The Data Driven Manager

1

One Sample Tests

Learning Objectives

- Characterize different types of sampling
- Use a sample to describe a population
- Maximize the probability that samples are an accurate representation of the population
- Create a vector of random numbers

Learning Objectives

- Describe the concept of sampling error
- Explore the concept of random sampling distributions in RStudio
- Describe the Central Limit Theorem
- Estimate probability using the Random Sampling Distribution of the mean

Introduction to One Sample Tests

One Sample Tests

- When a single sample is drawn to make inferences about some population parameter a “one-sample test” will be performed

$$H_0: \mu = 500$$

$$H_0: \sigma^2 = 12$$

One Sample Examples

- Has the failure rate in field use for a particular model of wiper motor changed from its past level?
- Has the mean of the outer diameter of a particular type of shaft changed from its historical value?
- Has the piece-to-piece variability of the output voltage from an electronic engine control module increased or decreased from past levels?
- Is it reasonable to assume that a significant relationship exists between mold box temperature and end curl for an aluminum casting operation?

One Sample Tests

- Interval or Ratio Data
  - One Sample Test for Mean
  - One Sample Test for Variance

- Nominal Data
  - One Sample Proportion Test

- Ordinal Data
  - Sign Test for Location

Hypothesis Testing Procedure

Step 1

1

- Title or Name of the Study
- State the Business (Research) Question
- State the Statistical Question

Step 2

2

- Identify the
  - Dependent Variable:
  - Its Criterion Measure:
  - Level of Data (Scale of Measurement)
    - Nominal, Ordinal or Continuous
  - Performance Criterion:
    - Bigger is Better, Smaller is Better, Target or nominal is Best

Step 3

3

- State the Statistical Hypotheses.
  - H_0: Status Quo
  - H_1: Research Hypotheses

Step 4

4

- Select the Statistical Test and Identify its RSD when H_0 is true

Step 5

5

- Select the Effect Size and Type I and a Type II Error Rates and Decision for reject H_o
  - The effect size (and how it was obtained)
  - Type I Error and Its Consequence:
  - Type II Error and Its Consequence:
  - Type I Error Rate: α =
  - Type II Error Rate: □ =
  - Decision Rule for Rejecting H_o: $p \leq \alpha$

Step 5

5

- Calculate the sample size and develop the sampling plan
- Collect the data

Step 6

6

- Validate the Underlying Assumptions (list them and check them off) and Perform a Basic Descriptive Analysis
  - Graphic
  - Numeric

Step 7

7

- Perform the Statistical Test and Obtain Its Probability (p-value)

Step 8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_o. Provide Appropriate Estimates and Compute Power if needed.
  - Reject H_o or Fail to Reject H_o for each Hypothesis:
  - Report the p value(s) (for each Hypothesis):
  - WHSSETIT: We Have Sufficient Statistical Evidence To Infer That:__
  - Importance (Power if relevant) Calculations:

Step 8

8

- Illustration (Graphic and Tabular result of key statistical values: Means):
- Final Conclusions Related to the Statistical Hypotheses in This Set:

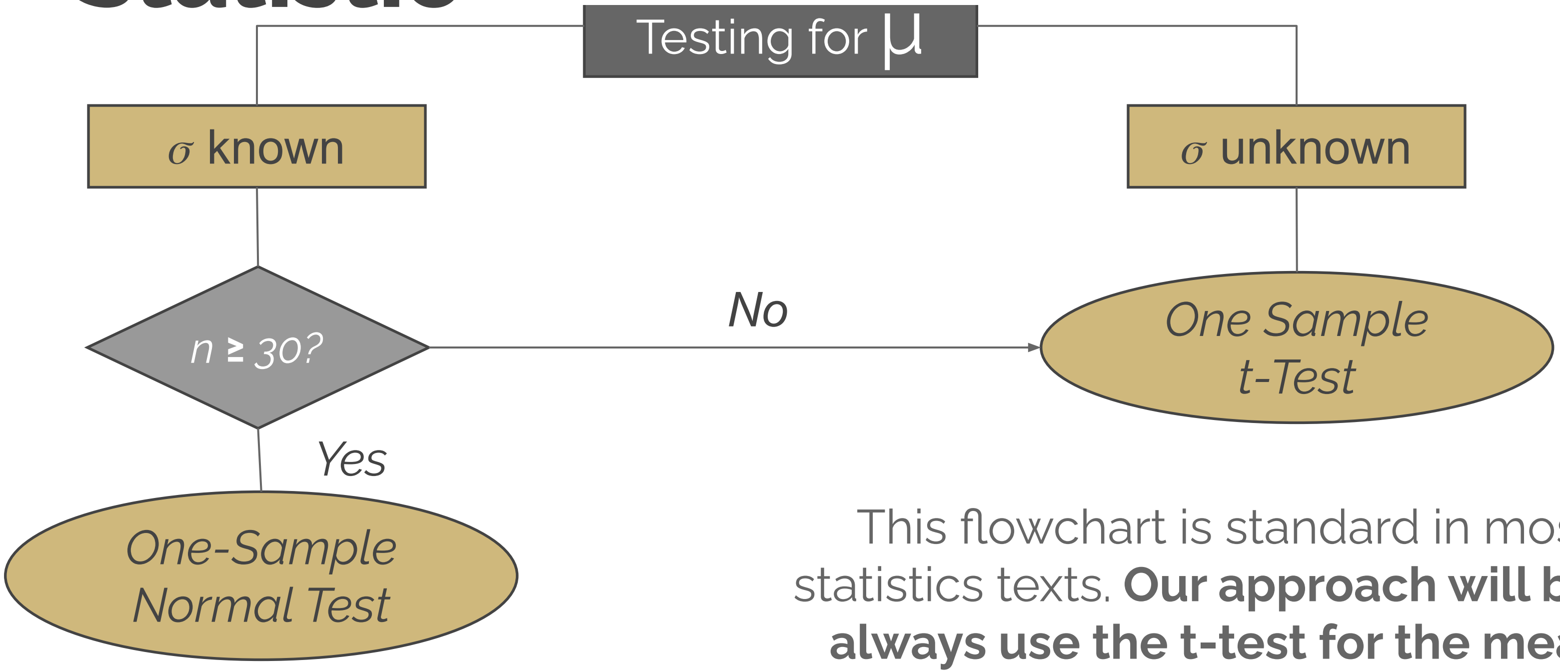
Step 9

9

- Interpretation of the Results in Terms of the Business (Research) Question.

One Sample Tests for the Mean

Selecting the Right Test Statistic



This flowchart is standard in most statistics texts. **Our approach will be to always use the t-test for the mean.**

The One Sample Normal Test

3 Underlying Assumptions

- The population is normally distributed with a mean of μ and a standard deviation of σ
- σ is known
- Independence (the sample was randomly drawn from the research population)

The One Sample Normal Test Example

- Axles for trucks must be able to withstand 80,000 lbs./sq. inch in a stress test. Excessive strength is costly. Standard deviation of strength is known to be 4,000 lbs/sq.inch. A sample of 100 axles produced a mean stress capacity of 79,600. At a significance level of 0.05, do the axles meet the stress requirement?

The One Sample **Normal** Test Example

- μ_0 or μ_{Ho} = 80,000 — hypothesized value of the population mean
- σ = 4,000 — population standard deviation
- n = 100 — sample size
- $\bar{X}$ = 79,600 — sample mean

Step 1

1

- State the Research Question:
  Does current production of axles have a mean stress capacity equal to the historical mean of 80,000 lb / sq inch?

Step 2

2

- Dependent Variable: ***Safety***
- Criterion Measure: ***Stress Capacity***
- Level of Data: ***Continuous***
- Performance Criterion: ***Target or Nominal is Best***

Step 3

3

- State the Statistical Hypotheses.
  - H_0: $\mu_0 = 80{,}000$
  - H_1: $\mu_1 \neq 80{,}000$

Step 4

4

- Select the Statistical Test and Identify its RSD when H_0 is true
  - ***One-sample z-test for a population mean***
  - ***$Z_{\bar{x}} \sim N(0,1)$ when H_0 is true.***

$$Z_{\bar{X}} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Step 5

5

- Select the Type I and a Type II Error Rates and Decision for reject H_o
  - Type I Error and Its Consequence:
    Rejecting H_o when it is true, will lead to the conclusion that the axles are either not strong enough, or are too strong, when it is, in fact, not true. This could lead to work related design modifications that is not necessary, wasting resources.

