The Data Driven Manager

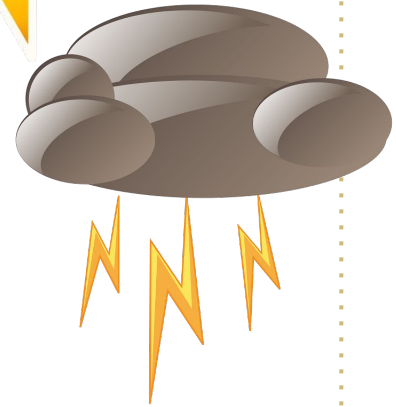
3

Best Case / Worst Case Analysis

Learning Objectives

- Use statistical software to properly compute confidence intervals for means and standard deviations
- Determine the best and worst case scenarios using confidence interval values for a given problem
- Calculate a confidence level to control the joint probability of worst mean and worst standard deviation

Best / Worst Case Engineering Analysis



- Best / Worst Case Analysis has to do with combining the results of confidence interval values for the mean and standard deviation, and checking to find the best and worst possible cases given the confidence interval values used.

Best / Worst Case Engineering Analysis

- For example, a shift in the average towards either of the specification limits, moving away from the nominal or target value, would constitute a negative change, while a shift towards the nominal or target value would be considered a positive change.

Best / Worst Case Engineering Analysis

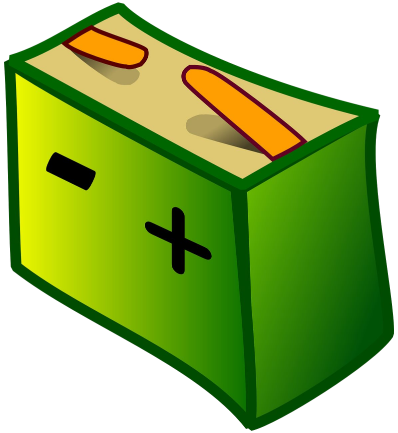
- For example, a shift in the **average** towards either of the specification limits, moving away from the nominal or target value, would constitute a negative change, while a shift towards the nominal or target value would be considered a positive change.

Best / Worst Case Engineering Analysis

- Likewise, if the standard deviation (or variance) increases, indicating higher variability, it is usually considered a negative change, whereas a decrease in standard deviation (or variance), indicating lower variability, is generally regarded as a positive change.

Warranty Example

- A warranty group wishes to determine the **average life** of a new battery design for a new car model.
- This is of particular concern because batteries failing **earlier than 36 months** are covered by a manufacturer's warranty and cost the company a lot of money.



Warranty Example

- They need to arrive at an estimate regarding the uncertainty of the point estimates for the mean and standard deviation.
- They would like to be 95% confident that the interval estimate contains the population mean, and likewise, the population standard deviation.

Warranty Example

- Based upon a sample of 200 users, the mean battery life has been found to be 38 months, with a standard deviation of 4 months.

Warranty Example

- Based upon these data, should the warranty group be worried ?

Step 1

- Define the specification limits
  - USL = Upper Specification Limit
  - LSL = Lower Specification Limit
- Define the sample size used to obtain the estimates
- Determine the confidence level

Step 1

- Define the specification limits
  - USL = Not Applicable (N/A)
  - LSL = **36 months**
- Define the sample size used to obtain the estimates
  - **n = 200**
- Determine the confidence level = **95%**

Step 2

- Obtain the point estimate for the mean
- Obtain the point estimate for the standard deviation
- Define these values in the spreadsheet or RStudio

Step 2

Estimator	Point Estimate
$\overline{X}$	38
s	4

Step 2

- Spreadsheet demonstration
    - https://tinyurl.com/bcwcgs
- RStudio demonstration

Step 3

- Obtain the interval estimate for the mean (using the t distribution)
- Define the upper confidence limit and lower confidence limit in the spreadsheet or RStudio

Interval Estimate for the Mean

In RStudio

- `t.test.onesample.simple( )`
- `t.test.onesample( ) #use with data file`

Interval Estimate for the Mean

In ROIStat

- One-and-Two-Sample Tests

Step 3

95% Confidence Interval

Estimator	Point Estimate	Interval Estimate
$\overline{X}$	38	**37.442 - 38.558**
s	4	

