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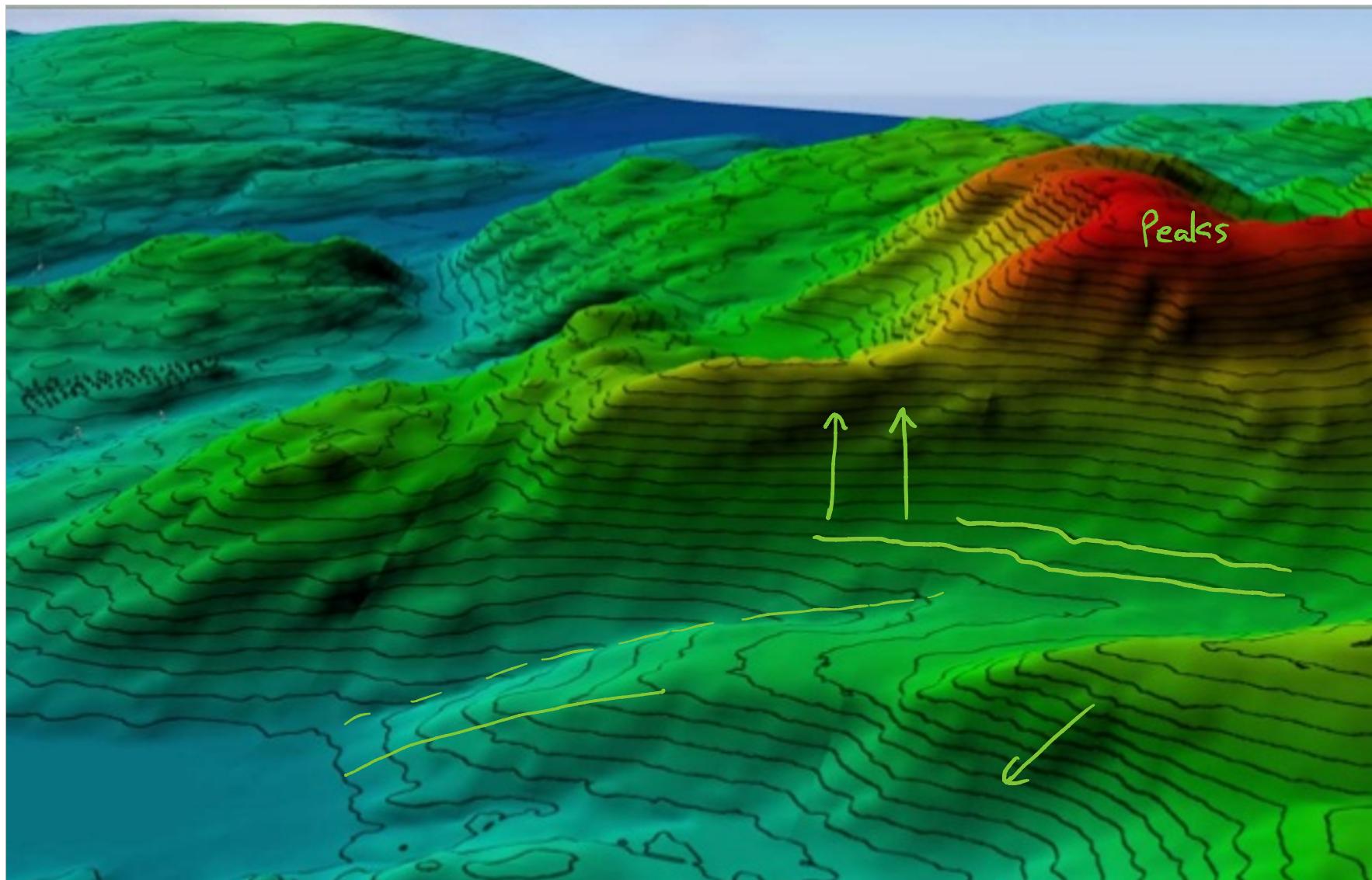
Deep Learning Applications for Computer Vision

Lecture 7: Gradients and Linear Filters



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Terrain Concepts



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Terrain Concepts

Basic notions:

- Uphill / downhill
- Contour lines (curves of constant elevation)
- Steepness of slope
- Peaks/Valleys (local extrema) : \max, \min

More mathematical notions:

- Tangent Plane
- Normal vectors
- Curvature

Gradient vectors (vectors of partial derivatives) will help us define/compute all of these.



