Gerrymandering New Mexico (OPTIONAL)

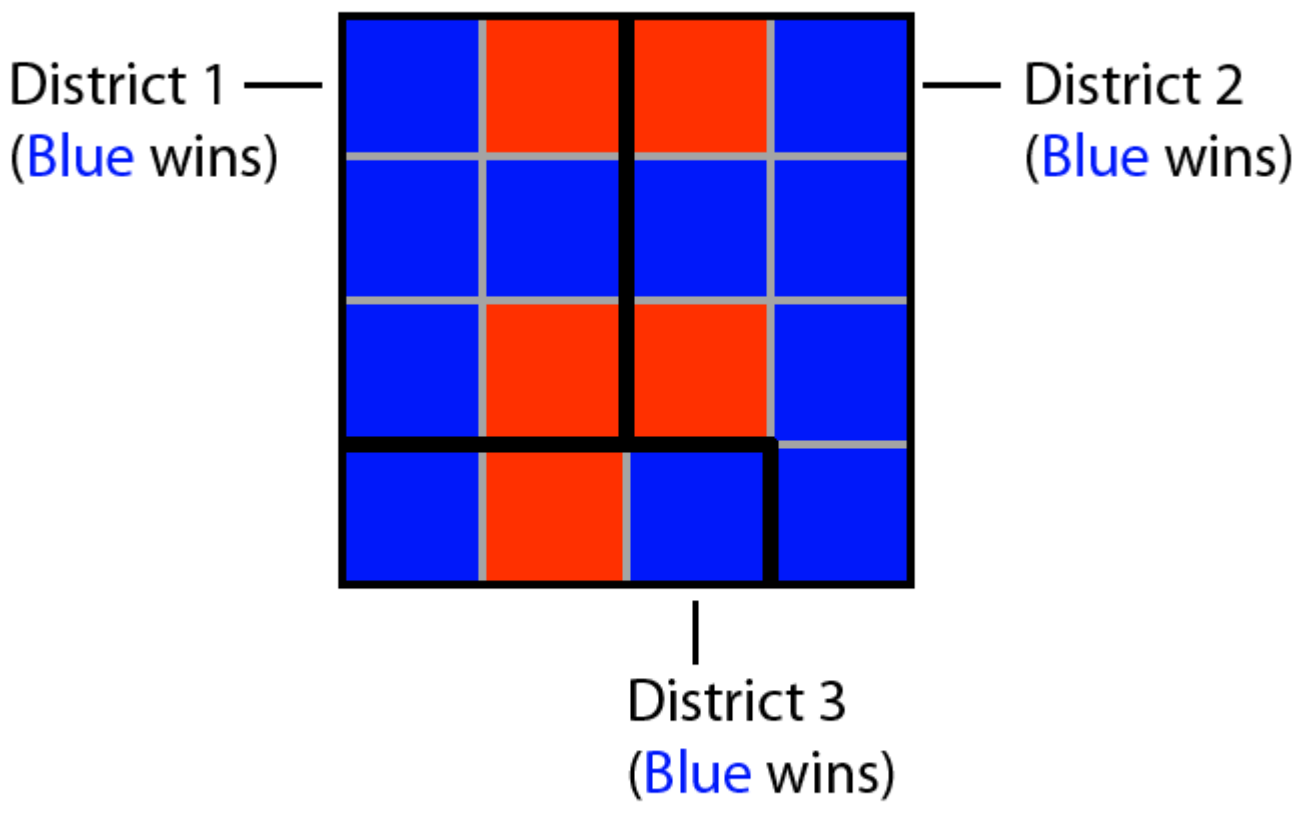
IMPORTANT NOTE: This problem is optional, and will not count towards your grade. We have created this problem to give you extra practice with the topics covered in this unit.

