Dennis Kuzminer
CSCI-UA 310-001 PS4

5. Let i be a bit's place/index such that $i \in \{0, ... n-1\}$. If $\widehat{x}$ is the complement of x, such that if $\widehat{x}_i = 0$ then $x_i = 1$ and if $\widehat{x}_i = 1$ then $x_i = 0$, we can add each digit of place/index i to see a relationship between the two numbers. Because $\widehat{x}_i$ and x_i will always be opposite their bit sum will always be 1 + 0 = 1. Therefore, for each place i, $\widehat{x}_i + x_i = 1$. More specifically, this also shows that for $i \in \{0, ... n-1\}$, $\widehat{x} + x = 1_{2^{n-1}} \dots 1_{2^2}\ 1_{2^1}\ 1_{2^0} \rightarrow \widehat{x} + x = 1$ (repeated n times).
   This implies that $\widehat{x} + x = 2^n - 1 \rightarrow \widehat{x} + 1 = 2^n - x$
   We know that $x + ny \equiv x(mod\ n),\ y \in \mathbb{Z} \rightarrow 2^n - x \equiv - x(mod\ 2^n) \rightarrow \widehat{x} + 1 \equiv - x(mod\ 2^n)$