Step 4

- Obtain the interval estimate for the standard deviation (using the χ^2 distribution)*
- Define the upper confidence limit and lower confidence limit in the spreadsheet or RStudio

*Requires normality

Interval Estimate for the Standard Deviation

In RStudio

- `variance.test.onesample.simple( )`
- `variance.test.onesample( ) #use with data file`

Interval Estimate for the Standard Deviation

In ROIStat

- One-and-Two-Sample Tests

Step 4

95% Confidence Interval

Estimator	Point Estimate	Interval Estimate
$\overline{X}$	38	37.442 - 38.558
s	4	**3.643 - 4.435**

Step 5

- Determine the worst case combination of mean and standard deviation
    - Highest total % out of specification

- Determine the best case combination of mean and standard deviation
    - Lowest total % out of specification

In RStudio

		Est. Population Mean		
Est. Pop. Std Dev	% out of spec	Lower Conf Limit μ	Point Estimate μ	Upper Conf Limit μ
Upper Conf Limit σ	1. % above USL			
	2. % below LSL	37.25%	32.60%	28.21%
	3. Total %	37.25%	32.60%	28.21%
Point Estimate σ	4. % above USL			
	5. % below LSL	35.92%	30.85%	26.13%
	6. Total %	35.92%	30.85%	26.13%
Lower Conf Limit σ	7. % above USL			
	8. % below LSL	34.61%	29.15%	24.13%
	9. Total %	34.61%	29.15%	24.13%

In Google Sheets

			Best / Worst Case Analysis				
			Estimated Population Mean μ			Lower Spec Limit =	36.000000
			Lower Confidence Limit	Point Estimate	Upper Confidence Limit	Upper Spec Limit =	
			37.4422465	38.0000000	38.5577535		
Estimated Population St Dev s	Upper Confidence Limit						
		4.4356997	37.25%	32.60%	28.21%		
			37.25%	32.60%	28.21%		
	Point Estimate						
		4.0000000	35.92%	30.85%	26.13%		
			35.92%	30.85%	26.13%		
	Lower Confidence Limit						
		3.6426495	34.61%	29.15%	24.13%		
			34.61%	29.15%	24.13%		

The body of the table contains:

% above USL
% Below LSL
Total % Out of Specification

Remember to confirm the Normality of the data before using the calculations in this workbook!

Enter Values only in the Yellow highlighted cells: The mean and standard deviation and their upper and lower confidence limit values, and the Specification Limits.

Worst Case

μ

37.44224 **Lowest Mean**

σ

4.435699 **Highest SD**

One or Two Tails?

- (●) One-Tail
- ( ) Two-Tails

Point of Interest

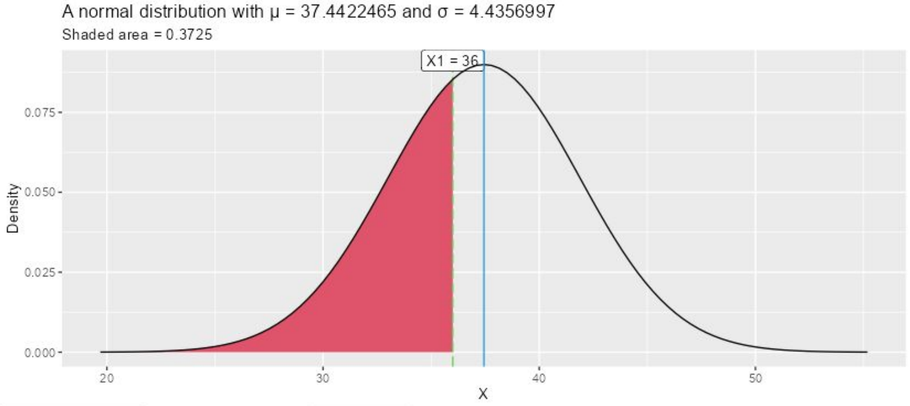
36

Area above or below?

- ( ) Above
- (●) Below

- [x] Label Graph?

Decimals



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Format: svg

	Result
μ =	37.4422
σ =	4.4357
X =	36
Area above X	0.6275
Area below X	0.3725

Best Case

μ

38.55775 **Highest Mean**

σ

3.642649 **Lowest SD**

One or Two Tails?

