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CSCI-UA 310-001 PS8

1. Show that the number of keys that satisfy this equation

$$\sum_{i=1}^{s}(a_{2i-1}+\lambda_{2i-1})(a_{2i}+\lambda_{2i})-(b_{2i-1}+\lambda_{2i-1})(b_{2i}+\lambda_{2i})=0$$

is at most $m^{t-1} \rightarrow$

$$\sum_{i=1}^{s}(a_{2i-1}a_{2i}+a_{2i-1}\lambda_{2i}+a_{2i}\lambda_{2i-1}\ +\lambda_{2i}\lambda_{2i-1})-(b_{2i-1}b_{2i}+b_{2i-1}\lambda_{2i}+b_{2i}\lambda_{2i-1}\ +\lambda_{2i}\lambda_{2i-1}) \rightarrow$$

$$\sum_{i=1}^{s} a_{2i-1}a_{2i}+a_{2i-1}\lambda_{2i}+a_{2i}\lambda_{2i-1}\ -b_{2i-1}b_{2i}-b_{2i-1}\lambda_{2i}-b_{2i}\lambda_{2i-1}\ \rightarrow$$

$$\sum_{i=1}^{s} a_{2i-1}a_{2i}-b_{2i-1}b_{2i}+\sum_{i=1}^{s} a_{2i-1}\lambda_{2i}+a_{2i}\lambda_{2i-1}\ -\sum_{i=1}^{s} b_{2i-1}\lambda_{2i}+b_{2i}\lambda_{2i-1}=0\ \rightarrow$$

Rearanging $\rightarrow \sum_{i=1}^{t} a_i\lambda_i-\sum_{i=1}^{t} b_i\lambda_i = 0$ (we do not need to consider the constant as there is no random variable associated with it)

$c_i = a_i - b_i \rightarrow \sum_{i=1}^{t} c_i\lambda_i \rightarrow \lambda_1 =- c_1^{-1}\sum_{i=2}^{t} c_i\lambda_i \rightarrow$ There are m^{t-1} ways of choosing $\lambda_2, ..., \lambda_t$, and each yields one solution. So $N = m^{t-1}$ where $N \leq |\Lambda|/m = m^{t-1}$

This shows $\mathcal{H}$ is universal.