Optimizing Business Decisions

A manager of a retail store and wants to build the work schedule for the upcoming work week (Monday-Friday). To do this, the manager observes the amount of customers that attended the store last week and plans accordingly. Every day, there are two shifts that need to be filled. For each shift, the manager observes the amount of customers that arrived and chooses which type of employees to schedule on that shift. In deciding which type of employees to schedule, the manager has three options:

1. Experienced employees

2. New employees

3. Interns

There are 8 experienced employees, 14 new employees, and 3 interns.

For every 4 customers, there has to be 1 experienced employee.

For every 2 customers, there has to be 1 new employee.

For every customer, there has to be one intern.

For example, if the shift is expected to have 8 customers, the following are possible configurations of employee schedulings for the shift:

1. 2 experienced

2. 1 experienced, 2 new

3. 1 experienced, 1 new, 2 interns

As an addition example, if the shift is expected to have 11 customers, one possible configurations is to have 3 experienced employees in the shift.

There can be a maximum of 3 intern slots scheduled per day.

There must be at least 8 experienced employee slots scheduled during each day (either in the morning or evening shift).

The following is last week's customer arrivals that the manager uses to plan:

	Monday	Tuesday	Wednesday	Thursday	Friday
Morning Shift (Shift 1)	20	30	40	42	56
Evening Shift (Shift 2)	60	44	50	60	60

Every cell represents the amount of customers expected at each shift. For example, there are 20 customers expected to arrive during Monday's morning shift.

The following is the salary of each option on each day in units of 10 dollars (morning and evening shift do not differ):

	Monday	Tuesday	Wednesday	Thursday	Friday
Experienced	7	7	7	9	9
New	3	3	3	5	5
Intern	1	1	1	2	2

For example, an experienced employee costs the store $70 dollars for a

morning shift on Monday and $70 dollars for an evening shift on Monday. If 5 experienced employees are scheduled to work on Monday morning, this will cost the store 5*$70 = $350. Alternatively, 1 experienced employee and 8 new employees can be scheduled. This option would cost the store 1*$70+8*$30 = $310.

The manager's task is to decide how to provide the upcoming week's work schedule. The goal is to minimize the upcoming week's costs. Assume that the store's costs include only the salaries of the employees scheduled to work during the shifts.

Define the following variables:

- x_ij (i=1,2, j=1,2...,5) The amount of experienced employees scheduled for shift i on day j.
- y_ij (i=1,2, j=1,2...,5) The amount of new employees scheduled for shift i on day j.
- z_ij (i=1,2, j=1,2...,5) The amount of interns scheduled for shift i on day j.

You can solve this problem with the spreadsheet software of your choice: LibreOffice, Excel, Google Sheets with OpenSolver, or OpenOffice.

Problem 1 - Objective Function

4.0/4.0 points (graded)
Before you set up the optimization problem in your spreadsheet software (Excel or other), we will walk you through the formulation.

Note: The (c_ij*x_kj + c_ij*x_kj + ... + c_ij*x_kj) term is the SUMPRODUCT of the 1x5 cost block of the ith option and jth day and the 1x5 x variables of the kth event on day j.

For example, (c_11*x_11 + c_12*x_12 + ... + c_15*x_15) is the SUMPRODUCT of an experienced employee's salary during the week and the amount of experienced employees working each shift.

Which of the following is a correct expression for the store's costs? Denote this expression to be the cost.

- ◯ (c_11*x_11 + c_12*x_12 + ... + c_15*x_15) + (c_11*y_11 + c_12*y_12 + ... + c_15*y_15) + (c_11*z_11 + c_12*z_12 + ... + c_15*z_15)
- ◯ (c_31*z_11 + c_32*z_22 + ... + c_35*z_15) + (c_31*z_21 + c_32*y_22 + ... + c_35*z_25)
- ◯ (c_11*x_11 + c_12*x_12 + ... + c_15*x_15) + (c_11*y_11 + c_12*y_12 + ... + c_15*y_15) + (c_11*z_11 + c_12*z_12 + ... + c_15*z_15) + (c_11*x_21 + c_12*x_22 + ... + c_15*x_25) + (c_11*y_21 + c_12*y_22 + ... + c_15*y_25) + (c_11*z_21 + c_12*z_22 + ... + c_15*z_25)
- ◉ (c_11*x_11 + c_12*x_12 + ... + c_15*x_15) + (c_11*x_21 + c_12*x_22 + ... + c_15*x_25) + (c_21*y_11 + c_22*y_12 + ... + c_25*y_15) + (c_21*y_21 + c_22*y_22 + ... + c_25*y_25) +(c_31*z_11 + c_32*z_12 + ... + c_35*z_15) + (c_31*z_21 + c_32*z_22 + ... + c_35*z_25)

✔

Which of the following is a correct expression for the objective function?

- ◯ maximize cost
- ◉ minimize cost
- ◯ minimize avg(cost) where avg is the average cost
- ◯ maximize min(cost) where min is the minimum cost

✔

Under the assumption that there is a feasible solution, will the objective value of this function ever be negative?

- ◯ Yes, because the store has to pay the salaries, the objective value will be negative.
- ◉ No, regardless of the decisions taken by the model, the cost will always be positive.
- ◯ Yes, the cost can be negative.
- ◯ There is not enough information.