Step 5

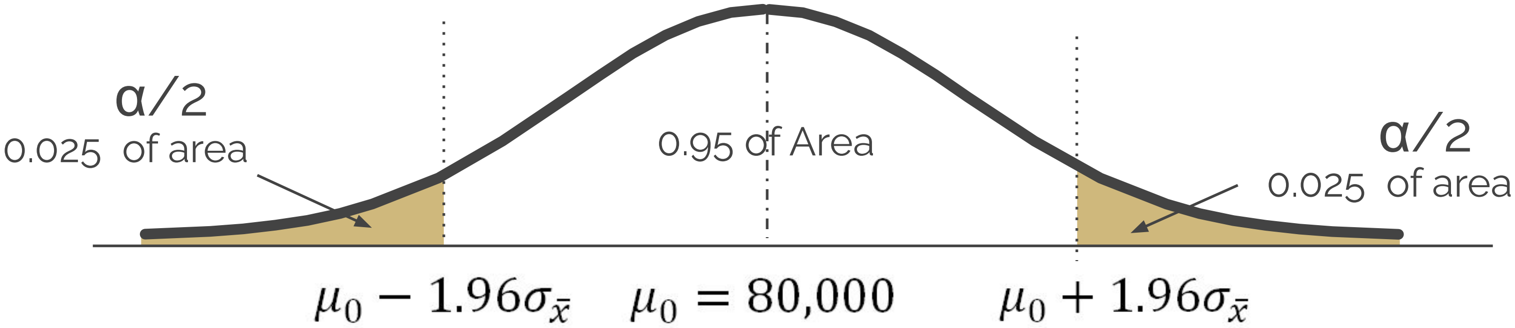
5

- Type II Error and Its Consequence:
  Not rejecting H_o when we should, would mean that we would not have detected the change in axle stress and any adverse effects that it may cause in use or in manufacturing cost. Furthermore, it is likely that any changes will only be detected as a result of ongoing problems in the field.

Step 5

5

- Type I Error Rate: ***α = 0.05***
- Decision Rule for Rejecting H_0: **p ≤ α**



Reject H_0 if ABS z > 1.96 or if P(z) < α

Step 6

6

- Validate the Underlying Assumptions
  - ***Independence of the individual specimens in the sample (accomplished by random sampling)***
  - ***Normality of the population of the scores (assertion provided by the instructor)***
  - ***Known population standard deviation, σ***

Step 6

6

- Perform a Basic Descriptive Analysis
  - Graphic - ***Histogram***
  - Numeric

$\sigma = 4{,}000$

$n = 100$

$\bar{X} = 79{,}600$

Step 7

7

- Perform the Statistical Test and Obtain Its Probability (p-value)

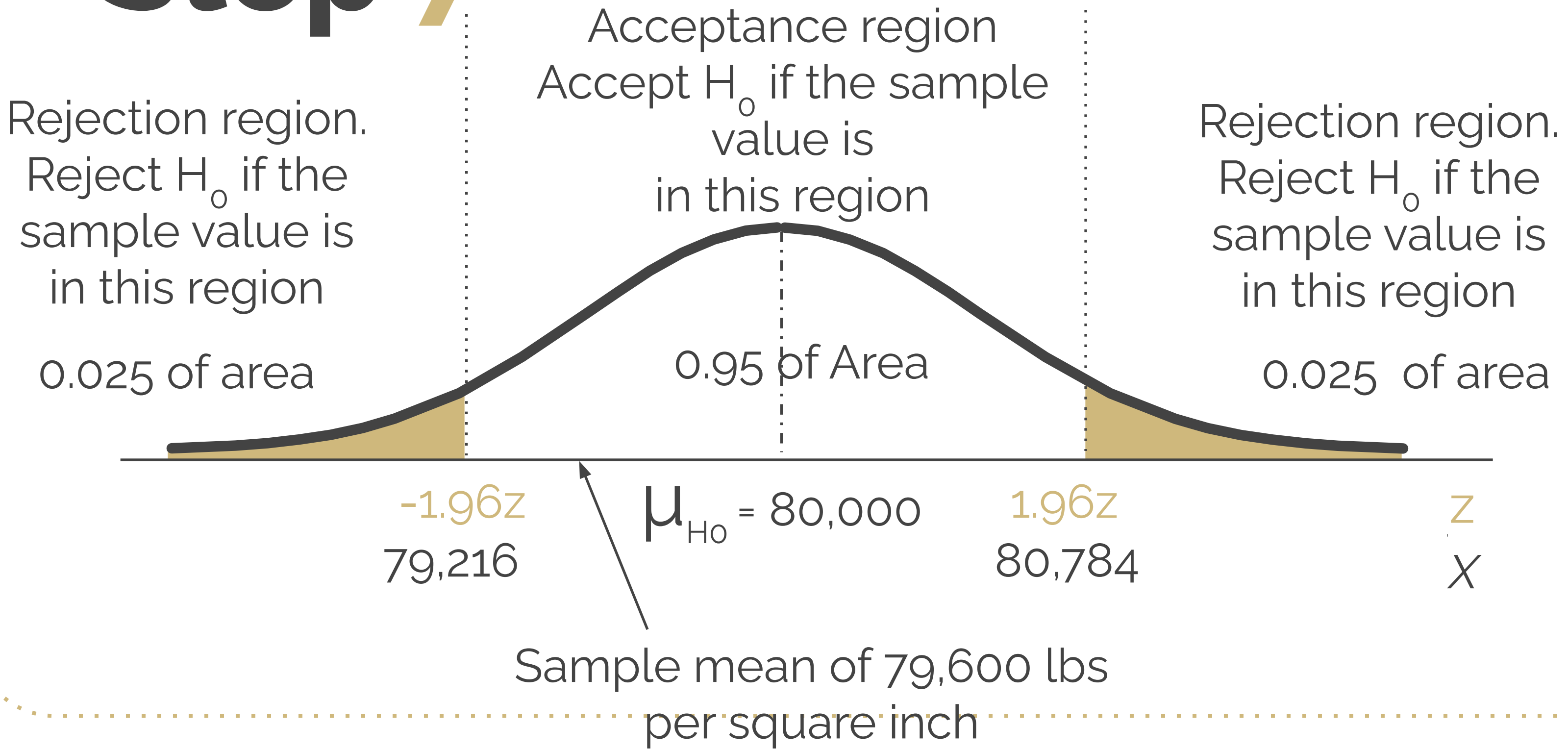
Step 7

7

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4000}{\sqrt{100}} = 400$$

Lower Critical Value	Upper Critical Value
$\mu_0 - 1.96\sigma_{\bar{x}}$	$\mu_0 + 1.96\sigma_{\bar{x}}$
80,000 - (1.96)(400)	80,000 + (1.96)(400)
80,000 - 784	80,000 + 784
79,216	**80,784**

Step 7



Acceptance region
Accept H_o if the sample value is in this region
Rejection region. Reject H_o if the sample value is in this region
Rejection region. Reject H_o if the sample value is in this region
0.025 of area
0.95 of Area
0.025 of area
-1.96z
79,216
μ_{Ho} = 80,000
1.96z
80,784
z
X
Sample mean of 79,600 lbs per square inch

Step 7

7

- Calculate the value of the test statistic, Z

$$Z_{\bar{X}} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{79600 - 80000}{400} = -1.00$$

In RStudio

```
z.test.onesample.simple()
```

Step 8

8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_o. Provide Appropriate Estimates and Compute Power if needed.
    - ***Fail to Reject H_o***
    - Report the p value(s) (for each Hypothesis):

      p = 0.3173
      The probability (p-value) of randomly drawing a sample mean of 79,600 if the population is still at a μ of 80,000 is 31.74% (0.1587*2)

