Python for Data Analytics

Module 4: Inferential statistics







Module 4 introduction

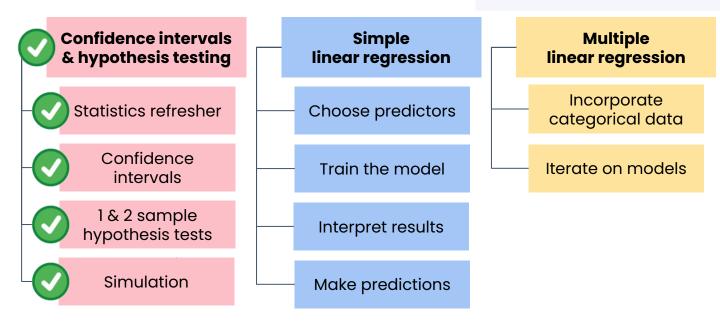


Module 4 outline









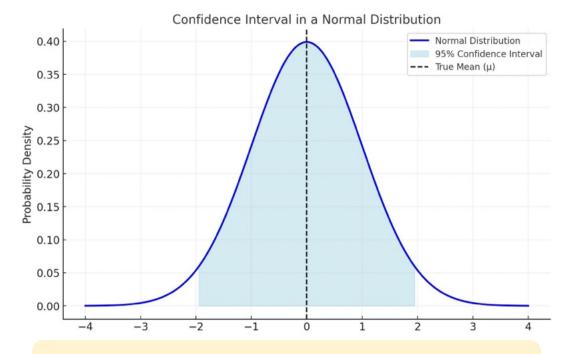


Confidence intervals



Confidence intervals

- Provides a range estimating a particular population parameter:
 - Calculated from sample data
 - Expected to capture true value with a confidence level
- To calculate, you'll need four values:
 - p̂ or x̄ sample statistic
 - o n Sample size
 - o s Sample standard deviation
 - Desired confidence level



95% of intervals contain true mean if you repeatedly sampled population and calculated intervals



Scenario



- **Goal**: Help retailer understand and predict distributions of diamonds
- Task: Pricing new diamonds acquired by the company
- **Dataset**: Past sales

Diamonds are evaluated based on the 4 C's:

- **Cut** quality of diamond's form
- Color color of the stone
- Clarity number of imperfections on stone or within
- Carat measure of weight used for gems



Data Analyst

Recap: Confidence intervals



To calculate a confidence interval:

1. Calculate core descriptive statistics

```
n = df["price"].count()  # Sample size
xbar = df["price"].mean()  # Mean
s = df["price"].std()  # Standard deviation
```

2. Calculate standard error (SEM)

```
SEM = s / np.sqrt(n)
```

3. Use norm.interval:

```
interval = stats.norm.interval(confidence=conf, loc=xbar, scale=SEM)
```



One-sample t-tests



One-sample t-test

- Hypothesis Testing: Test whether there is sufficient evidence in sample to conclude a hypothesis about larger population
- **Example**: Diamonds with a "Premium" cut have a mean price above \$4,500
 - Involves one-sample t-test
 - Comparing a single sample against a hypothesized value
- Review **Applied Statistics for Data Analytics** course if you'd like a refresher

1 Defining your hypotheses

Null

Alternative

- H_n: Premium cut diamonds have a price ≤ \$4500
- H_i: Premium cut diamonds have a price > \$4500

2 Choose your significance level α

- Complement of confidence (1 confidence level)
- Lower α makes it harder to reject H_n
- $\alpha = 0.05$ is common

3 Perform the test and calculate the p-value

- If p < α → Reject the null hypothesis
 Significant evidence Premium cut diamonds > \$4500.
- If $p \ge \alpha \to \text{Fail}$ to reject the null hypothesis Don't have evidence for this claim.

