The Data Driven Manager

5

One Way
Analysis of Variance

Learning Objectives

- Define ANOVA
- Differentiate between within-group and between-group variability
- Calculate and interpret the degrees of freedom (df), between-group variance (MSB), within-group variance (MSW), and the F-ratio (F) for the one-way ANOVA.

Learning Objectives

- Perform an ANOVA to test hypotheses about means
- Create an ANOVA summary table for the one-way ANOVA (fixed effects model)
- Use Welch's ANOVA to perform an analysis when there are unequal variances in groups

Learning Objectives

- Create data visualizations representing the ANOVA results
- Calculate statistical importance
- Perform an ANOVA to test hypotheses about dispersion
- Perform a Post Hoc Analysis to determine differences between groups

Learning Objectives

- Produce a complete One Way Analysis of Variance - Fixed Effects

Introduction to ANOVA

The Analysis of Variance

Terminology in Experimental Design

- Factor
- Levels
- Fixed and Random Factors

Factor

- A term used to describe an independent and/or treatment variable of interest in an experiment.

Factor Example

- Suppose that a study is being conducted to determine which of two designs for soldering parts on a ceramic substrate yield the lowest plant fallout.

Factor Example

- The **dependent variable** is plant fallout
- The **criterion measure** (the way we choose to measure the dependent variable) might be end of line defective rate
- The **factor** studied would be design

Level

- A term related to the number of values that a factor reflects in a given experiment.

Level Example

- For example, in the first experiment mentioned in the factor example, the comparative analysis consisted of two levels
  - Design Type A
  - Design Type B

Fixed and Random Factors

- Factors studied in an experiment may be either fixed or random.
- A **fixed** factor may be thought of as a factor where all the levels of interest to the researcher are included in the experiment.

Fixed and Random Factors

- A **random** factor is a factor where the levels included in the study are random selections from a wide range of possible levels which could have been included in the study.

What is ANOVA?

- **AN**alysis **O**f **VA**riance
- A test of differences in the means among more than two groups (i.e. levels)
- A one way ANOVA is a one factor analysis

What is ANOVA?

- Allows us to test hypothesis with J > 2 levels

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_1: Not\ H_0$$

where:

J = total number of groups (treatment levels)

j = each of the J groups

n = number of observations in each of the j groups

i = each of the individual observations in the j groups

What is ANOVA?

- Number of pairwise tests:

$$\frac{J(J-1)}{2} = \frac{4 \times 3}{2} = 6$$

- Multiple tests **inflate** Type I error

- One-Way Analysis of Variance (ANOVA) allows us to test for differences between multiple means simultaneously

What is ANOVA?

- Allows us to compare the J group means by comparing the variation between the groups to the variation within the groups.

ANOVA Principles

ANOVA Principles

- Assume multiple (J) samples, of equal size n, are taken from a single population

- The population variance can be estimated using the average within group variance as follows (assuming equal sample size):

$$\hat{\sigma}^2 = \frac{\sum s_j^2}{J} = \overline{s_j^2}$$

Within Subgroup Variation

$$s_W^2 = \sum_{j=1}^{J} \frac{s_j^2}{J} = \frac{\sum_{j=1}^{J} \sum_{i=1}^{n} \left(X_{ij} - \bar{X}_j\right)^2}{J(n-1)}$$

with J(n-1) = Jn-J df

ANOVA Principles

- The population variance can also be estimated using the variance of the means

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} \longrightarrow \sigma^2 = n\sigma_{\bar{X}}^2$$

- Therefore $$\hat{\sigma}^2 = n s_{\bar{X}}^2$$

Between Subgroup Variation

$$s_B^2 = n \sum_{j=1}^{J} \frac{\left(\bar{X}_j - \bar{X}_J\right)^2}{J-1}$$

with J-1df

Variance Components

- When the null hypothesis is true (no differences between the means of the groups), both variance components are estimates of σ^2.

ANOVA Principles

- We now have two ways to estimate the variance

$$\hat{\sigma}^2 = \overline{s_j^2} \quad \text{and} \quad \hat{\sigma}^2 = n s_{\bar{\bar{X}}}^2$$

Mean-Square Within (MS_W) Mean-Square Between (MS_B)

Variance Ratio

- The ratio of these two estimates should follow an F distribution

$$F = \frac{s_B^2}{s_W^2} = \frac{MS_B}{MS_W}$$

- With J -1, and J (n-1) degrees of freedom
- The expected value of F is 1

Variance Components

- When the null hypothesis is NOT true, $s_B{}^2$ will be an estimate of σ^2 **plus** the increase due to differences **between** the treatment.

