Two Sample Hypothesis Tests for Proportions

Data Science for Quality Management: Two Sample Hypothesis Testing

with **Wendy Martin**

Learning objective:

Perform a statistical test for differences in proportions for both independent and dependent groups

Two Independent Sample Proportion Tests

- Example:
- Is there a difference in the proportion of bottles that are not properly filled as related to an “old” versus “new” filler valve design?

Testing Hypotheses for Proportions

- Fisher's Exact two sample proportion test (Independent Groups)
- McNemar's Test for Change (Dependent Groups)

Fisher's Exact Test – Assumptions

- The two processes from which the sample data are drawn are inherently independent in nature, and are both based upon the Bernoulli process

- The samples are randomly selected from the underlying processes being investigated

Fisher's Exact Test

- Hypotheses

$$H_0: \pi_1 = \pi_2$$

$$H_1: \pi_1 \neq \pi_2$$

- p value $p = \dfrac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{a!\,b!\,c!\,d!\,N!}$

Fisher’s Exact Test

Where a, b, c and d are frequencies (counts) in a 2x2 contingency table, and N is the total count

	Group 1	Group 2	Row Total
Pass	a	b	a + b
Fail	c	d	c + d
Column Total	a + c	b + d	a + b + c + d = N

Fisher's Exact Test Problem

- A systems engineer is anxious to determine whether two recently installed pieces of equipment are operating on an equivalent basis
- The machines are blow molders, and the canisters they produce are assessed on an attribute basis

Fisher's Exact Test Problem

- Specifically, each canister is evaluated only on a pass/fail basis.

- Nonconformities include leaks/doesn't leak and cracked/not cracked, etc.

Fisher's Exact Test Problem

- A random sample of 750 canisters is selected from the initial production run of each machine. The results were as follows.

$p_1 = 0.18$ $p_2 = 0.12$

$n_1 = 750$ $n_2 = 750$

- Test an appropriate hypothesis. Assume $\alpha = 0.01$.

Fisher’s Exact Test Problem

- In RStudio

```
> proportion.test.twosample.exact.simple
```

McNemar's Test for Change

- Suppose we test 100 randomly-selected units of product and find that 20% are defective. Then, imagine that we apply some type of treatment to the units; and on a post-test, we find again that 20% are defective.

McNemar's Test for Change

- It is possible that the 20 units that were defective originally were still defective.

- But it is also possible that the 20 units that were defective on the second test were a completely different set of 20 units! It makes a difference.

McNemar's Test for Change

- McNemar's Test employs two unique features for testing the difference between two dependent sample proportions:
  - A special fourfold (2x2) contingency table
  - A special-purpose chi-square (χ^2) test statistic (the approximate test).

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McNemar's Test for Change

H_0: $Pass_1Fail_2 = Fail_1Pass_2$

		After Condition: Pass	After Condition: Fail
Before Condition	Pass	a	**b**
	Fail	**c**	d

where

(a+b) + (c+d) = (a+c) + (b+d) = n = number of pairs of units evaluated and where df = 1

McNemar's Test for Change

- Test Statistic

$$\chi^2 = \frac{\{ABS(b-c)-1\}^2}{b+c}$$

McNemar’s Test for Change Example

- An operations manager in a manufacturing plant wishes to determine whether a new maintenance procedure is likely to improve the repeatability of a particular test at a test station.

McNemar’s Test for Change Example

- They select a random sample of 120 electronically tuned radios, which contain nonconforming as well as conforming units at the same level as daily production levels.

McNemar's Test for Change Example

- They select a random sample of 120 electronically tuned radios, which contain nonconforming as well as conforming units at the same level as daily production levels.
- The entire sample is then tested.

McNemar’s Test for Change Example

- The maintenance procedure is performed and the test is repeated on the same sample of 120 radios.

McNemar’s Test for Change Example

- In both tests, the radios are tested in a random order. They are also numbered with a unique identifier so the results of the two tests may be recorded for the proper units. Note that this is a repeated assessment on the same radios.

McNemar’s Test for Change Example

- The summary data from the study appear as follows.

Number of Units	Status Before Maintenance	Status After Maintenance
4	Fail	Fail
4	Pass	Fail
56	Fail	Pass
56	Pass	Pass

McNemar's Test for Change Example

- Place these data in the proper cells of the 2x2 contingency table before we demonstrate the test.

		After Condition	
		Pass	Fail
Before Condition	Pass	56	**4**
	Fail	**56**	4

McNemar's Test for Change Example

Create a vector of the frequencies (counts)	`ct<-(a,c,b,d)`
Create a 2x2 contingency table	`matrix(ct,nrow = 2)`
Perform McNemar's Test	`proportion.test.mcnemar.simple` `mcnemar.test`

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982