Recap: One-sample t-tests

• To conduct a one-sample test:

Sample of data

test_results = stats.ttest_1samp(df[df["cut"] == "Premium"]["price"], popmean = 4500)

• Returns a sequence of three values

```
p_value = test_results[1]  
— Use to determine whether you are able to reject or fail to reject the null hypothesis
```



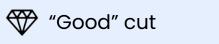
Two-sample t-tests



Scenario



- Goal: Help online retailer understand which cut of diamonds to market to which customers based on their price
- **Task**: Comparing average prices among different diamond cuts
 - Use **two-sample t-test** to determine whether two groups of diamonds have significantly different prices on average





Recap: Two-sample t-tests

To conduct a two-sample test with independent samples:

```
test_results = stats.ttest_ind( good_prices, very_good_prices)

First Second sample
sample
```

- Returns the same type of result as the ttest_1samp function TTestResults
- Access p-value:

```
p_value = test_results[1]
```

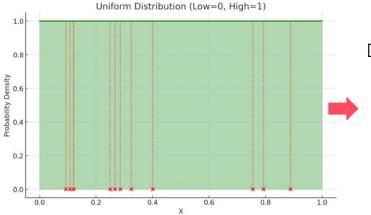


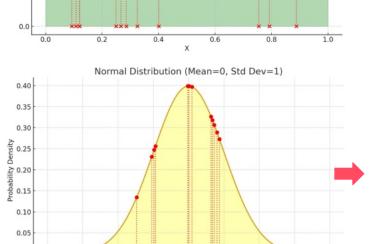
Simulation: uniform



Simulation

- Model how data behaves in the real world
- This approach helps:
 - Explore how factors affect confidence intervals
 - Make better-informed decisions when data is scarce





0.00

Random samples

```
[ 0.79215183, 0.10762506, 0.32422101, 0.09219966, 0.28585869, 0.26611665, 0.88797959, 0.12027102, 0.24911214, 0.11972559, 0.75448402, 0.40031947 ]
```

```
[-1.475415, 0.67372994,
-1.0467017, 0.00958942,
0.09162331,-0.02767743,
0.80450185, 0.87167706,
-0.97679177, 0.63349646
-0.94127279, 0.72664442]
```

Collecting large datasets

Real-world



- Challenging due to time, cost, or logistical constraints
- **Example:** Experiment pricing strategy Difficult to:
 - Interview many customers
 - Gather enough data to make precise estimates

Simulation **•**



- Approximate underlying distribution by estimating parameters
 - Mean
 - Standard deviation
- Generate random samples that model scenarios based on assumptions

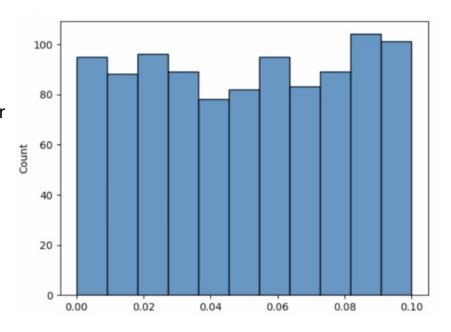
Scenario



- Goal: Assess the potential impacts of a new pricing strategy
- **Task**: Develop simulation of random discount prices within fixed range:
 - o Discounts from 0 to 10% on diamonds
 - Retailers plan to use this as first step towards assessing impact on customer purchasing habits

Using simulation for business insights

- Present this simulation to help your clients:
 - Understand how discounts might be delivered
 - What different scenarios they should prepare for
- Example: Maximum impact on revenue if many of the discounts cluster at higher end
 - Use simulation as starting point to understand likelihood of hitting this threshold



Recap: Simulation

• To generate a large random sample from a uniform distribution:

```
sample = np.random.uniform( low = 0. | high = 0.1 | size = n )
```

• To construct a confidence interval based on random sample:

```
interval = stats.norm.interval(confidence = conf, loc = xbar, scale = SEM)
```

• Used an LLM to write code to repeat the simulation



Simulation: normal



Scenario



- Task: Model potential competitor prices
 - o Hypothesis: Roughly normally distributed around mean price
- **Dataset**: Historical competitor data
 - o Prices vary relatively narrowly around client mean price
 - Standard deviation ≈ \$750
- Simulation could help:
 - Estimate how often prices may be undercut by competitors
 - o Determine discount levels that maintain competitiveness

Recap: Simulation - normal

• To generate random samples from a normal distribution:



What is linear regression?