ANOVA Principles

- Now assume the J samples are from **different** populations with **different** means
- The expected variance of the means would be as follows

$$\sigma^2_{Means} = \frac{\sigma^2_{Within}}{n} + \sigma^2_{Between}$$

ANOVA Principles

- The expected variance of the means times n, or the "expected mean-square between" would be as follows

$$MS_B = \sigma^2_{Within} + n\sigma^2_{Between}$$

ANOVA Principles

- The expected F ratio would be as follows

$$F = \frac{MS_B}{MS_W} = \frac{\sigma^2_{Within} + n\sigma^2_{Between}}{\sigma^2_{Within}}$$

- If there is no between variance the expected F ratio would be 1
- If there is a between variance the expected F ratio would be greater than 1

Standard ANOVA Table

Source of Variation	Sum of Squares	Df	Mean Square	F	Prob (F) or p-value
Between Groups (Treatment Levels)	$n\sum_{j=1}^{J}(\bar{X}_j - \bar{X}_J)^2$	$J-1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_W}$	Calculated Probability of the obtained F
Within Groups (Residual)	$\sum_{j=1}^{J}\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2$	$J(n-1)$	$\frac{SS_W}{df_W}$		
Total Variation	$\sum_{j=1}^{J}\sum_{i=1}^{n}(X_{ij} - \bar{X}_J)^2$	$Jn-1$			

One Way ANOVA for Means

Equal Variance

ANOVA for Means Example

- A design engineer wishes to test the effectiveness of three new methods of soldering components on a ceramic substrate.

- The component materials are expensive, and the procedures are time-consuming, so the engineer begins with a sample size of 6 at each level of the treatment factor.

ANOVA for Means Example

Dependent Variable	Solder Joint Strength
Criterion Measure	Push-off force required to remove test components
Independent Variable	Method of soldering components on substrate
Factor Levels	4 (Three new methods, one current method)

Data for the Solder Experiment

- Import the data file: Solder.txt

- Load the following packages:
    - lolcat
    - car

Hypotheses

Testing for equality of means

- H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
- H_1: Not H_0

Testing for equality of variances

- H_0: $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$
- H_1: Not H_0

One-Way ANOVA: Basic Computations

1.	Compute the mean for each of the 4 groups:				
2.	Compute the variance for each of the 4 groups:				
3.	Compute the variance between the 4 group means				
4.	Compute the mean (average) of the 4 group variances				
5.	Multiply the variance between the means by the sample size, n				
6.	Divide the between variance from step 5 by the within variance from step 4				

One-Way ANOVA: Basic Computations

1.	Compute the mean for each of the 4 groups:	**11.3333**	**13.1667**	**24.6667**	**11.3333**
2.	Compute the variance for each of the 4 groups:	**0.2667**	**0.5667**	**5.8667**	**0.6667**
3.	Compute the variance between the 4 group means:				**41.211**
4.	Compute the mean (average) of the 4 group variances:				**1.8417**
5.	Multiply the variance between the means by the sample size, n:				**247.2659**
6.	Divide the between variance from step 5 by the within variance from step 4:				**134.2596**

The ANOVA Source Table

Standard ANOVA Table

Source of Variation	Sum of Squares	Df	Mean Square	F	Prob (F) or p-value
Between Groups (Treatment Levels)	$n\sum_{j=1}^{J}(\bar{X}_j - \bar{X}_J)^2$	$J-1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_W}$	Calculated Probability of the obtained F
Within Groups (Residual)	$\sum_{j=1}^{J}\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2$	$J(n-1)$	$\frac{SS_W}{df_W}$		
Total Variation	$\sum_{j=1}^{J}\sum_{i=1}^{n}(X_{ij} - \bar{X}_J)^2$	$Jn-1$			

One-Way ANOVA

```
> solder.aov<-aov(formula = push~method, data =
Solder)
> summary(solder.aov)
             Df    Sum Sq    Mean Sq  F value     Pr(>F)
method        3  741.7917  247.26389 134.2609 2.0314e-13
Residuals    20   36.8333    1.84167
```