Gerrymandering New Mexico (OPTIONAL)

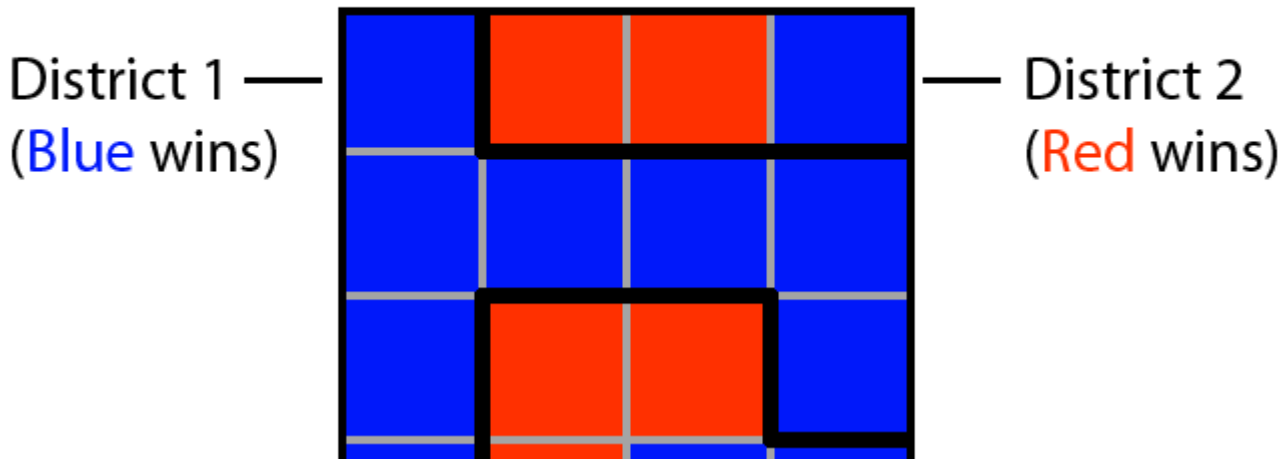
In the United States, each state is divided into small regions called *districts*. In every even-numbered year, the citizens who reside in each district can vote in an election to determine the *representative* of that district. The representative is a member of the House of Representatives, which is one of the two chambers of the Congress of the United States. Representatives hold great power, as they can propose and vote on bills that later can become laws.

Each representative typically is affiliated with one of the two major political parties in the United States: the Democratic Party or the Republican Party. Each party naturally wants to ensure that they have as many representatives in Congress as possible. One way of achieving this is through *gerrymandering*.

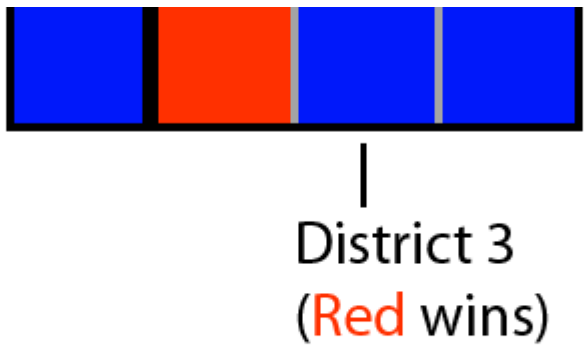
Gerrymandering refers to the process of redrawing district boundaries so as to favor a particular political party. To illustrate this, suppose we have the hypothetical state below, with three districts. Each district is further subdivided along a grid into smaller subregions, where each subregion votes unanimously for either party. Suppose that in this hypothetical example, there is only one voter in each subregion.



Based on the current district boundaries, the blue party has a majority in each district, so each district elects a blue representative. However, suppose we decide to redraw the boundaries in the following way:



Calculator

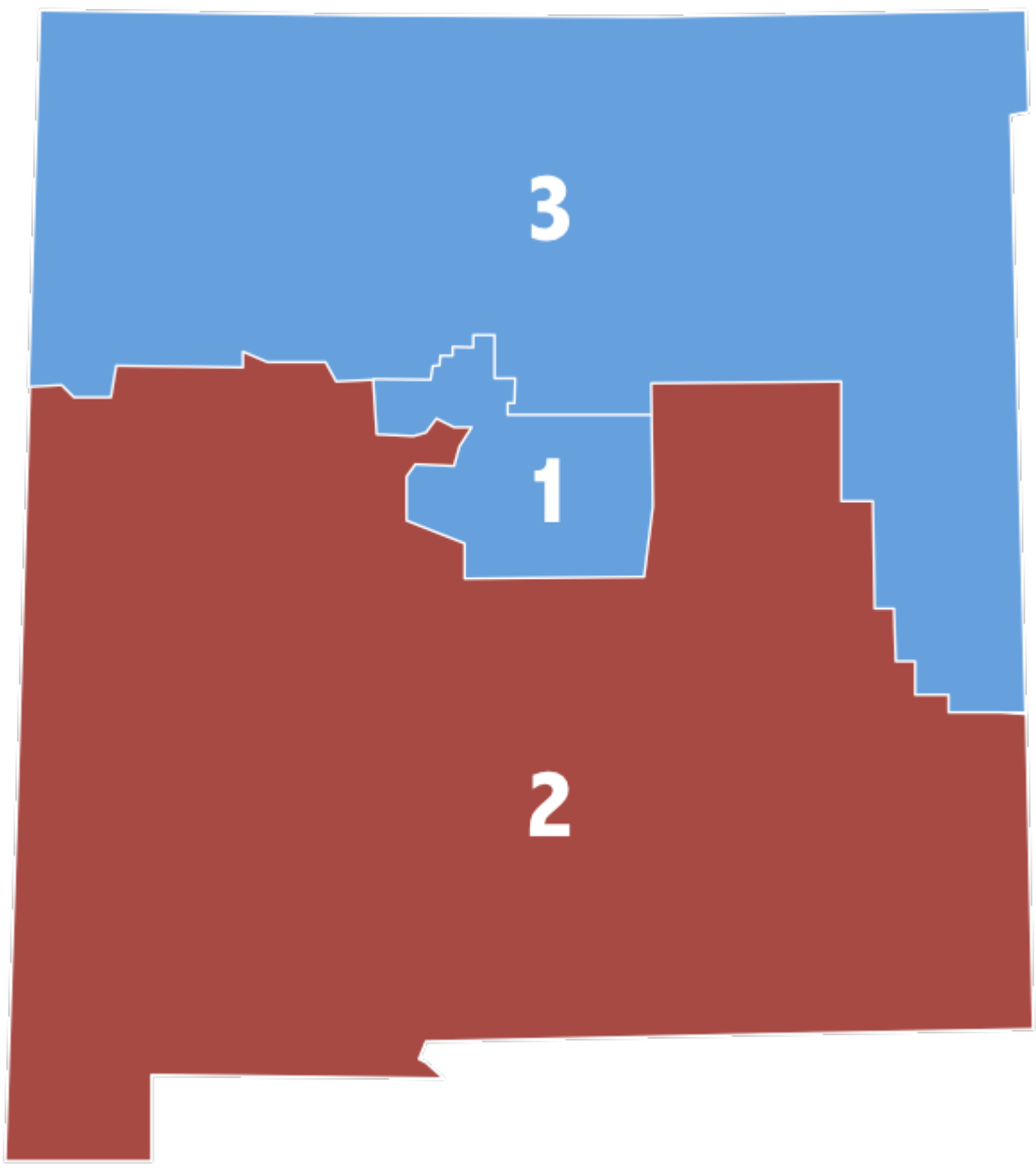


Now the blue party does not win in every district; in fact, the red party wins two of the three districts.

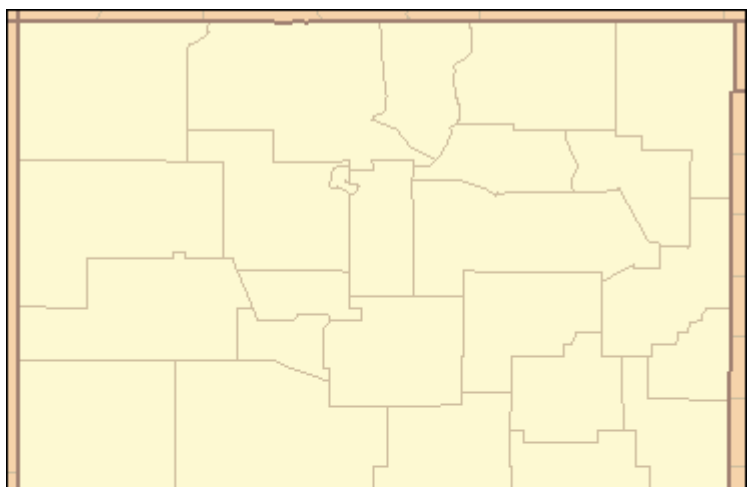
In this problem, we will be exploring how to systematically manipulate these kinds of boundaries. We will be doing this specifically for the state of New Mexico, which is one of the fifty states of the United States.

