

1.

- a. $\lim_{n \rightarrow \infty} \frac{n(\log_2 n)^2}{n^2 \log_2 n} \rightarrow \lim_{n \rightarrow \infty} \frac{\log_2 n}{n} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow \frac{\frac{1}{n \ln(2)}}{1} \rightarrow \frac{1}{n \ln(2)} \rightarrow \text{D.S.} = \frac{1}{\infty} = 0$
 \rightarrow This implies that g grows faster than f and **f=o(g)**
- b. $\lim_{n \rightarrow \infty} \frac{n^2}{n(\log_2 n)} \rightarrow \lim_{n \rightarrow \infty} \frac{n}{(\log_2 n)} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow \frac{1}{\frac{1}{n \ln(2)}} \rightarrow n \ln(2) \rightarrow \text{D.S.} = \infty$
 \rightarrow This implies that f grows faster than g and **g=o(f)**
- c. $\lim_{n \rightarrow \infty} \frac{n(\log_2 n)^4}{n^{1.2}} \rightarrow \lim_{n \rightarrow \infty} \frac{(\log_2 n)^4}{n^{.2}} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow \lim_{n \rightarrow \infty} \frac{4(\log_2 n)^3}{.2n^{.2} \ln(2)} \rightarrow \text{L'Hopital} \rightarrow$
 $\lim_{n \rightarrow \infty} \frac{12(\log_2 n)^2}{.2n^{.2} \ln(2) \ln(2) \cdot 2} \rightarrow \text{L'Hopital} \rightarrow \lim_{n \rightarrow \infty} \frac{24(\log_2 n)}{.2n^{.2} \ln(2) \ln(2) \cdot 2 \ln(2) \cdot 2} \rightarrow \text{L'Hopital} \rightarrow$
 $\lim_{n \rightarrow \infty} \frac{24}{.2n^{.2} \ln(2) \ln(2) \cdot 2 \ln(2) \cdot 2 \ln(2) \cdot 2} \rightarrow \text{D.S.} = 0 \rightarrow$ This implies that g grows faster than f and **f=o(g)**
- d. $\lim_{n \rightarrow \infty} \frac{200n^2 + n^{1.5}}{(1/500)n^2} \rightarrow \lim_{n \rightarrow \infty} \frac{200n + n^{.5}}{(1/500)n} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow \frac{200 + .5/n^{.5}}{(1/500)} \rightarrow \text{D.S.} =$
 $\frac{200+0}{(1/500)} = 200 * 500 = 100000 \rightarrow$ This implies that f grows at the same rate as g and **f = θ(g)**
- e. $\lim_{n \rightarrow \infty} \frac{\log_7 n}{\log_5 n} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow \frac{\frac{1}{n \ln(7)}}{\frac{1}{n \ln(5)}} \rightarrow \frac{n \ln(5)}{n \ln(7)} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow$
 $\frac{\ln(5)}{\ln(7)} \rightarrow$ This implies that f grows at the same rate as g and **f = θ(g)**
- f. $\lim_{n \rightarrow \infty} \frac{n(\log_2 n)^{-1}}{n^5 \log_2 n} \rightarrow \lim_{n \rightarrow \infty} \frac{n^{-5}}{(\log_2 n)^2} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow \text{L'Hopital} \rightarrow \frac{.5n^{-5} \ln(2)}{2 \log_2 n} \rightarrow \text{D.S.} = \frac{\infty}{\infty} \rightarrow$
 $\text{L'Hopital} \rightarrow \frac{.5 \ln(2)}{2} \frac{n^{-5}}{\log_2 n} \rightarrow \frac{.5 \ln(2)}{2} \frac{.5/n^5}{1/n \ln(2)} \rightarrow \frac{.5^2 \ln(2)^2}{2} \frac{n}{n^5} \rightarrow \frac{.5^2 \ln(2)^2}{2} n^{-5} \rightarrow \text{D.S.} \rightarrow \infty \rightarrow$
This implies that f grows faster than g and **g=o(f)**
- g. $\lim_{n \rightarrow \infty} \frac{5^n}{7^n} \rightarrow \lim_{n \rightarrow \infty} \left(\frac{5}{7}\right)^n \rightarrow \lim_{n \rightarrow \infty} e^{n \ln(\frac{5}{7})} \rightarrow \lim_{n \rightarrow \infty} e^{n(-0.336472237)} \rightarrow \lim_{n \rightarrow \infty} 1/e^{n(0.336472237)} \rightarrow \text{D.S.}$
 $= 0 \rightarrow$ This implies that g grows faster than f and **f=o(g)**
- h. $\lim_{n \rightarrow \infty} \frac{7^n}{5^{(n^2)}} \rightarrow \lim_{n \rightarrow \infty} \left(\frac{7}{5^n}\right)^n \rightarrow \text{D.S.} = \left(\frac{7}{\infty}\right)^\infty \rightarrow (0)^\infty = 0 \rightarrow$ This implies that g grows faster than f and **f=o(g)**
- i. $\lim_{n \rightarrow \infty} \frac{n!}{(n+1)!} \rightarrow \lim_{n \rightarrow \infty} \frac{1}{(n+1)} \rightarrow \text{D.S.} = \frac{1}{\infty} = 0 \rightarrow$ This implies that g grows faster than f and **f=o(g)**
- j. $\lim_{n \rightarrow \infty} \frac{n}{2n + (-1)^n n^{-.5}} \rightarrow$ Divide all terms by n $\rightarrow \lim_{n \rightarrow \infty} \frac{1}{2 + (-1)^n n^{-.5}} \rightarrow (-1)^n n^{-.5} = 0$ as $n \rightarrow \infty$.
Therefore, $\lim_{n \rightarrow \infty} = 1/2 \rightarrow$ This implies that f grows at the same rate as g and **f = θ(g)**