ANOVA Source Table

ANOVA Source Table					
Source	**Df**	**Sum Sq**	**Mean Sq**	**F value**	**Pr(>F)**
Method	3	741.7917	247.2639	134.2609	0.0000
Residuals	20	36.8333	1.8417		
Total	23	778.6250			

ANOVA in RStudio and ROIStat

One-Way ANOVA: Assumptions

- The measurement scale is at least interval level
- Each treatment population is normally distributed
- The treatment populations have equal variances
- Experimental units or specimens are independent
- The groups are independent

One-Way ANOVA: Violation of Assumptions

- Tests which can withstand a violation of an assumption are **robust** to that assumption
- ANOVA is robust to
  - Normality assumption if n is large (>10-15)
  - Equal variances if there is equal n per group
- **No** statistical test is robust to a violation of the independence of experimental units
- ANOVA is **not** robust to outliers
- In some cases you can substitute the Kruskal-Wallis One-Way ANOVA as a nonparametric approach

One-Way ANOVA

```
solder.aov<-aov(formula = push~method, data = Solder)
summary(solder.aov)
          Df  Sum Sq   Mean Sq  F value     Pr(>F)
method     3 741.7917 247.26389 134.2609 2.0314e-13
Residuals 20  36.8333   1.84167
```

Welch's ANOVA in RStudio

Unequal Variance

What is Welch's ANOVA?

- Welch's ANOVA is a one-way ANOVA that doesn't require homogeneity of variances.
- Calculates sources of variability differently than the regular (Fisher) ANOVA.
- Uses an approximate F value
- More conservative (protects against Type I error)

When to use Welch's ANOVA

- Use the Welch ANOVA for **means** when there are unequal variances

- Use the Welch ANOVA for **dispersion** when there are unequal sample sizes

One-Way ANOVA

```
welch.out <- oneway.test(push ~ method, data =
Solder)
One-way analysis of means (not assuming equal
variances)

data:  push and method
F = 56.821129, num df = 3.000000, denom df =
10.518098, p-value = 0.0000008562077
```

Data Visualization

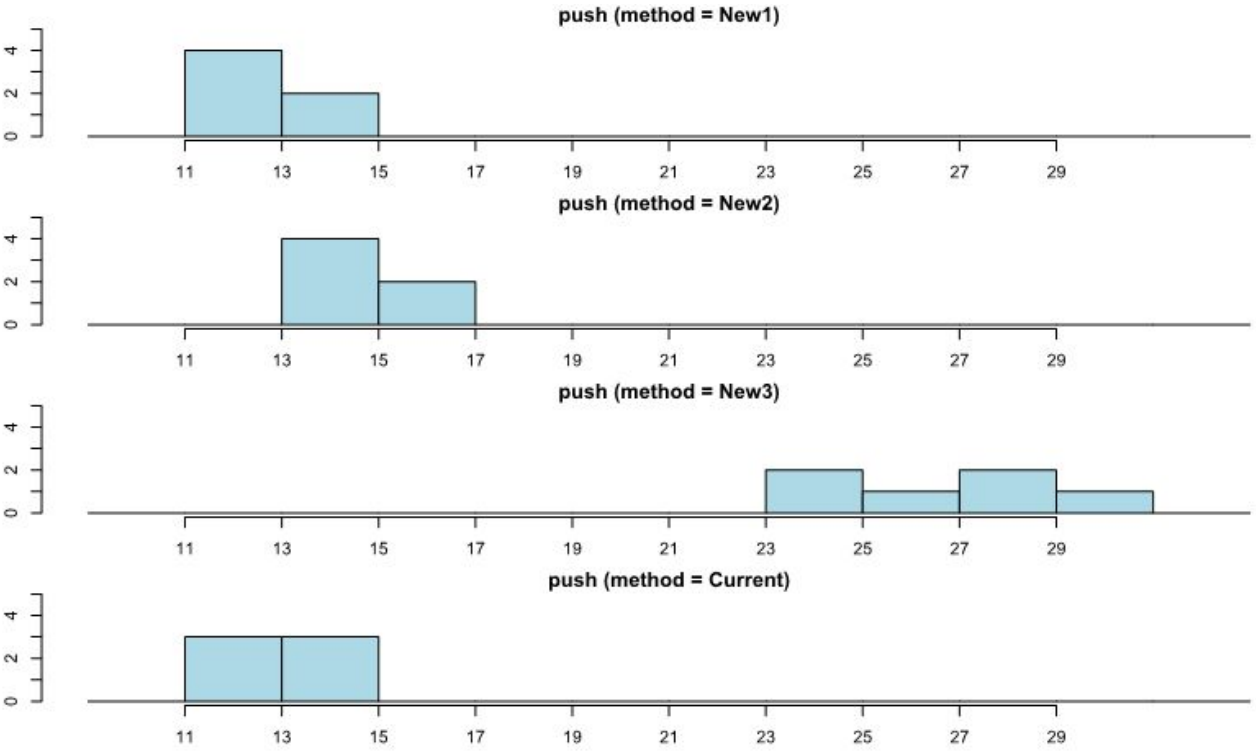
One-Way ANOVA: Descriptive Summary

```
(so <- ro(summary.continuous(push ~ method, Solder)))
```

