this course.).

# 4 Going Beyond the Master Theorem

(a)