Dennis Kuzminer
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6.

a. If r is a root, then the in-degree must be zero. This is because we have 2 presumptions here: 1) the root must be able to reach every V in the graph 2) G is acyclic.
Counterexample: Suppose there is an edge that connects v to r. This implies that there is a path from v to r. Similarly, we already know that r must have a path to v as the condition states that the root must be able to reach every V in the graph. Hence, we are creating a path from r to v and one from v to r. This is a cycle and would break our second condition that G is acyclic, meaning we cannot have a root with any incoming edges, only outgoing.
b. If r can reach any vertex in the graph, and r has an in-degree of 0, then no other point can have an in-degree of 0. This is because if some v has an in-degree of 0, then r will not be able to reach it (as there are no incoming edges). If r cannot reach v, then r and v cannot be roots as they have no connection, thus breaking the second condition.
c. If there is more than one node with an in-degree of 0, this will imply that r is not a root and that the graph is somehow disconnected. If the graph is connected and r has an in-degree of 0, this implies that r can reach every other node by some path, meaning that r must be a root, as stated by the first condition.