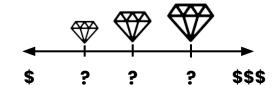


Scenario



You Data Analyst

- Goal: Pricing new diamonds acquired by the company
- **Task**: Predict market price for each diamond based on:
 - Size
 - Cut
 - Clarity



- Predict new prices for individual diamonds
- Start with simple model using just one factor, then add more later

Linear regression

- Enables you to **quantify** relationships
- Best for relationships that are linear
- Able to say:
 - "Bigger diamonds are associated with higher prices"
 - "1 carat increase corresponds to a \$10,000 increase in price"
- Involves two steps:
 - Training
 - Prediction

1. Training

- Quantify the relationship between two features
- Goal: Create a line using the form y=mx+b
 - Determine values for m and b that fit the data best
- Example:
 - price = m * carat + b
 - price = 10,000 * carat + 2000

2. Prediction

- Using trained model to predict y based on x
- o Example: Diamond is 0.5 carats →

Correlation vs. linear regression

Correlation

- Quantify the strength and direction of the relationship
- Example: Diamond carat and price
 - Correlation is 0.92
 - Carat explains 92% of variation in price
 - 8% is controlled by other factors
 - Can't say how much price goes up for increase in carat
- Used to identify most predictive features

Linear Regression

 Generating a equation for "line of best fit" to predict new prices

• Example:

- "A 1 carat increase leads to a \$10,000 increase in price"
- "A half carat diamond is estimated to cost \$7,000"
- If using one independent variable, choose strongest correlation with outcome variable



Important terms

- Inputs (e.g. carat) → Features
- Outputs (e.g. price) → **Outcomes**

In linear regression:

- Independent variable
 - Feature causing part of outcome
- Dependent variable
 - Outcome to predict

Cause	Effect
X Price	X Carat
✓ Carat	☑ Price



Building a linear regression model

- 1. Identify dependent variable
- 2. Identify best independent variable:
 - Calculate correlations
 - Scatter plots
 - Intuition
- 3. Pick one variable to develop first model
 - Independent variables with strongest correlation to outcome
- Train model to identify coefficients of line of best fit: y = mx+b



Independent variable



Choosing an independent variable



Scenario



Task: Pricing new diamonds acquired by your client

You need to predict new prices:

Train linear regression model



- Identify dependent variable
- Identify best independent variable
- Examine correlations and scatter plots

Recap: Choosing independent variables

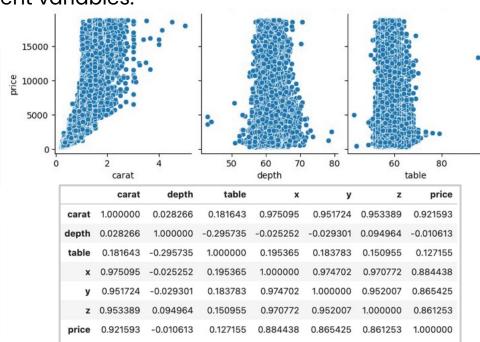
To identify the most promising independent variables:

Visual methods

- Pairplot
- More advanced methods: heatmap

Statistical methods

Correlations





Training the model



Scenario



Task: Predicting diamond prices based on other features of the diamond





Use a new Python module to train your linear regression model

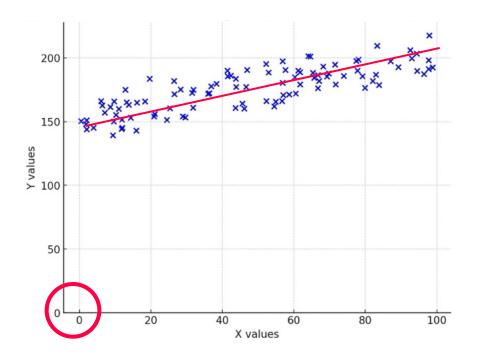
Training the model

 Involves determining the equation of the line of best fit for data:

Slope intercept form:

$$y = mx + b$$

- m slope of the line
 - Represent amount of change in y for each increase of 1 in x
- b intercept 🛑
 - Value of y when x is zero
- Model will identify the best slope and intercept for the data



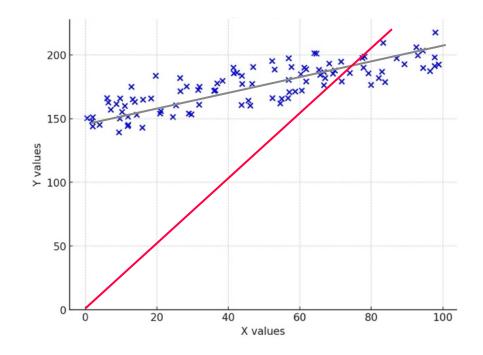
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- m slope of the line
 - Represent amount of change in y for each increase of 1 in x
- b intercept
 - Value of y when x is zero
- Model will identify the best slope and intercept for the data



- If you don't include intercept term b:
 - The intercept is 0
 - Limits the flexibility to best fit your data

