Two Sample Hypothesis Tests for Variances

Data Science for Quality Management: Two Sample Hypothesis Testing

with **Wendy Martin**

Learning objective:

Perform a statistical test for differences in variances for both independent and dependent groups when the underlying distribution is normal

Testing Hypotheses for Variances - Purpose

- To determine which t test to use when testing hypotheses for means
- To determine whether “treatments” applied to two groups differentially affect dispersion

Testing Hypotheses for Variances

- F Test for Variance (Independent Groups)
- The Dependent Sample t Test for Variances (Dependent Groups)

Two Independent Sample F Test for Variances

- Underlying assumptions
  - The samples are randomly selected from two independent populations or processes.
  - The underlying processes are normally distributed.

Two Independent Sample F Test for Variances

- Hypotheses

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

- Test Statistic

$$F_{(n_1-1,\, n_2-1\ df)} = \frac{s_1^2}{s_2^2}$$

Example 1 - F Test Problem

- A design engineer finds that for two different designs of the same motor, the brush box wear when tested appeared as follows:

Example 1 - F Test Problem

$\bar{X}_1 = 0.0060$ $\bar{X}_2 = 0.0090$

$s_1 = 0.0015$ $s_2 = 0.0013$

$n_1 = 25$ $n_2 = 30$

Example 1 - F Test Problem

- In RStudio

```
> variance.test.twosample.independent
> variance.test.twosample.independent.simple
```

The Dependent Sample t Test for Variances

- Hypotheses

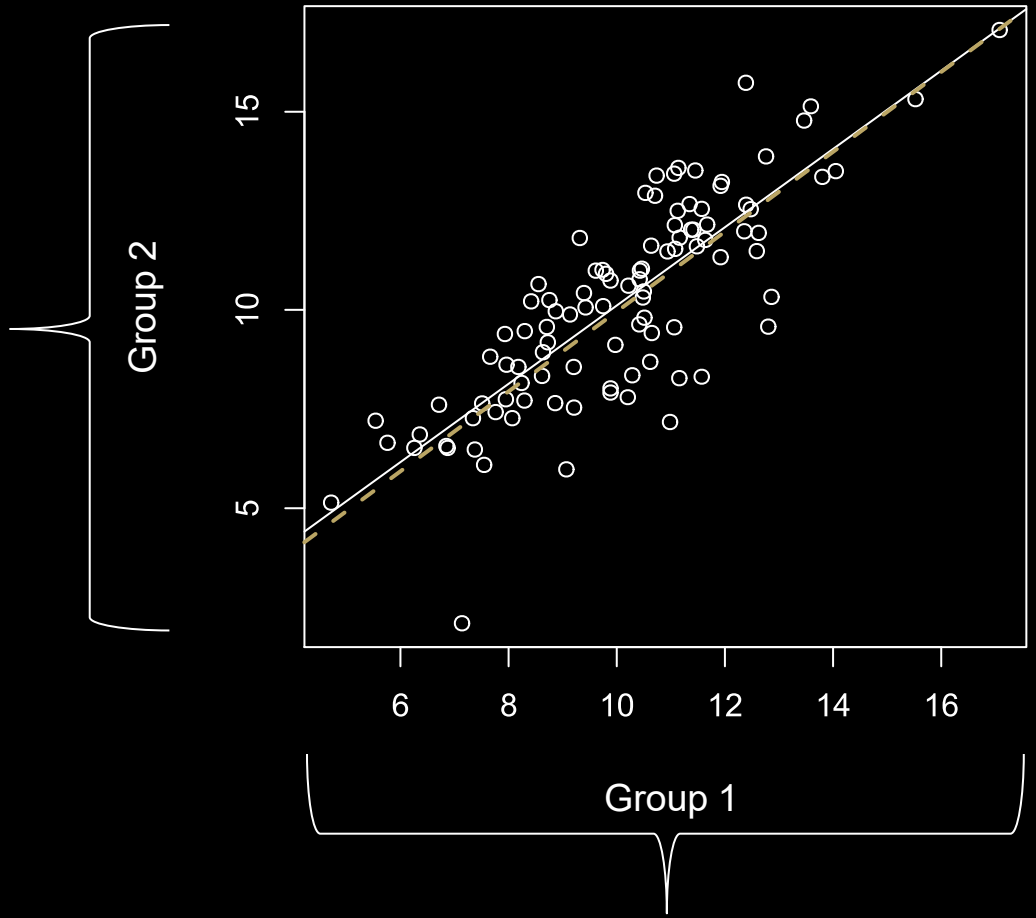
$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

- Test Statistic

$$t = \frac{s_1^2 - s_2^2}{2s_1 s_2 \sqrt{\frac{1 - r^2}{n - 2}}}$$

ISO Plot



Equal variation in both directions in the null case

The Dependent Sample t Test for Variances – Assumptions

- The pairs of scores are independent of one another (critical)
- The sample data are either dependent by nature, or dependent by design (critical, therefore you may be required to test the correlation)
- The underlying process distributions are normally distributed (not critical if n is large)

Example 2 – t Test for Variances

- It is quite possible that the two blanking presses (see Sample Problem - Dependent by Design) could be producing product which have equivalent means, but are different in terms of dispersion

Example 2 – t Test for Variances

- Assume you would like to know whether the processes are different in terms of piece-to-piece variability, and test an appropriate hypothesis for that data

Example 2 – t Test for Variances

- The summary statistics for the two groups of data are as follows.

$\bar{X}_1$	= 35.24	$\bar{X}_2$	= 38.02
s_1	= 5.18	s_2	= 5.63
r_{12}	= 0.60		

- Test an appropriate hypothesis at $\alpha = 0.05$

Example 2 – t Test for Variances

- In RStudio

```
> cor.pearson.r.onesample.simple
> variance.test.twosample.dependent.simple
```

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982