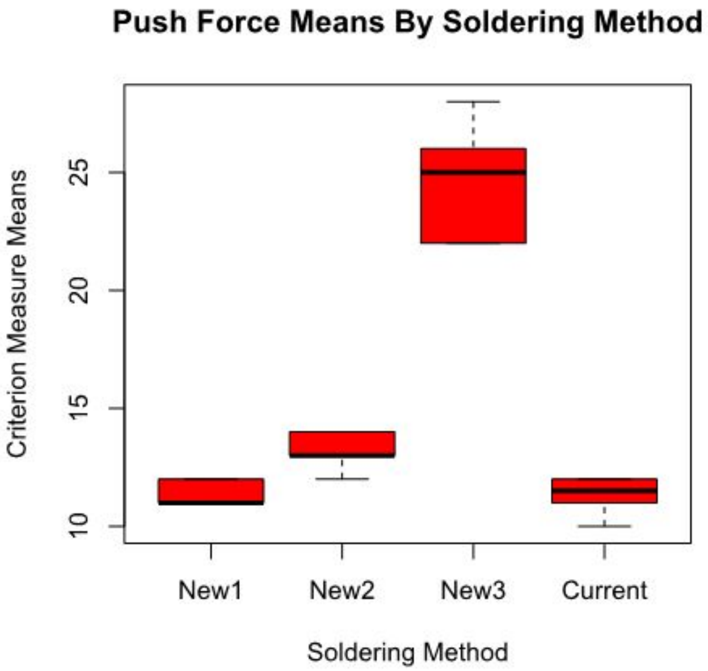
	method	n	missing	mean	var	adtest.AA	adtest.p	swtest.W	swtest.p
1	New1	6	0	11.3333	0.2667	1.2961	0.0023	0.6399	0.0014
2	New2	6	0	13.1667	0.5667	0.5660	0.1428	0.8663	0.2117
3	New3	6	0	24.6667	5.8667	0.3748	0.4149	0.9067	0.4150
4	Current	6	0	11.3333	0.6667	0.6465	0.0917	0.8216	0.0911

```
process.group.plot(fx = push ~ method, data = Solder)
```