Recap: Training the model

• Create your dependent variable:

```
Y = df["price"]
```

• Assemble your independent variable X:

```
X = sm.add_constant(df["carat"])
```

Create model:

```
model = sm.OLS(Y, X)
```

Train model on data:

```
results = model.fit()
```

• Print results of the regression model:

```
results.summary()
```



Interpreting the output of a regression model



R-squared

- Proportion of variance in the dependent variable that is predictable from the independent variable
- How reliably can carat predict price?
- Value between 0 and 1
- The higher, the more the independent variable explains the variation in dependent variable
- Higher is generally better

OLS Regression Results

Dep. Varia Model: Method: Date: Time: No. Observ Df Residua Df Model: Covariance	ations: ls:	price OLS Least Squares Fri, 03 Jan 2025 21:27:36 53938 1 nonrobust	Adj. F-sta Prob Log-L AIC: BIC:	wared: R-squared: atistic: (F-statist Likelihood:	ic):	0.849 0.849 3.041e+05 0.00 -4.7273e+05 9.455e+05 9.455e+05
	coef	std err	t	P> t	[0.025	0.975]
const carat	-2256.3606 7756.4256	13.055 -1 14.067 5		0.000		-2230.772 7783.996
Omnibus: Prob(Omnib Skew: Kurtosis:	us):	14025.341 0.000 0.939 11.035	Jarqu Prob():	0.986 153030.525 0.00 3.65

Carat explains **84.9%** of the variability in price.



P-values

- Tell you whether the coefficients are statistically significant
- Interpret same way the same way as for hypothesis tests
 - **H**₀: Regression coefficient = 0
 - H₁: Regression coefficient ≠ 0
- Is p-value for coefficient > 0.05?
 - YES → Independent variable doesn't predict the dependent variable well
 - NO → Independent variable predicts the dependent variable well

OLS Regression Results

Dep. Varial Model: Method: Date: Time: No. Observa Df Residua Df Model: Covariance	ations: ls:	pric OL Least Square Fri, 03 Jan 202 21:27:3 5394 5393 nonrobus	Adj. s F-st. F-st. Frob Log- AIC: 88 BIC:	wared: R-squared: atistic: (F-statist Likelihood:	ic):	0.849 0.849 3.041e+05 0.00 -4.7273e+05 9.455e+05 9.455e+05
	coef	std err	t	P> t	[0.025	0.975]
const	-2256.3606 7756.4256		-172.830 551.408	0.000	-2281.949 7728.855	-2230.772 7783.996
Omnibus: Prob(Omnibu Skew: Kurtosis:	us):	14025.34 0.00 0.93 11.03	00 Jarq 89 Prob	in-Watson: ue-Bera (JB (JB): . No.):	0.986 153030.525 0.00 3.65



P values for both are close to 0, so they are statistically significant

Coefficients

- Use these values to construct the equation for line of best fit
- Equation:

• 1 carat:

• 2 carats:

OLS Regression Results

========			=====				
Dep. Varia Model: Method: Date: Time: No. Observ Df Residua Df Model: Covariance	ations: ls:	Least Squ Fri, 03 Jan 21:2	2025 7:36 3940 3938 1	Adj. F-sta Prob	mared: R-squared: tistic: (F-statist ikelihood:	ic):	0.849 0.849 3.041e+05 0.00 -4.7273e+05 9.455e+05 9.455e+05
	coe	f std err		t	P> t	[0.025	0.975]
const	-2256.360 7756.425			2.830 1.408	0.000	-2281.949 7728.855	-2230.772 7783.996
Omnibus: Prob(Omnib Skew: Kurtosis:	us):	0	.341 .000 .939 .035):	0.986 153030.525 0.00 3.65



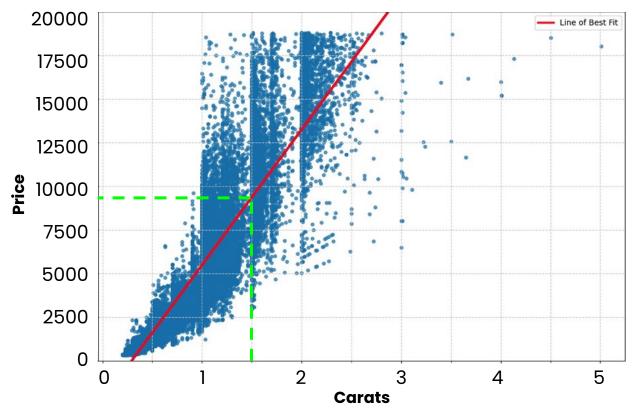
Prediction



Prediction

- **Task**: Predict the price of a new diamond that's 1.5 carats
- **Answer**: Around \$9000