The Data

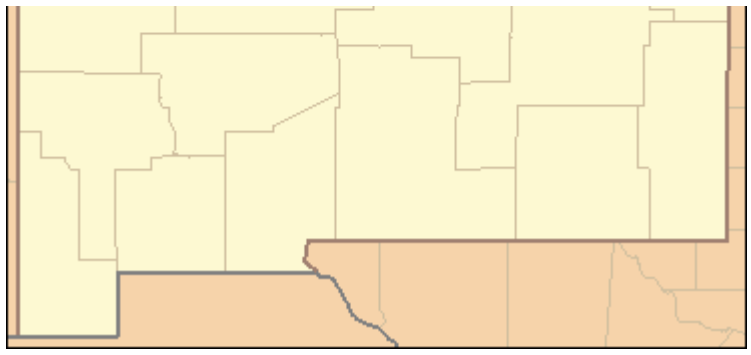
The state of New Mexico, located in the south of the US, currently has three districts:



The state of New Mexico is also divided into counties:



Calculator



Counties are administrative units that are typically smaller than districts. In many states counties are split across districts, but in this problem we will assume that the new districts we will design will be built from the existing counties.

We have the voting record from the 2012 presidential election for each county in New Mexico. We will use the presidential election voting record of each county in 2012 as a proxy for how the county will vote in the next election for the house of representatives. This data is provided in Gerrymandering.ods for LibreOffice and OpenOffice, or Gerrymandering.xlsx for Excel.

The Problem

In the 2012 House of Representatives election, the Democratic party won in New Mexico's 1st and 3rd districts, while the Republican party won in the 2nd district.

Suppose that we have the opportunity to gerrymander New Mexico, so that we still have three districts. Is there a way to redesign the three districts so that the Democratic party takes all three districts?

This is the topic of our problem — and we will tackle it using integer optimization. Let's formulate this problem as follows.

Our principal decision variables are defined for each district and each county , where is 1 if county is assigned to district and 0 otherwise.

The first scenario we will consider is selecting our objective to maximize the number of votes that the Democratic party wins district 2 by (remember that the Democratic party lost in the 2nd district). If we can get the Democratic party to win district 2 by a margin of at least 100 votes while still winning districts 1 and 3 by a margin of at least 100 votes in each district, the Democratic party will win all three districts.

With regard to constraints, we would like:

- Each county to be assigned to exactly one district;
- Each district to consist of at least one county; and
- The Democratic party to still win districts 1 and 3 by a margin of at least 100 votes.

Problem 1.1 - The Objective

0 points possible (ungraded)

Which of the following is the correct objective function for this problem?

- O1: maximize , where and are the numbers of Democratic and Republican votes, respectively, cast in county ✔
- O2: maximize
- O3: maximize , where and are the numbers of Democratic and Republican votes, respectively, cast in county
- O4: maximize

Calculator

Explanation
O1 is the correct objective function; the sum over all counties of will be the number of votes that the Democratic party wins district 2 by.
O2 is not correct. O2 merely sums the assignment variables -- this expression does not capture by how much the Democratic Party wins district 2.
O3 is also incorrect. O3 is the same as O1, except that it computes the number of votes that the Democratic Party wins district 1 by.
O4 is also incorrect, because it just sums up the decision variables for district 2.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Problem 1.2 - Assignment Constraints

0 points possible (ungraded)

Which of the following sets of inequalities should we add to our model to capture the constraints that each county should be assigned to exactly one district?

- C1:
- C2: , for ✔
- C3: , for
- C4: for , and

Which of the following sets of inequalities should we add to our model to capture the constraints that each district should contain at least one county?

- C1:
- C2: , for
- C3: , for ✔
- C4: for , and

Explanation
C2 is correct for the first question, and C3 is correct for the second question. C2 ensures that every county is assigned to one -- and only one -- district.
C3 says that the sum of is at least one; since the 's are binary, this means that for each district, there is at least one county j such that is 1. This is exactly the constraint that at least one county is assigned to every district.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Calculator

Problem 1.3 - District 1 and 3 Constraints

0 points possible (ungraded)
We would also like to ensure that the Democratic Party still wins districts 1 and 3 (with a margin of 100 votes).

Remember that our data gives us for each county the difference , where is the number of votes cast for the Republican Party, and is the number of votes cast for the Democratic Party.

Which of the following is the correct constraint to ensure that the Democratic Party will win district 1 by at least 100 votes?

- C1: ✔
- C2:
- C3: For each county j,

We'll need to add a similar constraint to our model for district 3.