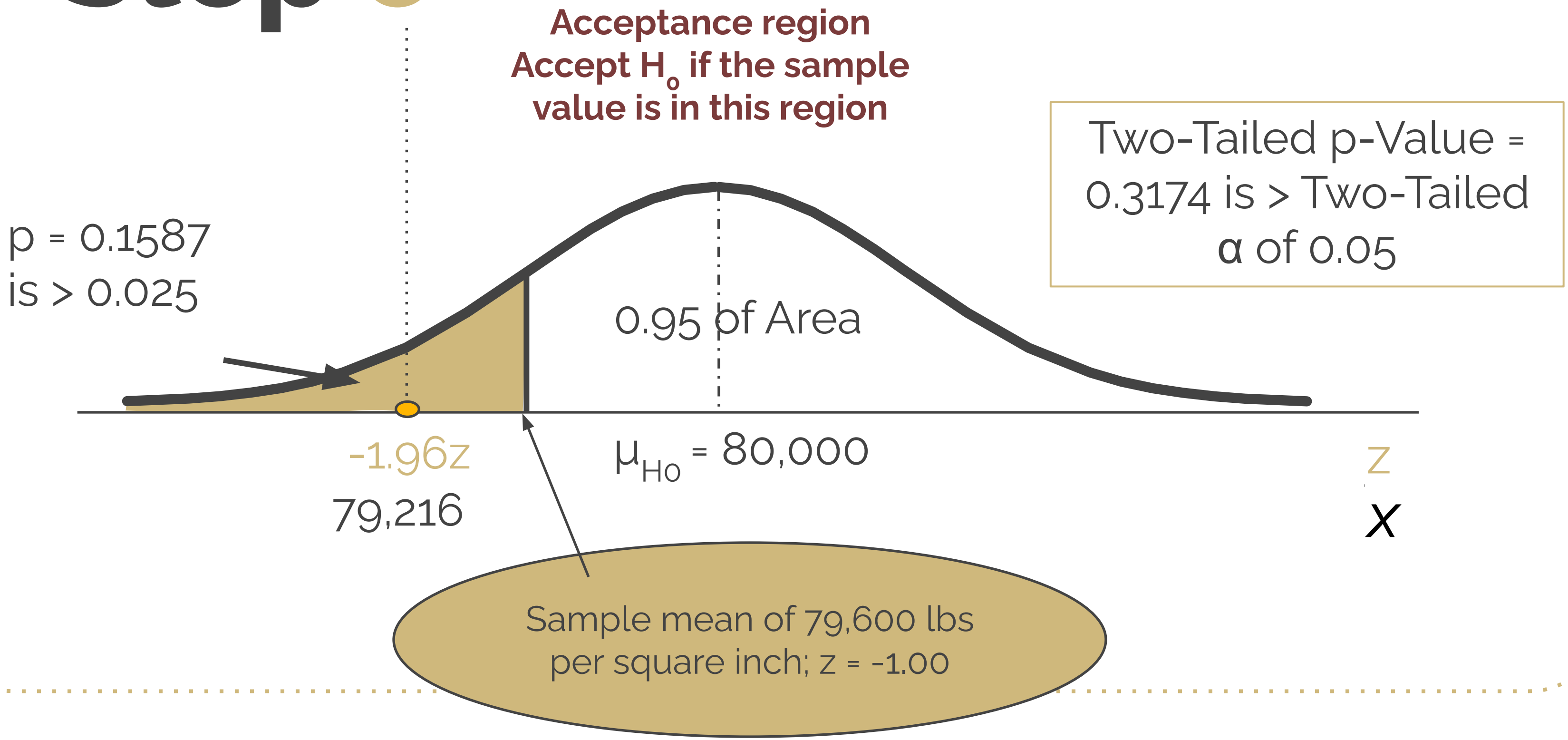
Step 8

8

- WHSSETIT: ***μ is still 80,000 (which would be a point estimate if we had to provide one)***

Step 8

8



Acceptance region
Accept H_o if the sample value is in this region
Two-Tailed p-Value = 0.3174 is > Two-Tailed α of 0.05
p = 0.1587
is > 0.025
0.95 of Area
-1.96z
79,216
μ_{Ho} = 80,000
z
X
Sample mean of 79,600 lbs per square inch; z = -1.00

Step 9

9

- Interpretation of the Results in Terms of the Research Question.
  Current production of axles has a mean stress capacity equal to the historical mean of 80,000 lb / sq inch

The One Sample t Test

3 Underlying Assumptions

- The population is normally distributed with a mean of μ and a standard deviation of **σ**
- σ is **un**known
- Independence (the sample was randomly drawn from the research population)

The One Sample t Test Example

- An engineer is attempting to determine whether it is reasonable to assume that output voltage on an EEC module will consistently average 12.50 millivolts, which is the nominal or target for a new design.

- Given that this is a new product, there is no historical value for σ. Using an initial production run of 60 modules, the tested output voltage yields a mean of 12.31, with an associated standard deviation (s) of 0.2.

The One Sample t Test Example

- Is it reasonable to assume that the modules are representative of a process that will yield a μ at the Target value at the 0.01 Significance Level?

Step 1

1

- State the Research Question:
  Does the circuit output voltage for electronic control modules have a mean value at the target of 12.50?

Step 2

2

- Dependent Variable: ***Circuit Quality***
- Criterion Measure: ***Output Voltage***
- Level of Data: ***Continuous***
- Performance Criterion: ***Target or Nominal is Best***

Step 3

3

- State the Statistical Hypotheses.
  - H_0: $\mu_0 = 12.50$
  - H_1: $\mu_1 \neq 12.50$

Hypothesized Value

Non-Directional

Step 4

4

- Select the Statistical Test and Identify its RSD when H_0 is true
  - ***One-sample t-test for a population mean***
  - ***$t \sim t\,(n-1)$ df when H_0 is true.***

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

5

Step 5

- Select the Type I and a Type II Error Rates and Decision for reject H_o
  - Type I Error and Its Consequence:
    Rejecting H_o when it is, in fact, true will lead to the conclusion that the circuit is not producing the proper output when, in fact, it is. This could lead to the belief that the new product is in need of further development and possible (unnecessary) design modifications leading to added expense and delays in introducing the product.

Step 5

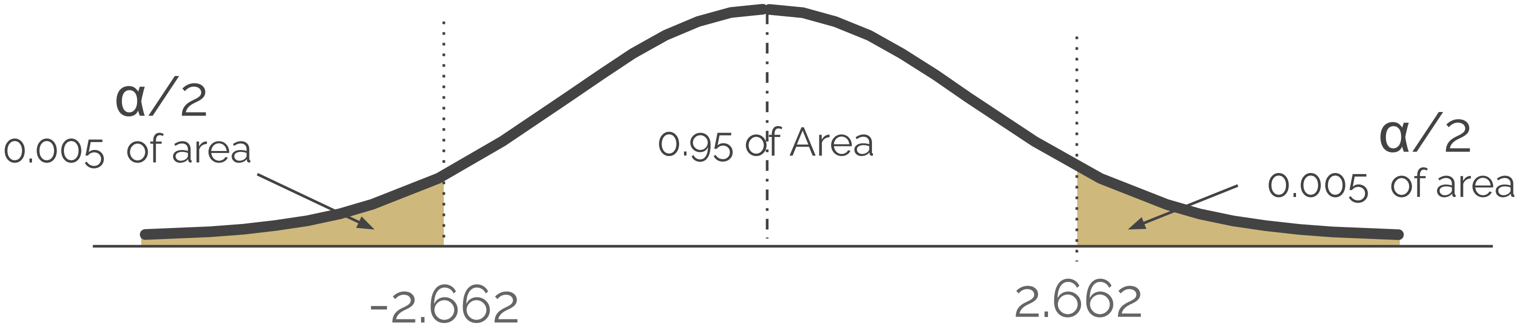
5

- Type II Error and Its Consequence:
  Not rejecting H_o when it should be would mean that the engineer would not have properly detected a poor-performing circuit. This could lead to releasing the new product when it is not meeting the design specification.

Step 5

5

- Type I Error Rate: ***α = 0.01***
- Decision Rule for Rejecting H_0: **p ≤ α**



Reject H_0 if ABS t > 2.662 or if P(z) < α

Step 6

6

- Validate the Underlying Assumptions
  - ***Independence of the individual specimens in the sample (accomplished by random sampling)***
  - ***Normality of the population of the scores (assertion provided by the instructor)***
  - ***Unknown population standard deviation, σ***

Step 6

6

- Perform a Basic Descriptive Analysis
  - Graphic - ***Histogram***
  - Numeric

$s = 0.20$

$n = 60$

$\overline{X} = 12.31$

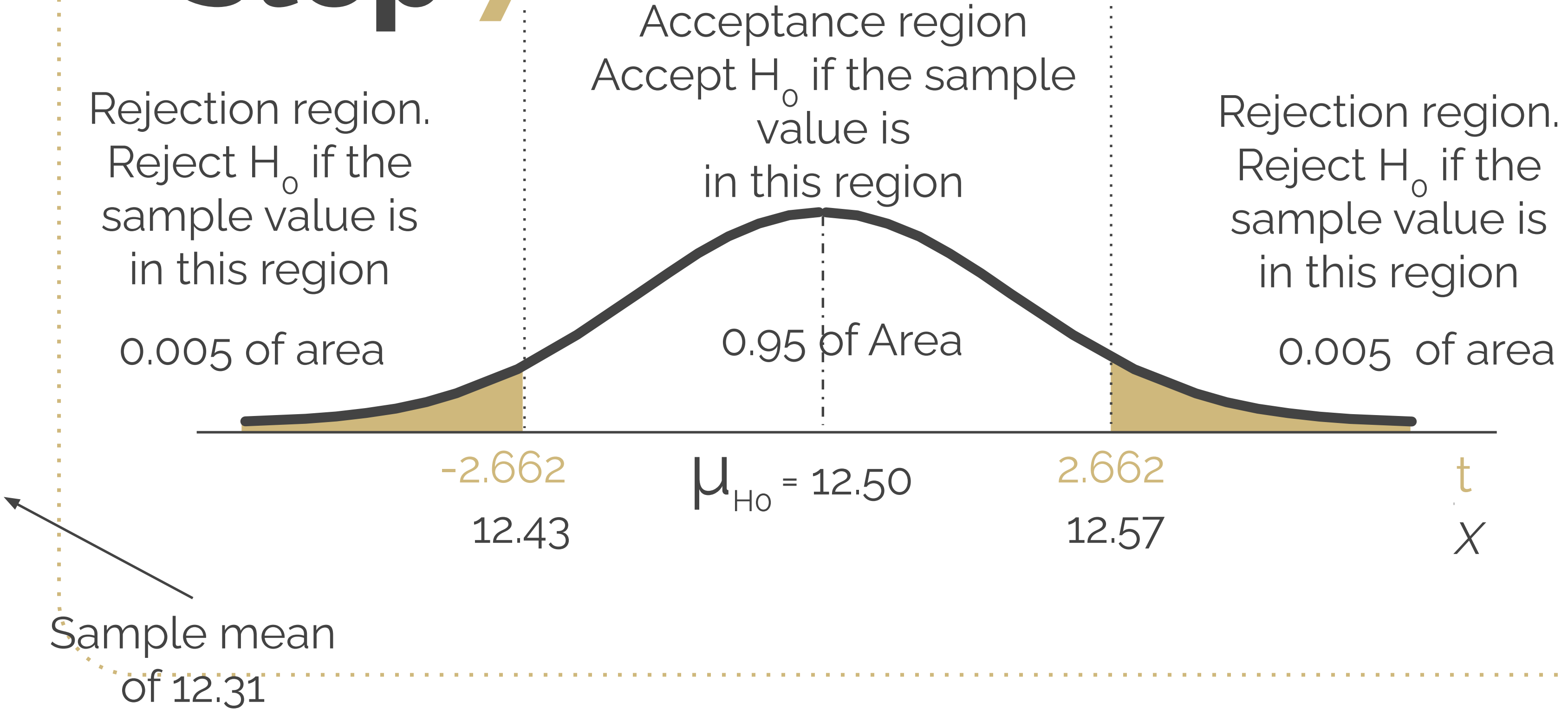
Sample Statistic

Step 7

7

- Perform the Statistical Test and Obtain Its Probability (p-value)