Price vs. Carat





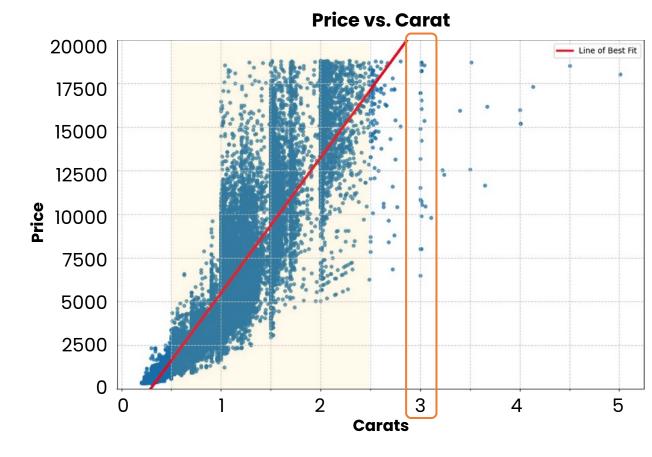
Next steps

• You can:

- Adjust to only simulate between 0.5 and 2.5 carats
- Be upfront by presenting data between 0.5 and 2.5 carats

• Your client can:

 Use it as a starting point to estimate prices





Recap: Prediction

Accessed the calculated m and b values:

```
m = results.params["carat"]
b = results.params["const"]
```

Using new value for carat, predict a price with:

```
carat = 1.5
price = m * carat + b
```

Predict many values by swapping a Series for the single value

```
carats = np.random.uniform(low=0, high=5, size=20)
prices = m * carats + b
```



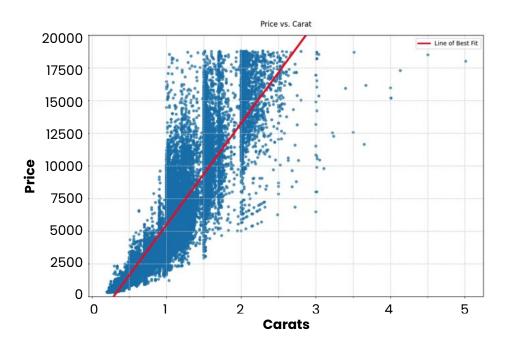
Multiple linear regression



Simple linear regression

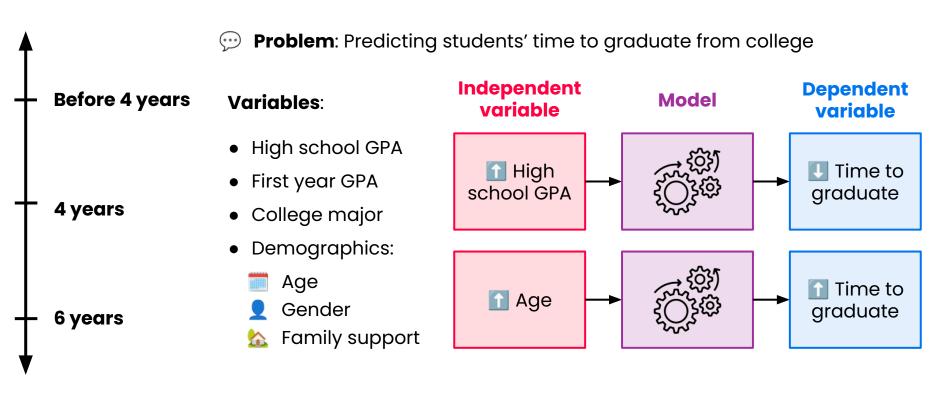
- Linear regression with one independent variable
- Useful starting point in inferential analysis
- Choose strong predictor to build good baseline
- For many problems:
 - Multiple independent variables improves predictive power of model

Example: Carat predicts 85% variability in prices





Multiple linear regression



Example

Simple linear regression model using:

- High school GPA → R² ≈ 0.2
 20% of variability in time to graduate
- Age → R² ≈ 0.1
 10% of variability in time to graduate
- High school GPA and age $\rightarrow R^2 > 0.2$ More predictive power to explain time to graduate

Use combination of variables with most reliable prediction

- High school GPA
- Complement each other

Age

Build complete picture of factors affecting time to graduate

- Freshman GPA
- Explain some of the same variation in time to graduate
- Try it anyway!

