Dennis Kuzminer
CSCI-UA 310-001 PS6

2. $E[S^2] \rightarrow E[(X_1 + ... + X_n)^2] \rightarrow (\sum_{i=1}^{n} X_i)^2 = (\sum_{i=1}^{n} X_i)(\sum_{j=1}^{n} X_j) = \sum_{i=1}^{n} X_i^2 + \sum_{i \neq j} X_i X_j$ (Example 30 in Probability Primer) $\rightarrow \sum_{i=1}^{n} E[X_i^2] + \sum_{i \neq j} E[X_i X_j]$. This is because of linearity of expectation.

   To find $\sum_{i=1}^{n} E[X_i^2]$, we must also consider that the variables are all *independently* and *uniformly* distributed; therefore, regardless of the value of *i,* all $E[X_i]$ will have the same values and all $E[X_i^2]$ will have the same value. Meaning, $\sum_{i=1}^{n} E[X_i^2] = nE[X_1^2]$. X_1^2 is distributed on the set of the squares of the original set = $\{0, 1, 1, 4, 4, 9, 9\}$. The mean/expected value of this set is 4 ([0+1+1+4+4+9+9]/7). This shows that $\sum_{i=1}^{n} E[X_i^2] = 4n$.

   To find $\sum_{i \neq j} E[X_i X_j]$, we use linearity of expectation once more. $E[X_i X_j] = E[X_i]E[X_j] \rightarrow$ $E[X_i] = (-3 + -2 + -1 + 0 + 1 + 2 + 3)/7 = 0$.

   This means $\sum_{i \neq j} E[X_i X_j] = E[X_i X_j] = E[X_i]E[X_j] = 0$.

   Finally, we combine the terms to get $E[S^2] = 4n + 0$ or just $4n$.