Step 7



Acceptance region
Accept H_0 if the sample value is in this region
Rejection region. Reject H_0 if the sample value is in this region
Rejection region. Reject H_0 if the sample value is in this region
0.005 of area
0.95 of Area
0.005 of area
-2.662
2.662
t
μ_{H_0} = 12.50
12.43
12.57
X
Sample mean of 12.31

Step 7

7

Test Statistic

- Calculate the value of the test statistic, t

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{12.31 - 12.50}{0.2/\sqrt{60}} = -7.3587$$

In RStudio

```
t.test.onesample.simple()
```

Step 8

8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_o. Provide Appropriate Estimates and Compute Power if needed.
  - ***Reject H_o***
  - Report the p value(s) (for each Hypothesis):

    p = 0.000

    The probability (p-value) of randomly drawing a sample mean of 12.31 if the population is still at a μ of 12.50 is 0.

Step 8

8

- WHSSETIT: ,
  We have sufficient statistical evidence to infer that the process mean of the modules' output voltage is not 12.50. Furthermore, our best point estimate of the value of μ is 12.31 and the 99% confidence interval for the mean is 12.2413 to 12.3787.

Step 9

9

- Interpretation of the Results in Terms of the Research Question.
  The output voltage of the circuits is less than the design target. Additional work will need to be done on the circuits to bring the output voltage to the design target.

t test for Mean

Image Source: www.wwpinc.com

- A new plastic injection molded cover was recently designed for a lipstick case assembly.
- The product designer identified the major characteristics for the toolmaker.
- When the tool and die department completed and installed the new mold, 50 parts were produced and set aside following the warm-up period.
- The machine operator measured each of the critical dimensions with an appropriate gauge.

t test for Mean

- The target value for one of the dimensions was **0.7330".**
- Determine whether the process mean may be reasonably assumed to be on target if the process is subsequently shown to be in a state of control over the long term.
- (In other words, is it reasonable to infer that μ is equal to the design target value if control can be established?)
- The pertinent data are provided for your use in the file titled **PlstCase.txt.**
- Use a significance level of **α = 0.10** in order to increase power for detecting a difference.

t test for Mean

1. Are the proper Null and Alternative Hypothesis Directional or Non Directional?
   a. Directional
   b. Non-directional

2. What is the value of the proper Sample Statistic?
   a. 0.7327
   b. 0.7330
   c. -1.556
   d. 0.0015

t test for Mean

3. What is the hypothesized value being tested?
   a. 0.7327
   b. 0.7330
   c. -1.556
   d. 0.0015

4. What is the value of the proper Test Statistic?
   a. 0.7327
   b. 0.126
   c. -1.556
   d. 0.7323 to 0.7330

t test for Mean

5. What is the p-value of the proper Test Statistic?
   a. 0.063
   b. 0.126
   c. 0.937
   d. 0.863

6. Do you Reject or Fail to Reject the Null Hypothesis?
   a. Reject the Null Hypothesis
   b. Fail to Reject the Null Hypothesis

One Sample Tests for the Variance / Dispersion

The Chi-Square (χ^2) Test of Variance

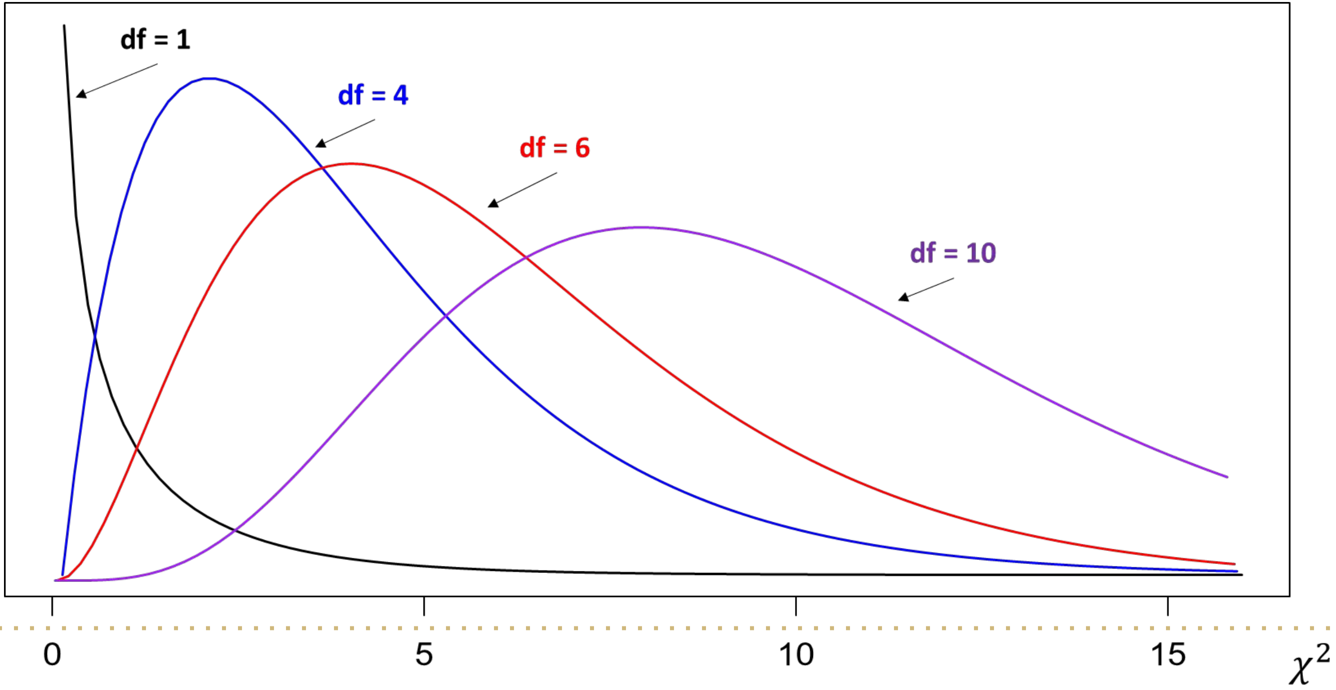
- Intended to determine whether a change has occurred in the dispersion of a population, as measured by the variance of the data; or

- Whether it is reasonable that a sample with a particular s^2 value could have been randomly drawn from a population with a hypothesized value of X

The Chi-Square (χ^2) Test of Variance

- Employs the χ^2 family of distributions
- The specific distribution to be employed depends on the degrees of freedom (*df*) for the test
- The χ^2 distribution originates at 0 and goes off to $+\infty$ and is positively skewed, but approaches normal as *df* increases

The Chi-Square Family of Distributions



The Chi-Square (χ^2) Test of Variance

Underlying Assumptions

- The population is **normally** distributed with a mean of μ and a standard deviation of σ (No Robustness to a violation of this assumption!)

- **Independence** of the specimens. (The sample was randomly drawn from the population associated with the hypothesis test / experiment / study)

Chi-Square (χ^2) Test of Variance Example

- A management professor has given careful thought to the design of examinations.
- In order to be reasonably certain that an exam does a good job of distinguishing the differences in achievement shown by the students, the standard deviation cannot be too small.
- On the other hand, if the standard deviation is too large, there will tend to be a lot of very low scores, which is bad for student morale.

Chi-Square (χ^2) Test of Variance Example

- Past experience has led the professor to believe that a standard deviation of 13 points on a 100-pt exam indicates that the exam does a good job of balancing these two objectives.

- The professor just gave an exam to their class of 31 students. The mean score was 72.7 and the standard deviation was 15.9. Does this exam meet the criterion at the 10% significance level?

Step 1

1

- State the Research Question:
  Does the current exam have a standard deviation at the target of 13?