Explanation
The first option is the correct answer. The left hand side models the difference of Democratic and Republican votes in district 1, and the constraint says that this difference has to be at least 100.
The second option is incorrect. It requires that the difference in Democratic and Republican votes, in district 1, is nonnegative: in words, the Democratic party wins district 1. This is not what we want.
The third option is also incorrect. This is immediate from the fact that it is specified for each county; the left hand side does not model the difference between the votes cast in district 1.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Problem 2.1 - Solving the Problem

0 points possible (ungraded)
Formulate the problem in LibreOffice and solve it. Use the decision variables, the objective and the constraints we defined above. For the vote difference , use the numbers given under the column "Scenario 1".

By how many votes does the Democratic Party win in district 2 under this redistricting?

Answer: 76197

Explanation
This is the optimal objective value of your model. Note that depending on your solver settings, your answer might be slightly different than the answer shown here (we get 76197 in LibreOffice/OpenOffice and 75909 in Excel).

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Problem 2.2 - Adjusting the Objective

0 points possible (ungraded)

Calculator

0 points possible (ungraded)

Given the data we have provided, it may seem that there are many ways to redistrict the state so that the Democratic Party wins in all three districts. However, some of the new proposals may not be very different from the existing districts, while some may require drastic changes to the boundaries.

In the spreadsheet, we have included information about the current districts, that is, which counties belong to which districts. (Note that this is an approximate assessment because in New Mexico, districts are not exactly made up of counties.)

Let be 1 if district i currently contains county j, and 0 otherwise. We wish to ensure that our proposed district assignments and the current district assignments are as "similar" as possible. Which of the following objectives allows us to correctly do this?

- O1: maximize
- O2: minimize
- O3: maximize ✔

Explanation

O3 is the correct answer. To see this, fix a county j. If we consider the terms for county j, we have

If the assignment of county j is the same in our proposal and in the current (2012) assignments, then this expression evaluates to one.

If the assignments of county j in our proposal and in the 2012 districts are different, then this expression evaluates to zero. In words, the sum is 0 if county j is assigned differently in the two assignments, and 1 if it is assigned the same way. If we now sum this over all i, as in O3, we get the total number of counties that we assigned the same as the existing 2012 districts. By maximizing this sum, we ensure that our proposal is as similar as possible to the existing 2012 districts.

O1 is not correct. The reason why is that for any solution, the value of this objective is the same.

O2 is not correct because it is minimizing instead of maximizing.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Problem 2.3 - Re-Solving the Problem

0 points possible (ungraded)

Modify your problem to include the new objective. The old objective should become a constraint like the ones we have for districts 1 and 3 - we want to ensure that the Democratic party wins by a margin of at least 100 votes in district 2 as well.

You should still be using the "Scenario 1" column for the vote differences.

Solve the problem. How many counties are NOT given new assignments (relative to the 2012 districts -- columns C through E in the spreadsheet) in this new solution?

Answer: 32

Explanation

After solving the problem in LibreOffice, the optimal objective is 32. Therefore, the number of counties that do not have new assignments is 32.

Submit You have used 0 of 2 attempts

Calculator

Answers are displayed within the problem

Problem 2.4 - Understanding the Solution

0 points possible (ungraded)

Which counties have been re-assigned relative to the 2012 assignments (columns C through E in the spreadsheet)? Select all that apply.

- [ ] 1 - Bernalillo
- [ ] 2 - Catron
- [ ] 3 - Chaves
- [ ] 4 - Cibola
- [ ] 5 - Colfax
- [ ] 6 - Curry
- [ ] 7 - DeBaca
- [ ] 8 - Dona Ana
- [ ] 9 - Eddy
- [ ] 10 - Grant
- [ ] 11 - Guadalupe
- [ ] 12 - Harding
- [ ] 13 - Hidalgo
- [ ] 14 - Lea
- [ ] 15 - Lincoln
- [ ] 16 - Los Alamos
- [ ] 17 - Luna
- [ ] 18 - McKinley
- [ ] 19 - Mora
- [ ] 20 - Otero
- [ ] 21 - Quay
- [ ] 22 - Rio Arriba
- [ ] 23 - Roosevelt

Calculator

- [ ] 24 - Sandoval
- [ ] 25 - San Juan
- [ ] 26 - San Miguel
- [ ] 27 - Santa Fe ✔
- [ ] 28 - Sierra
- [ ] 29 - Socorro
- [ ] 30 - Taos
- [ ] 31 - Torrance
- [ ] 32 - Union
- [ ] 33 - Valencia

Explanation
Simply compare the values in your assignment cells to the values in columns C through E. By doing this, the only county that has changed is Santa Fe.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

In addition to ensuring that the Democratic Party wins in each district, we also may have to take into account other considerations:

1. Exactly one of Santa Fe (county 27) or Dona Ana (county 8) must be in district 2.
2. Both Socorro (county 29) and Torrance (county 31) must be in the same district.