Recap: Multiple linear regression

- Linear regression model with more than one independent variable
- Choose independent variables strongly correlated with dependent variable

- Use intuition to:
 - Evaluate why each independent variable might affect the dependent variable
- Evaluate the model's strength using summary



Developing a multiple linear regression model



Scenario





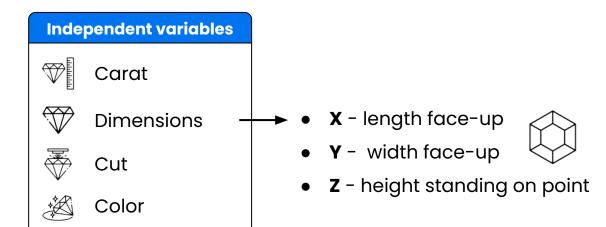
Goal: Need a more accurate model in order to adopt



Task: Add diamond's dimensions to model

• Start with X:





Recap: Multiple linear regression model

• Create a multiple linear regression model:

```
predictors = ["carat","x","y","z"]
Y = df["price"]
X = sm.add_constant(df[predictors])
model = sm.OLS(Y, X)
results = model.fit()
```

Predict new values by modifying equation:

```
m1 = results.params["carat"]
m2 = results.params["x"]
m3 = results.params["y"]
m4 = results.params["z"]
b = results.params["const"]
```

		OLS R	egression A	Results		
Dep. Variabl Model: Method: Date: Time: No. Observat Df Residuals Df Model: Covariance T	ions:	Least Squ Sun, 12 Jan 03:2 5	OLS Adj ares F-si 2025 Prol 7:26 Log- 3941 AIC 3936 BIC		:):	0.854 0.854 7.892e+04 0.00 -4.7188e+05 9.438e+05 9.438e+05
	coef	std err	t	P> t	[0.025	0.975]
carat x y	1921.0000 1.023e+04 -884.0663 166.0140 -576.3115	104.372 62.936 40.470 25.858 39.282	18.405 162.606 -21.845 6.420 -14.671	0.000 0.000 0.000 0.000 0.000		-804.746
Omnibus: Prob(Omnibus Skew: Kurtosis:):	0	.000 Jaro .743 Prol	oin-Watson: que-Bera (JB): o(JB): d. No.		2.002 336488.351 0.00 171.

 Use p-value to understand if variable is a significant predictor in presence of other variables in the model



Interpreting multiple linear regression



Interpreting multiple linear regression

1R-Squared reflects the whole model

- R-squared = $0.854 \rightarrow 85.4\%$ of price variation.
- Can't come to conclusions about how much each variables individually contributes

2P-Values & coefficients considered in context

- Carat's P-value ≈ 0 → Non-zero relationship with price
- **Interpretation**: If x, y, and z are held constant, changes in carat still affect price.

3 Carat's coefficient & impact on price

- Coefficient = \$10,230
 - o 1 carat increase = \$10,230 price increase

OLS Regression Results

Dep. Vari	iable:	pr	ice	R-squa	 red:		0.854
Model:			0LS	Adj. R-	-squared:		0.854
Method:		Least Squa	res	F-stat:	istic:		7.892e+04
Date:	Sa	at, 11 Jan 2	025	Prob (F	F-statisti	c):	0.00
Time:		18:06	:26	Log-Lik	kelihood:		-4.7187e+05
No. Obser	rvations:	53	940	AIC:			9.437e+05
Df Residu		53	935	BIC:			9.438e+05
Df Model:			4				
Covariand	ce Type:	nonrob	ust				
	coef	std err		t	P> t	[0.025	0.975]
const	1921.1740	104.373	18	.407	0.000	1716.601	2125.747
carat	1.023e+04	62.937	162	.607	0.000	1.01e+04	1.04e+04
X	-884.2091	40.470	-21	.848	0.000	-963.532	-804.887
у	166.0384	25.858	6	.421	0.000	115.356	216.721
Z	-576.2035	39.282	-14	.668	0.000	-653.197	-499.216
Omnibus:		14400.	===== 324	Durbin-	======= -Watson:	======	1.198
Prob(Omn	ibus):	0.	000	Jarque-	-Bera (JB)	:	336485.128
Skew:		0.	743	Prob(JE	3):		0.00
Kurtosis	:	15.	145	Cond. N	No.		171.
			=====				