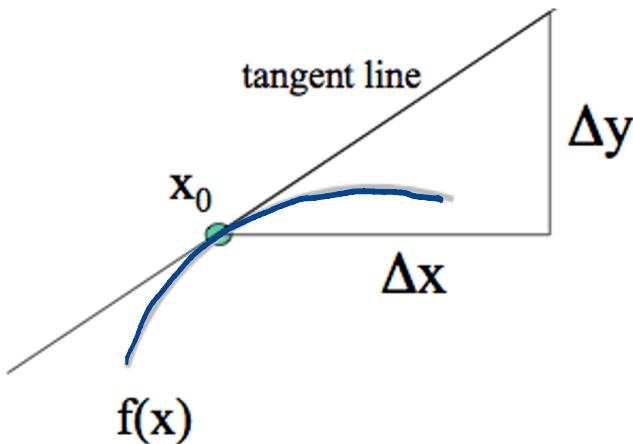
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Gradients – 1D

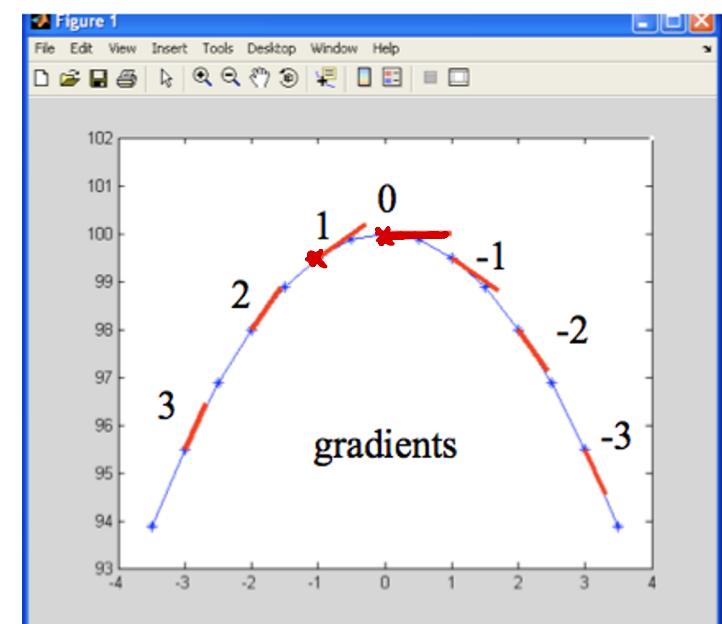
Consider function $f(x) = 100 - 0.5 * x^2$

Gradient: $\frac{d f(x)}{dx} = (-0.5) * 2 * x = -x$

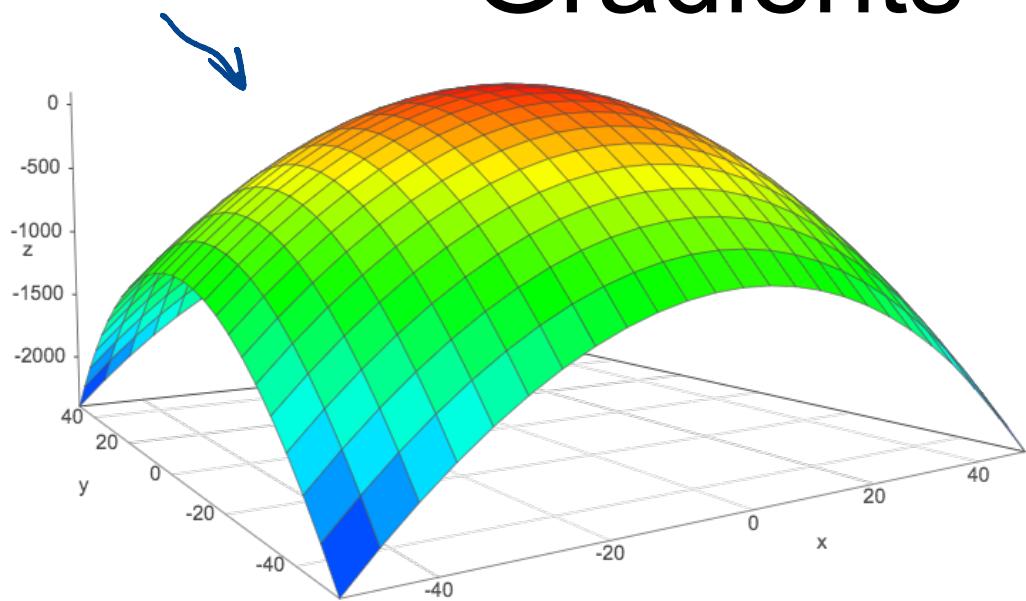
Geometric interpretation: the gradient at x_0 is the slope of tangent line to the curve at point x_0



