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CSCI-UA 310-001 PS3a

1.

1) Find in-degree of all nodes

A, B, C, D, E, F, G, H

[0, 0, 2, 1, 1, 2, 1, 1]

We will also keep track of the nodes with in-degree of 0 with a queue, and we will decrease the in-degree of all affected nodes. Begin:

Queue: A, B → Move A to top sort → top sort = A, in-degree of C = 1

Queue: B → Move B to top sort → Top sort = A, B, in-degree of C = 0

Queue: C → Move C to top sort → Top sort = A, B, C, in-degree of D, E = 0

Queue: D, E → Move D to top sort → A, B, C, D, in-degree of F = 1

E → E → A, B, C, D, E, indegree of F = 0

F → F → A, B, C, D, E, F, in-degree of G, H = 0

G, H → G → A, B, C, D, E, F, G

H → H → A, B, C, D, E, F, G, H

Resulting top sort

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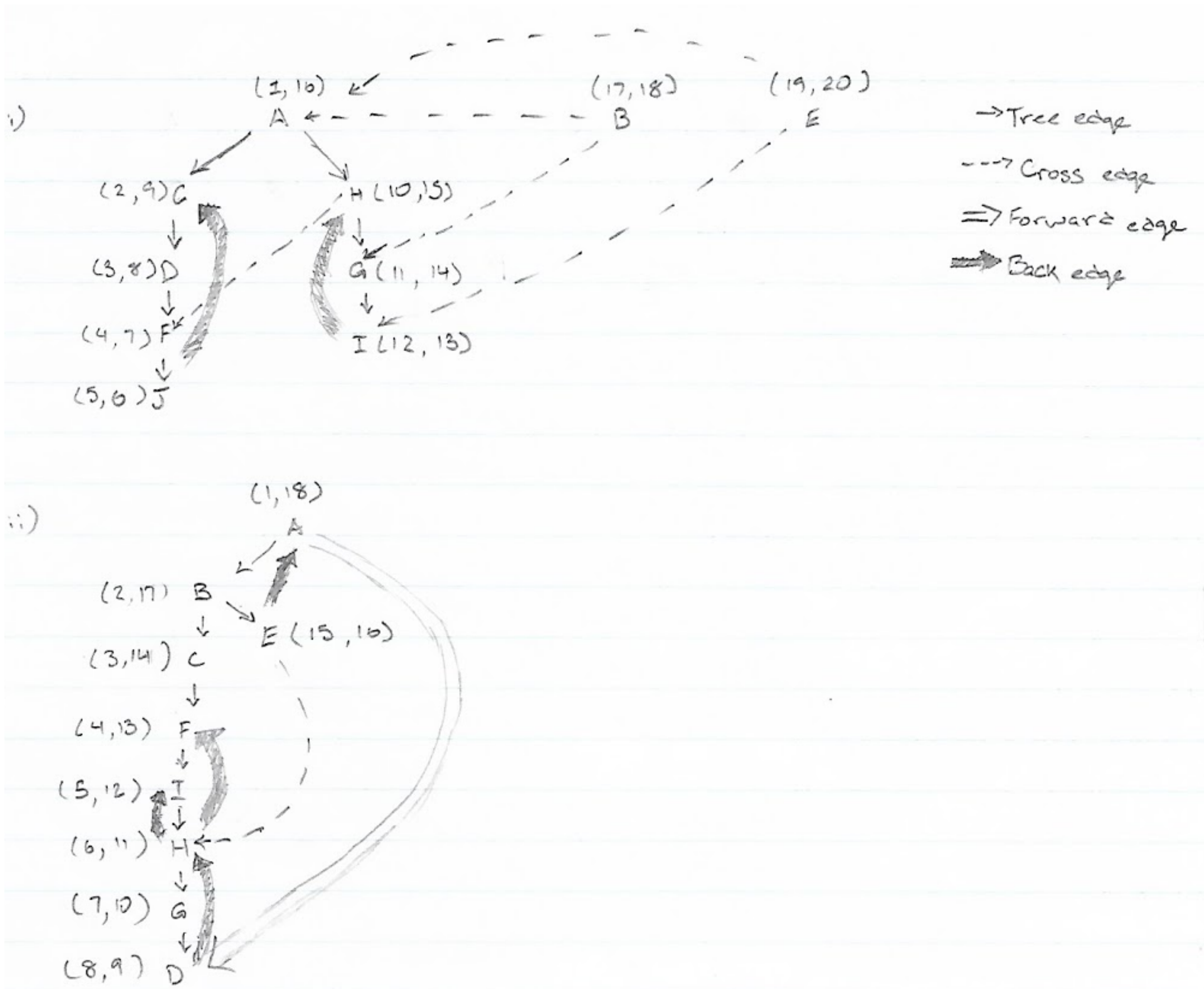
2.



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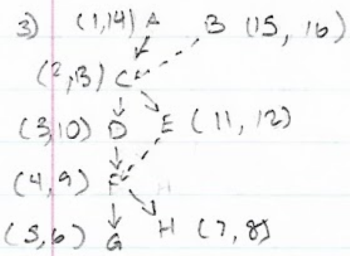
3.

