

6. n =leaves and m =internal nodes

Q0 \rightarrow For a 2-3 tree of height zero, $n = 1$ and $m = 0$; therefore, the assertion holds for $h = 0$, as $0 \leq 0$.

Inductive step: Assume that $m \leq n - 1$ holds for all $h - 1 \geq 0 \rightarrow$ Show it holds for h .

For a tree of height h , this would mean that the number of internal nodes would increase by n and the number of leaves would increase by at least $2n_{h-1}$ and at most $3n_{h-1}$.

New internal nodes: $m_{h-1} + n_{h-1} = m_h \rightarrow m_{h-1} = m_h - n_{h-1} \rightarrow$

$$m_h - n_{h-1} \leq n_{h-1} - 1 \rightarrow m_h \leq 2n_{h-1} - 1$$

New leaves: $2n_{h-1} \leq n_h \leq 3n_{h-1} \rightarrow 2n_{h-1} - 1 \leq n_h - 1 \leq 3n_{h-1} - 1 \rightarrow$

$$m_h \leq 2n_{h-1} - 1 \leq n_h - 1 \leq 3n_{h-1} - 1 \rightarrow m_h \leq n_h - 1$$

$\therefore m_h \leq n_h - 1$ holds for all $h \geq 0$.