

7.

- a. It is important to note that a path in  $G$  is called  $k$ -alternating if it changes color *at least*  $k$  times (not necessarily exactly  $k$ ).
  - i. Run a topological sorting algorithm such that the runtime is  $O(|V| + |E|)$  (Kahn)
  - ii. Run DFS starting on the first node with in-degree = 0  
During DFS: If a node in the path changes the color from the starting color, increment a counter variable associated with that particular tree branch. (Cross edges will already be accounted for)
  - iii. Suppose  $i$  is the number of tree branches  
If  $i > 1$   
For 0 to  $i-1$   
 $m[v] = \max(\text{counter at branch } i, \text{counter at branch } i+1)$   
Return  $m[v] \geq k$  //boolean
- b. An arbitrary graph implies that there could be a cycle(s) in the graph.
  - i. Therefore, run the SCC algorithm
  - ii. If a particular SCC contains 2 or more nodes with different colors, then return true. This is because we know that there is a cycle in each SCC, and we can achieve at least  $k$  by just remaining in the loop. Once  $k$  is achieved, we continue to our path.
  - iii. Else if all of the nodes in the SCC have the same color, check for a  $k$ -alternating path the same way we did in part a.