CS180 — Algorithms and Complexity

HW#5 — Network Flow

Due: 11:55pm Friday March 13, 2015

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Using CCLE, please upload your answers as a PDF document by the deadline.

1. **Mobile Computing**
   Consider a set of mobile computing clients in a certain town that each need to be connected to one of several possible *base stations*. There are n *clients*, with the position of each client specified by its (x, y) coordinates. There are also k base stations with (x, y) coordinates. We wish to connect each each client to exactly one of the base stations.

   The choice of connections is constrained in two ways: (1) there is a *range parameter* r: a client can only be connected to a base station that is within distance r. (2) there is also a *load parameter* L: no more than L clients can be connected to any single base station.

   Your goal is to design a polynomial-time algorithm for the following problem: Given the positions of a set of clients and a set of base stations, decide whether every client can be connected simultaneously to a base station, subject to the range and load constraints.

2. **Simple Maximum Flow**
   Suppose that we have an airport network in which the link values represent the number of flights along that link. For example, if the link value from *MDL* to *DEL* is 5, then the 'capacity' of air travel between MDL and DEL is 5 planeloads of people. Suppose we have the network shown in Figure 1.

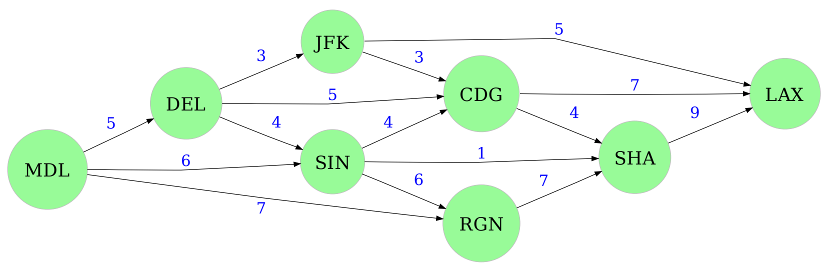

Figure 1: Airport network flow problem.

   (a) Find a maximum flow; how many planeloads of people can we move from MDL to LAX?

   (b) Find a minimum cut for this network, and give its capacity.

   (c) Is the minimum cut unique? (Is there any other cut with the same capacity?)

3. **Min Flow / Max Cut**
   Let $G = (V, E)$ be a flow network with source s and sink t — but instead of edge capacities $c(e)$ for $e \in E$ G has <u>*minimum*</u> *flow requirements* $0 \leq \ell(e)$ for $e \in E$. That is, any flow $f : E \mapsto \Re^+$ on G must satisfy the constraint $0 \leq \ell(e) \leq f(e)$.

   (a) Give a polynomial time algorithm that finds the minimum possible flow on G.

   (b) Consider any <u>minimum</u> flow f. Is it true that it corresponds to some 'maximum cut'? That is, for networks like this, is there a *Min Flow / Max Cut* theorem? If so, prove it; if not, give a counterexample.

4. **Directed (not necessarily acyclic) Hamiltonian Path**
   Show that the following problem is NP-complete:

   DIRECTED HAMILTONIAN PATH
   given a directed graph G and two distinct vertices u and v, determine whether G contains a path that starts at u, ends at v, and visits every vertex of the graph exactly once.

   Notice that the graph does not have to be acyclic (as in the problem on the midterm). Show that this problem is in NP, and prove that it is NP-hard by reduction from some other problem.

   Hint: Consider reduction from HAMILTONIAN CYCLE (8.17 in KT) or HAMILTONIAN PATH (8.19 in KT).