Step 2

2

- Dependent Variable: ***Course Quality***
- Criterion Measure: ***Variability of Exam Scores***
- Level of Data: ***Continuous***
- Performance Criterion: ***Target or Nominal is Best***

Step 3

- State the Statistical Hypotheses.
  - H_0: $\sigma = 13$ or $\sigma^2 = 169$
  - H_1: $\sigma \neq 13$ or $\sigma^2 \neq 169$

Hypothesized Value

Non-Directional

Step 4

4

- Select the Statistical Test and Identify its RSD when H_0 is true
  - ***One-sample χ^2 test for a population variance***
  - ***$\chi^2 \sim \chi^2$ (n-1) df when H_0 is true.***

$$\chi^2 = \frac{s^2(n-1)}{\sigma_0^2}$$

Step 5

5

- Select the Type I and a Type II Error Rates and Decision for reject H_0
  - Type I Error and Its Consequence:
    Rejecting H_0 when it is, in fact, true will lead to the conclusion that the standard deviation of the exam is not 13 points when, in fact, it is. This could lead to the belief that the exam is in need of further development and possible (unnecessary) modifications leading to added expense and frustration.

Step 5

5

- Type II Error and Its Consequence:
  Not rejecting H_0 when it should be would mean that the professor would not have properly detected a change in the standard deviation of the exam. This could lead to low student morale if the dispersion is too large, or the inability to discern performance differences if too small.

Step 5

5

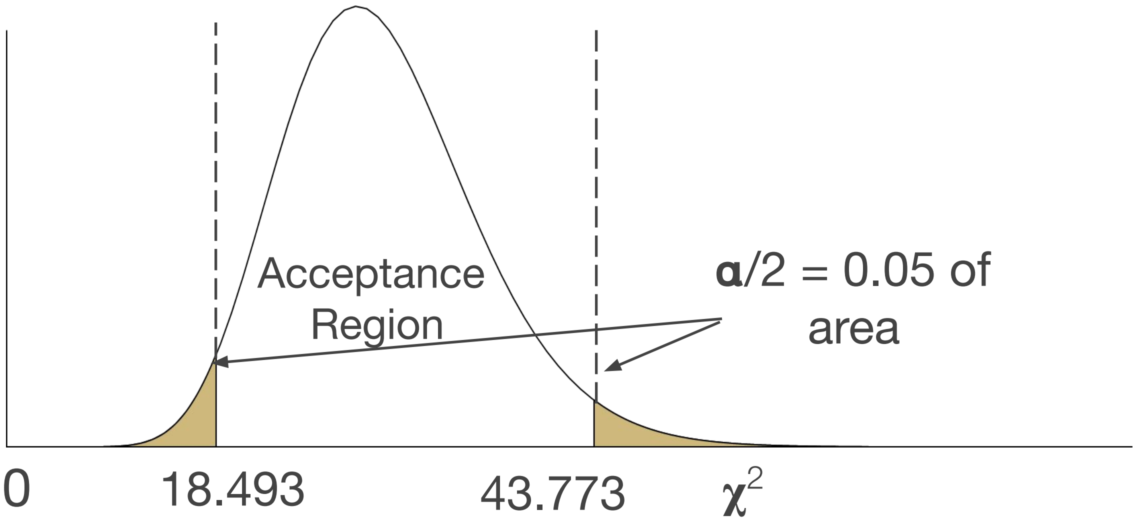
- Type I Error Rate: $\alpha = 0.10$
- Decision Rule for Rejecting H_0: $p \leq \alpha$

Step 5

5

$$\chi^2_{1-\alpha/2,\, df} = \chi^2_{1-0.10/2,\, n-1}$$
$$= \chi^2_{0.95,\, 31-1}$$
$$= \chi^2_{0.95,\, 30}$$
$$= \mathbf{18.493}$$

$$\chi^2_{\alpha/2,\, df} = \chi^2_{0.10/2,\, n-1}$$
$$= \chi^2_{0.05,\, 31-1}$$
$$= \chi^2_{0.05,\, 30}$$
$$= \mathbf{43.773}$$



Step 6

6

- Validate the Underlying Assumptions
  - ***Random Sample, Independence of experimental units***
  - ***Normality***

Step 6

6

- Perform a Basic Descriptive Analysis
  - Graphic - ***Histogram***
  - Numeric

Sample Statistic

s = 15.9

n = 31

Step 7

7

- Perform the Statistical Test and Obtain Its Probability (p-value)

Step 7

7

Test Statistic

- Calculate the value of the test statistic, χ^2

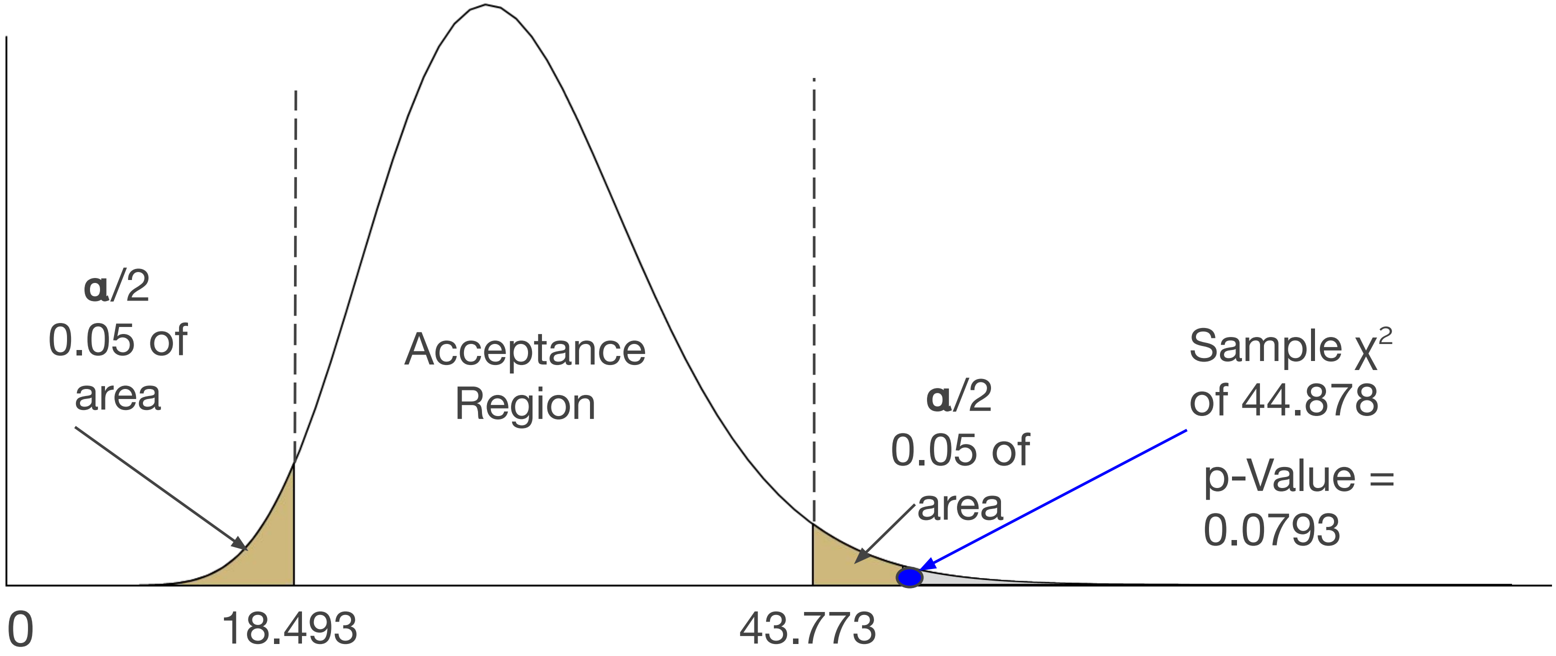
$$\chi^2 = \frac{s^2(n-1)}{\sigma_0^2} = \frac{15.9^2(31-1)}{13^2} = 44.878$$

In RStudio

```
variance.test.onesample.simple()
```

Step 7

7


α/2
0.05 of
area
Acceptance
Region
α/2
0.05 of
area
Sample χ2
of 44.878
p-Value =
0.0793
0
18.493
43.773

Step 8

8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_o. Provide Appropriate Estimates and Compute Power if needed.
  - ***Reject*** $\boldsymbol{H_o}$
  - Report the p value(s) (for each Hypothesis):
    ***p* = 0.07925**

Step 8

8

- WHSSETIT: ***We have sufficient statistical evidence to infer that the standard deviation of the exam is not 13. Furthermore, our best point estimate of the value of σ is 15.9 and the 90% confidence interval for the standard deviation is 13.163 to 20.25.***

Step 9

9

- Interpretation of the Results in Terms of the Research Question.
  The standard deviation of the exam is more than the target of 13. Additional work will need to be done reduce the standard deviation of the exam.

Image Source: www.directindustry.com

Chi-Square Test for Variance

- The module-to-module variability for a particular delay characteristic has been relatively stable over time, with an estimated **standard deviation of 0.03 seconds**.
- A process engineer recently discovered that the purchasing department had made a ***change*** in the supplier base that provides components for assemblies.
- The engineer has reason to believe that this change will have an effect on the variability of the process.

Chi-Square Test for Variance

- To test this assumption (hypothesis), the engineer drew a random sample of **20 modules** from a lot that was recently manufactured with the new components.
- These data are in the file called **switch.txt**
- Test an appropriate hypothesis given the information presented.
- If a significant difference exists, what action should the engineer take? Assume **α = 0.05**.