Problem 3.1 - A New Constraint

0 points possible (ungraded)
Which of the following constraints models constraint 1 listed above?

- ○ C1:
- ◉ C2: ✔
- ○ C3:

Explanation
C2 is the correct answer. There are only two ways that C2 can be satisfied: if and (Santa Fe is in district 2, and Dona Ana is not in district 2), or if and (Santa Fe is not in distric and Dona Ana is in district 2)

Calculator

and Dona Ana is in district 2).
C1 is incorrect. The meaning of C1 is that at most one of counties 27 and 8 is in district 2.
C3 is also incorrect. C3 can only be satisfied if both Santa Fe and Dona Ana are in district 2. Clearly this is not what we want, so C3 is not appropriate.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Problem 3.2 - A New Constraint

0 points possible (ungraded)
Which of the following models constraint 2 listed above?

- C1:
- C2: ✔
- C3:
- This constraint cannot be modeled using the variables of our model.

Explanation
C2 is the correct answer. To see this, consider what the value of the left hand side will be. If county 29 is assigned to district 1, then is 1, and and are both 0, so the value of the left hand side is 1. If county 29 is assigned to district 2, then , and , so the value is 2. If county 29 is assigned to district 3, then , so the value is 3. In words, the left hand side exactly models the district to which county 29 is assigned. The right hand side similarly models the district to which county 31 is assigned. By setting these two values to be equal to each other, we ensure that county 29 and county 31 end up in the same district.
C1 is not correct. It can be satisfied by assigning counties 29 and 31 to different districts; for example, if (so {) and if (so), then the left hand side ends up being 1 and the right hand side ends up being 1, even though county 29 is in district 1 and county 31 is in district 2.
C3 is not correct. C3 requires that either both county 29 and 31 are in district 1, or both county 29 and 31 are not in district 1. This is in the right direction, but not quite what we want.
C4 is clearly not correct, as C2 is a valid way to model this constraint.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Problem 3.3 - Re-Solving the Problem

0 points possible (ungraded)
Add these two constraints and re-solve the problem. How many counties have been re-assigned (relative to the 2012 assignments in columns C through E of the spreadsheet)?

Answer: 2

Explanation
The optimal objective is 31; since the optimal objective counts the number of assignments that are not different relative to the 2012 assignments, the answer is 2.

Calculator

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Answers are displayed within the problem

Problem 3.4 - Re-Assigned Counties

0 points possible (ungraded)

Which counties have been re-assigned relative to the 2012 assignments (columns C through E in the spreadsheet)? Select all that apply.

- [x] 1 - Bernalillo
- [ ] 2 - Catron
- [ ] 3 - Chaves
- [ ] 4 - Cibola
- [ ] 5 - Colfax
- [ ] 6 - Curry
- [ ] 7 - DeBaca
- [ ] 8 - Dona Ana
- [ ] 9 - Eddy
- [ ] 10 - Grant
- [ ] 11 - Guadalupe
- [ ] 12 - Harding
- [ ] 13 - Hidalgo
- [ ] 14 - Lea
- [ ] 15 - Lincoln
- [ ] 16 - Los Alamos
- [ ] 17 - Luna
- [ ] 18 - McKinley
- [ ] 19 - Mora
- [ ] 20 - Otero
- [ ] 21 - Quay
- [ ] 22 - Rio Arriba

Calculator

- [ ] 23 - Roosevelt
- [ ] 24 - Sandoval
- [ ] 25 - San Juan
- [ ] 26 - San Miguel
- [ ] 27 - Santa Fe
- [ ] 28 - Sierra
- [x] 29 - Socorro
- [ ] 30 - Taos
- [ ] 31 - Torrance
- [ ] 32 - Union
- [ ] 33 - Valencia

Explanation
Simply compare the values in the cells containing your assignment decision variables to the values in columns C through E of the spreadsheet.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Problem 4.1 - Voting Considerations

0 points possible (ungraded)
So far, we have been using only one voting scenario to design our districts. In this scenario, we've assumed that each county will vote in the representative election of its designated district the same way it voted in the 2012 presidential election (scenario 1 in the spreadsheet). For example, Bernalillo county will vote for the Democratic candidate of its district, with a margin of 42,941 more voters (i.e., the number of Democratic votes from Bernalillo is 42,941 higher than the number of Republican votes).

