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CSCI-UA 310-001 PS5

2.

a. n > m → The number of jobs is more than the number of machines.
If there are m machines, and m is strictly less than n, then there must be at least one machine with 2 jobs (pigeonhole principle). If we order in descending order, then the first job to be assigned to a machine with a pre-existing job is the m + 1 job. Since the ordering is descending, we know that the time of the m + 1 job (t_{m+1}) is at least as great as all of the jobs that came before it $(t_1 \geq t_2 \geq ... \geq t_{m+1})$. Because there are now at least 2 jobs assigned to this machine, and the first job was at least as long as the second, we can say that $T^* \geq 2t_{m+1}$.

b. Let *l* be the last job scheduled on machine *k*. Right before l was scheduled: the load on machine k was $T_k - t_l$ and the load on k was minimal.
From class, we derived that $T_k \leq t_l + \frac{1}{m}\sum_j t_j$. We also know that $2t_{m+1} \leq T^* \rightarrow$ $t_{m+1} \leq T^*/2$. Since l was the last job, we can say that $t_l \leq t_{m+1}$.
Combining: $t_l \leq t_{m+1} \leq T^*/2$, showing that t_l is bounded ($t_l \leq T^*/2$).
By Lemma 2, $\frac{1}{m}\sum_j t_j$ is also bounded $\frac{1}{m}\sum_j t_j \leq T^*$.
Substituting: $T_k \leq T^*/2 + T^* \rightarrow T \leq 3T^*/2$