Histograms

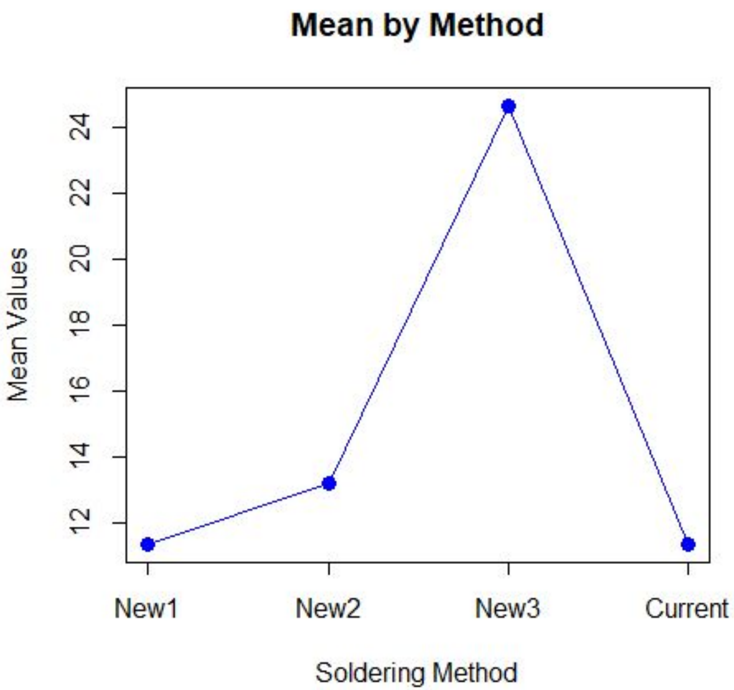


Boxplot




```
boxplot(push ~ method,data=Solder,
  xlab="Soldering Method",
  ylab="Criterion Measure Means",
  main="Push Force Means By Soldering Method",
  col="red")
```

Line Plot




```
plot(so$mean, xaxt="n"
, type = "o", lty = 1
, pch = 19, cex = 1.1, lwd= 1.7
, col="blue", xlab = "Soldering
Method", ylab = "Mean Values",
main = "Mean by Method")
```

Statistical Importance

One-Way ANOVA: Importance

- Statistical significance does not mean statistical importance

- Importance requires statistical significance

One-Way ANOVA: Importance

- There are two types of importance
  - **Statistical importance** is a calculation yielding the percent of variability explained
  - **Practical importance** is an engineering or technological assessment of the ability to affect the dependent variable

Statistical Importance

$$\hat{\omega}^2_{effect} = \frac{SS_{effect} - (df_{effect})MS_{AET}}{SS_{Total} + MS_{AET}}$$

$$\hat{\omega}^2_{Method} = \frac{SS_{Method} - (df_{Method})MS_W}{SS_{Total} + MS_W} = \frac{741.79 - (3)1.84}{778.62 + 1.84} = 0.9434$$

Statistical Importance

In sjstats

```
model<-lm(formula = push~method, data = Solder)
anova.stats<-anova_stats(model, digits = 4)
anova.stats$omegasq[1] = 0.9434
```

Statistical Importance - Welch

$$\widehat{\omega}^2_{effect} = \frac{df_{effect} * (F - 1)}{df_{effect} * (F - 1) + n_{total}}$$

$$\widehat{\omega}^2_{Method} = \frac{df_{effect} * (F - 1)}{df_{effect} * (F - 1) + n_{total}} = \frac{(3)56.821}{(3)56.821 + 24} = 0.8766$$

Levels of Importance

ω^2 value (%)	Importance Level
70 - 100%	High Importance
50 – 69%	Moderate Importance
25 – 49%	Low Importance
< 25%	Unimportant

One-Way ANOVA

- After Significance is found ALWAYS do the following Preparation for Post Hoc analysis.

- After Importance, You should ALWAYS provide:
  - Means Plot
  - Table of Means

  (Both needed following a significant effect)

Testing for Homogeneity of Dispersion

Testing for Equal Variance or Equal Dispersion

- Hypotheses for Variance
  - H_0: $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$
  - H_1: H_0 is not true

- Hypotheses for Dispersion
  - H_0: The treatment groups have equal dispersion
  - H_1: H_0 is not true

Reasons for Testing Variance or Dispersion

1. To validate the underlying assumption of homogeneity of variance for ANOVA

2. To determine the proper post-hoc test procedures to use

3. To determine the effect that the treatment has on variability

Test Procedures

- With data are sampled from a normally distributed populations use:
  - ADA (Levene Test)

- When the data are not from normally distributed populations use:
  - ADM, or (when $n \leq 10$)
  - ADM(n-1) Test (when $n > 10$)

Alternate Measures of Dispersion

- ADA = Absolute Deviation from the Average
- ADM = Absolute Deviation from the Median
- ADMn1 = Absolute Deviation from the Median (dropping a redundant score)

Calculating ADA

- Must be calculated within each level of each factor (cell)
- In lolcat
    - `compute.group.dispersion.ADA()`

Calculating ADM/ADMn1

- Must be calculated within each level of each factor (cell)
- In lolcat
  - `compute.group.dispersion.ADM()`
  - `compute.group.dispersion.ADMn1()`

Levene Test (ADA)

- Compute absolute deviations from the group **mean** for each score and perform a one-way ANOVA on these scores

```
            Df   Sum Sq  Mean Sq F value     Pr(>F)
method       3 9.536945 3.178982 8.91768 0.00059334
Residuals   20 7.129615 0.356481
```