This is a problematic feature of the model, because voters will not vote in this exact way in future elections. In fact, if they vote sufficiently differently, the democratic party may not be able to win all of its representative elections.

To illustrate this, consider the margins under "Scenario 2" in the spreadsheet. Use these values in place of the original margins to compute how many votes more than the Republican party the Democratic Party gets in the districts, given the optimal solution from Problem 3.4. Which districts does the Democratic Party lose? Select all that apply.

- [x] District 1
- [ ] District 2
- [ ] District 3

Calculator

Explanation
Apply the SUMPRODUCT command using the assignments and the new margins. Doing this, we see that the difference between the Democratic votes and Republican votes in each of the districts are
District 1: -489 votes
District 2: 24899 votes
District 3: 51662 votes
Since district 1 is the only one where the difference is negative, this is the only district the Democrats lose.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

Problem 4.2 - Voting Scenarios

0 points possible (ungraded)
Let's change our formulation to make it more robust to changes in voter behavior. Suppose that in addition to the data we have been using so far (based on the 2012 presidential election numbers), we also wish to account for two other scenarios: scenario 2 (which we just used in Problem 12) and scenario 3. These scenarios are based on forecasts obtained from a separate prediction model. Furthermore, we want to make sure that the Democratic party wins by a large margin, so we will change the constraints to ensure that the Democratic party wins at least 12,000 more votes than the republicans.

To do this, we need to revisit our constraints that ensure that the Democratic party wins each district. In particular, the Democratic Party should win each district with a margin of at least 12,000 votes in every scenario; so instead of three constraints (one for each district), we should have nine constraints (one for each district and scenario pair).

Add these constraints to the model, and re-solve it. How many counties have been re-assigned relative to the existing 2012 assignments (columns C through E in the spreadsheet)?

Answer: 4

Explanation
After solving the problem, the objective value is 29; since the objective value is the number of counties that have not changed, the answer is 4.

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Answers are displayed within the problem

Problem 4.3 - Understanding the New Solution

0 points possible (ungraded)
Which counties have been re-assigned? Select all that apply.

- [ ] 1 - Bernalillo
- [ ] 2 - Catron
- [ ] 3 - Chaves
- [ ] 4 - Cibola
- [ ] 5 - Colfax
- [ ] 6 - Curry

Calculator

- [ ] 7 - DeBaca
- [x] 8 - Dona Ana
- [ ] 9 - Eddy
- [ ] 10 - Grant
- [ ] 11 - Guadalupe
- [ ] 12 - Harding
- [ ] 13 - Hidalgo
- [ ] 14 - Lea
- [ ] 15 - Lincoln
- [ ] 16 - Los Alamos
- [ ] 17 - Luna
- [ ] 18 - McKinley
- [ ] 19 - Mora
- [ ] 20 - Otero
- [ ] 21 - Quay
- [x] 22 - Rio Arriba
- [ ] 23 - Roosevelt
- [ ] 24 - Sandoval
- [ ] 25 - San Juan
- [ ] 26 - San Miguel
- [x] 27 - Santa Fe
- [ ] 28 - Sierra
- [x] 29 - Socorro
- [ ] 30 - Taos
- [ ] 31 - Torrance
- [ ] 32 - Union

Calculator

☐ 33 - Valencia

Explanation
Simply compare the values in the cells containing your assignment decision variables to the values in columns C through E of the spreadsheet.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Problem 4.4 - Margin of Victory

0 points possible (ungraded)
By what margin does the Democratic Party win in district 3 in Scenario 2?

Answer: 18054

By what margin does the Democratic Party win in district 1 in Scenario 3?

Answer: 12095

Explanation
You can find these numbers by looking at the left-hand-sides of the constraints.

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Answers are displayed within the problem

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