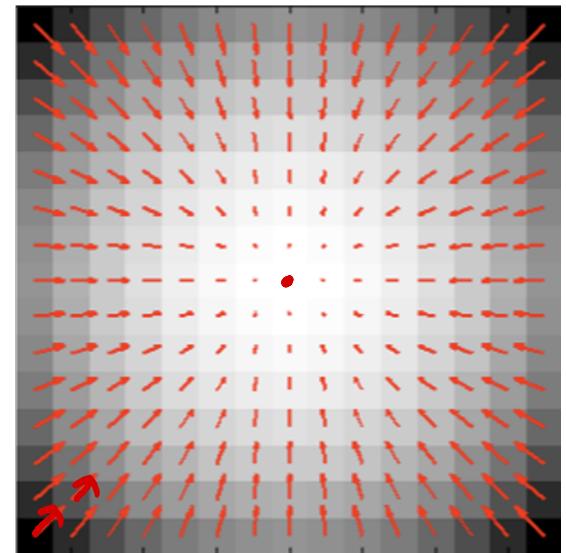
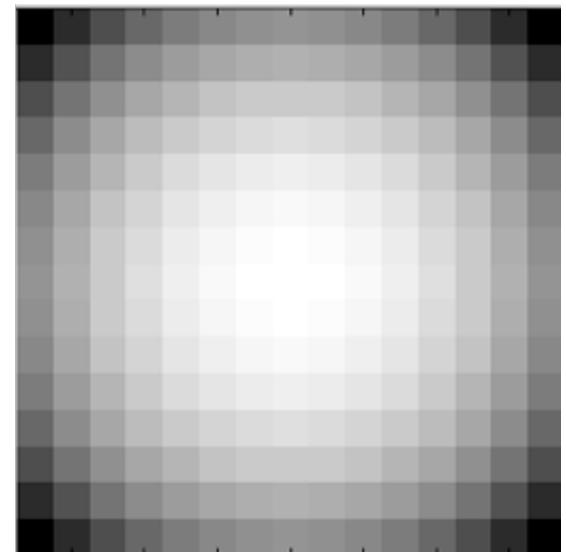
$$\begin{aligned}\text{slope} &= \Delta y / \Delta x \\ &= \left. \frac{df(x)}{dx} \right|_{x_0}\end{aligned}$$



Gradients – 2D_{col}



row



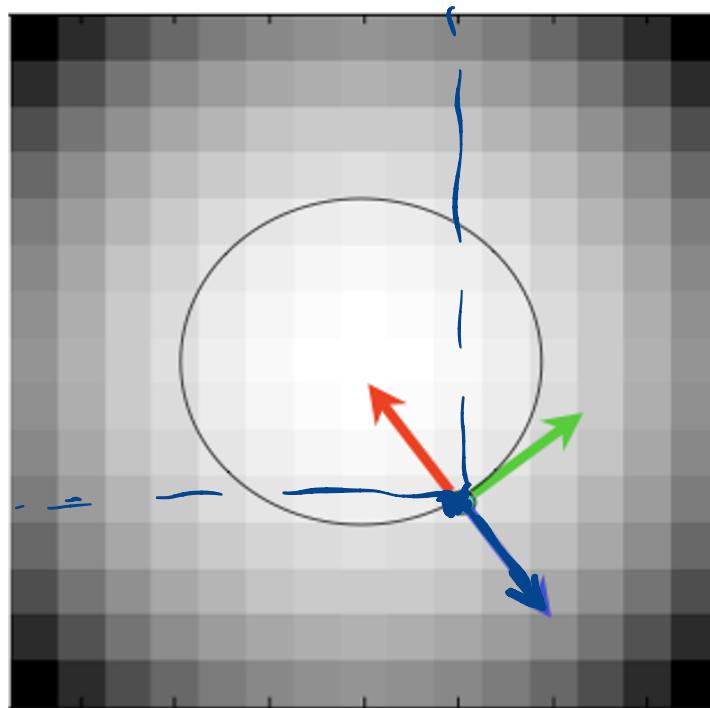
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2D Gradients