✔

Submit You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

Problem 2.1: Constraints

2.0/4.0 points (graded)

Let's organize our constraints in the following list:

- There are at most 8 experienced employee slots scheduled in each shift.

- There are at most 14 new employee slots scheduled in each shift.

- There must be at least 8 experienced employee slots scheduled each day.

- All experienced employees must be shceduled 7 times. Since we are not scheduling each person, this means that there must be at least 8*7=56 slots scheduled for experienced employees.

- There must be at most 3 intern slots scheduled each day.

- The amounts of employee slots scheduled are nonnegative.

- 4*x_ij + 2*y_ij + z_ij = a_ij for every i = 1,2 and j = 1,...5 where a_ij is the expected amount of customers for shift i on day j

How many constraints does this list correspond to?

50 ✖ **Answer:** 71

50

Which of the following is the correct constraint for "At most 8 experienced employees can be scheduled to work on Monday morning"?

- (●) x_11 is less than or equal to 8
- ( ) c_11 * x_11 is less than or equal to 8
- ( ) y_11 is less than or equal to 8
- ( ) y_11 is greater than or equal to 8

✔

Which of the following is the correct constraint for "All experienced employees must be scheduled at least 7 times"?

- ( ) x_11 + x_21 is greater than or equal to 56
- (●) x_11 + x_12 + x_13 + x_14 + x_15 is greater than or equal to 56
- ( ) x_11 + x_12 + x_13 + x_14 + x_15 + x_21 + x_22 + x_23 + x_24 + x_25 is greater than or equal to 7
- ( ) x_11 + x_12 + x_13 + x_14 + x_15 + x_21 + x_22 + x_23 + x_24 + x_25 is greater than or equal to 56 ✔

✖

Which of the following is the correct constraint for "there must be at least 8 experienced employees scheduled on Tuesday"?

- ○ x_11 + x_21 is greater than or equal to 8
- ○ x_12 + x_22 is greater than or equal to 16
- ◉ x_12 + x_22 is greater than or equal to 8
- ○ z_12 + z_22 is greater than or equal to 8

✔

Submit You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

Problem 2.2: Constraints (cont'd)

3.0/3.0 points (graded)

Let's observe the last two constraints:

- The amounts of type of employee scheduled are nonnegative

- 4*x_ij + 2*y_ij + z_ij = a_ij for every i = 1,2 and j = 1,...5 where a_ij is the expected amount of customers for shift i on day j

Why do we constrain the variables to be nonnegative? Select the best explanation.

- ○ No reason, this is a redundant constraint in the model
- ◉ By definition, these variables are nonnegative - it is not possible to schedule a negative number of employees to the shift
- ○ Because all variables should be nonnegative

✔

Why do we constrain 4*x_ij + 2*y_ij + z_ij for each shift i on each day j to be equal to the expected amount of customers? Select the best explanation.

- ◯ No reason, this is a redundant constraint in the model
- ◯ Because all variable that are fractions need to sum up to one.
- ◉ Every additional employee costs the store money. Therefore, the manager will try to schedule the exact amount of employees needed for the amount of customers.

✔

If we change the last constarint to be "greater than or equal to the amount of customers" instead of "equal to the amount of customers", will this affect the optimal value of the model?

- ◉ No, since the since every additional employee costs money, no additional employees will be scheduled to work.
- ◯ Yes, this will allow the manager to reduce the costs.
- ◯ There is not enough information.

✔

Submit You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

Problem 3 - Sanity Check

2.0/4.0 points (graded)

As you set up your model, it may be helpful to check that the output is what you expect before submitting your answers. Answer the following before setting up your model to check your understanding.

How many cutomers are expected in the upcoming week?

0

✖ **Answer:** 462

0

Ignoring the constraints, assume the manager chooses to only employ new employees this week. What is the objective value in such a scenario? Denote this solution to be the new employee solution and the value to be the new employee value.

0

✖ **Answer:** 911

0

Returning to our original problem which includes all of the constraints formulated, is the new employee problem feasible?

- ◯ Yes
- ◉ No
- ◯ Not enough information

✔

Assuming there exists a feasible solution to our original problem, will the optimal revenue be lower than that of the new employee value?

- ◯ Yes
- ◉ No
- ◯ Not enough information

✔

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

Problem 4 - Solving the Model

3.0/6.0 points (graded)
Formulate the model in LibreOffice/Excel and solve.

What is the optimal objective value?

7 ✖ **Answer:** 836

7

How many interns are working on Monday during the eveing shift?

3 ✔ **Answer:** 3

3

We note that some of the variable's values in the optimal solution are not integers and we realize the manager cannot schedule 2.5 employees for a shift (All employees must work full shifts). What is a correct way to solve this problem?

- (●) Round all values up
- ( ) Round all values down
- ( ) Round all values to their neart integer

✔

After altering the optimal solution such that all variables are integers, what is the objective value?

9

✖ **Answer:** 862

9

Submit

You have used 5 of 5 attempts

Answers are displayed within the problem

Problem 5 - Sensitivity Analysis

2.0/2.0 points (graded)

Suppose there were only 2 interns.

How will this impact the amount of new employees being scheduled? Answer this without re-solving the model.

- ○ It will stay the same
- ○ It will be higher
- ◉ It could be either of the above

✔

How will this impact the optimal objective value? Answer this without re-solving the model.

- ○ It will stay the same
- ◉ It will be higher
- ○ It could be either of the above

✔

Submit You have used 2 of 2 attempts