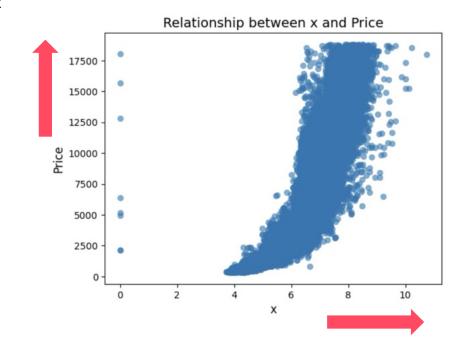


Coefficients & multicollinearity

- Coefficients help understand magnitude of impact
- Why they are impacting is difficult to interpret

Example: x vs. Price graph

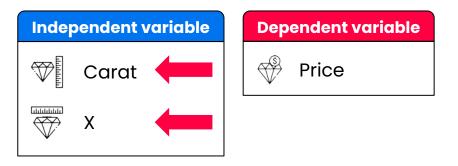
- Positive relationship
- Model coefficient: -884 for x
- Multicollinearity: Both are strongly correlated with dependent variable and each other
- In practice, this can look like:
 - One variable having a positive coefficient
 - Other having a negative coefficient





Multicollinearity

- Two or more independent variables are highly correlated with each other and dependent variable
- Difficult to determine which is driving changes in dependent variable
- Often encounter datasets with many variables that overlap
- Doesn't affect predictive power of model
 - Makes it harder to understand impact of each independent variable



Task	Multicollinearity
Interpreting coefficients and p values	Matters
Predicting new data points	Won't matter as much

carat

table

Measures of size:

Multicollinearity

 Two or more independent variables are highly correlated with each other and dependent variable



Independent variable Carat X

Dependent variable



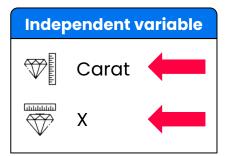
Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
 - [2] The condition number is large, 1.41e+05. This might indicate that there are strong multicollinearity or other numerical problems.
 - Makes it harder to understand impact of each independent variable

Task	Multicollinearity
Interpreting coefficients and p values	Matters
Predicting new data points	Won't matter as much

Multicollinearity

- Two or more independent variables are highly correlated with each other and dependent variable
- Difficult to determine which is driving changes in dependent variable
- Often encounter datasets with many variables that overlap
- Doesn't affect predictive power of model
 - Makes it harder to understand impact of each independent variable
- To address multicollinearity:
 - Remove highly correlated independent variables, keeping one of them
 - Creating composite features of multiple of these variables combined





Measures of size:

carat

table

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Z

Task	Multicollinearity
Interpreting coefficients and p values	Matters
Predicting new data points	Won't matter as much

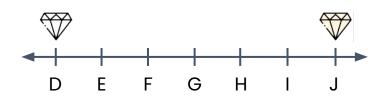


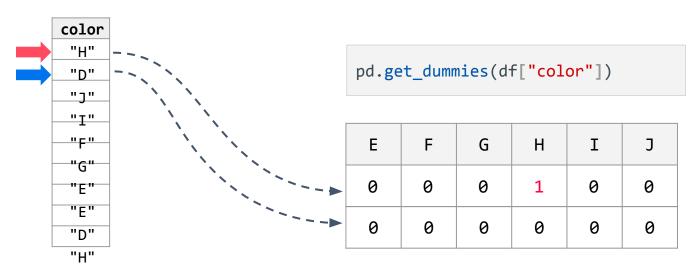
Encoding categorical data



Encoding categorical data

- sm.OLS(Y, X) does not accept non-numeric variables
- To use categorical variable as a predictor, you'll need to turn it into a number:





Recap: Encoding categorical data

• To encode categorical data:

```
pd.get_dummies(df[predictors], columns=["color"], drop_first=True, dtype=int)
```

- columns list of columns to encode
- drop_first=True remove redundant data
- **dtype=int** to get numbers rather than booleans



Modeling with categorical data



Results summary

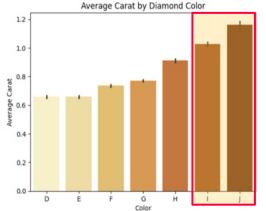
- R-squared: 0.864 → ~1.5% improvement
- P-values all appear significant
- Carat coefficient increased to ~\$8000 per carat