$$f(x, y) = 100 - 0.5 * x^2 - 0.5 * y^2$$

x component $\rightarrow \frac{d f(x, y)}{dx} = (-0.5) * 2 * x = -x$

y component $\rightarrow \frac{d f(x, y)}{dy} = (-0.5) * 2 * y = -y$



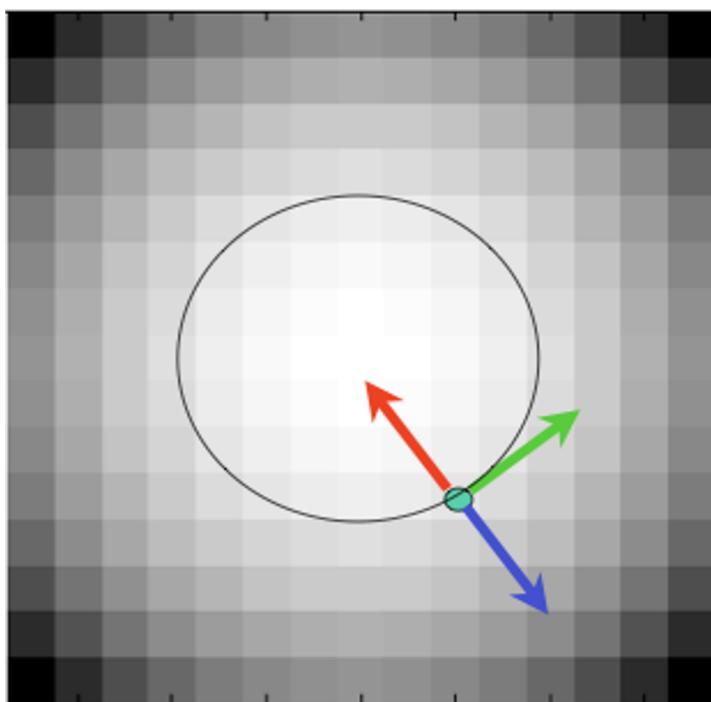
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2D Gradients

$$f(x, y) = 100 - 0.5 * x^2 - 0.5 * y^2$$

$$\frac{d f(x, y)}{dx} = (-0.5) * 2 * x = -x$$

$$\frac{d f(x, y)}{dy} = (-0.5) * 2 * y = -y$$



Let $\mathbf{g}=[g_x, g_y]$ be the gradient vector at point/pixel (x_0, y_0)

Vector \mathbf{g} points uphill
(direction of steepest ascent)

Vector $-\mathbf{g}$ points downhill
(direction of steepest descent)

Vector $[g_y, -g_x]$ is perpendicular to \mathbf{g}
(direction of constant elevation)

