Confidence Intervals for Proportions and Poisson Counts

Data Science for Quality Management: Sampling Distributions, Error and Estimation

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Learning objective:

Calculate interval estimates for proportions and Poisson counts

Calculating Confidence Intervals

- Confidence intervals may be calculated for various statistics
  - Proportion
  - Poisson Counts

Proportion (Exact Binomial)

- Many formulas exist to generate confidence intervals for proportions
- lolcat uses the formula is that based on the exact binomial distribution

Proportion (Exact Binomial)

- The exact binomial confidence interval for a proportion uses quantiles from the Beta distribution

$$\pi_{Lower} = \beta(\frac{\alpha}{2}; np, n - np + 1)$$

$$\pi_{Upper} = \beta(1 - \frac{\alpha}{2}; np + 1, n - np)$$

Example

- For example, assume a sample was taken with the following characteristics

$$n = 100, p = 0.12$$

Confidence Level Desired = 95%

Example

- np = 12
- n = 100
- α = 0.05

$$\pi_{Lower} = qbeta(0.025; 12,89) = 0.0636$$

$$\pi_{Upper} = qbeta(0.975; 13,88) = 0.2002$$

Interval Estimate for a Proportion

In RStudio

```
> proportion.test.onesample.exact
> proportion.test.onesample.exact.simple
```

Poisson Counts

- Even more formulas exist to generate confidence intervals for Poisson Counts

- lolcat uses the formula is that based on the exact Poisson distribution

Poisson Counts

- The exact Poisson confidence interval for a Poisson Count uses the quantile values from the Gamma distribution

$$\lambda_{Lower} = \frac{G(\frac{\alpha}{2}, x)}{n} \qquad \lambda_{Upper} = \frac{G(1 - \frac{\alpha}{2}, x + 1)}{n}$$

where x = λ*n

Example

- For example, assume a sample was taken with the following characteristics

$$n = 20, \lambda = 25.05$$

Confidence Level Desired = 95%

Example

- λ = 25.05
- n = 20
- $\lambda * \mathrm{n} = 501$
- α = 0.05

$$\lambda_{Lower} = qgamma(0.025; 501)/20 = 22.90$$

$$\lambda_{Upper} = qgamma(0.975; 502)/20 = 27.34$$

Example

- λ = 25.05
- n = 20
- $\lambda * \mathrm{n} = 501$
- α = 0.05

$$\lambda_{Lower} = qgamma(0.025; 501)/20 = 23.24$$

$$\lambda_{Upper} = qgamma(0.975; 502)/20 = 26.97$$

Interval Estimate for Poisson Count

In RStudio

```
> poisson.test.onesample.simple()
```

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982