The ADM(n-1) Procedure

- Compute absolute deviations from the group **median** for each score
  - Drop the middle score for odd data sets
  - Drop one of the two middle scores for even data sets
- Perform a regular one-way ANOVA on the ADM(n-1) scores
- Interpret the results the same way you would a regular ANOVA on the raw data

The ADM(n-1) Procedure

```
              Df   Sum Sq  Mean Sq F value    Pr(>F)
method         3 10.23753 3.412509 6.82502 0.0035725
Residuals     16  8.00000 0.500000
4 observations deleted due to missingness
```

The ANOVA Machine

- The ANOVA is an input-output "machine"
- It has one function: to give us information about the significance of the difference between the means of the groups in the metric of the criterion measure of the input data.

The ANOVA Machine

- The input data may be in any “measure”: height, weight, length, density, percents, concentrations, differences, variances, log variances, deviations from means or medians, ranks, proportions, etc.

ANOVA for Dispersion

- When sample sizes are equal, use Fisher's ANOVA
- When sample sizes are unequal, use Welch's ANOVA

Post-Hoc Analysis

Post-Hoc Analysis

- Conducted after **rejecting** the overall null hypothesis
- Purpose: To identify where the significant differences are, or to identify which groups are different from which other groups on the metric assessed

Post-Hoc Analysis: Key Terms

- Comparison: A procedure employed to assess the difference between two means
- Contrast: A procedure employed to assess the difference between two **sets** (or **groups**) of means

Post-Hoc Procedures

- Many procedures exist
  - Regular t-tests (Fisher's LSD Method): No Type I Error rate control
  - Bonferroni (Dunn) Procedure: Controlling Type I error rate per comparison at α_{OV}/c
  - Tukey HSD and the Games & Howell Procedure: All pairwise comparisons
  - Scheffé and the Brown-Forsythe Procedure: Complex contrasts

Post-Hoc Procedures for Central Tendency (Means)

Recommendation:

- For pairwise comparisons
- With homogeneity of variance:
    - Tukey HSD
- Without homogeneity of variance:
    - Games & Howell procedure

Post-Hoc Tests for Dispersion

- Use ADA or ADMn1 scores as raw data
- Perform a one-way ANOVA on the ADA or ADMn1 scores
- Use Tukey HSD if sample sizes are equal
- Use Games & Howell if sample sizes are unequal

ANOVA Roadmap

Rmarkdown

- `install.packages(“rmarkdown”)`

Roadmap for One-Way Fixed Factor ANOVA Analysis

When generated from an experiment, the following procedure may be used to perform the One-Way ANOVA analysis

Step 1. Import data into R, define factors and variables

Step 2. Review descriptive statistics
(n, Means, Std. Dev., Variance)

Step 3. Review normality tests within each group / level

Step 4. Review Histograms / Box and Whisker Plots by group / level

Roadmap for One-Way Fixed Factor ANOVA Analysis

Step 5. Conduct One-Way ANOVA for **Dispersion**

- If within cell **normality exists**, generate and use the **ADA**. Use the Fisher ANOVA if sample sizes are equal, and Welch's ANOVA if sample sizes are unequal

- If within cell **normality does not exist**, generate use the **ADMn1**. Use the Fisher ANOVA if sample sizes are equal, and Welch's ANOVA if sample sizes are unequal

Roadmap for One-Way Fixed Factor ANOVA Analysis

Step 6. Conduct Post-Hoc Tests if significant

- If **sample sizes** are **equal**, use Tukey HSD on either the ADA (normal) or ADMn1 (non-normal)

- If **sample sizes** are **unequal**, use Games and Howell on either the ADA (normal) or ADMn1 (non-normal)

Roadmap for One-Way Fixed Factor ANOVA Analysis

Step 7. Conduct Oneway ANOVA for **Means**

- If variance / dispersion is **equal**, use the Fisher ANOVA
- If variance / dispersion is **unequal**, use Welch's ANOVA

Roadmap for One-Way Fixed Factor ANOVA Analysis

Step 8. Conduct Post-Hoc Tests if significant

- If variance / dispersion is **equal**, use Tukey HSD on the means
- If variance / dispersion is **unequal**, use Games and Howell on the means