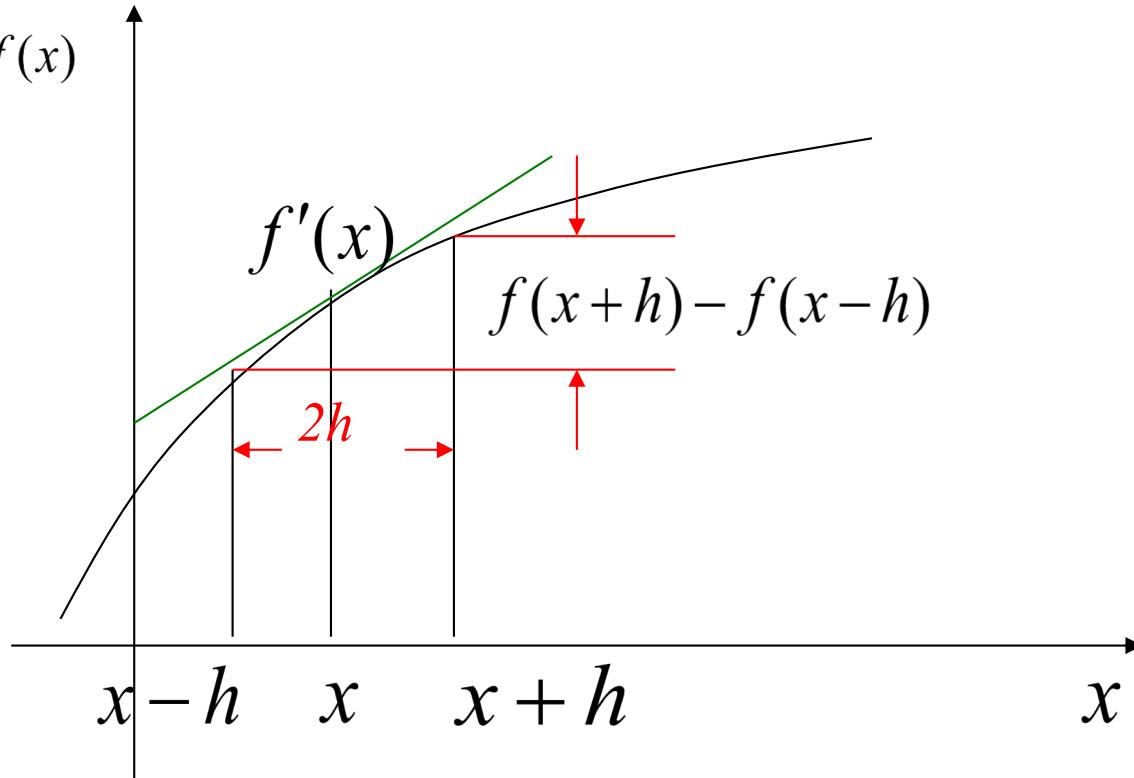


Central Difference Formula for $f'(x)$

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

h - small

Geometrically



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Introducing: Spatial Image Gradients



$$h=1$$

$$I_x = \frac{I(x+1, y) - I(x-1, y)}{2}$$

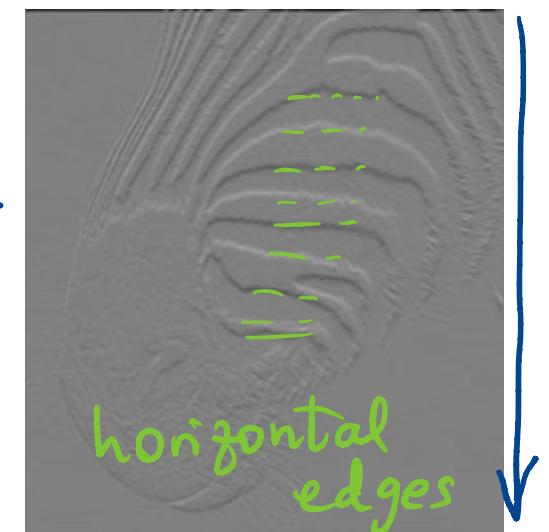
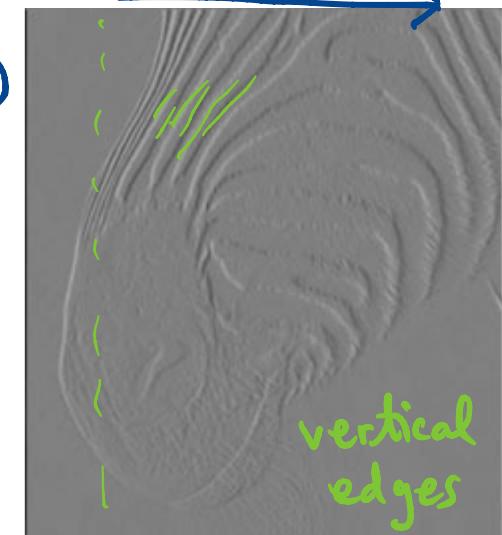
Partial derivative
wrt x

$$I$$

$$I(x, y)$$

$$I_y = \frac{I(x, y+1) - I(x, y-1)}{2}$$

Partial derivative
wrt y



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Example:

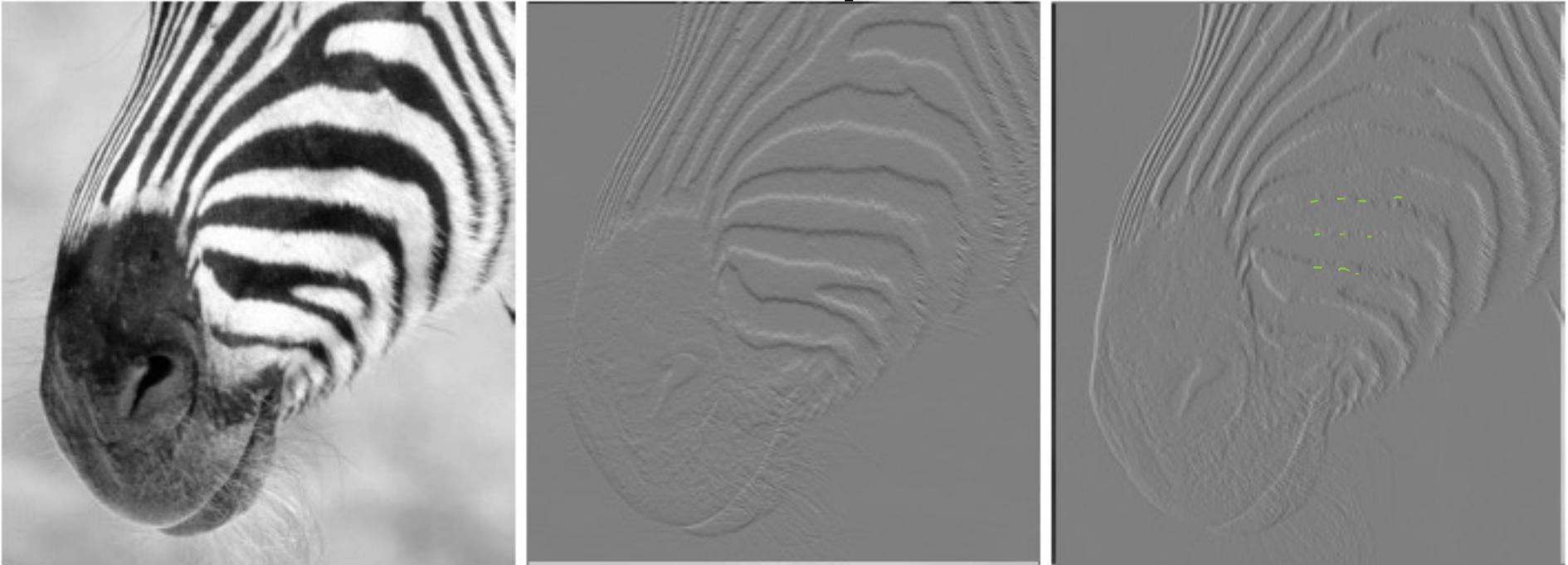
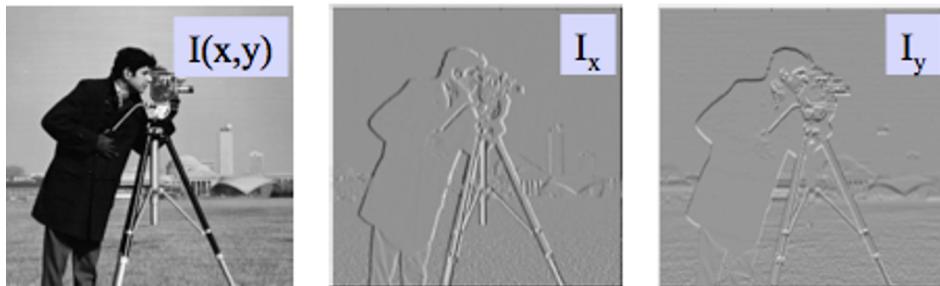


Figure 4.4 ... shows estimates of derivatives obtained by finite differences. The image at the left shows a detail from a picture of a zebra. The center image shows the *partial derivative in the y-direction*—which responds strongly to horizontal stripes and weakly to vertical stripes—and the right image shows the *partial derivative in the x-direction*—which responds strongly to vertical stripes and weakly to horizontal stripes.