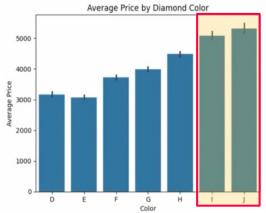
Color coefficients:

- Relative to D color diamonds
- $E \rightarrow ~\$94$ less expensive
- I → ~\$1054 less expensive
- Negative because D color are priciest, as long as carat of the diamond is the same

Dep. Variable: price R-squared:
Model: OLS Adj. R-squared:

=======						
	coef	std err	t	P> t	[0.025	0.975]
const	-2136.8108	20.269	-105.421	0.000	-2176.539	-2097.083
carat	8065.0644	14.164	569.426	0.000	8037.304	8092.825
color_E	-94.2070	23.418	-4.023	0.000	-140.106	-48.308
color_F	-77.4595	23.582	-3.285	0.001	-123.679	-31.239
color_G	-85.7091	22.832	-3.754	0.000	-130.459	-40.959
color_H	-729.6169	24.532	-29.742	0.000	-777.699	-681.535
color_I	-1054.8711	27.539	-38.304	0.000	-1108.848	-1000.894
color_J	-1914.1406	34.049	-56.217	0.000	-1980.877	-1847.405





0.864

0.864

Recap: Categorical data

• Used train/test split strategy to separate data:

```
X_test = X[:1000]  # for testing
Y_test = Y[:1000]  # for testing

X_train = X[1000:]  # for training
Y_train = Y[1000:]  # for training
```

- Coefficients are interpreted relative to the category that was dropped:
 - D was dropped, so all coefficients are relative to D
 - Coefficient was negative because D diamonds are most expensive, all else constant

```
color_E -94.2070

color_F -77.4595

color_G -85.7091

color_H -729.6169

color_I -1054.8711

color_J -1914.1406
```



Prediction: Multiple Linear Regression



Recap: Multiple linear regression

- To predict the **dependent variable** from:
 - Single set of independent variables:

```
predicted = results.predict(diamond1)
```

• Entire data frame at once:

```
predicted = results.predict(X_test)
```

Returns a Series containing one predicted price for each diamond

• **Remember**: X_test data frame must be formatted exactly as X_train



Evaluating your model



Evaluating your model

Compare predictions with the actual values:

- Visualize relationship using a scatter plot
 - Valuable for multiple linear regression
 - Once you have 3 or more variables, you get into hyperdimensional space
- Calculate correlation between prediction and actual values, called multiple r
 - Number between 0 and 1
 - Strength of predictive power
 - Higher value is better

2. Calculate the residuals

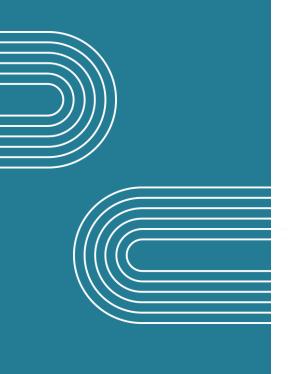
How much model would need to adjust prediction to be correct

Actual	Prediction	Residudis
\$1000	\$1100	-\$100
Actual	Prediction	Residuals

3. Calculate mean absolute error (MAE)

- Average size of the errors in predictions
- In the same unit as data (i.e. dollars)

Daaiduala



LLMs for model iteration





The linear regression process



Typical linear regression workflow

To train your model:

- 1. Select the dependent variable (Y)
- 2. Examine scatterplots and correlations between other features and the dependent variable
 - Identify strongly correlated features to use as independent variables (X)
- 3. Separate data into training and testing sets
 - Reserve 10-20% for test set

- 4. Start with simple linear regression
 - Model with most strongly correlated X
 - Use statsmodels to run regression and evaluate its fit
- 5. You've developed the first in series of models!

Model iteration

Evaluate fit of model:

- Examine **r-squared** understand predictive power (higher is better).
- Examine p-values of coefficients understand if each one is significant
- Other metrics to evaluate model's fit:
 - Graph predicted vs. actual values
 - Calculate multiple R
 - Calculate residuals and mean absolute error

Achieve higher r-squared and lower MAE:

- Add more features to model to create a multiple linear regression
- Use diverse set of independent variables that provide some predictive power
- Be mindful of multicollinearity



Making predictions

Once satisfied with model's explanatory power:

- 1. Use it to predict new values
 - Predict single value or series of values using statsmodels