- (•) One-Tail
- ( ) Two-Tails

Point of Interest

36

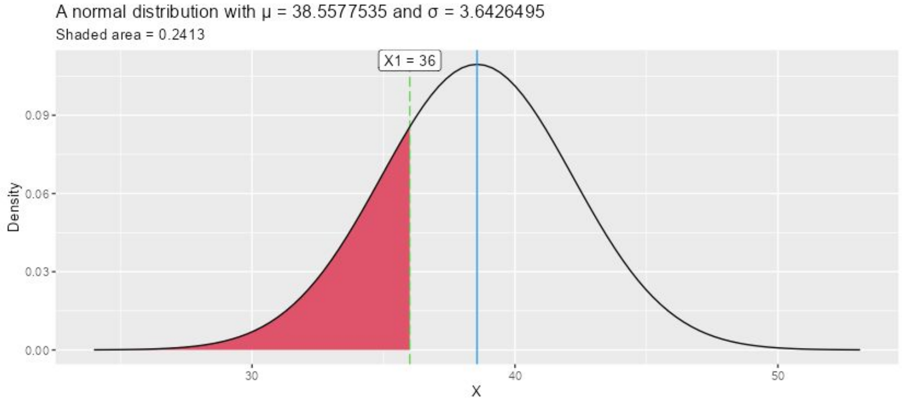
Area above or below?

- ( ) Above
- (•) Below

- [x] Label Graph?

Decimals

4



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Format: svg

	Result
μ =	38.5578
σ =	3.6426
X =	36
Area above X	0.7587
Area below X	0.2413

Joint Probability

- When we use a 95% confidence interval, there is a 5% chance that the parameter value will not fall within the interval.
- This is split into 2.5% chance of being above the upper limit and 2.5% chance of being below the lower limit.

Joint Probability

- With a normal distribution, the mean and standard deviation are independent, so the chance of having a population mean and standard deviation worse than their respective interval limits is calculated by multiplying the two individual probabilities together. This results in a very low chance of 0.0625%.

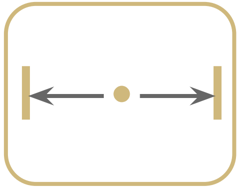
Best / Worst Case

Practice Activities

Best and Worst Case Analysis - Example 1

A test of new control modules has yielded a random sample with a mean (xbar) of 106.3 and a standard deviation (s) of 8.620, and it passed the normality test. The random sample consisted of 42 modules. Assume specifications of Target = 110 +/- 5, and a **95%** confidence interval.

Step 1



- Define the specification limits
  - USL = Upper Specification Limit
  - LSL = Lower Specification Limit
- Define the sample size used to obtain the estimates
- Determine the confidence level

Step 1

- Define the specification limits
  - USL = 110 + 5 = **115**
  - LSL = 110 - 5 = **105**
- Define the sample size used to obtain the estimates
  - **n = 42**
- Determine the confidence level = **95%**

Step 2

- Obtain the point estimate for the mean
- Obtain the point estimate for the standard deviation
- Define these values in the spreadsheet or RStudio

Step 2

Estimator	Point Estimate
$\overline{X}$	106.3
s	8.62

Step 3

- Obtain the interval estimate for the mean (using the t distribution)
- Define the upper confidence limit and lower confidence limit in the spreadsheet or RStudio

Step 3

95% Confidence Interval

Estimator	Point Estimate	Interval Estimate
$\overline{X}$	106.3	**103.614 - 108.986**
s	8.62	

Step 4

- Obtain the interval estimate for the standard deviation (using the χ^2 distribution)*
- Define the upper confidence limit and lower confidence limit in the spreadsheet or RStudio

*Requires normality

Step 4

95% Confidence Interval

Estimator	Point Estimate	Interval Estimate
$\overline{X}$	106.3	**103.614 - 108.986**
s	8.62	**7.093 - 10.992**

Step 5

- Determine the worst case combination of mean and standard deviation
  - Highest total % out of specification

- Determine the best case combination of mean and standard deviation
  - Lowest total % out of specification

Best and Worst Case Analysis - Example 1

For the "**Worst** Case", what is the **% below the LSL**?

A. 15.01%
B. 44.98%
C. 55.02%
D. 83.02%

Best and Worst Case Analysis - Example 1

For the "**Worst** Case", what is the **% above the USL**?