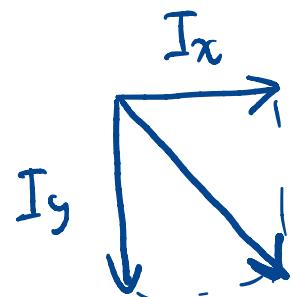


Functions of Gradients



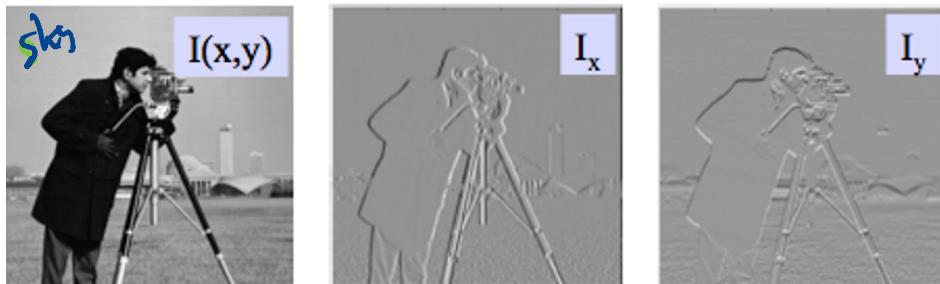
Magnitude of gradient

$$\sqrt{I_x^2 + I_y^2}$$



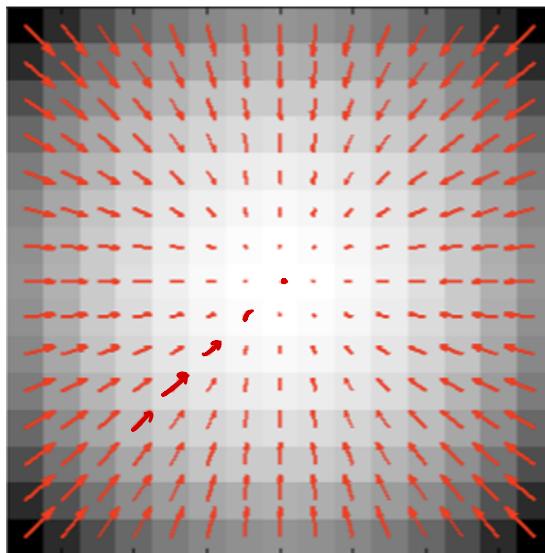
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Functions of Gradients



Magnitude of gradient

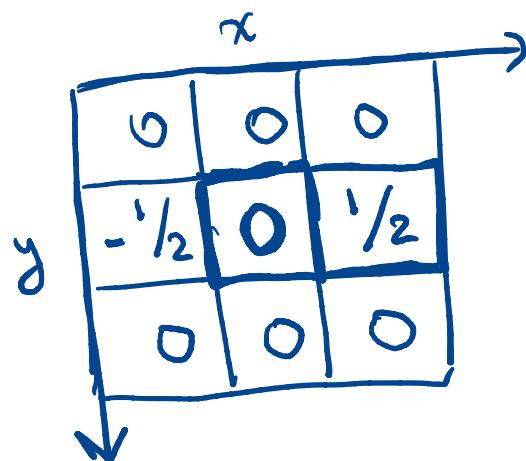
$$\sqrt{I_x^2 + I_y^2}$$



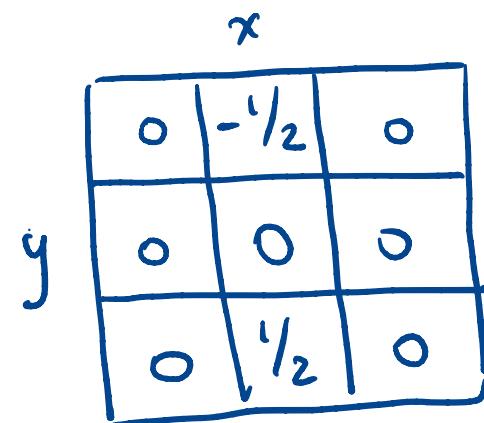
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Gradients as linear operators

Gradients are an example of **linear operators**, i.e. the value at a pixel is computed as a linear combination of values of neighboring pixels.



