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6. $E[T] = \sum_{i=1}^{n} E[T \mid X = i]\, Pr[X = i] \rightarrow$ The probability of X = i is uniform and will always be = 1/10 in this case $\rightarrow E[T] = \frac{1}{n}\sum_{i=1}^{n} E[T \mid X = i] \rightarrow$ If i = 1, we can say that we will expect 10 tries before we reach at most 1, as the distribution is uniform. Similarly, if we spin a 2, we expect it to take 5 tries to get to a number of at most 2 after (uniform and independent spins). Therefore, we can see that $\sum_{i=1}^{n} E[T \mid X = i] = \sum_{i=1}^{n} \frac{n}{i} \rightarrow$ Substituting back $\rightarrow E[T] = \frac{1}{n}\sum_{i=1}^{n} \frac{n}{i} \rightarrow \sum_{i=1}^{n} \frac{1}{i} \rightarrow$ We know that this sum is bounded by the integral of 1/x. More specifically, $\sum_{i=1}^{n} \frac{1}{i} \leq 1 + ln(n)$. Generally, because $E[T] = \sum_{i=1}^{n} \frac{1}{i}$, $|E[T] - ln(n)| \leq c$ for some positive constant c and all sufficiently large n. Therefore, $E[T] = ln(n) + O(1)$.