3)



→ Decreasing Finishing times a.k.a. last painted black to first painted black

↳ B, A, C, E, D, F, H, G

B A C E D F H G

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4.

STEP 1:

4) This graph is actually G^T of G presentee in problem 2 ::
Therefore, DFS for G^T is:::

STEP 2:

(1,18) A
(2,17) B
(15,16) E
(3,14) C
(4,13) F
(5,12) I
(6,11) H
(7,10) G
(8,9) D

Top sort: Decreasing finishing time
A, B, E, C, F, I, H, G, D

STEP 3:

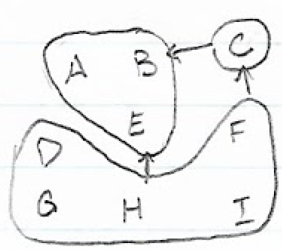
(1,6) A
(2,5) E
(3,4) B

(7,8) C

(9,18) F
(10,17) H
(11,12) I
D (13,16)
G (14,15)

$\hookrightarrow$ SCCs = (A, E, B), (C), (F, H, I, D, G)

G^{SCC}:



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5. `
    a. For each C_i (i in [1, k])
        For each v in C_i
            Map[v]=i
    Run time of O(n)
    b. For each C_i (i in [1, k])
        For each v in C_i up until the number of edges in C_i or m
            Find some node x that is connected to some node v by an edge
            If Map[v] != Map[x]
                Add v → x into L
    Run time on O(m)
    c. Extra: Remove all dependencies that satify the condition
    u → v is a duplicate to x → y if v is a different node from y and Map[u] == Map[x] and Map[v] == Map[y]

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6.

a. If r is a root, then the in-degree must be zero. This is because we have 2 presumptions here: 1) the root must be able to reach every V in the graph 2) G is acyclic.
Counterexample: Suppose there is an edge that connects v to r. This implies that there is a path from v to r. Similarly, we already know that r must have a path to v as the condition states that the root must be able to reach every V in the graph. Hence, we are creating a path from r to v and one from v to r. This is a cycle and would break our second condition that G is acyclic, meaning we cannot have a root with any incoming edges, only outgoing.

b. If r can reach any vertex in the graph, and r has an in-degree of 0, then no other point can have an in-degree of 0. This is because if some v has an in-degree of 0, then r will not be able to reach it (as there are no incoming edges). If r cannot reach v, then r and v cannot be roots as they have no connection, thus breaking the second condition.

c. If there is more than one node with an in-degree of 0, this will imply that r is not a root and that the graph is somehow disconnected. If the graph is connected and r has an in-degree of 0, this implies that r can reach every other node by some path, meaning that r must be a root, as stated by the first condition.

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7.

a. It is important to note that a path in G is called k-alternating if it changes color ***at least*** k times (not necessarily exactly k).
   i. Run a topological sorting algorithm such that the runtime is O(|V| + |E|) (Kahn)
   ii. Run DFS starting on the first node with in-degree = 0
   During DFS: If a node in the path changes the color from the starting color, increment a counter variable associated with that particular tree branch. (Cross edges will already be accounted for)
   iii. Suppose i is the number of tree branches
   If i > 1
   For 0 to i-1
   m[v] = max(counter at branch i, counter at branch i+1)
   Return m[v] >= k //boolean
b. An arbitrary graph implies that there could a cycle(s) in the graph.
   i. Therefore, run the SCC algorithm
   ii. If a particular SCC contains 2 or more nodes with different colors, then return true. This is because we know that there a cycle in each SCC, and we can achieve at least k by just remaining in the loop. Once k is achieved, we continue to our path.
   iii. Else if all of the nodes in the SCC have the same color, check for a k-alternating path the same way we did in part a.

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8.

a. First run reverse topological sort so that you can begin with the largest value
b. Run the following algorithm similar to the one presented in class with the coin problem
   P[v] is the max number off stones what can be on a path starting at v
   N[u] is the number of stones on a particular spot
   for i in reverse [0 . . n)
   u ← TopSort[i]
   m ← 0
   for each v ∈ Sccessor(u) do
   if P[v] > m then m ← min(P[v], c(e)), where e is the edge from v to its successor
   P[u] ← N[u] + m

c. However, c(e) must be considered. If c(e) is less than P[v] at a particular edge, then we must drop stones until we can cross the edge.

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9.

a. Run a Depth-First Search algorithm on some node that need not be a root with runtime O(|V| + |E|). If there is more than one tree in the DFS forest, this must mean that there is a particular intersection that we cannot reach from some point on a different tree using only one-way streets.

b. Run a DFS with runtime O(|V| + |E|) starting on the node with the smallest in-degree (if possible)
If all nodes are connected by tree edges, then the root of the tree is privileged
If there is a back edge from a node to the root, then all of the nodes including and above that node will also be privileged

c. First, compute G^T
Then, call DFS(G^T), and order the nodes 1, . . . , n in order of decreasing finishing time (as in DFSTopSort)
Lastly, call DFS(G) — but in the top-level loop, process in the order 1, . . . , n
The result will be all the safe spaces or Strongly Connected Components
This will run in time O(|V| + |E|).