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CSCI-UA 310-001 PS1

7.

a. Base case: k = 0
F2 = F1 + F0 → 0 + 1 = 1 → From definition
From sigma notation → 1 + 0 = 1, and 1 = 1, so the claim holds for k = 0.
Inductive step: k+1 holds. Assume that the claim is true. $F_{k+2} = 1 + \sum_{i=0}^{k} F_i$
$F_{k+3} = F_{k+1} + F_{k+2} = 1 + \sum_{i=0}^{k+1} F_i \rightarrow 1 + F_{k+1} + \sum_{i=0}^{k} F_i = F_{k+1} + F_{k+2} \rightarrow$ which can be simplified to by canceling out $F_{k+1} \rightarrow F_{k+2} = 1 + \sum_{i=0}^{k} F_i$
We can see that the claim holds for k+1. $\therefore F_{k+2} = 1 + \sum_{i=0}^{k} F_i$ holds for all k ≥ 0.

b. Base case 1: k = 0
F2 = F1 + F0 → 0 + 1 = 1 → From definition
ϕ^(0+1) → 1.61803398875 ≥ 1, so the claim holds for k = 0.
Base case 2: k = 1
F3 = F1 + F2 → 1 + 1 = 2 → From definition. ϕ^(1+1) = 2.61803399
Inductive step: k+1 holds. Assume that the claim is true. $F_{k+2} \leq \phi^{k+1}$
$F_{k+3} \leq \phi^{k+2} \rightarrow F_{k+1} + F_{k+2} \leq \phi * \phi^{k+1} \rightarrow F_{k+2}$ is at most ϕ^{k+1} and F_{k+1} is at most $\phi^{k} \rightarrow \phi^{k} + \phi^{k+1} \leq \phi * \phi^{k+1} \rightarrow \phi^{k}(1+\phi) \leq \phi^{2} * \phi^{k} \rightarrow (1+\phi) \leq \phi^{2} \rightarrow$
2.61803399≤2.61803399 → We can see that the claim holds for k+1.
$\therefore F_{k+2} \leq \phi^{k+1}$

c. Base case 1: k = 0
F2 = F1 + F0 → 0 + 1 = 1 → From definition
ϕ^(0) → 1 ≤ 1, so the claim holds for k = 0.
Base case 2: k = 1
F3 = F1 + F2 → 1 + 1 = 2 → From definition. ϕ^(1) = 1.61803398875
Base case 2: k = 2
F4 = F3 + F2 → 1 + 2 = 3 → From definition. ϕ^(2) = 2.61803399
The claim holds for base case 1 and 2
Inductive step: k+1 holds. Assume that the claim is true. $F_{k+2} \geq \phi^{k}$
$F_{k+3} \geq \phi^{k+1} \rightarrow F_{k+1} + F_{k+2} \geq \phi * \phi^{k} \rightarrow F_{k+2}$ is at least ϕ^{k} and F_{k+1} is at least $\phi^{k-1} \rightarrow \phi^{k-1} + \phi^{k} \geq \phi * \phi^{k} \rightarrow \phi^{k-1}(1+\phi) \geq \phi^{2} * \phi^{k-1} \rightarrow (1+\phi) \geq \phi^{2} \rightarrow$
2.61803399≥2.61803399 → We can see that the claim holds for k+1.
$\therefore F_{k+2} \geq \phi^{k}$