Chi-Square Test for Variance

1. Are the proper Null and Alternative Hypothesis Directional or Non Directional?
   a. Directional
   b. Non-directional

2. What is the value of the proper Sample Statistic?
   a. 0.0001
   b. 0.0003
   c. 0.0005
   d. 0.0009

Chi-Square Test for Variance

3. What is the hypothesized value being tested?
   a. 0.0001
   b. 0.0003
   c. 0.0005
   d. 0.0009

4. What is the value of the proper Test Statistic?
   a. 2.106
   b. 0.000
   c. 0.0009
   d. 171

Chi-Square Test for Variance

5. What is the p-value of the proper Test Statistic?
   a. 1.000
   b. 0.01
   c. 0.000
   d. 0.03

6. Do you Reject or Fail to Reject the Null Hypothesis?
   a. Reject the Null Hypothesis
   b. Fail to Reject the Null Hypothesis

One Sample Tests for Proportions

The One Sample Binomial Test (r)

- This test is intended to provide a method by which you can determine:
  - whether a population defective rate (or yield) is at a particular level; or
  - whether a population has changed in terms of its historical or hypothesized defective rate (or yield).

The One Sample Binomial Test (r)

Underlying Assumptions

- The distributional model associated with this test is the binomial, where
  - r = the count of the event of interest
  - $\mu_r = n\pi$ = expected value of the distribution; and
  - $\sigma_r = \sqrt{n\pi q}$ = standard error and where $q = (1 - \pi)$

The One Sample Binomial Test (r)

Underlying Assumptions

- Type I and II Error Levels may not be available at an exact and preferred level
- The sample data were randomly drawn from a research population

Binomial Test Example

- A video camera in a University classroom has failed in 50% of the one hour presentations provided over the last few months.
- After the company 'repairs' the device, the computer services support personnel operate the device at 10 (ten) random one hour periods throughout the first week after the projector has been returned.
- In two of the ten simulated presentations, the projector fails. Test an appropriate hypothesis.

Step 1

1

- State the Research Question:
  Has the proportion of failures of the multimedia projector changed from its historical value of 0.50?

Step 2

2

- Dependent Variable: ***Projector Performance***
- Criterion Measure: ***Proportion of Failures***
- Level of Data: ***Nominal***
- Performance Criterion: ***Smaller is Better***

Step 3

3

- State the Statistical Hypotheses.
  - H_0: $\pi = 0.5$
  - H_1: $\pi \neq 0.5$

Hypothesized Value

Non-Directional

Step 4

4

- Select the Statistical Test and Identify its RSD when H_0 is true
    - ***One-sample exact Binomial Test***
    - $r \overset{d}{=} \text{Binomial}(n = 10, \pi = 0.50)$ ***when H_0 is true.***

Step 5

5

- Select the Type I and a Type II Error Rates and Decision for reject H_0
  - Type I Error and Its Consequence:
    Rejecting H_0 when it is, in fact, true will lead to the conclusion that the failure rate of the projector is not 50% when, in fact, it is. This could lead to the belief that the projector has improved, when it has not, and it will continue to fail at the current rate. It could also lead to the conclusion that the projector has gotten worse, leading to further adjustments and cost.

Step 5

5

- Type II Error and Its Consequence:
  Not rejecting H_0 when it should be would mean that the projector would continue to fail at a rate of 50%, leading to further frustration in the classroom and increased cost related to repairs.

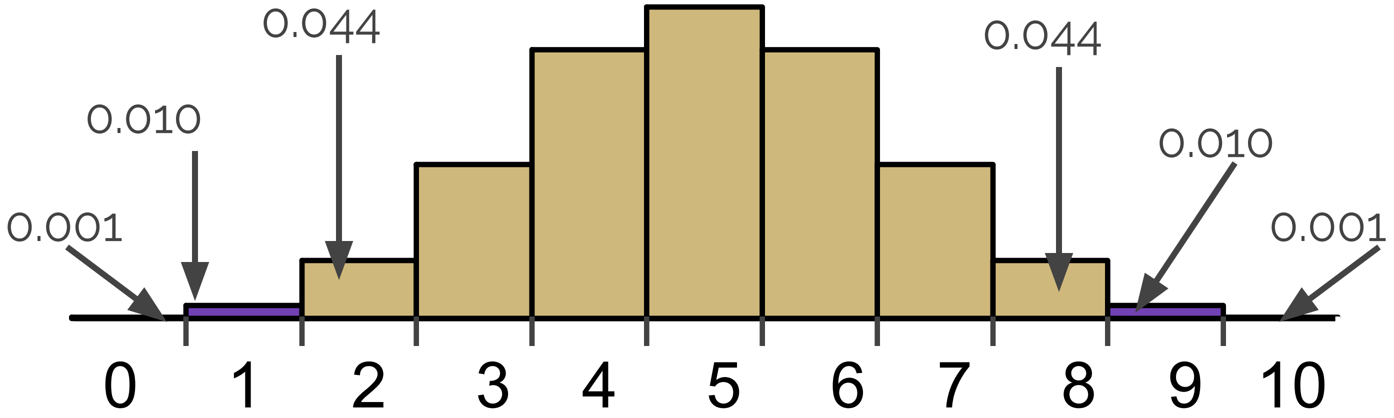
Step 5

5

- Type I Error Rate: $\boldsymbol{\alpha = 0.05}$
- Decision Rule for Rejecting H_0: $\mathbf{p \leq \alpha}$

Step 5

5



Therefore, reject H_0 if r = 0, 1, 9, or 10 with an actual α = 0.022

Step 6

6

- Validate the Underlying Assumptions
    - ***Random Sample, Independent specimens***
    - ***Two-outcome events***
    - ***Constant probability events***
    - ***Events are independent of every other event***

Step 6

6

- Perform a Basic Descriptive Analysis
  - Graphic - ***Binomial Table, Bar Chart***
  - Numeric

p **= 0.2**

n **= 10**

Sample Statistic

Step 7

7

- Perform the Statistical Test and Obtain Its Probability (p-value)

Step 7

7

- Calculate the value of the test statistic
  - This is the exact binomial

$$P(r) = \frac{n!}{(n-r)!\,r!} p^r q^{n-r}$$

Test Statistic

=

Sample Statistic

=

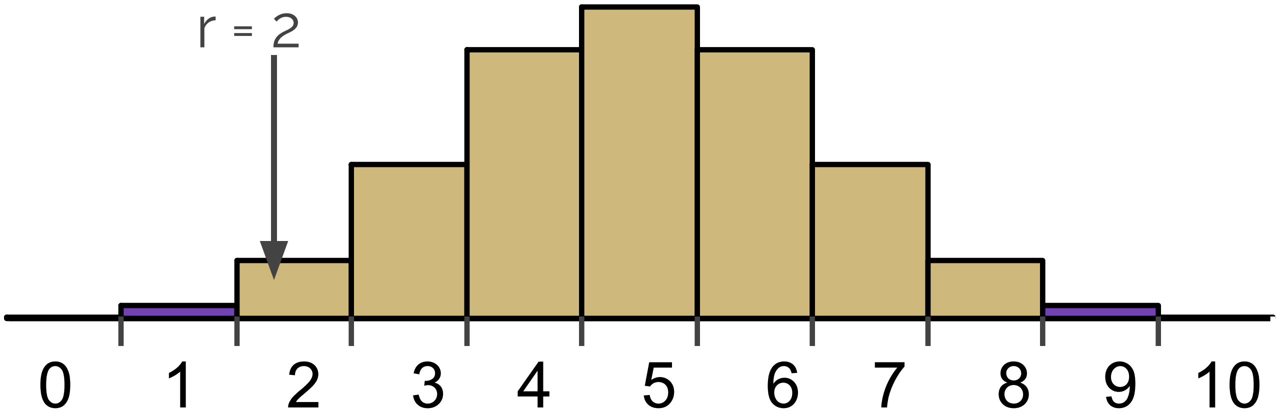
r or p

In RStudio

```
proportion.test.onesample.exact.simple()
```

Step 7

7



$p = 0.0547 * 2 = 0.1094$

Step 8

8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_0. Provide Appropriate Estimates and Compute Power if needed.
    - ***Fail to Reject H_0***
    - Report the p value(s) (for each Hypothesis):
      p = 0.1094

Step 8

8

- WHSSETIT:
  We have sufficient statistical evidence to infer that the proportion of failure of the projector is still 50%.

Step 9

9

- Interpretation of the Results in Terms of the Research Question.
  The failure proportion of the projector has not changed from 50%. Additional work will need to be done reduce the proportion of failures.

Exact Binomial Test on Proportions

- Spark plugs manufactured in a particular plant have commonly been ground as greenware after wet milling.
- In an effort to increase productivity, the raw material is pressed after dry milling on an experimental basis.
- In the past, the process defective rate has been in a state of statistical control at 3.42%.

Image Source: Wikimedia Commons

Exact Binomial Test on Proportions

- A random sample of 500 plugs from the first lot of 250,000 units reflects the current defective rate. The data are in a file called sparkplugs.txt. A value of zero represents not defective, a value of 1 represents defective.
- Has the **change** in the milling method had an effect on the process defective rate? Assume a confidence level of 95%.

