

3. Consider the size of the collision set $h_I(a) = h_I(b)$ over the size of the seed set.

$h_I(a) = h_I(b) \rightarrow$ How many times are $a \in I, b \in I$

$$\text{Collision set size} = {}_{n-2}C_{k-2} = \frac{(n-2)!}{(k-2)!(n-k)!}$$

$$\text{Seed set size} = {}_nC_k = \frac{n!}{k!(n-k)!}$$

$$\frac{(n-2)!}{(k-2)!(n-k)!} / \frac{n!}{k!(n-k)!} \rightarrow \frac{(n-2)!k!(n-k)!}{n!(k-2)!(n-k)!} \rightarrow \frac{k(k-1)}{n(n-1)} \rightarrow \frac{k(k-1)}{n(n-1)} \leq 1/n \rightarrow \frac{k(k-1)}{(n-1)} \leq 1 \rightarrow$$

$$k(k-1) \leq n-1 \rightarrow k(k-1) \leq k^2 \leq n-1 \rightarrow k \leq \sqrt{n-1}$$

This means that because $k \leq \sqrt{n-1}$, \mathcal{H} is universal.