## Dennis Kuzminer CSCI-UA 310-001 PS4

3.

a. If b/d and a/d are relatively prime, this implies that gcd(b/d, a/d) = 1. If b/d and a/d are not relatively prime, meaning that  $gcd(b/d, a/d) \neq 1$ , then  $gcd(b, a) \neq d$ , as there will always be some number (gcd(b, a) > d) larger than d that would be the real gcd. This number will continue to become larger until it satisfies the condition that gcd(b, a) = d, gcd(b/d, a/d) = 1.

## More formally,

By Bezout's Lemma, we know  $as + bt = d \rightarrow (a/d)s + (b/d)t = 1$ . We can rewrite the equation setting a' = (a/d),  $b' = (b/d) \rightarrow a's + b't = 1$ . By Theorem 1.7 (Corollary to Bezout's Lemma), we can see that a', b' are both relatively prime.

b. We can apply similar logic to show that s and t are relatively prime. By Bezout's Lemma, we know  $as + bt = d \rightarrow (a/d)s + (b/d)t = 1$ . We can rewrite the equation setting s' = (a/d), t' = (b/d), a' = s,  $b' = t \rightarrow a's' + b't' = 1$ . By Theorem 1.7 (Corollary to Bezout's Lemma), we can see that a', b' are both relatively prime, meaning that both s and t are relatively prime.