

5.

- a. The first for-loop creates a runtime of n . The second for-loop creates a runtime of n again, meaning that the first for-loop's run time will not matter as $n \rightarrow \infty$. The while-loop inside of the for-loop will consistently reduce the number of operations that need to be done in comparison $O(n)$. For instance, if $n = 10$, then the first iteration of the for-loop will perform $10 (* 2)$ operations within the while-loop. In the second iteration, $5 (* 2)$. Then $3 (* 2)$, $2 (* 2)$, $2 (* 2)$, $1 (* 2)$, and so on. We can see that the number of iterations/operations of the while-loop is decreasing by some proportion of n as n grows. Therefore, we can simplify the runtime of this algorithm to $n + n \log n$. We can simplify, we get **$O(n \log n)$** .
- b. If we run the program for an array of integers size 12, we will be adding one to indices $A[i]$ every for-loop iteration. However, we do not add it to every index. We add one to every i th index. For instance, if $i = 4$, then we would only add one to $A[4]$, $A[8]$, and $A[12]$. At the end of the 12th iteration of the algorithm, we can see that $A = [1, 2, 2, 3, 2, 4, 2, 4, 3, 4, 2, 6]$. Each value of $A[i]$ actually **represents the number of factors of a given number i** .