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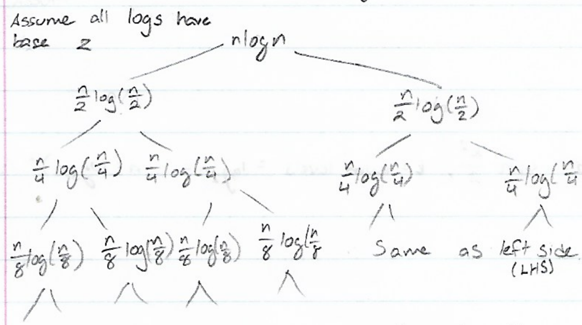
CSCI-UA 310-001 PS7

3. Assume that all logs have a base of 2

** Assume all logs have base 2



Cost per level

$c_2 n \log n$

$c_2 n \log(\frac{n}{2})$

$c_2 n \log(\frac{n}{4})$

$c_2 n \log(\frac{n}{8})$

$\rightarrow$ Total cost: $\sum_{j=0}^{k-1} c_2 n \log(\frac{n}{2^j})$

$\hookrightarrow k = \log_2 n = \#$ levels

Multiply by number of levels

$\log n \cdot c_2 n \log(\frac{n}{2^j})$

$\hookrightarrow$ We can reason that the "2^j" term makes the inequality smaller; therefore, we can say that removing this term will maintain the inequality correctness, as we are making this RHS term greater. So, we can bound $T(n) \leq d_2 n (\log_2 n)^2$. Next for the lower bound, assume n is a power of $2 = 2^k$.

$\sum_{j=0}^{k} c_1 n \log(\frac{2^k}{2^j}) \rightarrow \sum c_1 n (k-j) \rightarrow$ Rearrange $\rightarrow c_1 n \sum^{k} j \rightarrow c_1 n \left(\frac{k(k+1)^2}{2}\right) \rightarrow$ Using the same reasoning from the RHS, we can lower the LHS and still preserve correctness of the inequality. Therefore, for simplicity, we can lower the LHS to $c_1 n \frac{k^2}{2}$, $k =$ num levels $= \log_2 n = c_1 n (\log_2 n)^2 / 2 \rightarrow$ This bounds $d_1 n (\log_2 n)^2 \leq T(n)$