A. 15.01%
B. 29.22%
C. 55.02%
D. 0.00%

Best and Worst Case Analysis - Example 1

For the "**Worst** Case", what is the total % Out of Specification?

A. 83.02%
B. 74.20%
C. 66.73%
D. 70.03%

Best and Worst Case Analysis - Example 1

For the "**Best** Case", what is the **% below the LSL**?

A. 42.73%
B. 19.82%
C. 0.06%
D. 28.71%

Best and Worst Case Analysis - Example 1

For the "**Best** Case", what is the **% above the USL**?

A. 11.00%
B. 19.82%
C. 5.42%
D. 28.71%

Best and Worst Case Analysis - Example 1

For the "**Best** Case", what is the total % Out of Specification?

A. 48.53%
B. 0.06%
C. 53.73%
D. 52.06%

Best and Worst Case Analysis - Example 1

If you did not have the spreadsheet or code, what would you have to do to do these Best Case/Worst Case Problems?

A. I couldn't
B. I wouldn't
C. Figure out which combination of "mean" and "standard deviation" lead to the "Worst Case" (or "Best Case")
D. Use the Normal distribution to find respective probabilities above and below the corresponding specification limits
E. C and D above

Best and Worst Case Analysis - Example 2

- The file **Candy.txt** relates to a new production process that has been started by a global Candy Company.
- The candy is sold in bags with a label weight of 12 ounces, which is the Nominal (Target) value for this process. The characteristic of interest is Candy Bag Ounces.

Best and Worst Case Analysis - Example 2

- Because there are more serious consequences if the bags are lighter than they should be, versus heavier, the Nominal value is NOT centered between the two specifications; the **USL = 12.50** and the **LSL = 11.75.**

Best and Worst Case Analysis - Example 2

- Be sure draw yourself a picture and be careful when you answer the following questions for this problem.
- Use a Confidence Level of 90% for both Dispersion (Standard Deviation) and Location (Mean) in your calculations.

Step 1

- Define the specification limits
  - USL = Upper Specification Limit
  - LSL = Lower Specification Limit
- Define the sample size used to obtain the estimates
- Determine the confidence level
- **Test for normality**

Step 1

- Define the specification limits
  - USL = **12.50**
  - LSL = **11.75**
- Define the sample size used to obtain the estimates
  - **n =**
- Determine the confidence level = **90%**

Test for Normality

- Sample size < 25, use Anderson Darling and Shapiro Wilk, p value of 0.05

- Sample size ≥ 25, use the moment tests (skewness and kurtosis), p value of 0.05

Can we assume Normality?

- Yes
- No

Step 2

- Obtain the point estimate for the mean
- Obtain the point estimate for the standard deviation
- Define these values in the spreadsheet or RStudio

Step 2

Estimator	Point Estimate
$\overline{X}$	12.018
s	0.2459

Step 3

- Obtain the interval estimate for the mean (using the t distribution)
- Define the upper confidence limit and lower confidence limit in the spreadsheet or RStudio

Step 3

90% Confidence Interval

Estimator	Point Estimate	Interval Estimate
$\overline{X}$	12.018	**11.923 - 12.113**
s	0.2459	

Step 4

- Obtain the interval estimate for the standard deviation (using the χ^2 distribution)*
- Define the upper confidence limit and lower confidence limit in the spreadsheet or RStudio

*Requires normality

Step 4

90% Confidence Interval

Estimator	Point Estimate	Interval Estimate
$\overline{X}$	12.018	**11.923 - 12.113**
s	0.2459	**0.1952 - 0.3370**

Step 5

- Determine the worst case combination of mean and standard deviation
  - Highest total % out of specification

- Determine the best case combination of mean and standard deviation
  - Lowest total % out of specification

Best and Worst Case Analysis

For the "**Worst** Case", what is the **% below the LSL**?

Best and Worst Case Analysis

For the "**Worst** Case", what is the **% above the USL**?

Best and Worst Case Analysis

For the "**Worst** Case", what is the total % Out of Specification?

Best and Worst Case Analysis

For the "**Best** Case", what is the **% below the LSL**?

Best and Worst Case Analysis

For the “**Best** Case”, what is the **% above the USL**?

Best and Worst Case Analysis

For the "**Best** Case", what is the total % Out of Specification?