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CSCI-UA 310-001 PS4

8. By modular multiplication, we know that $(A * B) \ mod \ C = (A \ mod \ C * B \ mod \ C) \ mod \ C$.
Therefore, $3^{99} \ mod \ 100$ can be shown as
((some combination of numbers that multiply to 3^{99}) *each mod* 100) *mod* 100 .
We can have the combination of numbers be powers of 2 using base 2 and the repeated squaring algorithm.
$e = 99_{10} = 1100011_2$
$\beta \leftarrow [1]$ // 0
$\beta \leftarrow \beta^2,\ \beta \leftarrow \beta * \alpha$ $//1 \rightarrow 3^1 \ mod \ 100 = 3$
$\beta \leftarrow \beta^2,\ \beta \leftarrow \beta * \alpha$ $//11 \rightarrow 3^3 \ mod \ 100 = 27$
$\beta \leftarrow \beta^2$ $//110 \rightarrow 3^6 \ mod \ 100 = 27^2 \ mod \ 100 = 29$
$\beta \leftarrow \beta^2$ $//1100 \rightarrow 3^{12} \ mod \ 100 = 29^2 mod \ 100 = 41$
$\beta \leftarrow \beta^2$ $//11000 \rightarrow 3^{24} \ mod \ 100 = 41^2 mod \ 100 = 81$
$\beta \leftarrow \beta^2,\ \beta \leftarrow \beta * \alpha$ $//110001 \rightarrow 3^{49} \ mod \ 100 = 81^2 mod \ 100 * 3 = 83$
$\beta \leftarrow \beta^2,\ \beta \leftarrow \beta * \alpha$ $//1100011 \rightarrow 3^{99} \ mod \ 100 = 83^2 mod \ 100 * 3 =$ **67**