Exact Binomial Test on Proportions

1. Are the proper Null and Alternative Hypothesis Directional or Non Directional?
   a. Directional
   b. Non-directional

2. What is the value of the proper Sample Statistic?
   a. 0.0020
   b. 0.0342
   c. 0.0280
   d. 0.0062

Exact Binomial Test on Proportions

3. What is the hypothesized value being tested?
   a. 0.0020
   b. 0.0342
   c. 0.0280
   d. 0.0062

4. What is the value of the proper Test Statistic?
   a. 0.0342
   b. 0.0280
   c. 0.0020
   d. 0.0062

Exact Binomial Test on Proportions

5. What is the p-value of the proper Test Statistic?
   a. 0.483
   b. 0.000
   c. 0.242
   d. 0.537

6. Do you Reject or Fail to Reject the Null Hypothesis?
   a. Reject the Null Hypothesis
   b. Fail to Reject the Null Hypothesis

One Sample Tests for Ordinal Data

Wilcoxon Signed Ranks Test for Location

- The Wilcoxon Signed Ranks Test for Location
  - Uses the signs and the ordered magnitude of the deviations above and below the ***hypothesized*** median
  - Deviation from hypothesized median
  - Ranks the scores in absolute magnitude, then applies a sign to the scores
  - Compares the sums of ranks that are positive vs the sums of ranks that are negative

Wilcoxon Signed Ranks Test for Location

Underlying Assumptions

- The specimens in the sample are independent
- The measurement scale is at least ordinal (critical).
- The underlying property being studied is continuous.
- The probability of values falling on the median is low (Important).
- The distribution is symmetrical around the population median (critical). This assumption has implications for the interpretation of the results.

Wilcoxon Signed Ranks Example

- A survey, with 96 respondents, has been conducted which included the following item.

- Rate our product on the characteristic of "Formability"

Poor	Fair	Good	Very Good	Best
1	2	3	4	5

Wilcoxon Signed Ranks Example

- The following frequencies were observed for each (ordered) rating category.

1	2	3	4	5
16	8	26	18	28

- See file named **FormItEach.txt**

Step 1

1

- State the Research Question:
  Is it reasonable to conclude that the process (population) median for Formability is actually 4?

Step 2

2

- Dependent Variable: ***Metal Quality, Formability***
- Criterion Measure: ***Formability Rating (5 Point Scale)***
- Level of Data: ***Ordinal Scale***
- Performance Criterion: ***Bigger is Better***

Step 3

3

- State the Statistical Hypotheses.
  - H_0: $\mathbf{M_o = 4}$
  - H_1: $\mathbf{M_o \neq 4}$

Hypothesized Value

Non-Directional

Step 4

4

- Select the Statistical Test and Identify its RSD when H_o is true
  - ***One-sample Median Test for Location (Wilcoxon Signed-Ranks Test***
  - ***z ~ appx $N(0,1)$ when H_o is true***
  - ***Asymptotically Normally distributed***

Step 5

5

- Select the Type I and a Type II Error Rates and Decision for reject H_o
    - Type I Error and Its Consequence:
      Rejecting H_o when it is, in fact, true would lead to the conclusion that the product did not have a formability median of 4. Depending on the outcome, that might lead to the conclusion that the product was better than "Very Good," the median rating. If this were the case, complacency might set in because the conclusion would be that the median was greater than a 4 rating. Or, if the result were in the other direction, the conclusion might be that the product was worse than the median. This could lead to taking some kind of action when, in fact, none is needed.

Step 5

5

- Type II Error and Its Consequence:
  Failing to reject H0 when it is False and should be rejected might lead to a sense of acceptance with the status quo since a median of 4, "Very Good" is acceptable. Thus the organization could pursue other important projects, not knowing that they were either better or worse than they thought, depending on the specific outcome of the study.

Step 5

5

- Type I Error Rate: ***α = 0.05 (two-tailed)***
- Decision Rule for Rejecting H_0: **$p \leq \alpha$**

6

Step 6

- Validate the Underlying Assumptions
  - ***The specimens in the sample are independent.***
  - ***The measurement scale is at least ordinal.***
  - ***The underlying property being studied is continuous.***
  - ***The probability of values falling on the median is low.***
  - ***The distribution is symmetrical about the population median.***

Step 6

6

- Perform a Basic Descriptive Analysis
  - Graphic - ***Histogram***
  - Numeric

    $\tilde{X}$ **=3.0**

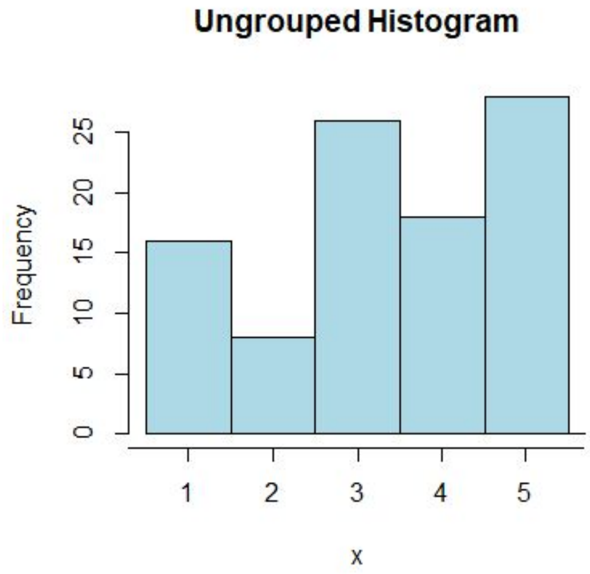
    or

    W+ / W-

Sample Statistic

Step 6

```
> summary(FormItEach$rating)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  1.000   2.750   3.000   3.354   5.000   5.000 
```



6

7

Step 7

- Perform the Statistical Test and Obtain Its Probability (p-value)

Step 7

7

- Calculate the value of the test statistic

$$W = \sum_{i=1}^{N_r} [\operatorname{sgn}(x_{2,i} - x_{1,i}) \cdot R_i]$$

W = test statistic

N_r = sample size, excluding pairs where x1 = x2

sgn = sign function

$x_{1,i}, x_{2,i}$ = corresponding ranked pairs from two distributions

R_i = rank i

From the web

The **test statistic** for the **Wilcoxon Signed Rank Test** is W, defined as the smaller of W+ (sum of the positive **ranks**) and W- (sum of the negative **ranks**). ...

https://sphweb.bumc.bu.edu › BS704_Nonparametric6

Test Statistic for the Wilcoxon Signed Rank Test - SPH

Test Statistic

=

min(W+, W-)

In RStudio

```
median.test.onesample.wilcoxon()
```

Step 8

8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_o. Provide Appropriate Estimates and Compute Power if needed.
  - ***Reject H_o***
  - Report the p value(s) (for each Hypothesis):
    p = 0.000

Step 8

8

- WHSSETIT:
  We have sufficient statistical evidence to infer that the population median M, does not equal 4. In fact, the process median appears to be less than 4!

Step 9

9

- Interpretation of the Results in Terms of the Research Question.
  Based on the sample data and the statistical test, we conclude that the population median is not 4. Unfortunately, the value of the median is determined to be less than 4. This means that the formability of the product is less than "Very Good" in terms of the median ratings.

Wilcoxon Signed-Ranks test for Location-Median

In the recent past, a plant's customers have become increasingly demanding that the product shipped to them has a "bright" appearance.

In an attempt to improve the cosmetic appearance of the surface of the rolled sheet product, a team has put in place procedures that are intended to increase brightness.

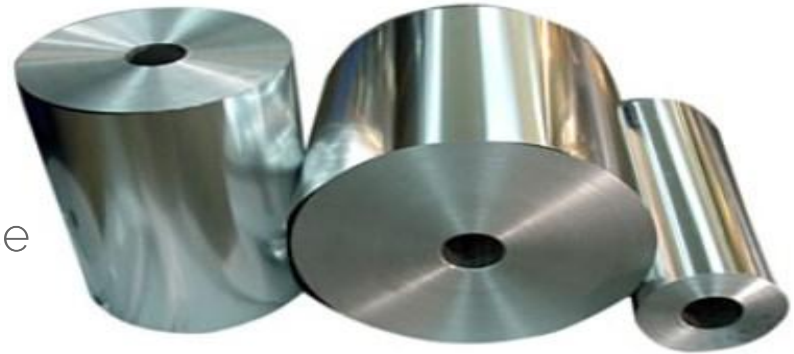


Image Source: www.globalmetals.com

Wilcoxon Signed-Ranks test for Location-Median

- After the changes have been in place for a short period of time, the improvement team conducts an assessment of the product.
- A random sample of 40 specimens of product, which were obtained to perform mechanical properties assessment, were also rated for brightness on their standard 7-point brightness scale as follows:

1	2	3	4	5	6	7
Very Dull	Moderately Dull	Slightly Dull	Neutral	Slightly Bright	Moderately Bright	Very Bright

Wilcoxon Signed-Ranks test for Location-Median

Past data indicates that the typical evaluation of the product is "Slightly Bright" (a rating value of 5), but this is only minimally acceptable per customer feedback. The team collected the following data. (Score values are in the top row and frequency of occurrence is in the bottom row of the following table.)