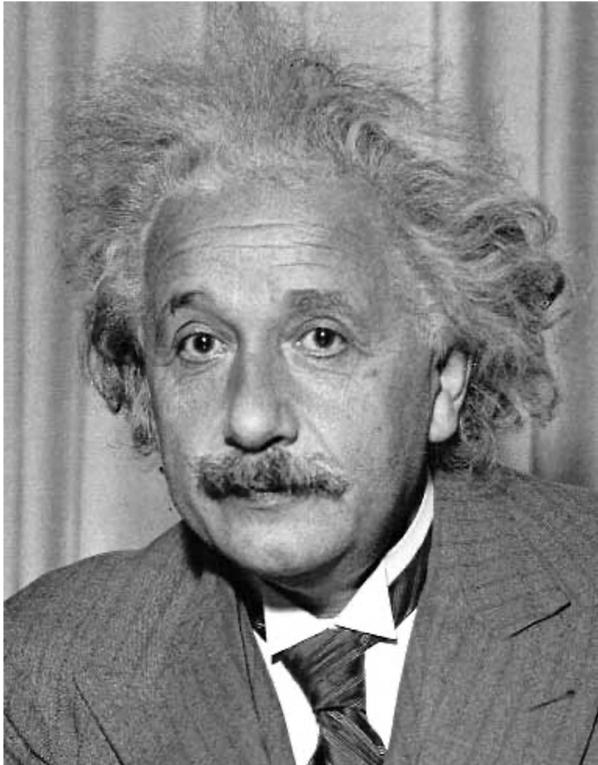
$$I_x = \frac{I(x+1, y) - I(x-1, y)}{2}$$



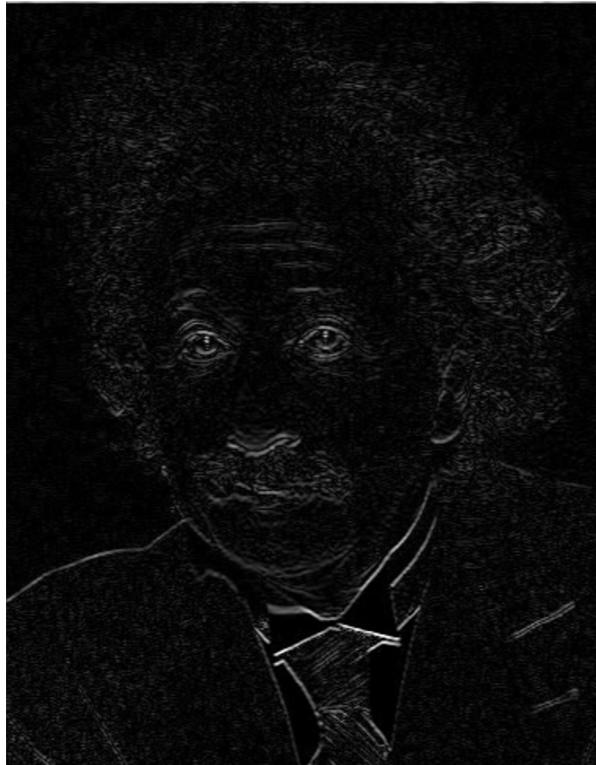
$$I_y = \frac{I(x, y+1) - I(x, y-1)}{2}$$



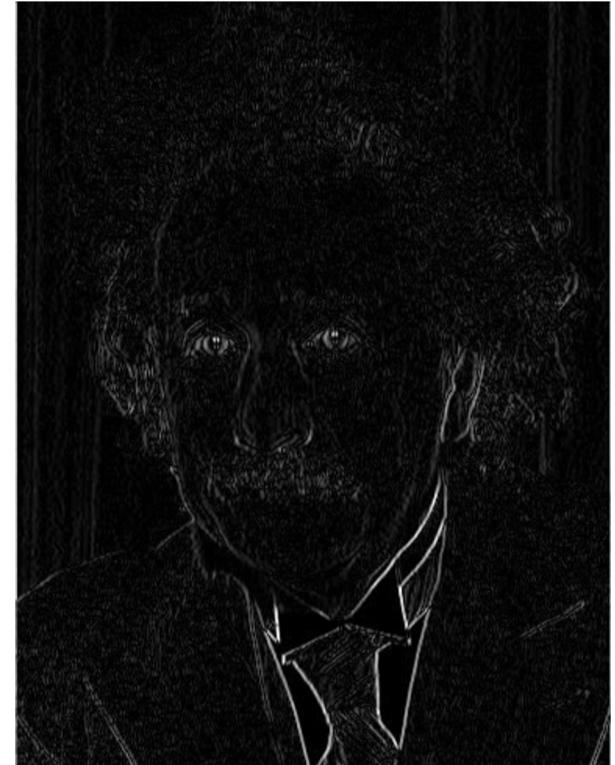
Examples – Sobel operator



original image



horizontal edges



vertical edges

Sobel
Prewitt
Roberts

1	2	1
0	0	0
-1	-2	-1

1	0	-1
2	0	-2
1	0	-1



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