1	2	3	4	5	6	7
Very Dull	Moderately Dull	Slightly Dull	Neutral	Slightly Bright	Moderately Bright	Very Bright
	2	3	6	10	11	8

Wilcoxon Signed-Ranks test for Location-Median

- The data is located in a file named **Brightness.txt**. Conduct this test using the Wilcoxon Signed Ranks Test.
- Have implemented process changes (specifically increased) the median rating of the quality of the product's appearance?

Wilcoxon Signed-Ranks test for Location-Median

1. Are the proper Null and Alternative Hypothesis Directional or Non Directional?
   a. Directional
   b. Non-directional
2. What is the value of the proper Sample Statistic?
   a. 10
   b. 5
   c. 6
   d. 5.225

Wilcoxon Signed-Ranks test for Location-Median

3. What is the hypothesized value being tested?
   a. 10
   b. 5
   c. 6
   d. 5.225

4. What is the value of the proper Test Statistic?
   a. 182
   b. 40
   c. 283
   d. 30

Wilcoxon Signed-Ranks test for Location-Median

5. What is the p-value of the proper Test Statistic?
   a. 0.285
   b. 1.000
   c. 0.154
   d. 0.857

6. Do you Reject or Fail to Reject the Null Hypothesis?
   a. Reject the Null Hypothesis
   b. Fail to Reject the Null Hypothesis

One Sample Poisson Exact Test for Rates

Underlying Assumptions

- The specimens are independent (critical).
- The data are discrete counts that follow a Poisson distribution (critical).

One Sample Poisson Exact Test Example

- As a result of recent customer complaints, a process engineer is attempting to improve the surface quality of rolled rod product.
- In particular the engineer has worked with a team of production operators and technicians to improve the cleanliness of (a) the handling of equipment, (b) the roll guides, and (c) the work stations, in order to reduce surface related defects.

One Sample Poisson Exact Test Example

- As the bars finish the final straightening operation they pass through an Eddy Current tester and the number of major and minor “indications” is recorded.
- In an attempt to assess the effect of cleanliness on surface quality, a random sample of 130 bars was selected from recent production and the number (count) of both major and minor Eddy Current indications were recorded for each bar.

One Sample Poisson Exact Test Example

- The team was interested in determining whether or not their activities involving cleanliness have made a difference in the (average) population rate (λ) of major Eddy Current indications.

- Past data from one important customer indicates that the historical rate for major indications was **λ_o = 1 per bar**. Given the data collected, test a relevant hypothesis. The data are in a file named **Eddy.txt**.

Step 1

1

- State the Research Question:
  Have the activities of an improvement team in the area of cleanliness resulted in an improvement (a reduction) in the number of "Major" Eddy Current indications concerning bar quality?

2

Step 2

- Dependent Variable: ***Metal Bar Quality***
- Criterion Measure: ***Number of major indications per bar***
- Level of Data: ***Absolute Scale***
  (Ratio Discrete, Count Data)
- Performance Criterion: ***Smaller is Better***

Step 3

3

- State the Statistical Hypotheses.
  - H_0: $\lambda_0 = 1$
  - H_1: $\lambda_0 \neq 1$

Hypothesized Value

Non-Directional

Step 4

4

- Select the Statistical Test and Identify its RSD when H_0 is true
  - ***One sample Poisson Exact test***
  - ***$c \sim \text{Poisson}(\lambda)$ when H_0 is true***
    c = total count of all indications detected in the sample

Step 5

5

- Select the Type I and a Type II Error Rates and Decision for reject H_o
  - Type I Error and Its Consequence:
    Rejecting H_o when it is, in fact, true would lead to the conclusion that the cleanliness activities have contributed to a reduction in surface defects as detected by the Eddy Current tester, when they have not. Hence we would ignore the issue until a major event caused us to direct our attention there, again.

Step 5

5

- Type II Error and Its Consequence:
  Failing to reject H0 when it is False and should be rejected will likely cause the team to continue to work on other things in order to reduce surface defects detected by the Eddy Current device when, in fact, there would be no need to do so.

Step 5

5

- Type I Error Rate: ***α = 0.01 (two-tailed)***
- Decision Rule for Rejecting H_0: **$p \leq \alpha$**

Step 6

6

- Validate the Underlying Assumptions
    - ***The specimens are independent (randomly sampled).***
    - ***The data are distributed as Poisson. (Verified by conducting the Poisson distribution test.)***

Step 6

6

- Perform a Basic Descriptive Analysis
  - Graphic - ***Histogram, Boxplot***
  - Numeric

Sample Statistic

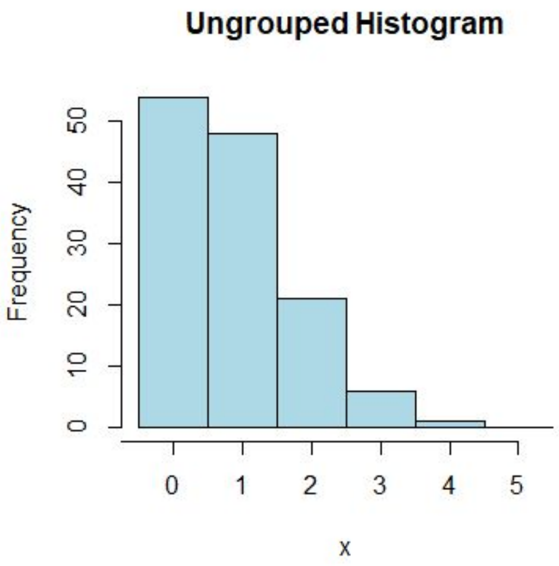
$\bar{c} = 0.8615$

$c = 0.8615 * 130 = 112$ ***(total count in the sample)***

Step 6

6

```
> summary(Eddy$Major)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.0000  0.0000  1.0000  0.8615  1.0000  4.0000
```



Step 7

7

- Perform the Statistical Test and Obtain Its Probability (p-value)

Step 7

7

- Calculate the value of the test statistic

Test Statistic
=
Number of events

In RStudio

```
poisson.test.onesample.simple()
```

Step 8

8

- State the Statistical Conclusion with Regard to the Null Hypothesis, H_o. Provide Appropriate Estimates and Compute Power if needed.
  - ***Fail to Reject H_o***
  - Report the p value(s) (for each Hypothesis):
    $p = 0.1245$

8

Step 8

- WHSSETIT:
  We have sufficient statistical evidence to infer that there has NOT been a change in the population Eddy Current rate for major indications.

Step 9

9

- Interpretation of the Results in Terms of the Research Question.
  The evaluation of the data following the cleanliness activities at the rolling and straightening process has not lead to a significant reduction in major Eddy Current indications.

One-Sample Poisson Test

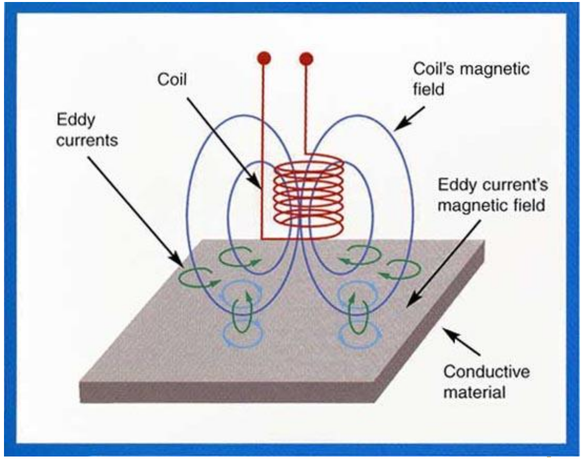


Image Source: www.nde-ed.org

- A team was interested in determining whether their activities involving cleanliness have made a difference in the (average) population rate (λ) of **minor Eddy Current** indications.
- Past data from the customer have indicated that the number of minor indications averaged **λ = 3 per bar**.
- Based on the data collected by the team (in the file **Eddy.txt**), can the team feel confident that their efforts have **changed** the number of minor indications per bar? Assume **α = 0.01**.

One-Sample Poisson Test

1. Are the proper Null and Alternative Hypothesis Directional or Non Directional?
   a. Directional
   b. Non-directional

2. What is the value of the proper <u>Sample Statistic</u>?
   a. 0.0154
   b. 0.0231
   c. 2
   d. 3

One-Sample Poisson Test

3. What is the hypothesized value being tested?
   a. 0.0154
   b. 0.0231
   c. 2
   d. 3

4. What is the value of the proper <u>Test Statistic</u>?
   a. 260
   b. 130
   c. 1.6950 to 2.3422
   d. 0.0154

One-Sample Poisson Test

5. What is the <u>p-value</u> of the proper Test Statistic?
   a. 0.846
   b. 0.423
   c. 0.000
   d. 0.673
6. Do you Reject or Fail to Reject the Null Hypothesis?
   a. Reject the Null Hypothesis
   b. Fail to Reject the Null Hypothesis